

Modeling galaxy halos using dark matter with pressure

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We investigate whether dark matter with substantial amounts of pressure, comparable in magnitude to the energy density, could be a viable candidate for the constituent of dark matter halos. We find that galaxy halo models, consistent with observations of flat rotation curves, are possible for a variety of equations of state with anisotropic pressures. It turns out that the gravitational bending of light rays passing through such halos is very sensitive to the pressure. We propose that combined observations of rotation curves and gravitational lensing can be used to determine the equation of state of dark matter. Alternatively, if the equation of state is known from other observations, rotation curves and gravitational lensing can together be used to test general relativity on galaxy scales.

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I. INTRODUCTION

The rotation curves of spiral galaxies [1] are one of the most direct probes of dark matter [2]. In these galaxies neutral hydrogen (HI) clouds are observed at large distances from the center, much beyond the extent of the luminous matter. The frequency shifts in the 21 cm HI emission from these clouds allow their velocities to be measured. These clouds move in circular orbits with velocity $v_c(r)$. The orbits are maintained by a balance of the centrifugal acceleration v_c^2/r and the gravitational pull $GM(r)/r^2$ of the total mass $M(r)$ contained within the orbit. This allows us to use the observed rotation curves (v_c as a function of r) to determine the mass profile $M(r) = rv_c^2(r)/G$ of the galaxy.

The rotational velocities are observed to increase near the center of the galaxy and then remain nearly constant at a value $v_c \sim 200$ km/s. This leads to a mass profile $M(r) = rv_c^2/G$ where the mass within a distance r from the center of the galaxy increases linearly with r even at large distances where there is very little luminous matter seen. This is usually explained by postulating the existence of some dark (invisible) matter distributed in a spherical halo around the galaxy. A modified law of gravity has been proposed as an alternative explanation [3]; we do not consider this here.

There is at present no clear picture as to what constitutes dark matter in galaxy halos. The prevalent belief is that it is a pressureless medium (cold dark matter) possibly made of weakly interacting massive particles [4] (WIMPs). Here we consider the possibility that dark matter in galactic halos has substantial pressure, comparable in magnitude to energy density.

The cold dark matter (CDM) model has been extremely successful in explaining the observed cosmic microwave background radiation (CMBR) anisotropies and large scale structures in the Universe [5]. The evolution of cosmological perturbations is drastically modified in models where dark matter is an ideal fluid with significant amounts of pressure,

and such models are ruled out by these observations. Our study considers the possibility that dark matter could be a medium which is not an ideal fluid and has significant pressure. Such a situation is generic if the dark matter is modeled as a scalar field (for references on the use of scalar fields in modeling dark matter or dark energy, see the papers cited in [7]), a scenario which has come up quite often in the various attempts to solve the problems being faced by the CDM model on small scales [6]. This, and the fact that there has, to date, been no direct detection of the CDM particles, motivates us to explore the possibility that dark matter in galaxy halos may have pressure. In this work, we do not restrict ourselves to any particular model for dark matter. We consider a general situation where the pressure inside the halo is anisotropic and is related to energy density through a linear equation of state. Other attempts at interpreting the flat rotation curves in the recent past are available in [8].

We investigate whether it is possible to explain the flat rotation curves seen in the outer parts of spiral galaxies using dark matter which has pressure. The Newtonian analysis of the rotation curves outlined in the beginning of this section fails when the pressure is comparable to the energy density, and a relativistic treatment is required. In the Newtonian analysis, the gravitational field inside the halo is described by a single function, the gravitational potential, which is completely determined by the observations of the rotation curve. In the relativistic treatment, the gravitational field inside a static, spherically symmetric dark matter halo is described by the two unknown metric coefficients $g_{00}(r)$ and $g_{rr}(r)$. In Sec. II of this paper we investigate to what extent the metric inside the halo can be determined by observations of the rotation curve. We show that while $g_{00}(r)$ is completely determined by just the rotation curve, inputs regarding the theory of gravity and the dark matter equation of state are required to determine $g_{rr}(r)$.

In Sec. III, we calculate the gravitational lensing properties of the dark matter halo. The usual analysis of gravitational lensing [9] does not take into account pressure. Here we analyze photon geodesics inside the halo and use these to calculate the gravitational lensing deflection angles. Subsequently, we study to what extent the lensing properties of dark matter halos change when pressure is introduced. We

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investigate whether it is possible to use combined observations of rotation curves and gravitational lensing to constrain the equation of state of the dark matter in galaxy halos. In Sec. IV, we discuss our results and present conclusions.

II. THE GEOMETRY WITHIN THE HALO

It is assumed that spiral galaxies are embedded in static, spherically symmetric halos of radius R . The gravitational field outside the halo is described by the exterior Schwarzschild metric with $M = Rv_c^2/G$. The gravitational field inside the halo is represented by two unknown functions $g_{00}(r) = e^{2\Phi(r)}$ and $g_{rr}(r) = e^{2\lambda(r)}$ which parametrize the metric, and the proper time interval $d\tau$ is

$$c^2 d\tau^2 = e^{2\Phi(r)} c^2 dt^2 - e^{2\lambda(r)} dr^2 - r^2 [d\theta^2 + \sin^2 \theta d\phi^2]. \quad (1)$$

We first investigate to what extent the gravitational field inside the halo can be determined from observations of the rotation curves.

The HI clouds from which the frequency-shifted 21-cm emission is observed are treated as test particles moving in circular orbits, in the plane of the disk of the galaxy under the gravitational influence of the halo. It is assumed that the HI clouds do not contribute significantly to the halo matter. We choose coordinates so that the disk of the galaxy is aligned with the equatorial plane of the halo ($\theta = \pi/2$). The orbits of the HI clouds have two conserved quantities, $E = e^{2\Phi} c dt/d\tau$ and $L = r^2 d\phi/d\tau$, which, for circular orbits, are related as $E^2 = e^{2\Phi} [c^2 + L^2/r^2]$. The HI clouds at different radii will have different values of E and L , and we denote this by $E(r)$ and $L(r)$.

A distant observer in the plane of the disk receives HI emission from a cloud located along a line of sight which is tangential to the orbit of the cloud. The observed shift in the frequency of the radiation, when the cloud is moving away, is $\Delta\nu \approx -v_e L(r)/(cr)$, provided $e^{2\Phi(r)} \sim 1$ and $L(r)/(cr) \ll 1$. Later in this paper we show that these two approximations are valid for our analysis. An extra angular dependence comes into the frequency shift if the line of sight is at an angle to the direction of motion of the HI cloud. Observations of the frequency shifts in the HI radiation show that, barring the central parts of galaxies, the ratio $L(r)/(cr)$ is nearly constant at a value $L(r)/(cr) = v_c/c \sim 7 \times 10^{-4}$. The ratio v_c/c being extremely small, in our analysis, we retain only the leading-order terms in v_c/c , and ignore all the higher terms.

We next shift our attention to the equation of motion of the HI clouds. For circular orbits the geodesic equation reduces to

$$\Phi'(r) = \frac{L^2(r)}{r^3 E^2(r)}, \quad (2)$$

where $\Phi' = d\Phi/dr$. This can be integrated to obtain

$$\Phi(r) = \int \left(\frac{L(r)}{rc} \right)^2 \frac{dr}{r} + \text{const.} \quad (3)$$

This equation allows us to determine the metric coefficient $g_{00}(r)$ inside the halo using just the observed rotation curves. For a flat rotation curve this gives us

$$\Phi(r) = \left(\frac{v_c}{c} \right)^2 [\ln(r/R) - 1], \quad (4)$$

which gives the metric coefficient $g_{00}(r)$ to be

$$g_{00}(r) = e^{2\Phi(r)} = e^{-2v_c^2/c^2} \left(\frac{r}{R} \right)^{2v_c^2/c^2}. \quad (5)$$

Here the constant of integration in Eq. (3) has been chosen so that the metric coefficient $g_{00}(r)$ matches the exterior Schwarzschild metric at $r=R$.

The rotation curve determines only one, namely $\Phi(r)$ of the two unknown functions $\Phi(r)$ and $\lambda(r)$, which are required to describe the gravitational field inside the halo. We also see that the metric coefficient $g_{00}(r)$ varies very slowly with r , and $e^{2\Phi(r)} \sim 1$, the change being less than 0.5% when r changes from $0.01R$ to R . The fact that g_{00} can indeed be obtained from the rotation curves has been *partially* noted in the papers in [8] and in some of the references on halo dark matter [7].

Next we address the question of how to determine $\lambda(r)$. We proceed by solving Einstein equations for gravity $G_{\mu\nu} = (8\pi G/c^4) T_{\mu\nu}$ to determine $\lambda(r)$ inside the halo. Solving this requires the equation of state (or equivalently the energy momentum tensor $T_{\mu\nu}$) for the matter which makes up the halo. We assume that the dark matter which makes up the halo is a fluid with energy density $c^2 \rho(r)$, radial pressure $P_r(r)$, and tangential pressure $P_T(r)$ in the θ and ϕ directions. Also, the gravitational field inside the halo is weak, i.e., $\Phi(r) \sim \lambda(r) \sim (v_c/c)^2$, which allows us to linearize the Einstein equations retaining only terms linear in $(v_c/c)^2$. Inside the halo, using Eq. (3) for $\Phi(r)$, the Einstein equations reduce to three equations for $\lambda(r)$,

$$\frac{(r\lambda)'}{r^2} = \frac{4\pi G}{c^2} \rho, \quad (6)$$

$$\frac{(v_c/c)^2 - \lambda}{r^2} = \frac{4\pi G}{c^4} P_r, \quad (7)$$

$$-\frac{\lambda'}{r} = \frac{8\pi G}{c^2} P_T, \quad (8)$$

which have to be solved with the boundary condition $\lambda(R) = (v_c/c)^2$ and the requirement $\rho(r) \geq 0$.

In the absence of any pressure ($P_r = P_T = 0$) we recover the usual Newtonian solution

$$\lambda_N(r) = \left(\frac{v_c}{c} \right)^2 \quad \text{and} \quad \rho_N(r) = \frac{v^2}{4\pi G r^2}. \quad (9)$$

This solution corresponds to the singular isothermal sphere which produces a flat rotation curve. We have solved Eqs. (6), (7), and (8) under various assumptions on the equation of state. It is convenient to express the results in terms of λ_N and $\rho_N(r)$, which are the solutions in the absence of pressure.

The most obvious possibility is to consider a medium with isotropic pressure (i.e., $P_r = P_T$). We find that the only solution where the metric matches the Schwarzschild metric at $r=R$ and the energy density is positive has $P_r = P_T = 0$.

We next consider two possible anisotropic equations of state: (a) $P_r = w_r \rho c^2$, where we use Eq. (8) to determine P_T , and (b) $P_T = w_T \rho c^2$, where we use Eq. (7) to determine P_r . As mentioned in the Introduction, such anisotropic pressures can indeed arise in the energy–momentum tensor of a real or complex scalar field. We recall that the energy momentum tensor for a real scalar field in a potential $V(\phi)$ is given as

$$T_{ij}^\phi = \partial_i \phi \partial_j \phi - \frac{1}{2} g_{ij} [\partial^k \phi \partial_k \phi + 2V(\phi)]. \quad (10)$$

When the gradient terms are small and the scalar field is frozen at a potential minimum $\phi_{\min} \neq 0$, it generates isotropic, negative pressures $P = -\rho c^2 = -V(\phi_{\min})$. On the other hand, the pressure is anisotropic, i.e., *the radial pressure T_{rr} is different from the tangential pressure $T_{\theta\theta}$* , if the radial derivative of the scalar field makes a significant contribution to the energy momentum tensor. Various scalar field configurations, including some with anisotropic pressures (for example, Matos and Nunez, astro-ph/0303455 in [7]), have been used extensively in cosmology to model dark matter and dark energy.

We now discuss the results for the two cases (a) and (b) mentioned above.

A. $P_r = w_r \rho c^2$

We find that solutions exist for $w_r \geq -1$. If we match $\lambda(r)$ to the Schwarzschild metric at the outer edge of the halo, the density turns out to be negative for $w_r < -1$ and this range is ruled out. The allowed solutions are tabulated below,

w_r	$\lambda(r) = \lambda_N \times$	$\rho(r) = \rho_N(r) \times$	$P_T(r) = (c^2/2) \rho_N(r) \times$
> -1	$\frac{1}{1+w_r} \left[1 + w_r \left(\frac{r}{R} \right)^{-w_r/(1+w_r)} \right]$	$\left(\frac{1}{1+w_r} \right) \left[1 + \frac{w_r}{1+w_r} \left(\frac{r}{R} \right)^{-w_r/(1+w_r)} \right]$	$\left(\frac{w_r}{1+w_r} \right)^2 \left(\frac{r}{R} \right)^{-w_r/(1+w_r)}$
-1	$[1 - \ln(r/R)]$	$\ln \left(\frac{R}{r} \right)$	1

B. $P_T = w_T \rho c^2$

We find that solutions exist for $w_T > -1/2$. For $w_T = -1/2$ we have an absurd situation where $\lambda(r) = 0$, and the density is negative if $w_T < -1/2$. The solution for the allowed range of w_T is shown below,

w_T	$\lambda(r) = \lambda_N \times$	$\rho(r) = \rho_N(r) \times$	$P_r(r) = c^2 \rho_N(r) \times$
$> -1/2$	$\left(\frac{r}{R} \right)^{-2w_T/(1+2w_T)}$	$\frac{1}{1+2w_T} \left(\frac{r}{R} \right)^{-2w_T/(1+2w_T)}$	$\left[1 - \left(\frac{r}{R} \right)^{-2w_T/(1+2w_T)} \right]$

III. GRAVITATIONAL LENSING

We now move on to calculate the bending of light by the halos. A light ray which goes past the halo without entering it propagates entirely in a Schwarzschild metric. The light ray is deflected by an angle $\Delta = 4GM/c^2 b = 4(v_c/c)^2 (R/b)$, where b is the impact parameter, and in this case $b \geq R$. The bending of a light ray which passes through the halo is determined by the metric inside the halo and this depends on the equation of state of dark matter.

The light ray is assumed to lie in the $\theta = \pi/2$ plane. Null geodesics in the halo metric are completely determined by the two conserved quantities E and L defined in Sec. II. The trajectory of the light ray is determined by the following equation [10] for $u(\phi) = 1/r(\phi)$:

$$\left(\frac{du}{d\phi} \right)^2 = e^{-2(\Phi+\lambda)} \left(\frac{E}{L} \right)^2 - e^{-2\lambda} u^2. \quad (11)$$

Retaining only terms to order $\sim (v/c)^2$, and using $1/b^2 = E^2/L^2$, we have

$$\left(\frac{du}{d\phi} \right)^2 = \left(\frac{1}{b^2} - u^2 \right) - 2 \left[(\Phi + \lambda) \frac{1}{b^2} - \lambda u^2 \right]. \quad (12)$$

It is convenient to replace u by a new variable y , where $y = u + \alpha(y)$ and

$$\alpha(y) = \frac{(\Phi + \lambda)/b^2 - \lambda y^2}{y}. \quad (13)$$

We can now integrate Eq. (12) to obtain

$$\phi = \int_0^{1/b} \left(1 - \frac{d\alpha}{dy} \right) \left[\frac{1}{b^2} - y^2 \right]^{-1/2} dy. \quad (14)$$

In Eq. (14), the limit $y=0$ is when the incoming photon is very far away from the center of the halo $y=1/b$ is when the photon is closest to the center of the halo and ϕ is the change in the angle subtended by the photon between these two positions. Setting $\alpha(y)=0$ allows us to calculate the photon

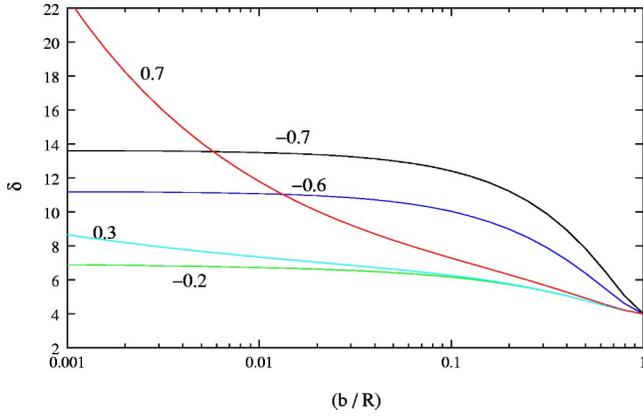


FIG. 1. This shows the deflection angle $\Delta = (v_c/c)^2 \delta$ as a function of the impact parameter b for a light ray passing through the dark matter halo of a galaxy which has a flat rotation curve with speed v_c . The equation of state of the dark matter is assumed to be $P_r = w_r \rho c^2$ and the values of w_r for which the deflection angles have been calculated are shown in the figure.

trajectory in the absence of any gravitation field, whereas the term involving α is the change produced due to the gravitational field of the halo. This gives the deflection angle to be

$$\Delta = -2 \int_0^{1/b} d\alpha \left(\frac{1}{b^2} - y^2 \right)^{-1/2} dy, \quad (15)$$

where the factor of 2 arises from the contributions of the incoming and the outgoing photon trajectory.

The integral in Eq. (15) is evaluated in two parts, where we use the Schwarzschild metric to calculate α outside the halo ($1/y > R$) and we use the solutions obtained in Sec. II inside the halo ($1/y < R$). The results for the deflection angle Δ are presented in units of $(v_c/c)^2$ using a variable $\delta = \Delta / (v_c/c)^2$. We recover the familiar result $\delta = 4(R/b)$ corresponding to the exterior Schwarzschild geometry when the impact parameter is larger than the halo radius ($b \geq R$). In the situation when the light ray does enter the halo ($b < R$), Eq. (15) can be evaluated analytically using MATHEMATICA for both the equations of state considered in Sec. II. The resulting expressions are rather lengthy and we do not present them here. In the absence of pressure, we recover the familiar result $\delta = 2\pi$ for an infinite, isothermal halo. We recover this result from Eq. (15) in the limit $(b/R) \rightarrow 0$. This can be interpreted as either the halo radius becoming very large for a fixed impact parameter, or the impact parameter approaching very close to the center of the halo with the halo radius remaining constant. The results for the deflection angle in the presence of pressure, under the two different equations of state assumed in Sec. II are shown in Figs. 1 and 2.

IV. RESULTS AND CONCLUSIONS

We have addressed the question of whether dark matter in the halos of spiral galaxies could have significant amounts of pressure, which in general could be anisotropic. This requires

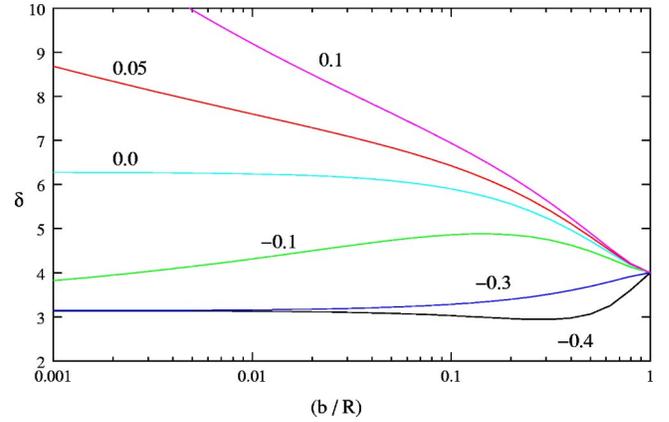


FIG. 2. This figure is the same as Fig. 1 except that the equation of state of the dark matter is assumed to be $P_T = w_T \rho c^2$.

a relativistic treatment where the gravitational field inside the halo is described by the two metric coefficients $g_{00}(r)$ and $g_{rr}(r)$. We find that observations of the rotation curve fully determine $g_{00}(r)$ irrespective of the equation of state of dark matter. As a matter of fact, $g_{00}(r)$ is determined without any reference to the theory for gravity, the only assumption being that the gravitational effects can be represented through the space-time metric.

Determining $g_{rr}(r)$ requires us to assume a theory for gravity, Einstein's theory of general relativity in this case, and an equation of state for the radial pressure P_r and the tangential pressure P_T of the dark matter. We consider two possibilities: (a) $P_r = w_r \rho c^2$, with P_T being determined from Einstein's equations, and (b) $P_T = w_T \rho c^2$, with P_r being determined from Einstein's equations. It may be noted that there is no solution matched to the Schwarzschild exterior metric at the boundary of the halo ($r = R$) if the pressure is assumed to be isotropic ($P_r = P_T$).

Solutions matched to a Schwarzschild exterior solution exist for (a) $w_r > -1$ and (b) $w_T > -1/2$. Making use of the calculated $g_{rr}(r)$, we determine the density and pressure profiles for the halos. The density profile is shallower than the r^{-2} Newtonian density profile only for $-1/2 < w_T < 0$.

For $P_r = -\rho c^2$, we have an interesting situation where $g_{rr}(r) = -g_{00}^{-1}(r)$ both inside and outside the halo. The density also goes to zero at the boundary, unlike in all the other cases. It should be noted that we do not attempt to match the density and pressures at the boundary of the halo, and in fact it is not possible to do this for a flat rotation curve and a Schwarzschild exterior.

We next investigate if it is possible to constrain g_{rr} or equivalently the equation of state using gravitational lensing. The bending of light rays was calculated for the cases (a) and (b) and the results are summarized below.

(a) For $|w_r| > 0$, the bending angle is always larger than the case where there is no pressure. When $w_r > 0$, the deflection angle progressively increases as the halo radius R is increased keeping the impact parameter b fixed, and it diverges in the limit $(b/R) \rightarrow 0$. On the other hand, for $w_r < 0$ the behavior of the deflection angle is similar to the

situation when there is no pressure, except that in the limit $(b/R) \rightarrow 0$ it saturates at a value larger than $\Delta_N = 2\pi(v_c/c)^2$, which is the deflection for an infinite halo with no pressure.

(b) The generic features of the deflection angle are different for the two cases $w_T > 0$ and $w_T < 0$. For $w_T > 0$, the deflection angle is larger than Δ_N , and it increases progressively as the halo radius is increased, diverging in the limit $(b/R) \rightarrow 0$. For $w_T < 0$, the deflection angle is less than Δ_N and it converges to around $0.5\Delta_N$ in the limit $(b/R) \rightarrow 0$.

We find that in case (b) with $w_T < 0$, the only situation where the deflection angle is less than Δ_N , the density profile is shallower than r^{-2} . The density has an r^{-2} behavior or is steeper in all the other cases. It may be worthwhile to remind the reader here that in all the cases considered here both the energy density and the pressure contribute to the bending of light. We next ask if there is any pattern seen in the form of $g_{rr}(r)$ [or effectively $\lambda(r)$] and the behavior of the deflection angle. We find that $w_T < 0$ is the only case

where $\lambda(r)$ is a monotonically increasing function of r . This leads to a picture where the gravitational deflection angle is Δ_N in the absence of pressure, where $\lambda(r)$ is a constant, the deflection is more than Δ_N if $\lambda(r)$ is a monotonically decreasing function of r and the deflection is less than Δ_N if $\lambda(r)$ is a monotonically increasing function of r .

In conclusion, the flat rotation curves observed in the outer parts of spiral galaxies can be equally well explained by a halo which is made up of dark matter with anisotropic pressures. The bending of light rays passing through the halo is found to be highly sensitive to the pressure content of the dark matter. This holds the possibility that combined observations of rotation curves and gravitational lensing can be used to determine the equation of state of the dark matter in galactic halos. Alternatively, if the equation of state of dark matter is determined from other observations, then combined observations of rotation curves and gravitational lensing can be used to test Einstein's theory of general relativity on galaxy scales.

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