PHYSICAL REVIEW D **68**, 021303(R) (2003)

You need not be afraid of phantom energy

Pedro F. González-Díaz

Centro de Física "Miguel A. Catalán," Instituto de Matemáticas y Física Fundamental, Consejo Superior de Investigaciones Científicas, Serrano 121, 28006 Madrid, Spain

(Received 13 March 2003; published 25 July 2003)

Phantom energy which violates the dominant-energy condition and is not excluded by current constraints on the equation of state may be dominating the evolution of the universe now. It has been pointed out that in such a case the fate of the universe may be a big rip where the expansion is so violent that all galaxies, planets and even atomic nuclei will be successively ripped apart in finite time. Here we show, however, that there are certain unified models for dark energy which are stable to perturbations in matter density where the presence of phantom energy does not lead to such a cosmic doomsday.

DOI: 10.1103/PhysRevD.68.021303 PACS number(s): 98.80.Es

I. INTRODUCTION

The Wilkinson Microwave Anisotropy Probe (WMAP) [1] has confirmed it with the highest accuracy: Nearly 70% of the energy in the Universe is in the form of dark energy—possibly one of the most astonishing discoveries ever made in science. Moreover, recent observations do not exclude, but actually suggest a value even smaller than -1 for the parameter of the equation of state ω, characterizing that dark energy [2]. That means that for at least a perfect-fluid equation of state the absolute value for negative pressure exceeds that for positive energy density, i.e., $\rho+p<0$, and hence it follows that the involved violation of the dominant-energy condition might allow the existence of astrophysical or cosmological wormholes. A most striking consequence from dark energy with $\omega<-1$ has been, however, pointed out [3]. It is that in a finite time the universe will undergo a catastrophic "big rip." Big rip is a term coined by Caldwell [3] that corresponds to a new cosmological model in which the scale factor blows up in a finite time because its cosmic acceleration is larger than what is induced by a cosmological constant, making in this way every component of the Universe to go beyond the horizon of all other universe's components in finite cosmic time. If dark energy is phantom energy, i.e., if dark energy is characterized by an equation of state with $\omega=p/\rho<-1$, and hence there is a violation of the dominant-energy condition, $\rho+p<0$, then the phantom-energy density is still positive, though it will first increase from a finite small initial value up to infinite in a finite time, thereafter steadily decreasing down to zero as time goes to infinity. A state with infinite energy density at finite cosmological time is certainly an unusual state in cosmology. To an observer on the Earth, this state coincides with the above-mentioned big rip where the Universe dies after ripping successively apart all galaxies, our solar system, the Earth itself, and finally molecules, atoms, nuclei and nucleons [3]. For a general cosmological model with phantom energy, the time at which that big rip would take place depends on both the initial size of the universe and the value of ω in such a way that the larger the absolute value of these quantities, the nearer the big rip will occur. The behavior of the Universe after the big rip is in some respects even more bizarre than the big rip itself, as its size then steadily decreases from infinite down to zero at infinite time. In case that a generic perfect-fluid equation of state, $p=\omega\rho$, with $\omega<-1$ is considered, the above new behaviors show themselves immediately. For flat geometry, the scale factor is then given by [4]

$$a(t)\propto[e^{C_1(1+\omega)t}-C_2e^{-C_1(1+\omega)t}]^{2/[3(1+\omega)]}, \qquad (1.1)$$

with $C_1>0$ and $0<C_2<1$. We note that for $\omega<-1$, in fact $a\to\infty$ as $t\to t_*=\ln C_2/[C_1(1+\omega)]$. This marks the time at big rip and the onset of the contracting phase for $t>t_*$.

This paper aims at showing that the above-described emergence of a cosmic doomsday at which the big rip occurred and the subsequent unconventional evolution of the Universe can both be avoided while keeping the phantom energy condition, $\rho+p<0$, $\rho>0$, $\omega<-1$, on the dark energy if, instead of a quintessential description of dark energy based on an equation of state $p=\omega\rho$, with $\omega<-1$, we consider a suitable generalization of the Chaplygin-gas model which, at sufficiently late times, does not show observable nonphysical oscillations and exponential enlargement in the matter density perturbations [5] that are present in current unstable Chaplygin-gas cosmic models [6]. The latter models describe a single substance which is characterized by an equation of state [6] $p=-A\rho^{-\alpha}$, where A is a positive-definite constant and α is a parameter which may take on any real positive values. This equation of state has been shown to represent the stuff that simultaneously describes dark matter and dark energy, but gives rise to instabilities stemming from the unobserved existence of oscillations and exponential enlargement in the perturbation power spectrum that arise whenever the speed of sound is nonzero [5].

The paper can be outlined as follows. In Sec. II we generalize the cosmic Chaplygin-gas models in such a way that the resulting models can be made stable and free from unphysical behaviors even when the vacuum fluid satisfies the phantom energy condition. The Friedmann equations for models which show and do not show unphysical behaviors are solved in Sec. III, checking that in the latter case the phantom energy condition does not imply any emergence of a big rip in finite time. We finally conclude in Sec. IV.

II. GENERALIZED COSMIC CHAPLYGIN-GAS MODELS

We introduce here some generalizations from the cosmic Chaplygin-gas model that also contains an adjustable initial parameter ω. In particular, we shall consider a generalized gas whose equation of state reduces to that of current Chaplygin unified models for dark matter and energy in the limit $\omega\to 0$ and satisfies the following conditions: (i) it becomes a de Sitter fluid at late time and when $\omega=-1$, (ii) it reduces to $p=\omega\rho$ in the limit that the Chaplygin parameter $A\to 0$, (iii) it also reduces to the equation of state of current Chaplygin unified dark matter models at high energy density, and (iv) the evolution of density perturbations derived from the chosen equation of state becomes free from the above-mentioned pathological behavior of the matter power spectrum for physically reasonable values of the involved parameters, at late time. We shall see that these generalizations retain a big rip if they also show unphysical oscillations and exponential enlargement leading to instability [i.e., if they do not satisfy condition (iv)], but if the latter effects are avoided then the evolution of the scale factor recovers a rather conventional pattern, without any big rip or contracting phase.

An equation of state that can be shown to satisfy all the above conditions (i)–(iv) is

$$p=-\rho^{-\alpha}[C+(\rho^{1+\alpha}-C)^{-\omega}], \tag{2.1}$$

where

$$C=\frac{A}{1+\omega}-1, \tag{2.2}$$

with A a constant which now can take on both positive and negative values, and $0>\omega>-\ell$, ℓ being a positive definite constant which can take on values larger than unity. By integrating the cosmic conservation law for energy we get for the energy density

$$\rho(a)=\left[C+\left(1+\frac{B}{a^{3(1+\alpha)(1+\omega)}}\right)^{1/(1+\omega)}\right]^{1/(1+\alpha)}, \tag{2.3}$$

where B is a positive integration constant. Let us now define the effective expressions of the state equation parameter and speed of sound, which, respectively, are given by

$$\omega^{\rm eff}=\frac{p}{\rho}=-\frac{C+D(a)^{-\omega}}{C+D(a)} \tag{2.4}$$

$$c_s^{\rm eff2}=\frac{\partial p}{\partial\rho}=\frac{\alpha C[D(a)^{1+\alpha}-1)+[C+D(a)][\alpha+\omega(1+\alpha)]}{[C+D(a)]D(a)^{1+\omega}} \tag{2.5}$$

with

$$D(a)=\left(1+\frac{B}{a^{3(1+\alpha)(1+\omega)}}\right)^{1/(1+\omega)}. \tag{2.6}$$

One can then interpret the model by taking the limit of these parameters as $a\to 0$ and $a\to\infty$, at which limits they, respectively, become $\omega^{\rm eff}\to 0$ and $c_s^{\rm eff2}\to 0$ [that correspond to the pressureless, cold dark matter (CDM) model], and $\omega^{\rm eff}\to -1$ and $c_s^{\rm eff2}\to\alpha+\omega(1+\alpha)$ (that correspond to a pure cosmological constant). Such as we have defined it so far, the present model does not satisfy condition (iv) above, as the evolution of density perturbations δ_k with wave vector k [5]

$$\delta_k''+F(\omega^{\rm eff},c_s^{\rm eff})\delta_k'-G(\omega^{\rm eff},c_s^{\rm eff},k)\delta_k=0, \tag{2.7}$$

where the prime denotes differentiation with respect to $\ln a$, and

$$F(\omega^{\rm eff},c_s^{\rm eff})=2+\xi-3(2\omega^{\rm eff}-c_s^{\rm eff2}), \tag{2.8}$$

$$G(\omega^{\rm eff},c_s^{\rm eff},k)=\frac{3}{2}(1-6c_s^{\rm eff2}+8\omega^{\rm eff}-3\omega^{\rm eff2})-\left(\frac{kc_s^{\rm eff}}{aH}\right)^2, \tag{2.9}$$

with H the Hubble parameter,

$$\xi=-\frac{2}{3}\left[1+\left(\frac{1}{\Omega_M}-1\right)a^{3(1+\alpha)(1+\omega)}\right]^{-1},$$

and Ω_M, the CDM density defined from Eq. (2.3) in the limit $a\to 0$, shows oscillations and exponential enlargement because $c_s^{\rm eff}$ is generally nonzero.

III. AVOIDING THE BIG RIP

In case that $\omega<-1$ for the large values of the scale factor for which dominance of the dark-phantom energy is expected, one can approximate the Friedmann equation that corresponds to the considered model as follows:

$$\left(\frac{\dot a}{a}\right)^2\simeq L^2a^3, \tag{3.1}$$

in which $L^2=8\pi GB^{-1/[(1+\alpha)(|\omega|-1)]}$. The solution to this equation is

$$a(t)\simeq\left(a_0^{-3/2}-\frac{3L(t-t_0)}{2}\right)^{-2/3}, \tag{3.2}$$

where a_0 and t_0 are the initial values of the scale factor and time, respectively. It is easy to see that in the considered case the phantom energy condition always satisfies $p+\rho<0$ and there will be a big rip, taking place now at a time

$$t_*\simeq\frac{2}{3a_0^{3/2}L}, \tag{3.3}$$

followed as well by a contracting phase where the size of the universe vanishes as $t\to\infty$. Note that, as it was pointed out before, the time at which the big rip occurs turns out to depend on the initial size of the universe a_0, in such a way that the big rip becomes nearer as a_0 is made larger. Thus, the above model, which is actually excluded because it shares the same kind of instabilities as the original cosmic Chaplygin-gas model [5], shows a cosmological big rip.

In order to allow for both stability and compatibility with observations, we consider next a model in which $c_s^{\rm eff}\to 0$ as $t\to\infty$ and the nonzero value of parameter B is small enough. The first of these conditions can be achieved by simply imposing $\alpha+\omega(1+\alpha)=0$, i.e.

$$1+\alpha=\frac{1}{1+\omega}. \tag{3.4}$$

The equation of state and the expression for the energy density are then reduced to read

$$p=-\rho^{-\alpha}[C+(\rho^{1+\alpha}-C)^{\alpha/(1+\alpha)}], \tag{3.5}$$

$$\rho(a)=\left[C+\left(1+\frac{B}{a^3}\right)^{1+\alpha}\right]^{1/(1+\alpha)}. \tag{3.6}$$

This equation of state satisfies then all the condition (i)–(iv) imposed above.

According to condition (3.4) the phantom-dark energy with $\omega<-1$ immediately implies that $\alpha<-1$ too. In such a case we can check that $p+\rho>0$ for all values of the scale factor and, in order to ensure positiveness of the energy density, we must have $A=-|A|$ and keep $B>0$. Then, for the large values of the scale factor for which phantom-dark energy is expected to dominate over matter, the energy density can be approximated to

$$\rho\simeq\left(\frac{|A|}{|\omega|-1}\right)^{-(|\omega|-1)}\left(1+\frac{B}{|A|(|\omega|-1)a^3}\right). \tag{3.7}$$

We can now write for the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2\simeq\tilde{A}\left(1+\frac{\tilde{B}}{a^3}\right), \tag{3.8}$$

where

$$\tilde{A}=\ell_p^2\left(\frac{|\omega|-1}{|A|}\right)^{|\omega|-1}>0 \tag{3.9}$$

$$\tilde{B}=\frac{B}{(|\omega|-1)|A|}>0, \tag{3.10}$$

with $\ell_p^2=8\pi G/3$. The solution to Eq. (3.8) is

$$a(t)\simeq D(C_0e^{-(3/2)\sqrt{\tilde{A}}(t-t_0)}-e^{(3/2)\sqrt{\tilde{A}}(t-t_0)})^{2/3}, \tag{3.11}$$

where

$$D=a_0\left(\frac{\mu}{4C_0}\right)^{1/3}, \tag{3.12}$$

$$C_0=\frac{\sqrt{1+\mu}-1}{\sqrt{1+\mu}+1} \tag{3.13}$$

and

$$\mu=\tilde{B}a_0^{-3}. \tag{3.14}$$

We notice that $a\to a_0$ as $t\to t_0$ and $a\to\infty$ as $t\to\infty$, and hence there is not a big rip for this solution.

IV. CONCLUSION

It appears then that if we choose a general equation of state for dark energy which is reasonably free from instabilities and unphysical effects, then a phantom energy can be predicted which does not show any big rip at finite time. The key difference between the scale factor given by Eq. (3.11) and that given by Eq. (1.1) is in the sign of the overall exponent of the right-hand side; while in Eq. (1.1) it is negative for $\omega<-1$, in Eq. (3.11) it is positive for the same case. Thus, cosmology can coexist with these phantoms in a quite safe manner.

ACKNOWLEDGMENTS

The author thanks Mariam Bouhmadi and Carmen L. Sigüenza for useful conversations. This work was supported by MCYT under Research Project No. BMF2002-03758.

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