Classical solutions in a Lorentz-violating Maxwell-Chern-Simons electrodynamics

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We take as a starting point the planar model arising from the dimensional reduction of Maxwell electrodynamics with the (Lorentz-violating) Carroll-Field-Jackiw term. We then write and study the extended Maxwell equations and the corresponding wave equations for the potentials. The solution to these equations shows some interesting deviations from the usual MCS electrodynamics, with background-dependent correction terms. In the case of a timelike background, the correction terms dominate over the MCS sector in the region far from the origin, and establish the behavior of a massless electrodynamics (in the electric sector). In the spacelike case, the solutions indicate the clear manifestation of spatial anisotropy, which is consistent with the existence of a privileged direction in space.

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I. INTRODUCTION

The intensive development of Lorentz- and *CPT*-violating theories in $1+3$ dimensions [1–7] has brought about the question of the structure of a similar model in $1+2$ dimensions and its possible implications. A dimensional reduction (to $D=1+2$) of Lorentz-violating Maxwell electrodynamics, based on the presence of the Carroll-Field-Jackiw term ($\epsilon^{\mu\nu\kappa\lambda}v_{\mu}A_{\nu}F_{\kappa\lambda}$), has been recently accomplished [8], resulting in gauge invariant planar quantum electrodynamics (QED_3) composed by a Maxwell-Chern-Simons gauge field (A_{μ}), by a Klein-Gordon massless scalar field (φ), and a *constant* 3-vector (v^{μ}). The MCS electrodynamics is supplemented by a mixing, Lorentz-violating term, consisting of the gauge field and the external background, v^{μ}. In this way, one has derived a Lorentz- and *CPT*-violating planar theory, whose structure stems from a known counterpart, previously defined in $1+3$ dimensions, by Carroll Field and Jackiw [1,2]. As for the physical consistency of this model, some of its general features have been investigated. One has then verified that the complete model is stable and preserves causality; unitarity is satisfied in the gauge sector without any restrictions. In the scalar sector, instead, unitarity is guaranteed only for a purely spacelike background. Therefore, the full model supports a consistent quantization in the spacelike case while its gauge sector may be consistently quantized for both timelike and spacelike backgrounds.

The main motivation to study a planar model that violates Lorentz covariance is twofold: we first intend to understand some of its consistency properties; next, we wish to set up a theoretical framework to address some peculiar features of two-dimensional systems in the presence of Lorentz symmetry breakdown. A common feature of condensed matter systems is that they do not obey Lorentz covariance, which may establish a natural connection with this class of Lorentz-violating field models. These may eventually describe some phenomena in low-dimensional systems, like the presence of anisotropy, common in a great deal of realistic situations.

In this paper, we focus attention on the issue of the classical equations of motion (the extended Maxwell equations) and wave equations (for the potential A^{μ}) derived from the reduced Lagrangian. The purpose here is to investigate the effects of the Lorentz-violating background on the field strengths and potentials generated in our planar QED_3. Initially, one verifies that these equations have a similar structure to the usual MCS case, supplemented by terms that depend on the background vector. Solving these equations, we obtain solutions that differ from the MCS ones also by v^{μ}-dependent correction terms both for timelike and spacelike v^{μ}. In the timelike case, qualitative physical changes appear when one investigates the asymptotic character of the solutions. The background seems to annihilate the screening characteristic of a massive electrodynamics, leading to a behavior typical of massless QED_3 (at least in the electric sector). Near the origin, no qualitative modification takes place. In this case, no signal of spatial anisotropy appears. On the other hand, adopting a spacelike v^{μ}, the spatial anisotropy becomes a manifest property of the solutions. Induced by the external background, the anisotropy arises in the form of terms (with a clear dependence on the angle relative to the fixed direction determined by the background, $\vec{v}$) that correct the MCS behavior. As for the screening property, the spacelike solutions do not exhibit any sensitive modification; actually, the absence of screening seems to be associated only with the timelike background.

In short, this paper is outlined as follows. In Sec. II, we present the basic features of the reduced model, previously developed in Ref. [8]. In Sec. III, the equations of motion, from which one derives the wave equations for potentials and field strengths, are displayed. In Sec. IV, we solve the equations (in the static limit) for the timelike and spacelike cases and discuss the results. In Sec. V, we conclude by presenting our final remarks.

II. THE LORENTZ-VIOLATING PLANAR MODEL

Our planar Lorentz-violating model is attained by means of a dimensional reduction of the Maxwell Lagrangian[1] (written in 1+3 dimensions) supplemented by the Carroll-Field-Jackiw term [1]:

$$\mathcal{L}_{1+3}=\{-\tfrac{1}{4}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}}-\tfrac{1}{4}\epsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}}v_{\hat{\mu}}A_{\hat{\nu}}F_{\hat{\kappa}\hat{\lambda}}+A_{\hat{\nu}}J^{\hat{\nu}}\}, \quad (1)$$

where v^{μ} represents the external background and $A_{\hat{\nu}}J^{\hat{\nu}}$ is an additional term considering the coupling between the gauge field and an external current. This model (in its free version) is gauge invariant but does not preserve Lorentz and *CPT* symmetries relative to the particle frame [1,3,8]. We remark that one adopts here a Carroll-Field-Jackiw term with opposite sign in relation to the one in Ref. [8]. Applying the prescription of the dimensional reduction, described in Ref. [8], on Eq. (1), one obtains the reduced Lagrangian,

$$\mathcal{L}_{1+2}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\partial_{\mu}\varphi\,\partial^{\mu}\varphi-\frac{s}{2}\epsilon_{\mu\nu k}A^{\mu}\,\partial^{\nu}A^{k}+\varphi\epsilon_{\mu\nu k}v^{\mu}\,\partial^{\nu}A^{k}-\frac{1}{2\alpha}(\partial_{\mu}A^{\mu})^{2}+A_{\mu}J^{\mu}+\varphi J, \quad (2)$$

where the gauge-fixing term was added up after the dimensional reduction. The scalar field, φ, which appears from the dimensional reduction of $A_{\hat{\nu}}$ ($A^{(3)}=\varphi$), is a Klein-Gordon massless field and also acts as the coupling constant that links the fixed v^{μ} to the gauge sector of the model, by means of the new mixing term: $\varphi\epsilon_{\mu\nu k}v^{\mu}\partial^{\nu}A^{k}$. In spite of being covariant in form, this term breaks the Lorentz symmetry in the particle-frame (since the 3-vector v^{μ} is not sensitive to particle Lorentz boost), behaving like a set of three scalars. This reduced model does not necessarily jeopardize the *CPT* conservation, which depends truly on the character of the *constant* vector v^{μ}: there will occur conservation if one works with a purely spacelike external vector $[v^{\mu}=(0,\vec{v})]$, or breakdown, if v^{μ} is purely timelike or otherwise [8]. Here, these results were established under the assumption φ is a scalar field.[2]

To evaluate the propagators related to Lagrangian (2), one defines some new operators that form a closed algebra:

$$Q_{\mu\nu}=v_{\mu}T_{\nu}, \quad \Lambda_{\mu\nu}=v_{\mu}v_{\nu},$$

$$\Sigma_{\mu\nu}=v_{\mu}\partial_{\nu}, \quad \Phi_{\mu\nu}=T_{\mu}\partial_{\nu}.$$

Lengthy algebraic manipulations yield the propagators as listed below:

$$\langle A^{\mu}(k)A^{\nu}(k)\rangle=i\Bigg\{-\frac{1}{k^2-s^2}\theta^{\mu\nu}-\frac{\alpha(k^2-s^2)\boxtimes(k)+s^2(v_{\alpha}k^{\alpha})^2}{k^2(k^2-s^2)\boxtimes(k)}\omega^{\mu\nu}+\frac{s}{k^2(k^2-s^2)}S^{\mu\nu}+\frac{s^2}{(k^2-s^2)\boxtimes(k)}\Lambda^{\mu\nu}$$
$$-\frac{1}{(k^2-s^2)\boxtimes(k)}T^{\mu}T^{\nu}-\frac{s}{(k^2-s^2)\boxtimes(k)}Q^{\mu\nu}+\frac{s}{(k^2-s^2)\boxtimes(k)}Q^{\nu\mu}+\frac{is^2(v_{\alpha}k^{\alpha})}{k^2(k^2-s^2)\boxtimes(k)}\Sigma^{\mu\nu}$$
$$+\frac{is^2(v_{\alpha}k^{\alpha})}{k^2(k^2-s^2)\boxtimes(k)}\Sigma^{\nu\mu}+\frac{is(v_{\alpha}k^{\alpha})}{k^2(k^2-s^2)\boxtimes(k)}\Phi^{\mu\nu}-\frac{is(v_{\alpha}k^{\alpha})}{k^2(k^2-s^2)\boxtimes(k)}\Phi^{\nu\mu}\Bigg\}, \quad (3)$$

$$\langle\varphi\varphi\rangle=\frac{i}{\boxtimes(k)}[k^2-s^2], \quad (4)$$

$$\langle\varphi A^{\alpha}(k)\rangle=-\frac{i}{\boxtimes(k)}\left[T^{\alpha}+sv^{\alpha}-\frac{s(v_{\mu}k^{\mu})}{k^2}k^{\alpha}\right], \quad (5)$$

$$\langle A^{\alpha}(k)\varphi\rangle=-\frac{i}{\boxtimes(k)}\left[-T^{\alpha}+sv^{\alpha}-\frac{s(v_{\mu}k^{\mu})}{k^2}k^{\alpha}\right],$$

[1]Here one has adopted the following metric conventions: $g_{\mu\nu}=(+,-,-,-)$ in $D=1+3$, and $g_{\mu\nu}=(+,-,-)$ in $D=1+2$. The Greek letters (with a hat), $\hat{\mu}$, run from 0 to 3, while the pure Greek letters, μ, run from 0 to 2.

[2]As discussed in Ref. [8], if this field behaves like a pseudoscalar, the *CPT* conversation will be assured for a purely time-like v^{μ}.

where $T_{\mu}=S_{\mu\nu}v^{\mu}$, $S_{\mu\nu}=\varepsilon_{\mu\kappa\nu}\partial^{\kappa}$, $\theta_{\mu\nu}=g_{\mu\nu}-\omega_{\mu\nu}$, $\omega_{\mu\nu}=\partial_{\mu}\partial_{\nu}/\Box$. The term $\boxtimes(k)=[k^4-(s^2-v\cdot v)k^2-(v\cdot k)^2]$ determines the pole structure associated with the poles of such propagators. In Ref. [8], a consistency analysis investigating the stability, causality, and unitarity of such a model was also performed. We should comment here on our gauge-fixing choice. We have taken the covariant gauge, but a modified form of the axial gauge, now written in terms of our constant background vector v^{μ} ($v^{\mu}A_{\mu}=0$), could be chosen. This would simplify the expressions above for the propagators. However, from now on, we shall be dealing with the classical field equations written for the field strengths, so that the question of the gauge fixing is not crucial to alter our extended Maxwell equations.

III. CLASSICAL WAVE EQUATIONS AND SOLUTIONS

Let us now consider the reduced model, given by Lagrangian (2), without the gauge-fixing term:

$$\mathcal{L}_{1+2}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\partial_{\mu}\varphi\,\partial^{\mu}\varphi-\frac{s}{2}\epsilon_{\mu\nu k}A^{\mu}\,\partial^{\nu}A^{k}+\varphi\varepsilon_{\mu\nu k}v^{\mu}\,\partial^{\nu}A^{k}+A_{\mu}J^{\mu}+\varphi J, \tag{6}$$

where one observes the Chern-Simons term (having s as topological mass) and the Lorentz-violating term that couples the constant background 3-vector v^{μ} to the gauge vector A^{μ}. We stress that the vector v^{μ} is not to be considered a Lagrange multiplier in spite of the absence of its derivatives in the action.

Associated with this Lagrangian, there are two Euler-Lagrangian equations of motion:

$$\partial_{\nu}F^{\mu\nu}=-\frac{s}{2}\varepsilon^{\mu\nu\rho}\,\partial_{\nu}A_{\rho}-\varepsilon^{\mu\nu\rho}v_{\nu}\partial_{\rho}\varphi-J^{\mu}, \tag{7}$$

$$\Box\varphi=\epsilon_{\mu\nu k}v^{\mu}\,\partial^{\nu}A^{k}+J. \tag{8}$$

The modified Maxwell equations associated with this Lagrangian read as below:

$$\vec{\nabla}\times\vec{E}+\partial_{t}B=0, \tag{9}$$

$$\partial_{t}\vec{E}-\nabla^{*}B=-\vec{j}+s\vec{E}^{*}+(\vec{v}^{*}\partial_{t}\varphi+v_{0}\vec{\nabla}^{*}\varphi), \tag{10}$$

$$\vec{\nabla}\cdot\vec{E}+sB=\rho-\vec{v}\times\vec{\nabla}\varphi, \tag{11}$$

$$\Box\varphi-\vec{v}\times\vec{E}=-v_{0}\vec{\nabla}\times\vec{A}+J, \tag{12}$$

where the first equation stems from the Bianchi identity[3] ($\partial_{\mu}F^{\mu *}=0$), while the two inhomogeneous ones come from the motion equation (7), and the last one is derived from Eq. (8). Explicitly, one notes that Eq. (8) can be written as two simpler equations whether the vector v^{μ} is purely space-like or timelike: $\Box\varphi=\vec{v}\times\vec{E}+J$ for $v^{\mu}=(0,\vec{v})$; $\Box\varphi=-v_{0}\vec{\nabla}\times\vec{A}+J$ for $v^{\mu}=(v_{0},\vec{0})$. Applying the differential operator, ∂_{μ}, in Eq. (7), there results the following equation for the gauge current: $\partial_{\mu}J^{\mu}=-\varepsilon^{\mu\nu\rho}\,\partial_{\mu}v_{\nu}\,\partial_{\rho}\varphi$, which reduces to the conventional current-conservation law, $\partial_{\mu}J^{\mu}=0$, when v^{μ} is constant or has a null rotational ($\varepsilon^{\mu\nu\rho}\,\partial_{\mu}v_{\nu}=0$). These conditions correspond exactly to the ones that lead to a gauge invariant theory [1]. Notice that no space-time derivatives act on v^{μ} in the modified Maxwell equation above, since v^{μ} is a constant Lorentz symmetry breaking vector.

Manipulating the Maxwell equations, one notes that the fields $B,\vec{E}$ satisfy inhomogeneous wave equations:

$$(\Box+s^2)B=s\rho+\vec{\nabla}\times\vec{j}-s\vec{v}\times\nabla\varphi-\partial_{t}(\nabla\varphi)\times\vec{v}^{*}+v_{0}\nabla^{2}\varphi, \tag{13}$$

$$(\Box+s^2)\vec{E}=-\vec{\nabla}\rho-\partial_{t}\vec{j}-s\vec{j}^{*}-s\vec{v}(\partial_{t}\varphi)-sv_{o}\vec{\nabla}\varphi+\vec{v}^{*}\,\partial_{t}^{2}\varphi+v_{0}\vec{\nabla}^{*}(\partial_{t}\varphi)+\vec{\nabla}(\vec{v}\times\vec{\nabla}\varphi), \tag{14}$$

which, in the stationary regime, are reduced to

$$(\nabla^2-s^2)B=-s\rho-\vec{\nabla}\times\vec{j}+s\vec{v}\times\nabla\varphi-v_{0}\nabla^{2}\varphi, \tag{15}$$

$$(\nabla^2-s^2)\vec{E}=s\vec{j}^{*}+\vec{\nabla}\rho+sv_{o}\vec{\nabla}\varphi-\vec{\nabla}(\vec{v}\times\vec{\nabla}\varphi). \tag{16}$$

Similarly to the behavior of the classical MCS model, here the potential components $(A_{0},\vec{A})$ obey fourth-order wave equations:

$$\Box(\Box+s^2)A_{0}=\Box\rho-\Box(\vec{v}\times\vec{\nabla}\varphi)-s\vec{\nabla}\times\vec{j}+s(\partial_{t}\vec{\nabla}\varphi)\times\vec{v}^{*}-sv_{o}\nabla^{2}\varphi, \tag{17}$$

$$\Box(\Box+s^2)\vec{A}=s\partial_{t}\vec{j}^{*}+s\vec{\nabla}^{*}\rho+s\vec{v}(\partial_{t}^{2}\varphi)+sv_{o}\vec{\nabla}(\partial_{t}\varphi)-s[\vec{\nabla}(\vec{v}\times\vec{\nabla}\varphi)]^{*}+\Box(\vec{j}-\vec{v}\,\partial_{t}\varphi-v_{0}\vec{\nabla}^{*}\varphi), \tag{18}$$

which are endowed with an inhomogeneous sector much more complex due to the presence of the terms $\vec{v}$ and φ in the Lagrangian (6). It is instructive to remark that wave equations (13), (14), (17), and (18) reduce to their classical MCS usual form [9,10] in the limit one takes $v^{\mu}\rightarrow 0$, namely

$$(\Box+s^2)B=s\rho+\vec{\nabla}\times\vec{j},\qquad (\Box+s^2)\vec{E}=-\vec{\nabla}\rho-\partial_{t}\vec{j}-s\vec{j}^{*}, \tag{19}$$

[3]In $D=1+2$ the dual tensor, defined as $F^{\mu *}=\frac{1}{2}\epsilon^{\mu\nu\alpha}F_{\nu\alpha}$, is a 3-vector given by $F^{\mu *}=(B,-\vec{E}^{*})$. Here one adopts the following convection: $\epsilon_{012}=\epsilon^{012}=\epsilon_{12}=\epsilon^{12}=1$. The symbol (*), in a general way, also designates the dual of a 2-vector: $(E^{i})^{*}=\epsilon_{ij}E^{j}\rightarrow\vec{E}^{*}=(E_{y},-E_{x})$.

$$\Box(\Box+s^2)A_0=\Box\rho-s\vec{\nabla}\times\vec{j},$$
$$\Box(\Box+s^2)\vec{A}=s\partial_t\vec{j}^*+s\vec{\nabla}^*\rho+\Box\vec{j}. \tag{20}$$

The above wave equations present the following solutions [9] (for a pointlike charge distribution and null current):

$$B(r)=(e/2\pi)K_0(sr),\quad \vec{E}=(e/2\pi)sK_1(sr)\hat{r}, \tag{21}$$

$$A_0(r)=(e/2\pi)K_0(sr),\quad \vec{A}(r)=(e/2\pi)[1/r-sK_1(sr)]\hat{r}^*. \tag{22}$$

Up to now, Eq. (8) was not still used in the derivation of the wave equations for the fields and potentials. It will be appropriately considered in the subsequent solutions.

IV. SOLUTIONS FOR THE SCALAR POTENTIAL AND THE STRENGTH FIELDS IN THE STATIC LIMIT

As for the wave equation (17), which rules the dynamics of the scalar potential, A_0, one notices that it is not entirely written in terms of A_0, since the scalar field φ is not a constant variable and exhibits its own dynamics described by Eq. (12). This information must now be taken into account to provide the correct solution to this wave equation. Furthermore, there will be two different solutions depending on the character of the fixed vector v^μ, as we will see below.

A. The external vector is purely timelike: $v^\mu=(v_0,0)$

Supposing the system reaches a stationary regime, Eq. (17) is reduced to

$$\nabla^2(\nabla^2-s^2)A_0=-\nabla^2\rho-s\vec{\nabla}\times\vec{j}-sv_o\nabla^2\varphi+\nabla^2(\vec{v}\times\vec{\nabla}\varphi). \tag{23}$$

In this case, the φ field satisfies the equation $\nabla^2\varphi=v_0B-J$. The use of Eq. (12) changes Eq. (23) to the form

$$\nabla^2(\nabla^2-s^2+v_0^2)A_0=-\nabla^2\rho-s\vec{\nabla}\times\vec{j}-v_0^2\rho+sv_0J. \tag{24}$$

Starting from a pointlike charge-density distribution, $\rho(r)=e\,\delta(r)$, taking a null current density, $\vec{j}=0,J=0$, and proposing a Fourier-transform expression for the scalar potential, $A_0(r)=[1/(2\pi)^2]\int d^2\vec{k}\,e^{i\vec{k}\cdot\vec{r}}\tilde{A}_0(k)$, the following solution is obtained:

$$A_0(r)=\frac{e}{(2\pi)w^2}[s^2K_0(wr)+v_0^2\ln r], \tag{25}$$

where $w^2=s^2-v_0^2$. Whenever $s^2>v_0^2$, this potential results repulsive. Moreover, it is trivial to see that in the limit $v_0\rightarrow 0$, one recovers the scalar potential associated with the MCS electrodynamics, given by Eq. (22). One can thus conclude that the term with dependence on $\ln r$ is then a contribution stemming from the background field. The electric field, derived from Eq. (25), is read as

$$\vec{E}(r)=\frac{e}{(2\pi)}\left[\frac{s^2}{w}K_1(wr)-\left(\frac{v_0^2}{w^2}\right)\frac{1}{r}\right]\hat{r}, \tag{26}$$

which compared with the MCS correspondent, that of Eq. (21), possesses the additional presence of the $1/r$ term, which also arises as a contribution of the background. In the limit of short distance $(r\ll 1)$, the scalar potential (25) and the electric field (26) are reduced to the form

$$A_0(r)=-\frac{e}{(2\pi)}\left[\ln r+\frac{s^2}{w^2}\ln w\right],$$
$$\vec{E}(r)=\left(\frac{e}{2\pi}\right)\frac{1}{r}\hat{r}, \tag{27}$$

which reveals the repulsive character of expression (25) and a radial $1/r$ electric field near the origin. At the same time, one notices that, at the origin, the correction terms induced in Eqs. (25) and (26) by the background exhibit the same functional behavior as the preexistent MCS terms. However, when one goes far away from the origin, the picture dramatically changes: the correction terms entirely dominate over the exponential-decaying Bessel functions, resulting in the following forms:

$$A_0(r)=\left[\frac{ev_0^2}{(2\pi)w^2}\right]\ln r,\quad \vec{E}(r)=-\left[\frac{e}{(2\pi)}\frac{v_0^2}{w^2}\right]\frac{1}{r}\,\hat{r}.$$

So, one has a substantial modification in the asymptotic behavior of the solutions, which indicates that one of the main roles of the background is to promote a sensitive decreasing in the screening (or decay factor) of the field solutions. Indeed, a logarithmic scalar potential and a $1/r$ electric field are usual asymptotic solutions in a massless QED_3.

In the absence of currents, the magnetic field is ruled by Eq. (15), which reads simply as $(\nabla^2-s^2+v_0^2)B=-s\rho$. This differential equation is fulfilled by a very simple solution:

$$B(r)=\left(\frac{es}{2\pi}\right)K_0(wr). \tag{28}$$

In comparing this magnetic field with that of Eq. (21), one does not observe any additional term. In this case, the influence of the background seems to be totally absorbed into the decay factor, w, here smoothly diminished by the effect of the background. Thus, one remarks that decisive effects concerning the vanishing of the screening (coming from the timelike background) are confined to the electric sector of the theory. Finally, one points out that the results here obtained do not exhibit any signal of spatial anisotropy, which is consistent with the adoption of a null vector $\vec{v}$, since this is the element responsible for the choice of a privileged direction in space. The anisotropy, therefore, must be manifest when v^μ is spacelike.

B. The external vector is purely spacelike: $v^\mu=(0,v)$

In this case, the equation fulfilled by the scalar field, $\nabla^2\varphi=-\vec{v}\times\vec{E}-J$, can be read in terms of the scalar potential: $\nabla^2\varphi=\vec{v}\times\vec{\nabla}A_0-J=(\vec{v}\cdot\vec{\nabla}^*)A_0-J$. Taking into account this relation, Eq. (17) in its stationary regime is reformulated as

$$[\nabla^2(\nabla^2-s^2)-(\vec{v}\cdot\vec{\nabla}^*)(\vec{v}\cdot\vec{\nabla}^*)]A_0 = -\nabla^2\rho-s\vec{\nabla}\times\vec{j}-(\vec{v}\cdot\vec{\nabla}^*)J, \qquad (29)$$

where the following relation was used: $\nabla^2(\vec{v}\times\vec{\nabla}\varphi)=(\vec{v}\cdot\vec{\nabla}^*)\nabla^2\varphi=(\vec{v}\cdot\vec{\nabla}^*)(\vec{v}\cdot\vec{\nabla}^*)A_0$, since $\vec{v}=cte$.

Starting from a pointlike charge density distribution, $\rho(r)=e\delta(r)$, $\vec{j}=J=0$, and proposing again a Fourier-transform expression for the scalar potential, one obtains

$$A_0(r)=-\frac{e}{(2\pi)^2}\int_0^\infty k\,dk \times\int_0^{2\pi}d\phi\frac{e^{ikr\cos\phi}}{[(\vec{k}^2+s^2)+\vec{v}^2\sin^2(\phi-\beta)]}, \qquad (30)$$

where $(\phi-\beta)$ is the angle defined by $\vec{v}$ and $\vec{k}$, namely $\vec{v}\cdot\vec{k}=vk\cos(\phi-\beta)$. An exact result was not found for this full integral, but an approximation can be accomplished in order to solve it algebraically. Indeed, considering $s^2\gg v^2$ an integration becomes feasible. Here, there is an external vector, $\vec{v}$, that fixes a direction in space and the coordinate position, $\vec{r}$, where one measures the fields. One then considers that the angle between $\vec{v}$ and $\vec{r}$ is given by $\vec{v}\cdot\vec{r}=vr\cos\beta$, where $\beta=cte$. Considering this information and working in the limit in which $s^2\gg v^2$, the integration is carried out (at first order on v^2/s^2), so that

$$A_0(r)\simeq\frac{e}{(2\pi)}\left[K_0(sr)-\frac{(1-\cos^2\beta)}{2s}v^2rK_1(sr) +\frac{v^2}{2s^2}(1-2\cos^2\beta)K_2(sr)\right]. \qquad (31)$$

In this expression, one notes a clear dependence of the potential on the angle β, which is an unequivocal sign of anisotropy determined by the ubiquity of the background vector on the system. Near the origin, the K_2 function dominates over the other terms, so that the short-distance potential behaves effectively as

$$A_0(r)=\frac{e}{(2\pi)}\left[(1-2\cos^2\beta)\frac{v^2}{s^2}\frac{1}{r^2}\right], \qquad (32)$$

which shows that the potential is always repulsive at origin. In spite of this fact, the expression (31) may exhibit an attractive well region, at larger r values, depending on the value of the s parameter. This fact brings into light the possibility of the occurrence of pair condensation concerning two particles interacting by means of this gauge field. This issue should be more properly investigated in the context of low-energy two-particle scattering [13], whose amplitude can be converted into the interaction potential by a Fourier transform.

Looking at the expression (18) for the vector potential, one observes the presence of the term $\vec{\nabla}(\vec{v}\times\vec{\nabla}\varphi)$, which cannot be written as a term depending directly on $\vec{A}$. This fact seems to prevent a solution for $\vec{A}$ starting from the static version of this differential equation, which also seems to be an impossibility for determining a solution for the magnetic field. However, one must indeed be interested in the magnetic field, and a simpler solution for it can arise from Eq. (10), which in the static regime is simplified to the form $\nabla B=-s\vec{E}-v_0\vec{\nabla}\varphi$. For a pure spacelike v^μ, this last equation reduces to $\nabla B=-s\vec{E}=s\nabla A_0$, an equation that links the magnetic field and the scalar potential: $B=sA_0+cte$. Based on Eq. (31), we achieve the following expression for the fields:

$$\vec{E}(r)=\frac{e}{(2\pi)}\left\{sK_1(sr)+(1-\cos^2\beta)\frac{v^2}{2}\left[r-\frac{2}{s^2r}\right]K_0(sr) +(1-\cos^2\beta)\frac{v^2}{2s}\left[1-\frac{4}{s^2r^2}\right]K_1(sr)\right\}\hat{r}, \qquad (33)$$

$$B(r)=\frac{e}{(2\pi)}\left\{sK_0(sr)-(1-\cos^2\beta)\frac{v^2}{2}rK_1(sr) +\frac{v^2}{2s}(1-2\cos^2\beta)K_2(sr)\right\}. \qquad (34)$$

Here, the effect of the background vector, $\vec{v}$, appears more clearly on the field solutions. As compared to the MCS fields (B and $\vec{E}$), there arise supplementary terms, proportional to $\cos^2\beta$, responsible for the spatial anisotropy. Despite the complexity of the expressions for the field configurations above, it can be readily seen that they exhibit screening (absent in the purely timelike case), once all the modified K-Bessel functions decay exponentially far from the origin. One thus concludes that the vanishing of the screening is associated only with a timelike background.

V. FINAL REMARKS

Starting from a dimensionally reduced gauge invariant, Lorentz, and *CPT*-violating planar model, derived from the Carroll-Field-Jackiw term (defined in 1+3 dimensions), we have studied the extended Maxwell equations (and the corresponding wave equations for the field strengths and potentials) stemming from the planar Lagrangian. While the field strengths satisfy second-order inhomogeneous wave equations, the potential components $(A_0,\vec{A})$ fulfill fourth-order wave equations, a clear similarity to the usual behavior inherent in the pure MCS sector. As expected, this structural resemblance is also manifest in the solutions to these equations. Indeed, in the case of a purely timelike background, one has attained solutions for the fields B and $\vec{E}$ that differ from the MCS counterparts just by correction terms (dependent on v^{μ}). These new terms do not bring about any remarkable physical change near the origin, where they present the same behavior as the MCS terms. Away from the origin, however, the panorama is new and intriguing: the correction terms, independently of the value of their coefficients, come to dominate over the MCS behavior, yielding an appreciable modification on the asymptotic solutions. In fact, at large distances, the electric sector of the massive MCS electrodynamics, characterized by strong screening (stemming from the topological mass), is smoothly replaced by a logarithmic behavior typical of a long-range massless electrodynamics. When the magnetic field is considered, no drastic modification takes place, but the decay factor suffers a softening (due to the presence of v_0^2). In this case, no signal of anisotropy was observed, as expected. This physical picture seems to be compatible with the absence of degrees of freedom associated with the pole $k^2=s^2$ of the propagator, which does not exhibit any dynamical content as concluded by analyzing the residue at the corresponding pole [8].

In the pure spacelike case, the background field appears more explicitly in the solutions in the form of spatial anisotropy, a consequence of the selection of a privileged spatial direction, given by $\vec{v}$. The solutions keep the MCS reference term, but at the same time present a complex form, which reflects the anisotropy induced by the background over the solutions, attained in the approximation $s^2 \gg v^2$. The reduction of the screening for large distances, observed in the timelike case, is absent here. The scalar potential obtained is always repulsive at the origin, but it may become attractive at an intermediary well-defined region, depending on the parameter s.

The attractiveness of this potential may be better explored in the realm of a nonrelativistic physical system. In fact, the verification of the consistency of this model (see Ref. [8]) in the case of a spacelike background shows that applications of this study to condensed-matter systems is a reasonable option. In this context, there arises the interesting possibility of investigating a Möller scattering in the low-energy (nonrelativistic) limit. For this task, following a usual procedure in QED_3 [11,12], one should include the Dirac sector and consider suitable couplings (electron-photon and electron-scalar ones). The electron-electron interaction would then be mediated by the scalar and gauge fields, whose action must appear in the form of an interaction potential (stemming from a tree-level calculation) [13].

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