$N=1$ supergravity with a cosmological constant and the AdS group

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It is shown that the supersymmetric extension of the Stelle-West formalism permits the construction of an action for $(3+1)$-dimensional $N=1$ supergravity with a cosmological constant genuinely invariant under the $OSp(4/1)$. Since the action is invariant under the supersymmetric extension of the AdS group, the supersymmetry algebra closes off shell without the need for auxiliary fields. The limit case $m\rightarrow 0$, i.e., $(3+1)$-dimensional $N=1$ supergravity, invariant under the Poincaré supergroup is also discussed.

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I. INTRODUCTION

In recent years it has been shown that in odd-dimensional supergravities [1,2] the fundamental field is always the connection A and, in their simplest form, these are pure Chern-Simons systems. In contrast with the standard cases, the supersymmetry transformations close off-shell without auxiliary fields.

The Chern-Simons construction fails in even dimensions for the simple reason that a characteristic class constructed with products of curvature in odd dimensions has not been found. This could be a reason why the construction of a (super)gravity in even dimensions invariant under the (anti–) de Sitter-group has remained an interesting open problem.

It is the purpose of this paper to show that the supersymmetric extension of the Stelle-West formalism [3], which is an application of the theory of nonlinear realizations to gravity, permits constructing a $(3+1)$-dimensional supergravity off-shell invariant under the (anti–) de Sitter group. The applications of the theory of nonlinear realizations to supergravity have been carried out by Chang and Mansouri [4] and by Gürsey and Marchildon [5]. These authors considered a nonlinear realization of the $OSP(1,4)$ in the context of the spontaneous breakdown of supergravity. Unlike the present work, they identified the corresponding coset parameters with the points of space-time itself.

In the present work, the Goldstone fields represent a point in an internal anti–de Sitter space. In describing the geometry of this internal space, we make use of some of the results of Ref. [6] on the nonlinear realization of supersymmetry in anti–de Sitter space.

An important stimulus for the interest in the construction of a supergravity invariant under the AdS superalgebra has come from recent developments in M theory [7,8]. In particular, some of the expected features of M theory are the following.

(1) Its dynamics should somehow exhibit a superalgebra in which the anticommutator of two supersymmetry generators coincides with the AdS superalgebra in eleven dimensions [9].

(2) The low-energy regime should be described by an eleven-dimensional supergravity of a new type which should stand on a firm geometric foundation in order to have an off-shell local supersymmetry [10].

The paper is organized as follows. In Sec. II, we shall review some aspects of the torsion-free condition in supergravity with cosmological constant. The supersymmetric extension of the Stelle-West formalism is carried out in Sec. III where the principal features of the nonlinear realizations are reviewed and the nonlinear fields vierbein, spin connection, and gravitino are derived. An action for supergravity genuinely invariant under the AdS superalgebra is constructed in Sec. IV, and its corresponding field equations as well as the limit $m\rightarrow 0$ are discussed. Section V concludes the work with a look forward to applications of the present results to supergravity in higher dimensions. Some technical details on the calculations are presented in the Appendix.

II. $N=1$ SUPERGRAVITY

In this section we shall review some aspects of the torsion-free condition in supergravity.

A. The torsion-free condition in $N=1$ supergravity

Supergravity is the theory of the gravitational field interacting with a spin 3/2 Rarita Schwinger field [11–13]. In the simplest case there is just one spin 3/2 Majorana fermion, usually called the gravitino ψ. The corresponding action is

$$S=\int \varepsilon_{abcd}e^a e^b R^{cd}+4\overline{\psi}\gamma_5 e^a\gamma_a D\psi, \qquad (1)$$

where e^a is the one-form vielbein, ω^{ab} is the one-form spin connection, and $D\psi=d\psi-\frac{1}{2}\ \omega^{ab}\gamma_{ab}\psi$ is the Lorentz covariant derivative.

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$D=3+1$, $N=1$ supergravity is based on the Poincaré supergroup whose generators P_a, J_{ab}, Q^α satisfy the following Lie superalgebra:

$$[P_a,P_b]=0,$$
$$[J_{ab},P_c]=i(\eta_{ac}P_b-\eta_{bc}P_a),$$
$$[J_{ab},J_{cd}]=i(\eta_{ac}J_{bd}-\eta_{bc}J_{ad}+\eta_{bd}J_{ac}-\eta_{ad}J_{bc}),$$
$$[J_{ab},Q_\alpha]=i(\gamma_{ab})_{\alpha\beta}Q_\beta,$$
$$[P_a,Q_\beta]=0$$
$$[Q_\alpha,\overline{Q}_\beta]=-2(\gamma^a)_{\alpha\beta}P_a. \tag{2}$$

Working in first-order formalism, the gauge fields e^a, ω^{ab},ψ are treated as independent. The key observation is that (e^a,ω^{ab},ψ), considered as a single entity, constitute a multiplet in the adjoint representation of the Poincaré supergroup. That is, we can write

$$A=A^AT_A=\tfrac{1}{2}i\omega^{ab}J_{ab}-ie^aP_a+\overline{\psi}Q, \tag{3}$$

where A is the gauge field of the Poincaré supergroup; P_a, J_{ab}, Q^α being the generators of the Poincaré translations, of the Lorentz transformations, and of the supersymmetry, respectively. Hence, supergravity is the gauge theory of the Poincaré supergroup.

The field strength associated with A^A is defined as the Poincaré Lie superalgebra-valued curvature two-form R^A. Splitting the index A, we get

$$R^{ab}=d\omega^{ab}-\omega^a_c\omega^{cd}, \tag{4}$$

$$\hat{T}^a=T^a-\tfrac{1}{2}\psi\gamma^a\psi, \tag{5}$$

$$\rho=D\psi. \tag{6}$$

The associated Bianchi identities are given by

$$DR^{ab}=0, \tag{7}$$

$$DT^a+R^{ab}e_b-i\psi\gamma^a\rho=0, \tag{8}$$

$$D\rho+\tfrac{1}{4}R^{ab}\gamma_{ab}\psi=0. \tag{9}$$

However, although $A^A\equiv(e^a,\omega^{ab},\psi)$ is a Yang-Mills potential and $R^A\equiv(R^{ab},\hat{T}^a,\rho)$ the corresponding field strength, action (1) is not of the Yang-Mills type. The main differences between an action of the Yang-Mills type and action (1) are the following.

(1) A Yang-Mills action is invariant under the whole gauge group of which the A^A are the Lie superalgebra-valued potentials.

(2) Action (1), instead, is not invariant under the whole gauge supergroup, but is invariant only under the Lorentz transformations.

The invariance under Lorentz gauge transformations is manifest. To show the noninvariance of Eq. (1) both under a supergauge translation and under supersymmetry we recall that, under any gauge transformation, the gauge connection A^A transforms as

$$\delta A=-D\lambda=d\lambda-[A,\lambda] \tag{10}$$

with

$$\lambda=\tfrac{1}{2}i\kappa^{ab}J_{ab}-i\rho^aP_a+\overline{\varepsilon}Q. \tag{11}$$

Using algebra (2) we obtain that e^a, ω^{ab}, and ψ, under the Poincaré translations, transform as

$$\delta e^a=D\rho^a,\quad \delta\omega^{ab}=0,\quad \delta\psi=0, \tag{12}$$

under the Lorentz rotations, as

$$\delta e^a=\kappa^a_be^b,\quad \delta\omega^{ab}=D\kappa^{ab},\quad \delta\psi=-\tfrac{1}{2}\kappa^{ab}\gamma_{ab}\psi, \tag{13}$$

and under supersymmetry transformations, as

$$\delta e^a=-2i\overline{\varepsilon}\gamma^a\psi,\quad \delta\omega^{ab}=0,\quad \delta\psi=D\varepsilon. \tag{14}$$

Action (1) is invariant under diffeomorphism, and under local Lorentz rotations, but it is not invariant under the neither the Poincaré translations nor the supersymmetry.

In fact, under the local Poincaré translations

$$\delta S_{pt}=2\int\varepsilon_{abcd}R^{ab}\left(T^c-\frac{1}{2}\overline{\psi}\gamma^c\psi\right)\rho^d+\text{surf. term},$$

$$\delta S=2\int\varepsilon_{abcd}R^{ab}\hat{T}^c\rho^d+\text{surf. term}. \tag{15}$$

Under local supersymmetry transformations

$$\delta S_{susy}=-4\int\overline{\varepsilon}\gamma_5\gamma_aD\psi\hat{T}^a+\text{surf. term}. \tag{16}$$

Thus, the invariance of the action requires the vanishing of the torsion

$$\hat{T}^a=0. \tag{17}$$

This means that the connection is no longer an independent variable. Rather, its variation is given in terms of δe^a and $\delta\psi$, and differs from the one dictated by group theory. An effect of the supertorsion-free condition on the local Poincaré superalgebra is that all commutators on e^a, ψ close except the commutator of two local supersymmetry transformations on the gravitino. For this commutator on the vierbein one finds

$$[\delta(\varepsilon_1),\delta(\varepsilon_2)]e^a=\tfrac{1}{2}\overline{\varepsilon_2}\gamma^aD\varepsilon_1-\tfrac{1}{2}\overline{\varepsilon_1}\gamma^aD\varepsilon_2=\tfrac{1}{2}D(\overline{\varepsilon_2}\gamma^a\varepsilon_1). \tag{18}$$

With $\rho^a=\tfrac{1}{2}\overline{\varepsilon_2}\gamma^a\varepsilon_1$, we can write

$$[\delta(\varepsilon_1),\delta(\varepsilon_2)]e^a=D\rho^a. \tag{19}$$

This means that, in the absence of the torsion-free condition, the commutator of two local supersymmetry transformations on the vierbein is a local Poincaré translation. However, the action is invariant by construction under general coordinate transformations, but not under the local Poincaré translation. The general coordinate transformation and the local Poincaré translation can be identified if we impose the torsion-free condition: since $\rho^a=\rho^\nu e^a_\nu$, we can write

$$D_\mu\rho^a=(\partial_\mu\rho^\nu)e^a_\nu+\rho^\nu(\partial_\nu e^a_\mu)+\tfrac{1}{2}\rho^\nu(\psi_\mu\gamma^a\psi_\nu)+\rho^\nu\omega_\nu^{ab}e_{\mu b}+\rho^\nu T^a_{\mu\nu}. \tag{20}$$

This means that, if $T^a_{\mu\nu}=0$, then the following commutator is valid:

$$[\delta_Q(\varepsilon_1),\delta_Q(\varepsilon_2)]=\delta_{GCT}(\rho^\mu)+\delta_{LLT}(\rho^\mu\omega_\mu^{ab})+\delta_Q(\rho^\nu\overline{\psi}_\nu), \tag{21}$$

where we can see that P in $\{Q,Q\}=P$, i.e., the local Poincaré translation, is replaced by general coordinate transformations besides two other gauge symmetries. The structure constants defined by this result are field dependent [13], which is a property of supergravity not present in the Yang-Mills theory.

The commutator of two local supersymmetry transformations on the gravitino is given by

$$[\delta(\varepsilon_1),\delta(\varepsilon_2)]\psi=\tfrac{1}{2}(\sigma_{ab}\varepsilon_2)[\delta(\varepsilon_1)\omega^{ab}]-\tfrac{1}{2}(\sigma_{ab}\varepsilon_1)\times[\delta(\varepsilon_2)\omega^{ab}]. \tag{22}$$

The condition $T\wedge^a=0$ leads to $\omega^{ab}=\omega^{ab}(e,\psi)$, which implies that the connection is no longer an independent variable, and its variation $\delta(\varepsilon)\omega^{ab}$ is given in terms of $\delta(\varepsilon)e^a$ and $\delta(\varepsilon)\psi$. Introducing $\delta(\varepsilon)\omega^{ab}(e,\psi)$ into Eq. (22) we see that, without the auxiliary fields, the gauge algebra does not close, as shows Eq. (10) of Ref. [13]. Therefore, the condition $T\wedge^a=0$ breaks not only the local Poincaré invariance but also the supersymmetry transformations.

B. The torsion-free condition in $N=1$ supergravity with cosmological constant

The action for supergravity with cosmological constant is given by [14]

$$S=\int \varepsilon_{abcd}R^{ab}e^ce^d+4\overline{\psi}\gamma_5\gamma_aD\psi e^a+2\alpha^2\varepsilon_{abcd}e^ae^be^ce^d+3\alpha\varepsilon_{abcd}\overline{\psi}\gamma^{ab}\psi e^ce^d, \tag{23}$$

where e^a is the one-form vielbein, ω^{ab} is the one-form spin connection, and $D\psi=d\psi-\frac{1}{2}\ \omega^{ab}\gamma_{ab}\psi$ is the Lorentz covariant derivative.

The anti–de Sitter version of $N=1$, $D=3+1$ supergravity is based on the graded extension of the AdS group, i.e., on the $OSp(1/4)$ whose generators P_a,J_{ab},Q^α satisfy the following Lie superalgebra:

$$[P_a,P_b]=-im^2J_{ab},$$

$$[J_{ab},P_c]=i(\eta_{ac}P_b-\eta_{bc}P_a),$$

$$[J_{ab},J_{cd}]=i(\eta_{ac}J_{bd}-\eta_{bc}J_{ad}+\eta_{bd}J_{ac}-\eta_{ad}J_{bc}),$$

$$[J_{ab},Q_\alpha]=i(\gamma_{ab})_{\alpha\beta}Q_\beta,$$

$$[P_a,Q_\alpha]=-\frac{i}{2}m(\gamma_a)_{\alpha\beta}Q_\beta$$

$$[Q_\alpha,\overline{Q}_\beta]=-2(\gamma^a)_{\alpha\beta}P_a-2m(\gamma^{ab})_{\alpha\beta}J_{ab}. \tag{24}$$

Working in the first-order formalism, the gauge fields e^a, ω^{ab},ψ are treated as independent. The key observation is that (e^a,ω^{ab},ψ), considered as a single entity, constitute a multiplet in the adjoint representation of the AdS supergroup; that is, we can write

$$A=A^AT_A=\tfrac{1}{2}i\omega^{ab}J_{ab}-ie^aP_a++\overline{\psi}Q, \tag{25}$$

where A is the gauge field of the AdS supergroup; P_a,J_{ab},Q^α being the generators of the AdS boosts, of the Lorentz transformations, and of the supersymmetry transformations, respectively. Hence, supergravity with cosmological constant is the gauge theory of the AdS supergroup.

The field strength associated with A^A is defined as the Poincaré Lie superalgebra-valued curvature two-form R^A. Splitting the index A, we get

$$\overline{R}^{ab}=R^{ab}+4\alpha^2e^ae^b+\alpha\psi\gamma^{ab}\psi, \tag{26}$$

$$\hat{T}^a=T^a-\tfrac{1}{2}\psi\gamma^a\psi, \tag{27}$$

$$\rho=D\psi-\alpha\gamma_a\psi e^a. \tag{28}$$

The associated Bianchi identities are given by

$$DR^{ab}-8\alpha^2T^ae^b+2\alpha\overline{\psi}\gamma^{ab}\rho=0, \tag{29}$$

$$DT^a+R^{ab}e_b-i\overline{\psi}\gamma^a\rho=0, \tag{30}$$

$$D\rho-i\alpha\gamma_a\psi T^a-\frac{1}{4}R^{ab}\gamma_{ab}\psi=0. \tag{31}$$

However, although $A^A\equiv(e^a,\omega^{ab},\psi)$ is a Yang-Mills potential and $R^A\equiv(R^{ab},\hat{T}^a,\rho)$ the corresponding field strength, action (23) is not of the Yang-Mills type. The main differences between an action of the Yang-Mills type and action (23) are the following.

(1) A Yang-Mills action is invariant under the whole gauge group of which A^A are the Lie superalgebra-valued potentials.

(2) Action (23), instead, is not invariant under the whole gauge supergroup, but is invariant only under the Lorentz transformations.

The invariance under the Lorentz gauge transformations is manifest. To show the noninvariance of Eq. (23) both under a supergauge translation and under supersymmetry we recall that, under any gauge transformation, the gauge connection A^A transforms as

$$\delta A = -D\lambda = d\lambda - [A,\lambda] \tag{32}$$

with

$$\lambda = \frac{1}{2} i\kappa^{ab} J_{ab} - i\rho^a P_a + +\overline{\varepsilon} Q. \tag{33}$$

Using algebra (24) we obtain that e^a, ω^{ab}, and ψ, under AdS boosts, transform as

$$\delta e^a = D\rho^a, \quad \delta\omega^{ab} = m^2(\rho^a e^b - \rho^b e^a), \quad \delta\psi = 0, \tag{34}$$

under the Lorentz rotations, as

$$\delta e^a = \kappa^a_b e^b, \quad \delta\omega^{ab} = D\kappa^{ab}, \quad \delta\psi = -\frac{1}{2}\kappa^{ab}\gamma_{ab}\psi, \tag{35}$$

and under the supersymmetry transformations, as

$$\delta e^a = -2i\overline{\varepsilon}\gamma^a\psi, \quad \delta\omega^{ab} = 0, \quad \delta\psi = D\varepsilon. \tag{36}$$

Action (23) is invariant under diffeomorphism and under the local Lorentz rotations, but it is not invariant under neither AdS boosts translations nor local supersymmetric transformation.

In fact, under the local Poincaré translations

$$\delta S_{AdS} = -2\int \varepsilon_{abcd}\overline{R}^{ab}\hat{T}^c\rho^d + \text{surf. term} \tag{37}$$

and under the local supersymmetry transformations

$$\delta S_{susy} = -4\int \overline{\varepsilon}\gamma_5\gamma_a D\psi\hat{T}^a + \text{surf. term.}$$

Thus, the invariance of the action requires the vanishing of the torsion

$$\hat{T}^a = 0. \tag{38}$$

This means that the connection is no longer an independent variable. Rather, its variation is given in terms of δe^a and $\delta\psi$, and differs from the one dictated by group theory. The condition $\hat{T}^a = 0$ breaks not only the local Poincaré invariance, but also the supersymmetry transformations.

III. SUPERSYMMETRIC EXTENSION OF THE STELLE-WEST FORMALISM

The basic idea of the Stelle-West (SW) formalism is founded on the mathematical definition [3,15] of the vielbein V^a. This vielbein, also called solder form [16], was considered as a smooth map from the tangent space to the spacetime manifold M at a point P with coordinates x^μ, and the tangent space to the AdS internal space at the point whose AdS coordinates are $\xi^a(x)$, as the point P ranges over the whole manifold M. Figure 1 of Ref. [3] illustrates that such a vielbein $V^a_\mu(x)$ is the matrix of the map between the space $T_x(M)$ tangent to the space-time manifold at x^μ, and the space $T_{\xi(x)}(\{G/H\}_x)$ tangent to the internal AdS space $\{G/H\}_x$ at the point $\xi^a(x)$, whose explicit form is given by Eq. (3.19) of Ref. [3]. In this section we consider the supersymmetric extension of the Stelle-West formalism

A. Nonlinear realizations of supersymmetry in AdS space

The nonlinear realizations in de Sitter space can be studied by the general method developed in Ref. [17,18]. Following these references, we consider a Lie (super)group G and a subgroup H.

Let us call $\{\mathbf{V}_i\}_{i=1}^{n-d}$ the generators of H. We assume that the remaining generators $\{\mathbf{A}_l\}_{l=1}^{d}$ can be chosen so that they form a representation of H. In other words, the commutator $[\mathbf{V}_i, \mathbf{A}_l]$ should be a linear combination of $\mathbf{A}_l$ alone. A group element $g \in G$ can be represented (uniquely) in the form

$$g = e^{\xi^l \mathbf{A}_l} h, \tag{39}$$

where h is an element of H. The ξ^l parametrize the coset space G/H. We do not specify here the parametrization of h. One can define the effect of a group element g_0 on the coset space by

$$g_0 g = g_0(e^{\xi^l \mathbf{A}_l} h) = e^{\xi'^l \mathbf{A}_l} h' \tag{40}$$

or

$$g_0 e^{\xi^l \mathbf{A}_l} = e^{\xi'^l \mathbf{A}_l} h_1, \tag{41}$$

where

$$h_1 = h' h^{-1}, \tag{42}$$

$$\xi' = \xi'(g_0, \xi),$$

$$h_1 = h_1(g_0, \xi).$$

If $g_0 - 1$ is infinitesimal, Eq. (41) implies

$$e^{-\xi^l \mathbf{A}_l}(g_0 - 1)e^{\xi^l \mathbf{A}_l} - e^{-\xi^l \mathbf{A}_l}\delta e^{\xi^l \mathbf{A}_l} = h_1 - 1. \tag{43}$$

The right-hand side of Eq. (43) is a generator of H.

Let us first consider the case in which $g_0 = h_0 \in H$. Then Eq. (41) gives

$$e^{\xi'^l \mathbf{A}_l} = h_0 e^{\xi^l \mathbf{A}_l} h_0^{-1}. \tag{44}$$

Since $\mathbf{A}_l$ form a representation of H, this implies

$$h_1 = h_0, \quad h' = h_0 h. \tag{45}$$

The transformation from ξ to ξ' given by Eq. (44) is linear. On the other hand, consider now

$$g_0 = e^{\xi_0^l \mathbf{A}_l}. \tag{46}$$

In this case, Eq. (41) becomes

$$e^{\xi_0^l \mathbf{A}_l} e^{\xi^l \mathbf{A}_l} = e^{\xi'^l \mathbf{A}_l} h. \tag{47}$$

This is a nonlinear inhomogeneous transformation on ξ. The infinitesimal form (43) becomes

$$e^{-\xi^l \mathbf{A}_l}\xi_0^i \mathbf{A}_i e^{\xi^j \mathbf{A}_j} - e^{-\xi^l \mathbf{A}_l}\delta e^{\xi^i \mathbf{A}_i} = h_1 - 1. \tag{48}$$

The left-hand side of this equation can be evaluated, using the algebra of the group. Since the results must be a generator of H, one must set equal to 0 the coefficient of $\mathbf{A}_l$. In this way one finds an equation from which $\delta\xi^i$ can be calculated.

The construction of a Lagrangian invariant under coordinate-dependent group transformations requires the introduction of a set of gauge fields $a = a^i_\mu \mathbf{A}_i dx^\mu$, $\rho = \rho^i_\mu \mathbf{V}_i dx^\mu$, $p = p^l_\mu \mathbf{A}_l dx^\mu$, $v = v^i_\mu \mathbf{V}_i dx^\mu$, associated, respectively, with the generators V_i and A_l. Hence, $\rho + a$ is the usual linear connection for the gauge group G, and the corresponding covariant derivatives is given by

$$D = d + f(\rho + a) \tag{49}$$

and its transformation law under $g \in G$ is

$$g:(\rho + a) \to (\rho' + a') = \left[g(\rho + a)g^{-1} - \frac{1}{f}(dg)g^{-1} \right], \tag{50}$$

where f is a constant which, as it turns out, gives the strength of the universal coupling of the gauge fields to all other fields.

We now consider the Lie algebra-valued differential form [17]

$$e^{-\xi^l \mathbf{A}_l}[d + f(\rho + a)]e^{\xi^l \mathbf{A}_l} = p + v. \tag{51}$$

The transformation laws for the forms $p(\xi, d\xi)$ and $v(\xi, d\xi)$ are easily obtained. In fact, using Eqs. (46),(47) one finds [6]

$$p' = h_1 p (h_1)^{-1}, \tag{52}$$

$$v' = h_1 v (h_1)^{-1} + h_1 d (h_1)^{-1}. \tag{53}$$

Equation (52) shows that the differential forms $p(\xi, d\xi)$ are transformed linearly by a group element of form (46). The transformation law is the same as by an element of H, except that now this group element $h_1(\xi_0, \xi)$ is a function of the variable ξ. Therefore, any expression constructed with $p(\xi, d\xi)$, which is invariant under the subgroup H, will be automatically invariant under the entire group G, the elements of H operating linearly on ξ, the remaining elements nonlinearly.

We have specified the fields p and v as well as their transformation properties, and now we make use of them to define the covariant derivative with respect to the group G:

$$D = d + v. \tag{54}$$

The corresponding components of the curvature two-form are

$$T = Dp, \tag{55}$$

$$R = dv + vv. \tag{56}$$

B. Supersymmetric Stelle-West formalism

We now take as G the graded Lie algebra (24) having as generators Q_α, P_a, and M_{ab}. It has as a subalgebra H that of the de Sitter group $SO(3,2)$ with generators P_a and M_{ab}. This, in turn, has as subalgebra L that of the Lorentz group $SO(3,1)$ with generators M_{ab}. An element of G can be represented uniquely in the form

$$g = e^{\bar{\chi} Q} h = e^{\bar{\chi} Q} e^{-i\xi^a P_a} l, \tag{57}$$

where $h \in H$ and $l \in L$. On can define the effect of a group element g_0 on the coset space G/H by

$$g_0 g = e^{\bar{\chi}' Q} h' = e^{\bar{\chi}' Q} e^{-i\xi'^a P_a} l' \tag{58}$$

or

$$g_0 e^{\bar{\chi} Q} = e^{\bar{\chi}' Q} h_1, \tag{59}$$

$$h_1 e^{-i\xi^a P_a} = e^{-i\xi'^a P_a} l_1, \tag{60}$$

$$l_1 l = l'. \tag{61}$$

Clearly, $h_1 = h_1(g_0, \chi)$ and $l_1 = l_1(g_0, \chi, \xi)$.

If $g_0 - 1$ and $h_1 - 1$ are infinitesimals, Eqs. (59),(60) imply

$$e^{-\bar{\chi} Q}(g_0 - 1)e^{\bar{\chi} Q} - e^{-\bar{\chi} Q}\delta e^{\bar{\chi} Q} = h_1 - 1, \tag{62}$$

$$e^{i\xi^a \mathbf{P}_a}(h_1 - 1)e^{-i\xi^a \mathbf{P}_a} - e^{i\xi^a \mathbf{P}_a}\delta e^{-i\xi^a \mathbf{P}_a} = l_1 - 1. \tag{63}$$

We consider now the following cases: If $g_0 = l_0 \in L$, Eqs. (59),(60) give

$$e^{\bar{\chi}' Q} = l_0 e^{\bar{\chi} Q} l_0^{-1}, \tag{64}$$

$$h_1 = l_1 = l_0, \tag{65}$$

$$e^{-i\xi'^a P_a} = l_0 e^{-i\xi^a P_a} l_0^{-1}. \tag{66}$$

Both χ and ξ transform linearly. If, on the other hand, we know only that $g_0 = h_0 \in H$, in particular, if

$$g_0 = e^{-i\rho^a \mathbf{P}_a} \tag{67}$$

is a pseudotranslation, Eq. (59) gives

$$e^{\bar{\chi}' Q} = h_0 e^{\bar{\chi} Q} h_0^{-1}, \tag{68}$$

$$h_1 = h_0, \tag{69}$$

while Eq. (60) gives

$$h_0 e^{i\xi^a \mathbf{P}_a} = e^{-i\xi'^a P_a} l_1(h_0, \xi). \tag{70}$$

In this case χ transforms linearly, but the transformation law (70) of ξ under pseudotranslations is inhomogeneous and nonlinear. Infinitesimally

$$e^{i\xi^a \mathbf{P}_a}(-i\rho^a \mathbf{P}_a)e^{-i\xi^a \mathbf{P}_a} - e^{i\xi^a \mathbf{P}_a}\delta e^{-i\xi^a \mathbf{P}_a} = l_1 - 1. \tag{71}$$

Finally, if

$$g_0 = e^{\bar{\varepsilon}Q} \tag{72}$$

is a supersymmetry transformation, one must use Eqs. (59) and (60) as they stand. Observe, however, that Eq. (60) has the same form as Eq. (70) except for the fact that h_1 is a function of χ, while h_0 is not. Therefore, the transformation law of ξ under a supersymmetry transformation has the same form as that under a de Sitter transformation but, with parameters which depend in a well-defined way on χ.

An explicit form for the transformation law of ξ^a under an infinitesimal AdS boost can be obtained from Eq. (71). The result is

$$\delta\xi^a = -\rho^a + \left(\frac{z\cosh z}{\sinh z} - 1\right)\left(\rho^a - \frac{\rho^b \xi_b \xi^a}{\xi^2}\right), \tag{73}$$

where $z = m\sqrt{(\xi^a \xi_a)} = m\xi$.

The transformation of ξ^a under an infinitesimal Lorentz transformation $l_0 = e^{(i/2)\kappa^{ab}J_{ab}}$ is

$$\delta\xi^a = \kappa^{ab}\xi_b, \tag{74}$$

and, under local supersymmetry transformation (72), ξ^a transforms as

$$\delta\xi^a = -i\left(1 + \frac{i}{6}m\bar{\chi}\chi\right)\bar{\varepsilon}\gamma^a\chi + i\left(\frac{z\cosh z}{\sinh z} - 1\right)\left(\delta^a_b - \frac{\xi_b\xi^a}{\xi^2}\right) \times\left(1 + \frac{i}{6}m\bar{\chi}\chi\right)\bar{\varepsilon}\gamma^b\chi - 2im\left(1 + \frac{i}{6}m\bar{\chi}\chi\right)\bar{\varepsilon}\gamma^{ab}\chi\xi_b. \tag{75}$$

Using Eq. (62) with $g_0 - 1 = \bar{\varepsilon}Q$, one finds that

$$\delta\chi = -\varepsilon - \frac{i}{6}m(5\bar{\chi}\chi + \bar{\chi}\Gamma_A\chi\Gamma^A)\varepsilon + \frac{1}{9}m^2(\bar{\chi}\chi)\varepsilon,$$

$$h_1 - 1 = \left(1 + \frac{i}{6}m\bar{\chi}\chi\right)(\bar{\varepsilon}\gamma^a\chi P_a + m\bar{\varepsilon}\gamma^{ab}\chi J_{ab}). \tag{76}$$

From Eq. (25) we know that the linear connections are given by (e^a, ω^{ab}, ψ). Then, based on these, we can define the corresponding nonlinear connections (V^a, W^{ab}, Ψ) from Eq. (51):

$$\tfrac{1}{2}iW^{ab}\mathbf{J}_{ab} - iV^a\mathbf{P}_a + \bar{\Psi}Q = e^{i\xi^a\mathbf{P}_a}e^{-\bar{\chi}Q}[d + \tfrac{1}{2}i\omega^{ab}\mathbf{J}_{ab} - ie^a\mathbf{P}_a + \bar{\psi}Q]e^{\bar{\chi}Q}e^{-i\xi^b\mathbf{P}_b}. \tag{77}$$

The corresponding transformation laws for V^a, W^{ab}, Ψ can be obtained from Eqs. (52),(53). In fact, one can verify that, under the AdS supergroup, the nonlinear connections transform as

$$\bar{\Psi}'Q = h_1(\bar{\Psi}Q)(h_1)^{-1}, \tag{78}$$

$$-iV'^a\mathbf{P}_a = h_1(-iV^a\mathbf{P}_a)(h_1)^{-1}, \tag{79}$$

$$\tfrac{1}{2}iW'^{ab}\mathbf{J}_{ab} = h_1(\tfrac{1}{2}iW^{ab}\mathbf{J}_{ab})(h_1)^{-1} + h_1 d(h_1)^{-1}. \tag{80}$$

The nonlinearity of the transformation with respect to the elements of G/H means that the labels associated with the parts of the algebra of G, which generate G/H, are no longer available as symmetry indices. In other words, the symmetry has been spontaneously broken from G to H. An irreducible representation of G will, in general, have several irreducible pieces with respect to H. Since, in constructing invariant actions, one only needs index saturation with respect to the subgroup H, as far as the invariance is concerned it is possible to select a subset of nonlinear fields with respect to G, which form irreducible multiplets with respect to H.

Note that, if $G = OSp(1,4)$ and $H = SO(3,1)$, the gauge fields V^a form a square 4×4 matrix, which is invertible and can be identified with the vierbein fields. In the same way we have that W^{ab} is a connection and $\bar{\Psi}$ can be identified with the Rarita-Schwinger field. These fields can be obtained from Eq. (77). The details of the calculation of V^a, W^{ab}, Ψ are given in the Appendix; the result is

$$V^a = \Omega[\cosh z]^a{}_b e^b + \Omega\left[\frac{\sinh z}{z}\right]^a_b D\zeta^b + i\left(1 - \frac{i}{6}m\bar{\chi}\chi\right) \times\Big\{[\bar{\chi}\gamma^b d\chi + 2\bar{\psi}\gamma^b\chi]\Omega[\cosh z]^a_b - 2m\xi_b[\bar{\chi}\gamma^b d\chi + 2\bar{\psi}\gamma^b\chi]\frac{\sinh z}{z}\Big\} - \frac{i}{2}\left[1 - \frac{i}{12}(\bar{\chi}\gamma^f\chi)\gamma_f + \frac{i}{6}(\bar{\chi}\gamma^{fg}\chi)\gamma_{fg}\right](\gamma_{cd}\omega^{cd} - im\gamma_c e^c) \times\left[(\bar{\chi}\gamma^b\chi)\Omega[\cosh z]^a_b + 2m(\bar{\chi}\gamma^{ab}\chi)\xi_b\frac{\sinh z}{z}\right], \tag{81}$$

where

$$\Omega(A)^a_b = A\delta^a_b + (1-A)\frac{\xi_b\xi^a}{\xi^2}, \tag{82}$$

$$W^{ab} = \omega^{ab} + m^2\left[(\xi^a e^b - \zeta^b e^a)\frac{\sinh z}{z} - m^2(\zeta^a D\zeta^b - \zeta^b D\zeta^a)\frac{(\cosh z - 1)}{z^2}\right] - 2im\left(1 - \frac{i}{6}m\bar{\chi}\chi\right)\Big\{2\bar{\psi}\gamma^{ab}\chi + m\bar{\chi}\gamma^{ab}d\chi + m[\bar{\chi}\gamma^b d\chi + 2\bar{\psi}\gamma^b\chi]\xi^a\frac{\sinh z}{z} + 2m^2[\bar{\chi}\gamma^{cb}d\chi + 2\bar{\psi}\gamma^{cb}\chi]\xi^a\xi_c\frac{(\cosh z - 1)}{z^2}\Big\} + \frac{1}{2}im\left[1 - \frac{i}{12}(\bar{\chi}\gamma^f\chi)\gamma_f + \frac{i}{6}(\bar{\chi}\gamma^{fg}\chi)\gamma_{fg}\right](\gamma_{cd}\omega^{cd} - im\gamma_c e^c)\left[(\bar{\chi}\gamma^{ab}\chi) + m(\bar{\chi}\gamma^a\chi)\xi^b\frac{\sinh z}{z}\right.$$

$$+2m^2(\overline{\chi}\gamma^{fb}\chi)\xi_f\frac{(\cosh z-1)}{z^2}\Bigg], \quad (83)$$

$$\overline{\Psi}=\Bigg\{\Bigg[1-\frac{i}{4}m(5\overline{\chi}\chi+\overline{\chi}\Gamma_A\chi\Gamma^A)-\frac{5}{24}m^2(\overline{\chi}\chi)^2\Bigg]\overline{\psi} -\frac{1}{2}\Bigg[1-\frac{i}{6}m(\overline{\chi}\gamma^a\chi)\gamma_a+\frac{i}{3}m(\overline{\chi}\gamma^{ab}\chi)\gamma_{ab}\Bigg](\gamma_{cd}\omega^{cd} -im\gamma_c e^c)\overline{\chi}+\Bigg[1-\frac{i}{12}m(5\overline{\chi}\chi+\overline{\chi}\Gamma_A\chi\Gamma^A) -\frac{1}{24}m^2(\overline{\chi}\chi)^2\Bigg]d\chi\Bigg\}e^{(1/2)m\xi^d\gamma_d}. \quad (84)$$

We have specified the fields V^a, W^{ab}, and Ψ as well as their transformation properties, and now we make use of them to define a covariant derivatives with respect to the group G:

$$\mathcal{D}=d+W. \quad (85)$$

The corresponding components of a curvature two-forms are

$$\mathcal{T}^a=\mathcal{D}V^a, \quad (86)$$

$$\mathcal{R}^{\ a}_{b}=dW^a_b+W^a_cW^c_b. \quad (87)$$

IV. SUPERGRAVITY INVARIANT UNDER THE AdS GROUP

Within the supersymmetric extension of the Stelle-West formalism, the action for supergravity with cosmological constant can be rewritten as

$$S=\int \varepsilon_{abcd}\mathcal{R}^{\ ab}V^cV^d+4\overline{\Psi}\gamma_5\gamma_a\mathcal{D}\Psi V^a +2\alpha^2\varepsilon_{abcd}V^aV^bV^cV^d+3\alpha\varepsilon_{abcd}\overline{\Psi}\gamma^{ab}\Psi V^cV^d, \quad (88)$$

which is invariant under Eqs. (78),(79),(80). From such equations we can see that the vierbein V^a and the gravitino field transform homogeneously according to the representation of the AdS superalgebra, but with the nonlinear group element $h_1\in H$.

The corresponding equations of motion are obtained by varying the action with respect to $\xi^a,\chi,e^a,\omega^{ab},\psi$. The field equations corresponding to the variation of the action with respect to ξ^a and χ are not independent equations. Following the same procedure of Ref. [20], we find that equations of motion for supergravity genuinely invariant under super AdS are the following:

$$2\varepsilon_{abcd}\overline{\mathcal{R}}^{ab}V^c+4\overline{\Psi}\gamma_5\gamma_d\rho=0, \quad (89)$$

$$\hat{\mathcal{T}}^{\ a}=0, \quad (90)$$

$$8\gamma_5\gamma_a\rho V^a-4\gamma_5\gamma_a\Psi\hat{\mathcal{T}}^{\ a}=0, \quad (91)$$

where

$$\hat{\mathcal{T}}^{\ a}=\mathcal{T}^a-\frac{i}{2}\overline{\Psi}\gamma^a\Psi, \quad (92)$$

$$\overline{\mathcal{R}}^{ab}=\mathcal{R}^{\ ab}+4\alpha^2V^aV^b+\alpha\overline{\Psi}\gamma^{ab}\Psi, \quad (93)$$

$$\rho=\mathcal{D}\Psi-i\alpha\gamma^a\Psi V^a. \quad (94)$$

Supergravity invariant under the Poincaré group

Taking the limit $m\rightarrow 0$ in Eqs. (24),(73),(75),(76),(81),(83),(84) we find that superalgebra (24) takes the form of the superalgebra of Poincaré (2) and that the transformation laws of ξ^a under an infinitesimal Poincaré translation, under an infinitesimal Lorentz transformation, and under the local supersymmetry transformation are given respectively, by

$$\delta\xi^a=-\rho^a, \quad (95)$$

$$\delta\xi^a=\kappa^a_b\xi^b, \quad (96)$$

$$\delta\xi^a=-i\overline{\varepsilon}\gamma^a\chi; \quad (97)$$

the transformation laws of χ under an infinitesimal Poincaré translation, under an infinitesimal Lorentz transformation, and under the local supersymmetry transformation are given, respectively, by

$$\delta\chi=0, \quad (98)$$

$$\delta\chi=0, \quad (99)$$

$$\delta\chi=-\varepsilon. \quad (100)$$

In this limit $G=ISO(3,1)$ and $H=SO(3,1)$ and the fields vierbein V^a, the connection W^{ab} and the Rarita-Schwinger field $\overline{\Psi}$ are given by

$$V^a=e^a+D\zeta^a+i(2\overline{\psi}+D\overline{\chi})\gamma^a\chi, \quad (101)$$

$$W^{ab}=\omega^{ab}, \quad (102)$$

$$\overline{\Psi}=\overline{\psi}+D\overline{\chi}, \quad (103)$$

where now

$$D=d+\omega. \quad (104)$$

The corresponding components of the curvature two-form are now

$$\mathcal{T}^a=DV^a, \quad (105)$$

$$R^a_b=d\omega^a_b+\omega^a_c\omega^c_b. \quad (106)$$

The limit $m\rightarrow 0$ of the action 88 is obviously the action for $N=1$ supergravity in $(3+1)$-dimensions:

$$S=\int \varepsilon_{abcd}R^{ab}V^cV^d+4\overline{\Psi}\gamma_5\gamma_aD\Psi V^a, \quad (107)$$

which is genuinely invariant under the Poincaré supergroup. In fact, it is direct to verify that action (107) is invariant under Eqs. (95)–(100) plus the transformation law of e^a, ω^{ab}, ψ under infinitesimal Poincaré translations, under infinitesimal Lorentz transformations, and under local supersymmetry transformations, which are given by

$$\delta\omega^{ab}=0, \quad \delta e^a = D\rho^a, \quad \delta\psi=0,$$

$$\delta\omega^{ab}=D\kappa^{ab}, \quad \delta e^a=\kappa^a_b e^b, \quad \delta\psi=-\tfrac{1}{2}\kappa^{ab}\gamma_{ab}\psi,$$

$$\delta\omega^{ab}=0, \quad \delta e^a=-2i\overline{\varepsilon}\gamma^a\psi, \quad \delta\psi=D\varepsilon.$$

The corresponding field equations are given by

$$2\varepsilon_{abcd}R^{ab}V^c+4\overline{\Psi}\gamma_5\gamma_d D\Psi=0, \tag{108}$$

$$\hat{\mathcal{T}}^a=0, \tag{109}$$

$$8\gamma_5\gamma_a D\Psi V^a - 4\gamma_5\gamma_a\Psi\hat{\mathcal{T}}^a=0, \tag{110}$$

where

$$\hat{\mathcal{T}}^a=\mathcal{T}^a-\frac{i}{2}\overline{\Psi}\gamma^a\Psi. \tag{111}$$

In Ref. [19] we have clamed that the successful formalism used by Stelle-West and by Grignani-Nardelli [21] to construct an action for $(3+1)$-dimensional gravity invariant under the Poincaré group can be generalized to supergravity in $3+1$ dimensions. In fact, that is correct. Using the vierbein of Stelle-West and Grignani-Nardelli, one gets a supergravity action invariant under the Poincaré translation. However, the action of Ref. [19] is not invariant under supersymmetry transformations, as we can see from (16).

To obtain an action invariant both under the Poincaré translations and under supersymmetry transformations, we must carry out the supersymmetric extension of the Stelle-West formalism. The correct vierbein, spin connection, and gravitino field to construct a supergravity action [see Eq. (107)] genuinely invariant under the Poincaré supergroup are given in Eqs. (101),(102),(103).

V. COMMENTS AND POSSIBLE DEVELOPMENTS

The main results of this work can be summarized as follows.

(1) In order to construct a gauge theory of the supersymmetric extension of the AdS group, it is necessary to carry out the supersymmetric extension of the Stelle-West-Grignani-Nardelli formalism.

(2) The correspondence with the usual $N=1$ supergravity with cosmological constant formulation has been established by giving the expressions, in terms of the gauge fields, of the spin connection, the vierbein, and the gravitino. These fields are given by complicated expressions involving ξ^a, χ, ψ, ω^{ab}, and e^a.

(3) An action for $(3+1)$-dimensional $N=1$ supergravity with cosmological constant genuinely invariant under the supersymmetric extension of the AdS group has been proposed. The corresponding equations of motion reproduce the usual equation for $N=1$ supergravity with cosmological constant.

Several aspects deserve consideration and many possible developments can be worked out. An old and still unsolved problem is the construction of an eleven-dimensional supergravity off-shell invariant under the supersymmetric extension of the AdS group (work in progress). The construction of an action for supergravity in ten dimensions genuinelly invariant under the AdS superalgebra, and its relation to eleven-dimensional supergravity, could also be of interest.

Another interesting issue is the connection between the present paper and the supergravity in $(3+1)$-dimensions obtained via dimensional reduction from five-dimensional Chern-Simons supergravity (work in progress).

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APPENDIX

In this appendix, we discuss how to derive some of the results given in the text, in particular, the expressions for V^a, W^{ab}, Ψ. We use the techniques of Refs. [3,6], which we summarize here for convenience.

For any two quantities X and Y, we define

$$[X,Y]\equiv X\wedge Y, \tag{A1}$$

$$X^2\wedge Y=[X,[X,Y]]. \tag{A2}$$

The expression $f(X)\wedge Y$ is defined as a series of multiple commutators, obtained by expanding the function $f(X)$ as a power series in X. It is direct to verify that

$$g(X)\wedge[f(X)\wedge Y]=[g(X)f(X)]\wedge Y. \tag{A3}$$

As a consequence, the equation

$$f(X)\wedge Y=Z \tag{A4}$$

can be solved for Y in the form

$$Y=[f(X)]^{-1}\wedge Z. \tag{A5}$$

In particular, we have

$$e^X Y e^{-X}=e^X\wedge Y, \tag{A6}$$

$$e^{X}\delta e^{-X}=\frac{1-e^{X}}{X}\wedge\delta X, \tag{A7}$$

where δ is any variation.

When written in the above notation, Eq. (71) become

$$e^{i\xi^{a}\mathbf{P}_{a}}\wedge(-i\rho^{a}\mathbf{P}_{a})-\frac{1-e^{i\xi^{a}\mathbf{P}_{a}}}{i\xi^{b}\mathbf{P}_{b}}\wedge(i\delta\xi^{a}\mathbf{P}_{a})=l_{1}-1. \tag{A8}$$

Since this is a Lorentz generator, we must evaluate the AdS boost component of the left-hand side and set it equal to 0; only commutators of even order contribute to it. Therefore, we must take the even powers of $i\delta\xi^{a}\mathbf{P}_{a}$ of the functions occurring in Eq. (A8). This leads to

$$\delta\xi^{a}P_{a}=\rho^{a}P_{a}+\left(\frac{z\cosh z}{\sinh z}-1\right)\left(\rho^{a}-\frac{\rho^{b}\xi_{b}\xi^{a}}{\xi^{2}}\right)P_{a}. \tag{A9}$$

In a similar way, we can make use of Eq. (62) with $g_{0}-1=\bar{\varepsilon}Q$; one finds that

$$e^{-\bar{\chi}Q}\wedge(\bar{\varepsilon}Q)-\frac{1-e^{-\bar{\chi}Q}}{\bar{\chi}Q}\wedge(\delta\bar{\chi}Q)=h_{1}-1. \tag{A10}$$

Here, there is a simplification: any power of χ higher than 4 vanishes identically due to the anticommuting property of the spinor component. Therefore, we need only

$$\bar{\chi}Q\wedge\bar{\varepsilon}Q=-2m\bar{\chi}\gamma^{AB}\varepsilon J_{AB}, \tag{A11}$$

$$(\bar{\chi}Q)^{2}\wedge\bar{\varepsilon}Q=-\tfrac{5}{2}im\bar{\chi}\chi\bar{\varepsilon}Q-\tfrac{1}{2}im\bar{\chi}\Gamma_{A}\chi\bar{\varepsilon}\Gamma^{A}Q, \tag{A12}$$

$$(\bar{\chi}Q)^{3}\wedge\bar{\varepsilon}Q=4im^{2}\bar{\chi}\chi\bar{\chi}\gamma^{AB}\varepsilon J_{AB}, \tag{A13}$$

$$(\bar{\chi}Q)^{4}\wedge\bar{\varepsilon}Q=-5m^{2}(\bar{\chi}\chi)^{2}\bar{\varepsilon}Q, \tag{A14}$$

where the five matrices

$$\Gamma^{A}\equiv(\gamma_{a}\gamma_{5},\gamma_{5})$$

satisfy

$$\Gamma_{A}\Gamma_{B}+\Gamma_{B}\Gamma_{A}=2\eta_{AB},$$

$$\gamma_{AB}=\tfrac{1}{4}[\Gamma_{A},\Gamma_{B}],$$

$$2m\gamma^{AB}J_{AB}=2\gamma^{a}P_{a}-2m\gamma^{ab}J_{ab}.$$

If one sets equal to 0, in the left-hand side of Eq. (A10), the part with the even powers of $\bar{\chi}Q$, one finds

$$\cosh(\bar{\chi}Q)\wedge\bar{\varepsilon}Q-\frac{\sinh(\bar{\chi}Q)}{\bar{\chi}Q}\wedge\delta\bar{\chi}Q=0. \tag{A15}$$

Using Eq. (A4), we have

$$\delta\bar{\chi}Q=[1+\tfrac{1}{3}(\bar{\chi}Q)^{2}-\tfrac{1}{45}(\bar{\chi}Q)^{4}]\wedge\bar{\varepsilon}Q. \tag{A16}$$

If one now makes use of Eqs. (A11) to (A14), one obtains

$$\delta\chi Q=\left[\varepsilon-\frac{i}{6}m(5\bar{\chi}\chi+\bar{\chi}\Gamma_{A}\chi\Gamma^{A})\varepsilon+\frac{1}{9}m^{2}(\bar{\chi}\chi)^{2}\varepsilon\right]Q. \tag{A17}$$

On the other hand, using Eq. (A16), the part with the odd powers gives

$$h_{1}-1=\left(1+\frac{i}{6}m\bar{\chi}\chi\right)(\bar{\varepsilon}\gamma^{a}\chi P_{a}+m\bar{\varepsilon}\gamma^{ab}\chi J_{ab}).$$

The nonlinear fields V^{a},W^{ab},Ψ are evaluated from their definitions (51),(77), following the same procedure of Ref. [3].

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