Stability of neutral Fermi balls with multiflavor fermions

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A Fermi ball is a kind of nontopological soliton which is thought to arise from the spontaneous breaking of an approximate Z_2 symmetry and to contribute to cold dark matter. We consider a simple model in which fermion fields with multiflavors are coupled to a scalar field through Yukawa coupling and examine how the number of the fermion flavors affects the stability of the Fermi ball against the fragmentation. (1) We find that the Fermi ball is stable against fragmentation in most cases even in the lowest-order thin-wall approximation. (2) We then find that in the other specific cases the stability is marginal in the lowest-order thin-wall approximation, and the next-to-leading order correction determines the stable region of the coupling constants; we examine the simplest case where the total fermion number N_i and the Yukawa coupling constant G_i of each flavor i are common to the flavors, and find that the Fermi ball is stable in a limited region of the parameters and has the broader region for the larger number of flavors.

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I. INTRODUCTION

A Fermi ball [1,2], a kind of nontopological soliton [3,4], is composed of three parts: a false vacuum domain, a domain wall enveloping the domain, and zero-mode fermions [5] confined in the domain wall. The Fermi ball is stabilized owing to the dynamical balance between the shrinking force due to the surface energy, as well as the volume energy, and the expanding force due to the Fermi energy. The Fermi ball is thought to be a candidate for cold dark matter in the present universe [6,7].

Macpherson and Campbell pointed out that such stability holds good only for the spherical shape of the Fermi ball [1]. They further showed that the Fermi ball is not stable against the deformation of the spherical shape, and thus flattens and fragments into tiny Fermi balls. The destabilization is caused by the volume energy of the Fermi ball. In these analyses, the effect of the domain wall curvature is neglected.

We, in previous papers [8,9], pointed out that the perturbative correction due to the domain wall curvature can stabilize the Fermi ball when the volume energy is small enough compared to the curvature effect. We found, however, that the region of parameters where the Fermi ball becomes stable is quite narrow in a single fermion flavor model.

The purpose of the present paper is to examine how the fermion content of the model affects the stability of the Fermi ball. As an example, we consider an extended model in which fermions with multiflavors are coupled to a scalar field through Yukawa coupling. Since Pauli's exclusion principle does not apply to the different flavors of fermions, the stable region of the parameters is expected to broaden.

II. STABILITY OF THE FERMI BALL

We consider the following Lagrangian density for a scalar field ϕ and fermion fields Ψ_i with $i=1,2,\ldots,n$ being the flavor indices:

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi)^2+\sum_{i=1}^{n}\overline{\Psi}_i(i\gamma_i^\mu\partial_\mu-G_i\phi)\Psi_i-U(\phi),\quad (1)$$

where the scalar potential $U(\phi)$ is given by

$$U(\phi)=\frac{\lambda}{8}(\phi^2-v^2)^2+\Delta(\phi).\quad (2)$$

If the quantity $|\Delta(v)-\Delta(-v)|$ is zero, the Lagrangian density is invariant under the Z_2 transformation, $\phi\leftrightarrow-\phi$. There is, however, a small but finite quantity $|\Delta(v)-\Delta(-v)|\simeq\Lambda \ll\lambda v^4$, where the invariance is not a strict one.

We consider a spherical Fermi ball with radius R and assume that the wave function Ψ_i and boson ϕ are static and that ϕ depends only on the radial coordinate r. Let Ψ_i be the eigenfunction of the total angular momentum squared $\vec{\mathbf{J}}^2$, the z component $\mathbf{J}_z$, and the parity $\mathbf{P}$ with the eigenvalues of $J(J+1)$, M, and $(-1)^{J-\eta/2}$ ($\eta=\pm1$), respectively. Then, Ψ_i is written as

$$\Psi_i(\vec{x})=\frac{1}{r}\begin{pmatrix} f(r)\mathcal{Y}_{lJ}^M(\theta,\varphi)\\ g(r)\mathcal{Y}_{l'J}^M(\theta,\varphi)\end{pmatrix},\quad (3)$$

where $\mathcal{Y}_{lJ}^M$ and $\mathcal{Y}_{l'J}^M=(\vec{\sigma}\vec{x}/r)\mathcal{Y}_{lJ}^M$ are the spherical spinors having eigenvalues J and M, with $J=l+\eta/2=l'-\eta/2$. Substituting Eq. (3) into the Lagrangian $L=\int d^3x\mathcal{L}$, we obtain

$$L[\phi,\psi_i]=-\int_0^\infty \mathrm{d}r\left[4\pi r^2\left\{\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}r}\right)^2+U(\phi)\right\}+\sum_i\sum_{KM}\psi_i^\dagger H_f\psi_i\right], \tag{4}$$

where

$$H_f=\sigma_1\frac{1}{i}\frac{\mathrm{d}}{\mathrm{d}r}+\sigma_2\frac{K}{r}+\sigma_3 G_i\phi, \tag{5}$$

with $K=\eta(J+\frac{1}{2})$ and $\psi_i(r)=\binom{f(r)}{g(r)}$. Since the Fermi ball is a ground state with a fixed number of fermions,

$$N_i=\int \mathrm{d}^3x\Psi_i^\dagger\Psi_i, \tag{6}$$

we obtain the wave function ψ_i and the scalar field ϕ by extremizing

$$L_\epsilon[\phi,\psi_i]=L[\phi,\psi_i]+\sum_i\epsilon_i\left(\sum_{KM}\int_0^\infty \mathrm{d}r\,\psi_i^\dagger\psi_i-N_i\right), \tag{7}$$

with the Lagrange multipliers ϵ_i. This multiplier turns out to be the Fermi energy, since $\epsilon_i=\int_0^\infty \mathrm{d}r\,\psi_i^\dagger H_f\psi_i$ is derived by extremizing L_ϵ with respect to ϕ_i. The energy of the Fermi ball is expressed in terms of the fields as

$$E=\int_0^\infty \mathrm{d}r\left[4\pi r^2\left\{\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}r}\right)^2+U(\phi)\right\}\right]+\sum_i\sum_{KM}\epsilon_i, \tag{8}$$

where we have eliminated ψ_i by using the normalization $\int_0^\infty \mathrm{d}r\,\psi_i^\dagger\psi_i=1$. In order to estimate the energy of the Fermi ball, we take the thin-wall approximation and obtain the correction due to the finite curvature radius R by the perturbation with respect to $1/R$. We expand ϕ, ψ_i, and H_f in powers of $1/R$ [10],

$$\phi=\phi_0+\phi_1+\cdots,$$
$$\psi_i=\psi_{i0}+\psi_{i1}+\cdots,$$
$$H_f=H_0+H_1+H_2+\cdots, \tag{9}$$

where

$$H_0=\sigma_1\frac{1}{i}\frac{\mathrm{d}}{\mathrm{d}r}+\sigma_2\frac{K}{R}+\sigma_3G_i\phi_0,$$
$$H_1=-\sigma_2\frac{Kw}{R^2}+\sigma_3G_i\phi_1,$$
$$H_2=\sigma_2\frac{Kw^2}{R^3}, \tag{10}$$

with $w=r-R$. From $\delta L_\epsilon/\delta\phi=\delta L_\epsilon/\delta\psi_i^\dagger=0$, we obtain the equations of motion,

$$H_0\psi_{i0}=\epsilon_{i0}\psi_{i0}, \tag{11}$$

$$\frac{\mathrm{d}^2\phi_0}{\mathrm{d}w^2}=\left.\frac{\partial U}{\partial\phi}\right|_{\phi=\phi_0}+\sum_i\frac{G_i}{4\pi R^2}\sum_{KM}\psi_{i0}^\dagger\sigma_3\psi_{i0} \tag{12}$$

and

$$(H_0-\epsilon_{i0})\psi_{i1}=-(H_1-\epsilon_{i1})\psi_{i0}, \tag{13}$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}w^2}-\left.\frac{\partial^2U}{\partial\phi^2}\right|_{\phi=\phi_0}\right]\phi_1=-\frac{2}{R}\frac{\mathrm{d}\phi_0}{\mathrm{d}w}+\sum_i\frac{G_i}{2\pi R^2}\sum_{KM}\psi_{i0}\sigma_3\psi_{i1}. \tag{14}$$

Neglecting $\Delta(\phi)$ in the scalar potential $U(\phi)$ for simplicity, we have analytic solutions for ϕ_0 and ψ_{i0},

$$\phi_0(w)=v\tanh\frac{w}{\delta_b}, \tag{15}$$

$$\psi_{i0}(w)=\frac{1}{\sqrt{\mathcal{N}_i}}\frac{1}{\cosh^{\gamma_i}\frac{w}{\delta_b}}\chi_+, \tag{16}$$

where $\delta_b=2\lambda^{-1/2}v^{-1}$ is the thickness of the domain wall, $\gamma_i=2\lambda^{-1/2}G_i$ is the constant [11], $\mathcal{N}_i=\int_{-\infty}^{+\infty}\mathrm{d}w\cosh^{-2\gamma_i}w/\delta_b$ is the normalization factor, and $\chi_\pm$ is the eigenspinor of σ_2 with the eigenvalue ±1. The neglect of $\Delta(\phi)$ is allowed here in the case that the volume energy $[\sim\Delta(\phi)R^3]$ is much smaller than the perturbed energy $(\sim v/\sqrt{\lambda})$ to be obtained below. In Ref. [9], we find there is such a parameter region that satisfies this inequality and is consistent with cosmological constraints.

We note that the second term on the right-hand side (RHS) of Eq. (12) vanishes. The leading order of the eigenvalue is given by

$$\epsilon_{i0}=\frac{K}{R}, \tag{17}$$

where we take K positive $(\eta=+1)$. We have solutions for ϕ_1 and ψ_{i1},

$$\phi_1(w)=\frac{1}{\cosh^2\frac{w}{\delta_b}}\int_0^w \mathrm{d}w'\cosh^4\frac{w'}{\delta_b}\int_0^{w'}\mathrm{d}w''\frac{h(w'')}{\cosh^2\frac{w''}{\delta_b}}, \tag{18}$$

$$\psi_{i1}(w)=\frac{1}{\sqrt{\mathcal{N}_i}}\{c_{i+}(w)\chi_++c_{i-}(w)\chi_-\}, \tag{19}$$

where

$$h(w)=-\frac{2v}{\delta_b R\cosh^2\frac{w}{\delta_b}}+\frac{1}{2\pi R^4}\times\sum_i\sum_{KM}\frac{KG_i}{\mathcal{N}_i}\int_w^\infty dw'\frac{w'}{\cosh^{2\gamma_i}\frac{w'}{\delta_b}} \tag{20}$$

and

$$c_{i+}(w)=\frac{1}{\cosh^{\gamma_i}\frac{w}{\delta_b}}\left\{\frac{2K^2}{R^3}\int_0^w dw'\cosh^{2\gamma_i}\frac{w'}{\delta_b}\times\int_{w'}^\infty dw''\frac{w''}{\cosh^{2\gamma_i}\frac{w''}{\delta_b}}-G_i\int_0^w dw'\,\phi_1(w')\right\},$$

$$c_{i-}(w)=\frac{K}{R^2}\cosh^{\gamma_i}\frac{w}{\delta_b}\int_w^{+\infty}dw'\frac{w'}{\cosh^{2\gamma_i}\frac{w'}{\delta_b}}. \tag{21}$$

Substituting the solutions into Eq. (8), we obtain the energy of the Fermi ball,

$$E=E_0+\delta E, \tag{22}$$

where E_0 is the leading order contribution to the energy,

$$E_0=\frac{8\pi\lambda^{1/2}v^3R^2}{3}+\frac{2\sum_i N_i^{3/2}}{3R}, \tag{23}$$

and δE is the energy correction of the order of $E_0\times(\delta_b/R)^2$,

$$\delta E=\frac{\sum_i N_i^{1/2}}{12R}+\pi\lambda v^4\int_{-\infty}^{+\infty}dw\frac{w^2}{\cosh^4\frac{w}{\delta_b}}-2\pi\lambda^{1/2}v^2R\int_{-\infty}^{+\infty}dw\frac{1}{\cosh^4\frac{w}{\delta_b}}\int_0^w dw'\cosh^4\frac{w'}{\delta_b}\int_0^{w'}dw''\frac{h(w'')}{\cosh^2\frac{w}{\delta_b}}$$
$$+\frac{2}{3R^3}\sum_i\frac{N_i^{3/2}}{\mathcal{N}_i}\int_{-\infty}^{+\infty}dw\frac{w^2}{\cosh^{2\gamma_i}\frac{w}{\delta_b}}-\frac{4}{5R^5}\sum_i\frac{N_i^{5/2}}{\mathcal{N}_i}\int_{-\infty}^{+\infty}dw\frac{w^2}{\cosh^{2\gamma_i}\frac{w}{\delta_b}}\int_0^w dw'\cosh^{2\gamma_i}\frac{w'}{\delta_b}\int_{w'}^\infty dw''\frac{w''}{\cosh^{2\gamma_i}\frac{w''}{\delta_b}}$$
$$+\frac{2}{3R^2}\sum_i\frac{G_iN_i^{3/2}}{\mathcal{N}_i}\int_{-\infty}^{+\infty}dw\frac{w}{\cosh^{2\gamma_i}\frac{w}{\delta_b}}\int_0^w dw'\frac{1}{\cosh^2\frac{w'}{\delta_b}}\int_0^{w'}dw''\cosh^4\frac{w''}{\delta_b}\int_0^{w''}dw'''\frac{h(w''')}{\cosh^2\frac{w''}{\delta_b}}. \tag{24}$$

In the above equations, we use the relations

$$N_i=\sum_{KM}=\sum_{K=1}^{K_{max}}\sum_{M=-J}^{J}=\sum_{K=1}^{K_{max}}(2K)=K_{max}(K_{max}+1), \tag{25}$$

$$\sum_{KM}\epsilon_{i0}=\frac{1}{R}\sum_{KM}K=\frac{2}{3R}K_{max}(K_{max}+1)\left(K_{max}+\frac{1}{2}\right)\simeq\frac{2N_i^{3/2}}{3R}+\frac{N_i^{1/2}}{12R}\quad(N_i\gg 1). \tag{26}$$

We note that the states with angular momentum $0\leqslant l\leqslant K_{max}-1$ are filled.

(1) *Stability in the leading order approximation*. Let us examine the stability of the Fermi ball within the leading order approximation in the δ_b/R expansion. From $\partial E_0/\partial R=0$, we get the minimizing radius

$$R_{min}=\frac{\left(\sum_i N_i^{3/2}\right)^{1/3}}{2\pi^{1/3}\lambda^{1/6}v} \tag{27}$$

and the energy at the radius,

$$E_0=2\pi^{1/3}\lambda^{1/6}\left(\sum_i N_i^{3/2}\right)^{2/3}v. \tag{28}$$

We note $\partial^2E_0/\partial R^2>0$ at $R=R_{min}$.

We first examine the stability of the Fermi ball against emission of a free fermion [2]. Since the energy of the Fermi ball decreases by $E_0(N_i+1)-E_0(N_i)\sim\partial E_0/\partial N_i$ for large N_i by releasing a fermion and the fermion has a mass $m_f\sim G_iv$ in the vacuum, the condition $\partial E_0/\partial N_i<m_f$ is required. From Eq. (28) we obtain the condition [2]

$$G_i > \frac{2\pi^{1/3}\lambda^{1/6}N_i^{1/2}}{\left(\sum_j N_j^{3/2}\right)^{1/3}}. \tag{29}$$

We next examine the stability against the fragmentation [9]. We compare two states: a state $\mathcal{A}$ in which a single Fermi ball has fermion number N_i for ith flavor and a state $\mathcal{B}$ in which m Fermi balls have fermion number $N_i^{(a)}$ each and conserve the total fermion number as $\Sigma_{a=1}^{m}N_i^{(a)}=N_i$ for each flavor. States $\mathcal{A}$ and $\mathcal{B}$ have the energy $E_{\mathcal{A}}=E_0(N_i)$ and $E_{\mathcal{B}}=\Sigma_a E_0(N_i^{(a)})$, respectively. To compare the energy of the two states, we use Minkowski's inequality

$$\left(\sum_i (N_i^{(1)}+N_i^{(2)})^{3/2}\right)^{2/3} \leqslant \left(\sum_i (N_i^{(1)})^{3/2}\right)^{2/3} + \left(\sum_i (N_i^{(2)})^{3/2}\right)^{2/3}, \tag{30}$$

where the equality is valid only for $N_i^{(2)}=cN_i^{(1)}$ $(c\geqslant 0)$ with c being common for all i. Using the relation repeatedly, we have

$$\left(\sum_i (N_i)^{3/2}\right)^{2/3} \leqslant \sum_a \left(\sum_i (N_i^{(a)})^{3/2}\right)^{2/3}, \tag{31}$$

where the RHS is equal to the LHS only for $N_i^{(a)}=c^{(a)}N_i$ $(c^{(a)}\geqslant 0$ and $\Sigma_{a=1}^{m}c^{(a)}=1)$. This leads to the fact that except for the special case of $N_i^{(a)}=c^{(a)}N_i$, the energy of state $\mathcal{A}$ is lower than that of state $\mathcal{B}$, and thus the Fermi ball is stable against fragmentation in the leading order approximation. This situation—that the Fermi ball is stable in most cases—is characteristic of the case with a multiflavor of fermions and qualitatively different from the case of a single flavor [9]. In case of $N_i^{(a)}=c^{(a)}N_i$, the two states have the same energy in the leading order approximation, and the correction term δE determines the stability of the Fermi ball against the fragmentation.

(2) *Stability in the next-to-leading order approximation in the special case* $N_i^{(a)}=c^{(a)}N_i$. We examine the stability of the Fermi ball against the fragmentation in the case of $N_i^{(a)}=c^{(a)}N_i$. Substituting $R=R_{min}$ into Eq. (24) yields

$$\delta E = \mathcal{C}(\lambda, G_i, N_i)v, \tag{32}$$

where

$$\begin{aligned}\mathcal{C}(\lambda,G_i,N_i) = {} & \frac{\pi^{1/3}\lambda^{1/6}\left(\sum_i N_i^{1/2}\right)}{6\left(\sum_i N_i^{3/2}\right)^{1/3}} + \frac{8\pi(I_1-I_2)}{\lambda^{1/2}} \\ & + \frac{64\pi\left[\sum_i N_i^{3/2}\overline{\mathcal{N}_i}I_3(i)\right]}{3\lambda^{1/2}\left(\sum_i N_i^{3/2}\right)} \\ & - \frac{2048\pi^{5/3}\left[\sum_i N_i^{5/2}\overline{\mathcal{N}_i}I_4(i)\right]}{5\lambda^{7/6}\left(\sum_i N_i^{3/2}\right)^{5/3}} \\ & + \frac{128\pi\left[\sum_i G_iN_i^{3/2}\overline{\mathcal{N}_i}I_5(i)\right]}{3\lambda\left(\sum_i N_i^{3/2}\right)}.\end{aligned} \tag{33}$$

Here, I_1 to I_5 are given by

$$\begin{aligned} I_1 &= \int_{-\infty}^{+\infty} dx \frac{x^2}{\cosh^4 x}, \\ I_2 &= \int_{-\infty}^{+\infty} dx \frac{1}{\cosh^4 x}\int_0^x dx' \cosh^4 x' \int_0^{x'} dx'' \frac{\overline{h}(x'')}{\cosh^2 x''}, \\ I_3(i) &= \int_{-\infty}^{+\infty} dx \frac{x^2}{\cosh^{2\gamma_i} x}, \\ I_4(i) &= \int_{-\infty}^{+\infty} dx \frac{x}{\cosh^{2\gamma_i} x}\int_0^x dx' \cosh^{2\gamma_i} x' \\ &\quad \times \int_{x'}^{+\infty} dx'' \frac{x''}{\cosh^{2\gamma_i} x''}, \\ I_5(i) &= \int_{-\infty}^{+\infty} dx \frac{x}{\cosh^{2\gamma_i} x}\int_0^x dx' \frac{1}{\cosh^2 x'}\int_0^{x'} dx'' \cosh^4 x'' \\ &\quad \times \int_0^{x''} dx''' \frac{\overline{h}(x''')}{\cosh^2 x'''}, \end{aligned} \tag{34}$$

with $h(w)$ rescaled as $\overline{h}(x)=(R\delta_b/v)h(\delta_b x)$ and $\mathcal{N}_i$ as $\overline{\mathcal{N}_i}=(1/\delta_b)\mathcal{N}_i$. We compare state $\mathcal{A}$ of the single Fermi ball and state $\mathcal{B}$ of m Fermi balls with the total fermion number to be conserved for each flavor. States $\mathcal{A}$ and $\mathcal{B}$ have the energy $E_{\mathcal{A}}=E_0(N_i)+\mathcal{C}(\lambda,G_i,N_i)v$ and $E_{\mathcal{B}}=\Sigma_a E_0(N_i^{(a)})+\Sigma_a\mathcal{C}(\lambda,G_i,N_i^{(a)})v$, respectively. In the case of $N_i^{(a)}=c^{(a)}N_i$, we derive $\Sigma_a E_0(N_i^{(a)})=E_0(N_i)$ from Eq. (28) and $\mathcal{C}(\lambda,G_i,N_i^{(a)})=\mathcal{C}(\lambda,G_i,N_i)$ from Eq. (33), and thus find that

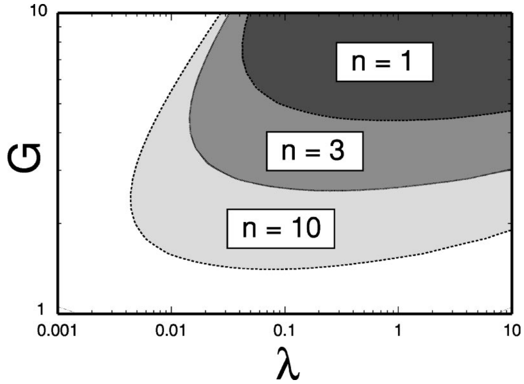

FIG. 1. The allowed regions (shadowed) of the scalar self-coupling constant λ and the Yukawa coupling constant G for the Fermi ball to be stable against fragmentation. We assume that the fermion $\Psi_i (1 \leqslant i \leqslant n)$ belongs to a multiplet and the boson ϕ to a singlet of the internal symmetry, and that the fermion number N_i is common to the flavor as $N_i = N$. The figure shows that the allowed region broadens as n increases.

state $\mathcal{B}$ has the energy $E_{\mathcal{B}} = E_0(N_i) + m\mathcal{C}(\lambda, G_i, N_i)v$. Therefore, if $\mathcal{C}(\lambda, G_i, N_i)$ is positive, the energy of state $\mathcal{A}$ is lower than that of state $\mathcal{B}$ by the magnitude of the correction term δE, and the Fermi ball is stable against fragmentation even in the special case of $N_i^{(a)} = c^{(a)} N_i$.

Let us consider the simplified model to examine how the number of the fermion flavors n affects the stability of the Fermi ball in the case of $N_i^{(a)} = c^{(a)} N_i$. We assume that Ψ_i belongs to a multiplet of internal symmetry with a common Yukawa coupling constant G and also assume that the fermion number is common to the flavor—i.e., $N_i = N$. Under these assumptions, the coefficient $\mathcal{C}$ is independent of N and dependent on λ, G, and n from Eq. (33). We evaluate Eq. (33) using numerical integrations and obtain the stable region of the parameters where $\mathcal{C}$ is positive (see Figs. 1 and 2). Figures 1 and 2 show that the allowed regions of parameters exist for the Fermi ball to be stable against fragmentation (the shadowed regions in the figures). We see in the figures that the allowed region broadens as the number of flavors, n, increases.

III. CONCLUSION

We have considered a model for the Fermi ball in which the fermions with multiflavors $\Psi_i (1 \leqslant i \leqslant n)$ are coupled to the scalar field ϕ and the total fermion number of each ith flavor is fixed as N_i. We have examined the region of parameters for the Fermi ball to be stable against fragmentation and observed how the number of fermion flavors n affects the stability.

We have considered the thin-wall Fermi ball where the radius R is much larger than the wall thickness δ_b. We have taken into account the effect due to the finite wall thickness by the perturbation expansion with respect to δ_b / R. In the leading-order thin-wall approximation, we have compared

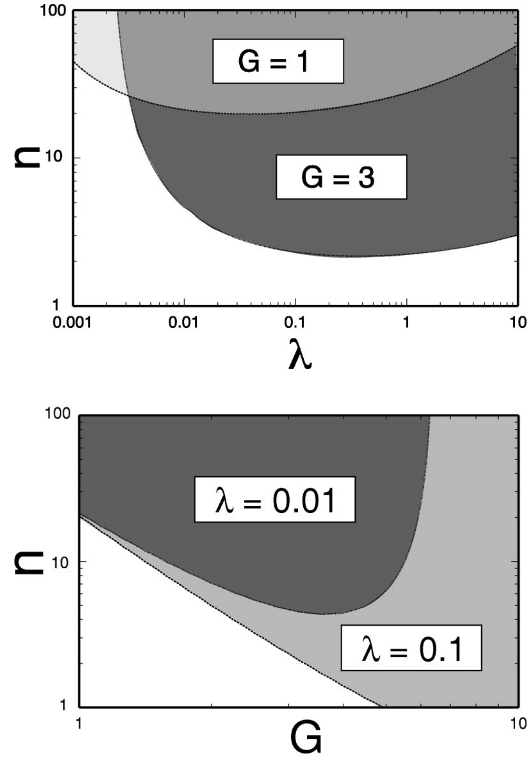

FIG. 2. The allowed region (shadowed) of the scalar self-coupling constant λ (left) and the Yukawa coupling constant G (right) for the Fermi ball to be stable. The assumptions are the same as those in Fig. 1. We see that the allowed regions broaden as n increases.

the energy of the initial state of a single Fermi ball and that of the final state of m Fermi balls after fragmentation, with the total fermion number N_i of each flavor i being conserved, $\Sigma_{a=1}^{m} N_i^{(a)} = N_i$. We have found that the former is smaller than the latter, and thus the Fermi ball is stable against fragmentation, except for the special case of $N_i^{(a)} = c^{(a)} N_i$ with $\Sigma_{a=1}^{m} c^{(a)} = 1$. This situation—that the Fermi ball is stable in most cases—is characteristic of the case with a multiflavor of fermions and qualitatively different from the case of a single flavor. In the special case of $N_i^{(a)} = c^{(a)} N_i$, the two states have the same energy in the leading order approximation and the next-to-leading order correction term δE determines the stability. There we have found that the energy of the initial state is $E_0 + \mathcal{C}v$ and that of the states after fragmentation is $E_0 + m\mathcal{C}v$, where v is a symmetry breaking scale and $\mathcal{C}$ is a coefficient dependent on the scalar self-coupling constant λ, the Yukawa coupling constant G_i, and the fermion number N_i. This tells us that even in that case the Fermi ball is stable when $\mathcal{C}$ takes a positive value in the region of parameters λ, G_i, and N_i.

In the simplified model in which a multiplet of fermions has a common G_i and a common N_i for each flavor i, we have found that the region of parameters for the Fermi ball to be stable broadens as the multiplet dimension n increases.

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[10] Here, the expansion parameter is $\delta_b/R = 4\pi^{1/3}\lambda^{-1/3}(\Sigma_i N_i^{3/2})^{-1/3}$. It is much smaller than unity for $N \gg 10^4$ even for such a small value as $\lambda = 10^{-3}$. Since we consider larger values of N_i in the cosmological context, we see it is small enough.
[11] The constant γ_i is equal to the squared ratio of the thickness of the domain wall to that of the distribution of the fermion confined in the wall, $\gamma_i = (\delta_b/\delta_f)^2$, where δ_f is given by $\delta_f = \sqrt{2}\lambda^{-1/4}G_i^{-1/2}v^{-1}$.