World-sheet stability of (0,2) linear sigma models

Anirban Basu* and Savdeep Sethi†
Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637, USA
(Received 2 April 2003; published 3 July 2003)

We argue that two-dimensional (0,2) gauged linear sigma models are not destabilized by instanton generated world-sheet superpotentials. We construct several examples where we show this to be true. The general proof is based on the Konishi anomaly for (0,2) theories.

DOI: 10.1103/PhysRevD.68.025003 PACS number(s): 11.27.+d

I. INTRODUCTION

One of the basic questions we ask about quantum field theory is whether the classical vacua are stable. Often, the structure of the quantum moduli space is significantly different from the classical moduli space. The theories about which we can usually say the most are supersymmetric. In these cases, we can often make exact statements, either perturbative or nonperturbative, because of nonrenormalization theorems. In cases where the vacuum structure is not renormalized at any finite order in perturbation theory, nonperturbative effects, such as instantons, can still generate superpotentials which modify or destabilize perturbative vacua. Numerous examples of this kind have been studied in various dimensions; for example, $N=1$ supersymmetric QCD in four dimensions [1].

The aim of this work is to study the stability of two-dimensional gauge theories, both massive and massless, with (0,2) world-sheet supersymmetry. On the string world sheet, the terminology (p, q) supersymmetry refers to theories with p left-moving and q right-moving supersymmetries. Conformal field theories with (0,2) supersymmetry are a key ingredient in building perturbative heterotic string compactifications (for a review, see Ref. [2]). Unlike their (2, 2) cousins, theories with (0,2) supersymmetry that are conformal to all orders in perturbation theory can still be destabilized by world-sheet instantons. The usual phrasing of this problem is that world-sheet instantons generate a *space-time* superpotential [3,4]. However, the general belief is that this destabilization is generic. Under special conditions described in Refs. [5,6] for nonlinear sigma models, there can be extra fermion zero modes in an instanton background which kill any nonperturbative superpotential.

We consider those (0,2) models which can be constructed as IR limits of gauged linear sigma models [7]. This is a rather nice class of models which can be conformal or nonconformal, and which can flow to theories with IR descriptions such as sigma models or Landau-Ginzburg theories. For perturbatively conformal cases, some criteria for the absence of a space-time superpotential have been described in Refs. [8–10]. Our interest is in whether a world-sheet superpotential is generated. In perturbatively conformal cases, the two questions should be related in a way that we will describe.

In the following section, we consider the stability of (0,2) theories without tree level superpotentials. We construct several examples of nonconformal (0,2) models without tree level superpotentials for which we show that no world-sheet superpotential is generated by instantons. This result surprised us initially since we were looking for a model with an instanton generated superpotential. In Sec. III, we give a general argument based on the Konishi anomaly [11,12] that this is true for all gauged linear sigma models without tree level superpotentials. This argument is inspired in part by recent progress in four-dimensional gauge theories [13,14]. We then extend the argument to cases with a tree level superpotential. In all cases, it appears that a nonperturbative superpotential is forbidden.

Lastly, we consider the implication of our results for the space-time superpotential. Based on the absence of a nonperturbative world-sheet superpotential, we argue that there is no corresponding space-time instability. Some related observations have appeared in Ref. [23].

II. SOME (0, 2) EXAMPLES

In this section we construct examples of (0,2) gauged linear sigma models [7] without tree level superpotentials. We will show that no superpotential is generated by nonperturbative instanton or anti-instanton effects. It is actually sufficient to consider one instanton contributions. Higher instanton numbers generate more fermion zero modes which obstruct the generation of a superpotential.

A. A bundle over $\mathbb{CP}^3$

The (0,2) superspace and superfield notations are reviewed in the Appendix. We begin by considering a $U(1)$ gauge theory. The (0,2) action is given by a sum of terms

$$S=S_g+S_{\rm ch}+S_F+S_{D\theta}+S_J, \tag{1}$$

where S_g, S_{ch}, S_F are canonical kinetic terms for the gauge fields, bosonic chiral superfields, and fermionic chiral superfields, respectively. The explicit form of these actions appear in the Appendix. The Fayet-Iliopoulos D term and the theta angle appear in $S_{D\theta}$ while S_J contains any tree level superpotential. For these models, we set $S_J=0$.

As our first example, we construct a linear sigma model whose IR limit is a nonlinear sigma model on $C\mathbb{P}^3$. This is a

*Email address: basu@theory.uchicago.edu
†Email address: sethi@theory.uchicago.edu

cousin of the (2,2) model studied in Ref. [15]. Apart from the $U(1)$ gauge superfields Ψ and V, we have bosonic superfields $\Phi_i = \phi_i + \cdots$, where $i=1,...,4$, and a single Fermi superfield Γ. Each Φ_i carries gauge charge 1 while Γ carries gauge charge -2. Since we do not have a tree level superpotential our action is

$$S = S_g + S_{\rm ch} + S_F + S_{D\theta}. \tag{2}$$

Solving for the auxiliary fields gives the following bosonic potential for the ϕ_i:

$$U = \frac{D^2}{2e^2} = \frac{e^2}{2}\left(\sum_i |\phi_i|^2 - r\right)^2. \tag{3}$$

We take r to be positive, and set $r=\eta^2$. After modding by the $U(1)$ gauge symmetry, we see that the target space is $C\mathbb{P}^3$.

The Fermi superfield determines the gauge-bundle over $C\mathbb{P}^3$ which, in this case, is the line bundle $\mathcal{O}(-2)$. These particular gauge charge assignments guarantee gauge anomaly cancellation, which is a basic consistency requirement. This can be seen either by computing the requisite one loop diagrams, or by checking that the condition for anomaly cancellation [5]

$${\rm ch}_2(TM) = {\rm ch}_2(V) \tag{4}$$

is satisfied. Here, TM is the tangent bundle of $C\mathbb{P}^3$, V is the $\mathcal{O}(-2)$ line bundle, and ${\rm ch}_2$ is the second Chern character. Using the definition

$${\rm ch}_2(X) = \frac{1}{2}c_1^2(X) - c_2(X)$$

we see that both sides of this equation gives $2J^2$, where J is the curvature two-form of the hyperplane bundle over $C\mathbb{P}^3$.

This theory is massive [as in the (2,2) $C\mathbb{P}^3$ model] because the sum of the gauge charges of the right moving fermions is nonzero. The theory does, at the classical level, have a chiral $U(1)$ symmetry under which $(\psi_+^i, \lambda_-, \chi_-)$ carry charges $(1,-1,q)$, where q is any integer. This symmetry is anomalous at one loop for any $q \neq -2$. The charge of the gaugino λ_- does not matter in the anomaly computation because it is not charged in a $U(1)$ theory. Since this chiral symmetry is anomalous, we can shift the theta angle to any value, and we choose to set it to zero.

Let us now consider how instantons modify the perturbative theory. First we construct the one instanton Bogomol'nyi-Prasad-Sommerfield (BPS) solution. In order to construct an instanton solution, we wick rotate to Euclidean space sending

$$y^0 \to -iy^2, \quad v_{01} \to -iv_{12}.$$

The Euclideanised bosonic action is

$$S = \int d^2y \left[\frac{1}{2e^2}v_{12}^2 + \sum_i |D_\alpha \phi_i|^2 + \frac{e^2}{2}\left(\sum_i |\phi_i|^2 - \eta^2\right)^2\right]. \tag{5}$$

We construct the well known vortex instanton solution [16,17] of the Abelian Higgs model in two dimensions: take $\phi_i = 0$ for $i=2,3,4$ and take nonzero ϕ_1 and gauge fields. From now on we refer to Φ_1 as Φ for brevity. We also set $e=1$. In polar coordinates, the one-instanton configuration is given by

$$v_r = 0, \quad v_\theta = v(r), \quad \phi = f(r)e^{i\theta}, \tag{6}$$

where for large r,

$$v(r) \sim \frac{1}{r} + {\rm const}\times\frac{e^{-\eta r}}{\sqrt{r}}, \tag{7}$$

$$f(r) \sim \eta + {\rm const}\times e^{-\sqrt{2}\eta r}, \tag{8}$$

and $v(0)=f(0)=0$. The Bogomolnyi equations are

$$(D_1 + iD_2)\phi = 0 \tag{9}$$

and

$$D + v_{12} = 0. \tag{10}$$

On evaluating Eq. (5) in this background, we easily obtain the usual instanton action $S = 2\pi\eta^2$. Next, we are interested in constructing the fermion zero modes in this instanton background. They are explicitly given by

$$\mu^0 = \begin{pmatrix} \overline{\psi}_+^0 \\ \lambda_-^0 \end{pmatrix} = \begin{pmatrix} -\sqrt{2}(\overline{D}_1 + i\overline{D}_2)\overline{\phi} \\ D - v_{12} \end{pmatrix}, \tag{11}$$

$$\overline{\chi}_-^0 = \phi^2, \tag{12}$$

and

$$\overline{\psi}_{+i}^0 = \overline{\phi} \tag{13}$$

for $i=2,3,4$. Note that these zero modes are normalizable because of the exponential fall off of the fields at large distances. The μ^0 fermion zero mode is actually the zero mode generated by the broken supersymmetry generator. In order to see this, we must examine the supersymmetry transformations in the instanton background.

The supersymmetry transformations become involved because we must also make gauge transformations to preserve Wess-Zumino gauge. The relevant supersymmetry transformations are given by

$$\delta\overline{\psi}_+ = -i\sqrt{2}(\overline{D}_0 + \overline{D}_1)\overline{\phi}\epsilon_-,$$

$$\delta\lambda_- = iD\epsilon_- + v_{01}\epsilon_-. \tag{14}$$

The supersymmetry parameter ϵ_- corresponds to Q_+. Wick rotating to Euclidean space gives the zero mode found in Eq. (11). Hence, Q_+ is the broken supersymmetry while the $\overline{Q}_+$ supersymmetry is still preserved by the instanton background. Using the Bogomol'nyi equations, it is not hard to check that the μ^0 zero mode does satisfy $i\not{D}\mu^0 = 0$, where $i\not{D}$ is the Dirac-Higgs operator

$$i\not{D}=\begin{pmatrix} -i(\overline{D}_1-i\overline{D}_2) & \sqrt{2}i\overline{\phi} \\ -\sqrt{2}i\phi & i\partial_1-\partial_2 \end{pmatrix}. \tag{15}$$

The field $\overline{\chi}_-$ is expanded in modes of the Dirac operator $(\overline{D}_1+i\overline{D}_2)$ (note that Γ has gauge charge -2). The $\overline{\psi}_{+i}$ ($i=2,3,4$) fields are expanded in modes of the Dirac operator $(\overline{D}_1-i\overline{D}_2)$. We ask again whether there are any zero modes for these fields. The existence of zero modes for these operators can be predicted using index theory and a vanishing theorem [18]. Note that in Minkowski space $\overline{\psi}=\psi^\dagger\gamma^0=(\overline{\psi}_+\overline{\psi}_-)$. However, in Euclidean space ψ and $\overline{\psi}$ are independent fermionic fields and not the conjugates of each other. Here, $\overline{\psi}=(\eta_-\eta_+)$ and so in the Euclidean formulation of the theory, the μ, $\overline{\psi}_{+i}$ ($i=2,3,4$) zero modes are zero modes of negative chirality while the $\overline{\chi}_-$ zero mode is a zero mode of positive chirality.

We can now ask what gauge invariant correlators are nonvanishing in this instanton background. There are only two possibilities

$$\langle\overline{\psi}_+\overline{\psi}_{+2}\overline{\psi}_{+3}\overline{\psi}_{+4}\overline{\chi}_-\phi^2\rangle, \quad \langle\lambda_-\overline{\psi}_{+2}\overline{\psi}_{+3}\overline{\psi}_{+4}\overline{\chi}_-\phi\rangle \tag{16}$$

which can have a nonzero vacuum expectation value in the instanton background. However, neither of these terms could be generated by a term in the (0,2) superpotential since there are far too many fermion zero modes. A superpotential term could absorb, at most, two fermion zero modes. Therefore, we see that there is no instanton generated superpotential. The same argument applies to instantons embedded in the other ϕ_i.

Next we show that there is no superpotential generated by a one anti-instanton contribution. The details are very similar to the one instanton case so we shall be brief. The anti-instanton configuration is similar to the instanton case except

$$\phi=f(r)e^{-i\theta} \tag{17}$$

and for large r,

$$v(r)\sim-\frac{1}{r}+\text{const}\frac{e^{-\eta r}}{\sqrt{r}}. \tag{18}$$

The Bogomol'nyi equations are now

$$(D_1-iD_2)\phi=0 \tag{19}$$

and

$$D-v_{12}=0 \tag{20}$$

leading to the anti-instanton action $S=2\pi\eta^2$. The normalizable fermion zero modes are now given by

$$\mu^0=\begin{pmatrix}\psi_+^0\\ \overline{\lambda}_-^0\end{pmatrix}=\begin{pmatrix}-\sqrt{2}(D_1+iD_2)\phi\\ D+v_{12}\end{pmatrix}, \tag{21}$$

$$\chi_-^0=\overline{\phi}^2, \tag{22}$$

and

$$\psi_{+i}^0=\phi \tag{23}$$

for $i=2,3,4$. Again the fermion zero mode analysis rules out the generation of a gauge invariant superpotential. Hence we see that there is no superpotential generated by a one anti-instanton contribution.

B. Changing the bundle

We can also consider the case where Γ carries gauge charge 2. This leads to a $\mathcal{O}(2)$ line bundle over $C\mathbb{P}^3$. The gauge anomaly cancellation condition is satisfied as the anomaly is proportional to the square of the charges. The model is still massive, and we again ask whether a superpotential is generated.

In this case, we find the same μ^0, $\overline{\psi}_{+i}^0$ zero modes (for $i=2,3,4$) in the instanton back-ground. However, there is now a zero mode for the field χ_- which is expanded in modes of the Dirac operator (D_1+iD_2). The zero mode is given by

$$\chi_-^0=\phi^2.$$

Again this is normalizable given the exponential decay of the fields at large distances. Once again a gauge invariant superpotential cannot be generated. Similar arguments hold for the case of the anti-instanton. Hence for both line bundles, $\mathcal{O}(\pm 2)$, over $C\mathbb{P}^3$, no world-sheet superpotential is generated. These theories are nonperturbatively stable.

C. A different route to stability

In the previous examples, a superpotential was forbidden because of the large number of zero modes in an (anti-)instanton background. We now turn to an example where we have the right number of fermion zero modes for a superpotential, but we will show that even in this case, there is no superpotential generated.

We consider a theory with one bosonic superfield Φ carrying gauge charge 1 and one Fermi superfield Γ carrying gauge charge -1. There are also the required gauge superfields Ψ and V. This gauge charge assignment causes the gauge anomaly to cancel. We can also set the theta angle to zero because of the nonzero chiral anomaly. We again construct vortex instanton solutions satisfying the Bogomol'nyi equations (9) and (10). The fermion zero modes are μ^0 as in Eq. (11) and $\overline{\chi}_-^0=\phi$. In this case, we see that

$$\langle\overline{\psi}_+\overline{\chi}_-\rangle$$

can get a nonzero vacuum expectation value in the instanton background. This would lead to the existence of a superpotential

$$S=-\frac{a}{\sqrt{2}}\int d^2y\, d\overline{\theta}^+\,\overline{\Gamma\Phi}|_{\theta^+=0}, \tag{24}$$

where a is a constant that can be determined. Hence the (0, 2) theory would be rendered unstable by this nonperturbative effect. However, the condensate has a vacuum expectation value proportional to

$$\int d^2x_0\phi(\bar{D}_1+i\bar{D}_2)\bar{\phi}, \tag{25}$$

where we have integrated over the two bosonic translational zero modes [19,20]. Using the identity

$$2i\phi(\bar{D}_1+i\bar{D}_2)\bar{\phi}+(\partial_1+i\partial_2)(D-v_{12})=0, \tag{26}$$

which can be proven using the Bogomolnyi equations, we see that this integral is actually zero. Yet again there is no instanton generated superpotential.

Lastly, we consider the case where Γ has a gauge charge 1. In this case, we can obtain Φ and Γ from a single (2,2) chiral superfield. The fermion zero modes are μ^0 as in Eq. (11) and $\chi_-^0=\phi$. However, the possibility $\langle\bar{\psi}_+\chi_-\rangle$ cannot be generated from a superpotential because of holomorphy of the superpotential. Therefore, no superpotential is generated at all. This certainly agrees with the (2,2) nonrenormalization theorem. From these examples, we see no nonperturbative superpotential generated. These results together with our other attempts at finding examples with nonvanishing superpotentials suggest that this phenomena is quite generic. In the next section, we give a general argument explaining why this happens.

III. THE (0,2) KONISHI ANOMALY

A. Deriving the anomaly

In this section, we obtain the Konishi anomaly [11,12] for the (0,2) linear sigma model with no tree level superpotential. Because of a perturbative nonrenormalization theorem, the only superpotential that can possibly be generated is a nonperturbative one. From the Konishi anomaly relation that we obtain, we argue that no such nonperturbative superpotential can be generated by instantons. This generalizes the results of the previous section.

Our derivation of the Konishi anomaly is along the lines of Ref. [12] which is a superspace generalization of Fujikawa's functional integral method [21]. We start with the linear sigma model with no tree level superpotential. We assume that a superpotential is generated nonperturbatively. Hence,

$$S=S_g+S_{\mathrm{ch}}+S_F+S_{D\theta}+S_J,$$

where S_J is the nonperturbative superpotential contribution. We want to prove that $S_J=0$ in the (anti-)instanton background. We denote all the chiral superfields by Σ, i.e., $\Sigma=\{\Phi_i^0,\Gamma_a^0\}$, and the corresponding antichiral superfields by $\bar{\Sigma}$. The partition function is given by

$$Z=\int[D\Phi_i^0 D\bar{\Phi}_i^0 D\Gamma_a^0 D\bar{\Gamma}_a^0 D\Psi DV]e^{iS}. \tag{27}$$

In the functional integral formalism, all the fields in the path integral measure

$$\Phi_i^0,\bar{\Phi}_i^0,\Gamma_a^0,\bar{\Gamma}_a^0,\Psi,V$$

are independent, and one can study their transformations separately.

We consider a global axial $U(1)$ transformation given by

$$\Sigma_m\to e^{i\alpha}\Sigma_m.$$

The subscript m signifies that only one of the fields in Σ transforms nontrivially. To extract a Ward identity in superspace, we consider the following transformation:

$$\Sigma_m\to\Sigma_m'=e^{iA}\Sigma_m, \tag{28}$$

where A is a chiral superfield satisfying $\bar{D}_+A=0$. This leads to a change in the measure and action

$$\begin{aligned} Z&=\int[D\Sigma_m\cdots]e^{iS}\\ &=\int[D\Sigma_m'\cdots]e^{iS'}\\ &=\int[D\Sigma_m\cdots]\mathcal{J}e^{iS+i\delta S}. \end{aligned}\tag{29}$$

For an infinitesimal transformation

$$\delta S=-\frac{i}{2}\int d^2y d^2\theta\bar{\Phi}_i(\mathcal{D}_0-\mathcal{D}_1)iA\Phi_i-\frac{1}{\sqrt{2}}\int d^2y d\theta^+ \times\sum_a \Gamma_a\frac{\delta J^a}{\delta\Phi_i}iA\Phi_i\bigg|_{\bar{\theta}^+=0} \tag{30}$$

if Σ_m is a bosonic chiral superfield and

$$\delta S=-\frac{1}{2}\int d^2y d^2\theta\bar{\Gamma}_a iA\Gamma_a-\frac{1}{\sqrt{2}}\int d^2y d\theta^+ iA\Gamma_a J^a\bigg|_{\bar{\theta}^+=0} \tag{31}$$

if Σ_m is a Fermi superfield. Also,

$$\mathcal{J}=\det{}_c\left(\frac{\delta\Sigma_m'}{\delta\Sigma_m}\right)=\det{}_c(-iA\bar{D}_+)=e^{\mathrm{tr}_c(-iA\bar{D}_+)}. \tag{32}$$

The reason for the subscript c, which means chiral, will be clear in a moment. The trace originally involves integration over y and $\theta^+,\bar{\theta}^+$. Using the relation

$$\int d\bar{\theta}^+=\frac{\partial}{\partial\bar{\theta}^+}=-\bar{D}_++i\theta^+(\partial_0+\partial_1), \tag{33}$$

we have replaced the integral over $\bar{\theta}^+$ by an insertion of $-\bar{D}_+$ in Eq. (32). The second term in Eq. (33) is a total derivative which we can drop since we integrate over y. The remaining superspace integral in the chiral trace only involves $\int d^2y d\theta^+$, and is therefore a chiral integral.

We regulate the trace in the following way:

$$\mathrm{tr}_c^{\mathrm{reg}}(-iA\bar{D}_+) = \lim_{M\to\infty}\mathrm{tr}_c(-iAe^{L/M^2}\bar{D}_+), \tag{34}$$

where

$$L = -\frac{i}{2}\bar{D}_+ e^{-i\Psi}(\mathcal{D}_0-\mathcal{D}_1)e^{-\Psi}D_+e^{2\Psi}. \tag{35}$$

Note that L respects manifest supersymmetry and is chiral because $\bar{D}_+L=0$. The $U(1)$ gauge transformation acts by

$$e^{\Psi}\to e^{-i\bar{\Lambda}}e^{\Psi}e^{i\Lambda}, \quad V\to V+(\partial_0-\partial_1)(\bar{\Lambda}+\Lambda). \tag{36}$$

So under a gauge transformation

$$L\to L' = e^{-2i\Lambda}Le^{2i\Lambda}. \tag{37}$$

Hence L is gauge covariant as well. We now proceed to compute the regulated trace in Eq. (34). We have

$$L = -\frac{i}{2}e^{-\Psi}\Upsilon e^{-\Psi}D_+e^{2\Psi} - \frac{i}{2}e^{-\Psi}(\mathcal{D}_0-\mathcal{D}_1)$$
$$\times[2i(\mathcal{D}_0+\mathcal{D}_1)-\mathcal{D}_+\bar{\mathcal{D}}_+]e^{\Psi}. \tag{38}$$

From the regulated trace in Eq. (34), it is clear that L always acts on $\bar{D}_+$. So we have a nonzero contribution only if we have a factor of D_+ along with $\bar{D}_+$ since

$$\langle D_+\bar{D}_+\rangle = -1.$$

This is possible when one factor of ΥD_+ is brought down from the exponential. To get a nonzero contribution, we have to set $\Psi=0$ in the first term in the expression for L. The last term in Eq. (38) involving $\mathcal{D}_+\bar{\mathcal{D}}_+$ does not contribute. Also the second term involving $(\mathcal{D}_0+\mathcal{D}_1)$ term contributes with $\Psi=0$. So acting on $\bar{D}_+$,

$$L = -\frac{i}{2}\Upsilon D_+ + (\mathcal{D}_0-\mathcal{D}_1)(\mathcal{D}_0+\mathcal{D}_1). \tag{39}$$

The leading term in the regulated trace is given by dropping the background gauge field terms in the second term in Eq. (39) leading to

$$L = -\frac{i}{2}\Upsilon D_+ + (\partial_0^2-\partial_1^2). \tag{40}$$

Hence, the regulated trace gives

$$\mathrm{tr}_c^{\mathrm{reg}}(-iA\bar{D}_+) = i\int d^2y\, d\theta + \frac{\Upsilon A}{8\pi}. \tag{41}$$

Finally, we obtain the Ward identity

$$\frac{1}{2}\bar{D}_+\bar{\Phi}_i(\mathcal{D}_0-\mathcal{D}_1)\Phi_i = -\frac{i}{\sqrt{2}}\sum_a \Gamma_a\frac{\delta J^a}{\delta\Phi_i}\Phi_i|_{\bar{\theta}^+=0} + \frac{\Upsilon}{8\pi} \tag{42}$$

for the bosonic chiral superfields and

$$\frac{1}{2}\bar{D}_+\bar{\Gamma}_a\Gamma_a = \frac{1}{\sqrt{2}}\Gamma_aJ^a|_{\bar{\theta}^+=0} + i\frac{\Upsilon}{8\pi} \tag{43}$$

for the Fermi superfields. They can be combined and written as

$$\bar{D}_+J = i\frac{\delta S_J}{\delta\Sigma_m}\Sigma_m|_{\bar{\theta}^+=0} + \frac{\Upsilon}{8\pi}. \tag{44}$$

Equation (44) and its conjugate obtained from considering antichiral transformations are the Konishi anomaly equations for the (0,2) linear sigma model.

B. Applying the Konishi equations

Now the relation (44) is a rather beautiful operator relation. We can take the expectation value of Eq. (44) in a BPS (anti-)instanton background. The left hand side is trivial in the chiral ring, and vanishes by fermion zero mode counting. We therefore obtain the general result

$$i\left\langle \frac{\delta S_J}{\delta\Sigma_m}\Sigma_m\bigg|_{\bar{\theta}^+=0}\right\rangle = -\left\langle\frac{\Upsilon}{8\pi}\right\rangle \tag{45}$$

for all m. Similarly from antichiral transformations, we obtain

$$i\left\langle \frac{\delta S_J}{\delta\bar{\Sigma}_m}\bar{\Sigma}_m\bigg|_{\theta^+=0}\right\rangle = \left\langle\frac{\bar{\Upsilon}}{8\pi}\right\rangle \tag{46}$$

for all m. From the component expansion for Υ and $\bar{\Upsilon}$, we see that the lowest component and the top component have vanishing vacuum expectation value because of Lorentz invariance: they involve the one point function of a fermion. The middle component of Υ is

$$2i\theta^+(D-iv_{01})$$

while that of $\bar{\Upsilon}$ is

$$-2i\bar{\theta}^+(D+iv_{01}).$$

Wick rotating to Euclidean space, we find that

$$\langle\Gamma_aJ^a|_{\bar{\theta}^+=0}\rangle = \frac{\theta^+}{2\sqrt{2}\pi}\langle D-v_{12}\rangle \tag{47}$$

and

$$\langle\bar{\Gamma}_a\bar{J}^a|_{\theta^+=0}\rangle = \frac{\bar{\theta}^+}{2\sqrt{2}\pi}\langle D+v_{12}\rangle \tag{48}$$

for all a. In the (anti-)instanton background, both $\langle D\rangle$ and $\langle v_{12}\rangle$ vanish because of fermion zero modes. Actually for theories with a broken chiral $U(1)$ symmetry, this vanishing also follows independently from the Bogomol'nyi equations. For example, in an instanton background we see that Eq. (48) vanishes using the Bogomol'nyi equation (10). The right hand side of Eq. (47) is proportional to $\langle v_{12}\rangle$ which, in turn, is proportional to θ [22]. However, because of the broken chiral $U(1)$ symmetry, all theta vacuua are equivalent and we can set theta to zero. The same analysis holds for the anti-instanton case leading to the final result that in an (anti-)instanton background

$$\Gamma_a J^a|_{\overline{\theta}^+=0} = \overline{\Gamma}_a \overline{J}^a|_{\theta^+=0} = 0. \tag{49}$$

We conclude that $S_J = 0$, and no nonperturbative superpotential is generated.

C. Cases with tree level superpotentials

What changes when we add a tree level superpotential? It appears that not a great deal changes in the preceeding argument. We replace S_J by the sum of the tree level superpotential S_J^0 and any nonperturbative superpotential S_J^{non}. The derivation just given goes through without further change, and we obtain the same equations (47) and (48). Evaluated in an instanton background, it again appears that the total superpotential must vanish. At first sight, this might appear to be a contradiction since, by construction, S_J^0 is nonzero.

However, the condition for an instanton to be BPS is now modified. In the presence of a superpotential, the BPS condition requires [7]

$$J_a^0 = 0 \tag{50}$$

so the Konishi relation is satisfied. Beyond multiplicatively renormalizing S_J^0, it seems that a nonperturbative superpotential is again ruled out.

D. The space-time superpotential

Lastly, for perturbatively conformal models, we want to address the question of whether the absence of a world-sheet superpotential implies the absence of a space-time superpotential. In models with no tree level superpotential, we can argue this relation as follows: a space-time superpotential implies that our perturbatively conformal theories, which we can label by the parameter $t = ir + \theta/2\pi$, do not flow to a family of superconformal field theories with a corresponding t modulus. Let us just consider the dependence on r. For example, they might flow to a trivial theory with $r \to \infty$.

How can r be renormalized? In the action, r appears in the term

$$-r\int \mathrm{d}^2 y D.$$

We need to ask whether D can be renormalized in an instanton background. Now D is bosonic, and we must absorb fermion zero modes. Where can they come from? The only place we see is a nonperturbative superpotential. The perturbative Lagrangian will not do because the zero modes are chiral. Since no world-sheet superpotential is generated, r remains an exactly marginal parameter and no space-time superpotential is generated.

What if there is a tree level world-sheet superpotential? In this case, fermion zero modes could be absorbed from the Yukawa terms generated from the superpotential. So fermion zero mode counting does not rule out renormalization of r. However, using the Bogomol'nyi equations, the remaining bosonic integral is always of the form

$$\int d^2 x_0 |\phi|^k (\partial_1 + i\partial_2)|\phi|^2,$$

where k is a non-negative integer, and we have embedded the instanton in ϕ. However, this integral over the two translational zero modes (with $\mathbb{R}^2$ as the Euclidean world-sheet) vanishes. Again, it appears that no space-time superpotential is generated.

ACKNOWLEDGMENTS

A.B. would like to thank J. Harvey for many useful discussions and A. Penin for useful email correspondence. We would also like to thank J. Distler for extensive discussions and D. Tong for helpful correspondence. The work of A.B. is supported in part by NSF Grant No. PHY-0204608. The work of S.S. is supported in part by NSF CAREER Grant No. PHY-0094328, and by the Alfred P. Sloan Foundation.

APPENDIX: THE STRUCTURE OF (0,2) SUPERSPACE

We review (0,2) superspace following [7]. We shall be dealing with abelian gauge theories. The superspace for (0,2) theories has bosonic coordinates y^0, y^1 and fermionic coordinates $\theta^+, \overline{\theta}^+$. The supersymmetry generators act in superspace in the following way:

$$Q_+ = \frac{\partial}{\partial\theta^+} + i\overline{\theta}^+(\partial_0 + \partial_1), \tag{A1}$$

$$\overline{Q}_+ = -\frac{\partial}{\partial\overline{\theta}^+} - i\theta^+(\partial_0 + \partial_1). \tag{A2}$$

On the other hand, the superspace covariant derivatives are given by

$$D_+ = \frac{\partial}{\partial\theta^+} - i\overline{\theta}^+(\partial_0 + \partial_1), \tag{A3}$$

$$\overline{D}_+ = -\frac{\partial}{\partial\overline{\theta}^+} + i\theta^+(\partial_0 + \partial_1). \tag{A4}$$

The following multiplets and the corresponding actions are used in various sections of the main text.

1. The gauge multiplet

The superspace gauge covariant derivatives $\mathcal{D}_+$, $\overline{\mathcal{D}}_+$, and $\mathcal{D}_\alpha$ ($\alpha = 1,2$) satisfy the algebra

$$\mathcal{D}_+^2 = \overline{\mathcal{D}}_+^2 = 0, \{\mathcal{D}_+, \overline{\mathcal{D}}_+\} = 2i(\mathcal{D}_0 + \mathcal{D}_1). \tag{A5}$$

The first two equations imply that $\mathcal{D}_+ = e^{-\Psi} D_+ e^{\Psi}$ and $\overline{\mathcal{D}}_+ = e^{\Psi} \overline{D}_+ e^{-\Psi}$, where Ψ takes values in the Lie algebra of the gauge group. In the Wess-Zumino gauge,

$$\Psi = \theta^+ \overline{\theta}^+ (v_0 + v_1)(y^\alpha).$$

We also have

$$\mathcal{D}_0 + \mathcal{D}_1 = \partial_0 + \partial_1 + i(v_0 + v_1), \tag{A6}$$

$$\mathcal{D}_+ = \frac{\partial}{\partial \theta^+} - i\overline{\theta}^+ (\mathcal{D}_0 + \mathcal{D}_1), \tag{A7}$$

$$\overline{\mathcal{D}}_+ = -\frac{\partial}{\partial \overline{\theta}_+} + i\theta^+ (\mathcal{D}_0 + \mathcal{D}_1), \tag{A8}$$

$$\mathcal{D}_0 - \mathcal{D}_1 = \partial_0 - \partial_1 + iV, \tag{A9}$$

where V is given by

$$V = v_0 - v_1 - 2i\theta^+ \overline{\lambda}_- - 2i\overline{\theta}^+ \lambda_- + 2\theta^+ \overline{\theta}^+ D. \tag{A10}$$

The gauge invariant field strength is $\Upsilon = [\overline{\mathcal{D}}_+, \mathcal{D}_0 - \mathcal{D}_1]$ which has a corresponding action

$$S_g = \frac{1}{8e^2} \int d^2y d^2\theta \overline{\Upsilon}\Upsilon$$

$$= \frac{1}{e^2} \int d^2y \left(\frac{1}{2} v_{01}^2 + i\overline{\lambda}_- (\partial_0 + \partial_1)\lambda_- + \frac{1}{2} D^2 \right). \tag{A11}$$

2. The chiral multiplet

There are bosonic chiral superfields Φ_i^0 satisfying $\overline{D}_+ \Phi_i^0 = 0$. Defining $\Phi_i^0 = e^{-\Psi}\Phi_i$, we see that $\overline{\mathcal{D}}_+ \Phi_i = 0$. Here Φ_i has the component expansion

$$\Phi_i = \phi_i + \sqrt{2}\theta^+ \psi_{+i} - i\theta^+ \overline{\theta}^+ (D_0 + D_1)\phi_i. \tag{A12}$$

This corresponding gauge invariant action is given by

$$S_{ch} = -\frac{i}{2} \int d^2y d^2\theta \sum_i \overline{\Phi}_i (\mathcal{D}_0 - \mathcal{D}_1)\Phi_i$$

$$= \int d^2y \sum_i \left(-|D_\alpha \phi_i|^2 + i\overline{\psi}_{+i}(D_0 - D_1)\psi_{+i} \right.$$

$$\left. - iQ_i\sqrt{2}\overline{\phi}_i \lambda_- \psi_{+i} + iQ_i\sqrt{2}\phi_i \overline{\psi}_{+i}\overline{\lambda}_- + Q_i D|\phi_i|^2 \right), \tag{A13}$$

where Φ_i has a $U(1)$ charge Q_i.

3. The Fermi multiplet

There are also fermionic chiral superfields Γ_a^0 with negative chirality satisfying

$$\overline{D}_+ \Gamma_a^0 = \sqrt{2} E_a^0,$$

where E_a^0 satisfies $\overline{D}_+ E_a^0 = 0$. Defining $\Gamma_a^0 = e^{-\Psi}\Gamma_a$ and $E_a^0 = e^{-\Psi} E_a$, the Fermi superfield has a component expansion

$$\Gamma_a = \chi_{-a} - \sqrt{2}\theta^+ G_a - i\theta^+ \overline{\theta}^+ (D_0 + D_1)\chi_{-a} - \sqrt{2}\overline{\theta}^+ E_a. \tag{A14}$$

We will consider cases where $E_a = 0$. In this case, the kinetic terms for the Fermi multiplet are given by

$$S_F = -\frac{1}{2} \int d^2y d^2\theta \sum_a \overline{\Gamma}_a \Gamma_a$$

$$= \int d^2y \sum_a [i\overline{\chi}_{-a}(D_0 + D_1)\chi_{-a} + |G_a|^2]. \tag{A15}$$

4. The $D\theta$ term

The terms in the action containing the Fayet-Iliopoulos D term and the theta term are given by

$$S_{D\theta} = \frac{t}{4} \int d^2y d\theta^+ \Upsilon|_{\overline{\theta}^+=0} + \text{H.c.} = \int d^2y \left(-rD + \frac{\theta}{2\pi} v_{01} \right), \tag{A16}$$

where $t = ir + \theta/2\pi$.

5. The superpotential term

The (0,2) superpotential is given by

$$S_J = -\frac{1}{\sqrt{2}} \int d^2y d\theta^+ \sum_a \Gamma_a J^a(\Phi_i)|_{\overline{\theta}^+=0} - \text{H.c.}$$

$$= -\int d^2y \sum_a \left(G_a J^a(\phi_i) + \sum_i \chi_{-\alpha}\psi_{+i} \frac{\partial J^a}{\partial \phi_i} \right) - \text{H.c.}, \tag{A17}$$

where the J^a are functions of the chiral superfields Φ^i.

[1] I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. **B241**, 493 (1984).
[2] J. Distler, "Notes on (0,2) superconformal field theories," hep-th/9502012.
[3] M. Dine, N. Seiberg, X. G. Wen, and E. Witten, Nucl. Phys. **B278**, 769 (1986).
[4] M. Dine, N. Seiberg, X. G. Wen, and E. Witten, Nucl. Phys. **B289**, 319 (1987).
[5] J. Distler, Phys. Lett. B **188**, 431 (1987).
[6] J. Distler and B. Greene, Nucl. Phys. **B304**, 1 (1988).
[7] E. Witten, Nucl. Phys. **B403**, 159 (1993).
[8] E. Silverstein and E. Witten, Nucl. Phys. **B444**, 161 (1995).
[9] J. Distler and S. Kachru, Nucl. Phys. **B430**, 13 (1994).
[10] J. Distler and S. Kachru, Phys. Lett. B **336**, 368 (1994).

[11] K. Konishi, Phys. Lett. **135B**, 439 (1984).
[12] K.-i. Konishi and K.-i. Shizuya, Nuovo Cimento A **90**, 111 (1985).
[13] R. Dijkgraaf and C. Vafa, ‘‘A perturbative window into non-perturbative physics,’’ hep-th/0208048.
[14] F. Cachazo, M. R. Douglas, N. Seiberg, and E. Witten, J. High Energy Phys. **12**, 071 (2002).
[15] A. A. Penin, Nucl. Phys. **B532**, 83 (1998).
[16] A. A. Abrikosov, JETP **5**, 1174 (1957).
[17] H. B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).
[18] N. K. Nielsen and B. Schroer, Nucl. Phys. **B127**, 493 (1977).
[19] J.-L. Gervais and B. Sakita, Phys. Rev. D **11**, 2943 (1975).
[20] E. Tomboulis, Phys. Rev. D **12**, 1678 (1975).
[21] K. Fujikawa, Phys. Rev. Lett. **42**, 1195 (1979).
[22] S. R. Coleman, Ann. Phys. (N.Y.) **101**, 239 (1976).
[23] C. Beasley and E. Witten, ‘‘Residues and World-Sheet Instantons,’’ hep-th/0304115.