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Magnetic monopoles from gauge theory phase transitions

A. Rajantie
DAMTP, CMS, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom
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Thermal fluctuations of the gauge field lead to monopole formation at the grand unified phase transition in the early Universe, even if the transition is merely a smooth crossover. The dependence of the produced monopole density on various parameters is qualitatively different from theories with global symmetries, and the monopoles have a positive correlation at short distances. The number density of monopoles may be suppressed if the grand unified symmetry is only restored for a short time by, for instance, nonthermal symmetry restoration after preheating.

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It is a generic property of grand unified theories (GUTs) that magnetic monopoles of mass of the order of $m_M \approx 10^{16}$ GeV exist [1,2], and these monopoles would have been produced in large numbers in the GUT phase transition at $T_{\rm GUT} \approx m_M$ [3]. Afterwards, pair annihilations can decrease the monopole density, but estimates show that the number density would still be comparable to baryons [4]. Because the monopoles are 10^{16} times heavier than protons, this would have caused the Universe to collapse under its own weight long ago.

This monopole problem, alongside several other cosmological puzzles, was wiped away by the theory of inflation [5], as the monopole density would have been diluted to a negligible level by a period of accelerating expansion. For this to solve the problem, the reheat temperature at which the Universe thermalizes must be lower than $T_{\rm GUT}$. These constraints are even stronger in models with nonperturbative effects such as preheating [6], since the GUT symmetry can be temporarily restored [7,8] and topological defects formed even if the reheat temperature is well below $T_{\rm GUT}$ [9,10]. It is therefore important to understand how monopoles are formed to estimate how strong the bounds imposed by the monopole problem really are.

In this paper, I will discuss monopole formation at a phase transition that starts from a complete thermal equilibrium. It is clear that this is not actually the case for the GUT transition, because of the high expansion rate and the non-equilibrium effects mentioned above. Nevertheless, the assumption of thermal equilibrium simplifies the problem significantly and makes it possible to identify the physical mechanisms that are responsible for monopole formation. Once these mechanisms are understood, their effects can be studied in more realistic non-equilibrium settings.

The symmetry broken at the GUT phase transition is a local gauge invariance, whereas most of the existing literature on monopole formation implicitly assumes a breakdown of a global symmetry. The Kibble (or Kibble-Zurek) mechanism [3,11], which forms the monopoles in the global case, is ultimately based on the observation that the direction of the order parameter cannot be correlated at infinitely long distances. Because the direction of the order parameter is not gauge invariant, this argument cannot be used in GUTs.

Moreover, gauge symmetries cannot be spontaneously broken [12]. For rather generic parameter values, there is actually no phase transition at all, but simply a smooth crossover between the phases [13,14]. Does this mean that the whole evolution could be adiabatic and thereby there would be no monopole formation at all?

I will present an argument that shows that monopoles are still formed. This result is based on causality and the conservation of magnetic charge. In fact, Weinberg and Lee [15,16] have used somewhat similar reasoning to constrain later annihilations of monopoles after the phase transition in the context of the Kibble mechanism. As I will argue, there are long-wavelength thermal fluctuations of the magnetic charge in the symmetric phase, and they will freeze out forming monopoles. These fluctuations are physical and well defined, because the high and low temperature phases are smoothly connected. As there is, in this sense, a high density of monopoles above the transition, one could say that we are describing annihilation rather than formation of monopoles. This is obviously a matter of taste, but in any case, the monopoles do not correspond to localized energy concentrations in the symmetric phase and cannot therefore be thought of as particles.

The mechanism presented in this paper is physically different from global theories. Both involve a freeze-out of long-wavelength degrees of freedom, but in global theories this happens when the scalar correlation length diverges at the transition point. In our case, everything is finite at the (approximate) transition point but the magnetic screening length diverges in the zero-temperature limit. The reason why this freeze-out leads to monopole formation is also different. As we shall see, the monopoles formed in a gauge theory have positive correlations at short distances, which is the opposite of what the Kibble mechanism predicts [17]. The number density of monopoles will also be qualitatively different from the Kibble mechanism.

Let us start by briefly reviewing the standard Kibble mechanism [3], which is valid in the case of global symmetries. For simplicity, we shall discuss the SU(2) symmetry group only, but the same arguments should apply to SU(5), SO(10) or other possible GUTs. The Lagrangian of the theory is

$$\mathcal{L} = {\rm Tr}\partial_\mu \Phi \partial^\mu \Phi - m^2 {\rm Tr}\Phi^2 - \lambda({\rm Tr}\Phi^2)^2, \tag{1}$$

where Φ is in the adjoint representation, and we are assuming that at zero temperature, the SU(2) symmetry is broken. To leading order, this means $m^2<0$.

We shall consider this theory at a non-zero temperature T. When the temperature is high enough, the SU(2) symmetry is unbroken. We are asking what happens if we start from thermal equilibrium in the symmetric phase and gradually decrease the temperature so that the symmetry gets broken. As long as couplings are weak, we can approximate the equilibrium and near-equilibrium dynamics reasonably well by a classical field theory with a temperature-dependent mass term $m^2(T)$ [18–21]. Although it is difficult to make this approach quantitatively very accurate, it gives us a way of thinking about the dynamics in terms of hot classical fields without quantum mechanical complications.

It only takes a small change in temperature near the critical temperature T_c to cause the phase transition, and this effect is mainly due to the changing of the effective mass parameter from positive to negative. Therefore, we shall simply consider keeping T fixed and varying m^2.

Let us now discuss the dynamics of the global theory (1). In the high-temperature phase, the field Φ vanishes on the average. In the broken phase, it would ideally have a non-zero constant value $\Phi(\vec{x})=\Phi_0$, where $\mathrm{Tr}\Phi_0^2=\phi^2/2=-m^2/2\lambda>0$, but this would require ordering of the field at infinite distances, which cannot be achieved in finite time. Instead, the scalar correlation length ξ grows as the transition point is approached, but freezes out to some finite value $\hat{\xi}$, which is determined by the critical dynamics of the system [11] and ultimately limited by causality.

After the transition, we can imagine that the system consists of domains of radius $\hat{\xi}$, between which Φ is totally uncorrelated. At each point where four of these domains meet, there is a fixed, non-zero probability that the field cannot smoothly interpolate between the domains without vanishing at a point. This point is a monopole, and therefore this scenario predicts a monopole density

$$n_M^{\mathrm{Kibble}}\approx\hat{\xi}^{-3}. \tag{2}$$

Furthermore, there is a strong negative correlation between monopoles at short distances [17,27]: Imagine a sphere centered at a monopole. If the radius is greater than $\hat{\xi}$, each point at the sphere is uncorrelated with its center and therefore insensitive to whether there is a monopole inside or not. Consequently, the average winding number must be zero and there must be an antimonopole within the distance $\approx\hat{\xi}$ from each monopole.

Having reviewed the Kibble-Zurek scenario, let us now turn our attention to the gauge theory. The gradients in the Lagrangian are replaced by covariant derivatives $D_\mu=\partial_\mu+igA_\mu$,

$$\mathcal{L}=-\tfrac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}+\mathrm{Tr}[D_\mu,\Phi][D^\mu,\Phi]-m^2\mathrm{Tr}\Phi^2-\lambda(\mathrm{Tr}\Phi^2)^2, \tag{3}$$

where $F_{\mu\nu}=(ig)^{-1}[D_\mu,D_\nu]$.

At a high temperature T and at weak coupling g, the phase structure of this theory is given to a good approximation by a three-dimensional effective theory [20], which depends on two parameters, $g^{-2}(T/T_c-1)$ and the ratio of the coupling constants λ/g^2. There is a line of first-order transitions at small λ/g^2 [22], which ends at a second-order point at around $\lambda/g^2\approx0.3$ [23]. To simplify the estimates, we actually assume that $\lambda/g^2\approx1$, which is above the critical value so that the two phases are smoothly connected [14] and all correlation lengths are finite. In particular, the scalar correlation length is microscopic [23], of the order of $1/g^2T$, and therefore we ignore scalar fluctuations. Even then, it is still possible to find an approximate crossover point, which separates a ‘‘symmetric’’ and a ‘‘broken’’ phase, and we will use this terminology although it is not quite precise.

In the broken phase, there is still an unbroken U(1) subgroup, and the corresponding magnetic field is given by the ’t Hooft operator [1],

$$\mathcal{B}_i=\frac{1}{2}\epsilon_{ijk}\left[\mathrm{Tr}\hat{\Phi}F_{jk}+\frac{1}{2ig}\mathrm{Tr}\hat{\Phi}(D_j\hat{\Phi})(D_k\hat{\Phi})\right], \tag{4}$$

where $\hat{\Phi}=\Phi\sqrt{2/\mathrm{Tr}\Phi^2}$ is well defined almost everywhere. In continuum, $\mathcal{B}_i$ is sourceless apart from points where Φ vanishes, and has sources of integer multiples of $4\pi/g$ at those points. Being sources of the magnetic field, these points are quantized magnetic charges, and therefore we call them magnetic monopoles, whether or not they resemble the ’t Hooft-Polyakov monopole solution [1,2]. It is straightforward to see that magnetic charge defined in this way is conserved, i.e., the world lines of these monopoles cannot end.

The same construction has also been done on lattice [24,25], and the conservation law of magnetic charge is preserved. Even though we will not discuss numerical simulations in this paper, this is very important, because classical field theory cannot be in thermal equilibrium in continuum. Because the monopoles are well defined and stable on lattice, we can consistently talk about them and the magnetic field at a non-zero temperature.

Deep in the broken phase, where $m_M>T$, we can treat the monopoles as point-like particles. Therefore, we have the standard expression for the equilibrium monopole density

$$n_M^{\mathrm{eq}}\approx(m_MT)^{3/2}\exp\left(-\frac{m_M}{T}\right). \tag{5}$$

The monopole mass m_M is roughly $m_M\approx\phi/g\approx(-m^2/\lambda g^2)^{1/2}$. When m^2 decreases further, m_M grows rapidly, which suppresses n_M^{eq}.

If the monopoles did not have long-range interactions, they would be essentially uncorrelated and behave very much like the magnetic field in the Abelian Higgs model [26,27], albeit in three rather than two dimensions. There is, however, a magnetic Coulomb interaction between the monopoles, and we shall see that it suppresses their production. This interaction gives rise to correlations, which are reflected in the screening of the magnetic field by the monopoles [13,28], in analogy with the Debye screening of the electric field.

We define the magnetic screening length ξ_B as the decay rate of the correlator of 't Hooft field strength operators $\mathcal{B}_i$. In equilibrium, it is approximately

$$\xi_B \equiv 1/m_B \approx \sqrt{\frac{T}{n_M q_M^2}} \approx \sqrt{\frac{g^2 T}{n_M}}, \tag{6}$$

where $q_M = 4\pi/g$ is the magnetic charge of a monopole. Correspondingly, if we define

$$\rho_M = \vec{\nabla} \cdot \vec{\mathcal{B}}, \tag{7}$$

the magnetic charge-charge correlator is

$$\langle \rho_M(\vec{x}) \rho_M(\vec{y}) \rangle \approx q_M^2 n_M \left(\delta(\vec{x}-\vec{y}) - \frac{m_B^2}{4\pi|\vec{x}-\vec{y}|} e^{-m_B|\vec{x}-\vec{y}|} \right). \tag{8}$$

Using Eq. (5), we find that the equilibrium screening length behaves as

$$\xi_B \approx \left(\frac{g^4}{T m_M^3} \right)^{1/4} e^{m_M/2T}. \tag{9}$$

Because there is no phase transition, the correlators of $\mathcal{B}_i$ or ρ_M cannot change qualitatively when we move to the symmetric phase. Otherwise, their behavior could be used to distinguish between the phases. Consequently, the screening length is always well defined, and we can actually use Eq. (6) to define the monopole density n_M in the symmetric phase. Furthermore, we expect that above the crossover, $\xi_B \approx (g^2 T)^{-1}$, because the only relevant scale for equal-time correlations is $g^2 T$ [14].[1]

If m^2 is decreased at a constant rate, ξ_B would have to grow exponentially fast to stay in equilibrium, but, obviously, it cannot grow faster than the speed of light. In practice, it would grow much slower than this. This means that sooner or later the growth rate $d\xi_B/dt$ needed for the system to stay in equilibrium exceeds the maximum value, and the system falls out of equilibrium. We shall denote the time when this happens by $\hat{t}$.

The screening length ξ_B can still keep on growing, but so slowly that we can ignore it if we are only interested in finding an order-of-magnitude estimate for the initial monopole density. Therefore, we define the freeze-out screening length $\hat{\xi}_B$ as ξ_B at the time when it falls out of equilibrium,

$$\hat{\xi}_B = \xi_B(\hat{t}). \tag{10}$$

At the time of the freeze-out, the monopole density is

$$\hat{n}_M \approx \frac{T}{q_M^2 \hat{\xi}_B^2} \approx \frac{g^2 T}{\hat{\xi}_B^2}. \tag{11}$$

Note that by cooling the system slowly, we can make $\hat{\xi}_B$ arbitrarily large. When $\hat{\xi}_B \gg (g^2 T)^{-1}$, the typical distance $\hat{d} \approx \hat{n}_M^{-1/3}$ between monopoles and antimonopoles is much shorter than the screening length.

Even after the freeze-out, the monopole density will keep on decreasing, but this is now due to pair annihilations at length scales shorter than $\hat{\xi}_B$. These annihilations smoothen the distribution of monopoles at short distances, but they cannot remove them completely [15,16]. To see this, consider a sphere of radius $\hat{\xi}_B$. The annihilations may reduce the number of monopoles inside the sphere to the minimum, but they cannot change its net magnetic charge significantly. While the net magnetic charge is zero on the average, it fluctuates with a root-mean-squared value of

$$Q_M(\hat{\xi}_B) = \sqrt{\left\langle \left(\int^{\hat{\xi}_B} d^3x \rho_M(\vec{x}) \right)^2 \right\rangle} \approx \sqrt{T\hat{\xi}_B}. \tag{12}$$

Since the annihilations cannot reduce the charge below this, the monopole density cannot fall below

$$n_M \approx \frac{Q_M(\hat{\xi}_B)}{q_M \hat{\xi}_B^3} \approx q_M^{-1} \sqrt{\frac{T}{\hat{\xi}_B^5}} \approx g \sqrt{\frac{T}{\hat{\xi}_B^5}}. \tag{13}$$

We have not shown how to estimate $\hat{\xi}_B$, but nevertheless, this expression is clearly different from the Kibble-Zurek result (2), because of the explicit appearance of g and T.

Moreover, as long as $Q_M(\hat{\xi}_B) \gg q_M$, there will be clusters of monopoles of equal sign, and the number of monopoles in each of them can be large if $T \gg \hat{\xi}_B^{-1}$. This means that there is a positive correlation between monopoles at short distances, very much in the same way as in the case of vortices in the Abelian Higgs model [26,27] and in stark contrast with the Kibble mechanism.

We can reach the same conclusions by studying the time evolution of the magnetic charge correlator in the Fourier space. We define the equal-time correlator $G(k)$ by

$$\langle \rho_M(\vec{k}) \rho_M(\vec{q}) \rangle = q_M^2 G(k) (2\pi)^3 \delta(\vec{k}+\vec{q}), \tag{14}$$

and from Eq. (8), we find

$$G(k) = \frac{T}{q_M^2} \frac{m_B^2 k^2}{k^2 + m_B^2}. \tag{15}$$

As there is no transition, we expect that

$$G(k) \approx \frac{Tk^2}{q_M^2} \tag{16}$$

in the symmetric phase where m_B is large.

[1]The scale $g^2 T$ is generic in hot gauge theories and is known as the magnetic screening scale. This magnetic screening refers to non-Abelian magnetic fields, and it is uncertain whether it can be thought of as originating from some kind of monopoles. In contrast, the field $\mathcal{B}_i$ is, by definition, screened by monopoles, because the monopoles were defined as sources of $\mathcal{B}_i$.

Deep in the broken phase, $G(k)$ approaches zero, but causality implies that very long-wavelength (low k) correlations can only change slowly [15,16]. We can give a rough upper bound for the rate of change,

$$\left|\frac{d\ln G(k)}{dt}\right| \lesssim k. \tag{17}$$

Using Eq. (15), this becomes

$$\frac{k^2}{k^2+m_B^2}\frac{d\ln m_B^2}{dt} \lesssim k. \tag{18}$$

Below the transition, $\ln m_B \approx \sqrt{-m^2}/g^2T$, and if we keep on decreasing m^2, then sooner or later Eq. (18) ceases to be satisfied for k less than some critical value $\hat{k}$. The modes with higher k keep on decreasing and we approximate the final correlator by

$$G(k) \approx \frac{Tk^2}{q_M^2}\exp\left(-\frac{k^2}{2\hat{k}^2}\right). \tag{19}$$

A Gaussian falloff such as this would follow naturally from diffusion, but our conclusions do not depend on the precise form of the correlator, as long as it has a relatively sharp cutoff at $\hat{k}$. The corresponding monopole density is given by

$$n_M \approx \left(\int \frac{d^3k}{(2\pi)^3} G(k)\right)^{1/2} \approx q_M^{-1}\sqrt{T\hat{k}^5}, \tag{20}$$

which agrees with Eq. (13) if we identify $\hat{k}=1/\hat{\xi}_B$.

We can also find the monopole-monopole correlator in coordinate space by taking the Fourier transform of Eq. (19),

$$G(r) \approx \frac{T\hat{k}^5}{q_M^2}\frac{e^{-r^2\hat{k}^2/2}}{(2\pi)^{3/2}}(3-r^2\hat{k}^2), \tag{21}$$

and it is indeed positive at distances $r \lesssim \sqrt{3}/\hat{k}$.

As a concrete example, let us now estimate the monopole density produced in the GUT phase transition using only causality to limit the growth of ξ_B. It is clear that causality leads to a freeze-out, because the current magnetic screening length would be proportional to $\exp(m_M/2T) \sim \exp(10^{28})$ and therefore enormously longer than the size of the observable Universe. This is still an oversimplification and the estimates should not be taken literally.

At high temperatures, the effective mass parameter of the theory is $m^2(T) \approx g^2(T^2-T_{\rm GUT}^2)$. Because of the expansion of the Universe, the temperature is decreasing at the rate $dT/dt \approx -T^3/M_P$, where $M_P \approx 10^{19}$ GeV is the Planck mass. Near $T_{\rm GUT}$, we can therefore approximate

$$m^2 \approx -g^2\frac{T_{\rm GUT}^4}{M_P}t. \tag{22}$$

Deep enough in the broken phase, the monopole mass grows as

$$m_M \approx \sqrt{\frac{t}{g^2M_P}}T_{\rm GUT}^2. \tag{23}$$

From Eq. (9) we see that the growth rate of ξ_B is

$$\frac{d\xi_B}{dt} \approx \frac{T_{\rm GUT}^{11/4}}{gm_M^{7/4}M_P}e^{m_M/2T_{\rm GUT}} = \frac{T_{\rm GUT}}{gM_P}x^{-7/4}e^x, \tag{24}$$

where we have introduced the dimensionless variable $x = m_M/2T_{\rm GUT}$. We require that this is equal to 1 for the freeze-out scale, and find $x \approx \ln(gM_P/T_{\rm GUT})$, and consequently $\hat{\xi}_B \approx g^2M_P/T_{\rm GUT}^2$. Then, Eq. (13) tells us that the monopole density is

$$n_M \approx \frac{1}{g^4}\left(\frac{T_{\rm GUT}^{11}}{M_P^5}\right)^{1/2}, \tag{25}$$

which we can compare with the prediction of the Kibble mechanism under the same circumstances [29],

$$n_M^{\rm Kibble} \approx \frac{g^2T_{\rm GUT}^4}{M_P}. \tag{26}$$

The two results differ by a factor of $g^6(M_P/T_{\rm GUT})^{3/2}$, which is not particularly large for realistic GUTs, but could in principle have any value.

According to Eq. (12), the typical number of monopoles in a cluster is

$$N_M^{\rm net} = \frac{Q_M}{q_M} \approx g^2\sqrt{\frac{M_P}{T_{\rm GUT}}}. \tag{27}$$

This combination is, again, of the order of 1, which means that there is a possibility of forming small clusters.

As already mentioned, the estimate in Eq. (25) is not very precise. The main factor in this is that the magnetic charges are likely to move diffusively rather than at the speed of light. The true freeze-out scale $\hat{\xi}_B$ is necessarily shorter than our estimate and therefore Eq. (25) can be thought of as an approximate lower bound and Eq. (27) as an upper bound. Furthermore, if the transition is fast enough, which may actually be the case in the GUT transition, the approximation in Eq. (5) that the monopoles are point particles is not justified and one should instead use a field theory description. Nevertheless, this simplified calculation shows the places where more accurate physical input is needed to improve the estimates.

It is also interesting to apply this same picture to cases where the GUT symmetry is restored only briefly after inflation, either because of "nonthermal" fluctuations [7,8,10,9] or because the reheat temperature is slightly above $T_{\rm GUT}$. The estimated monopole density depends on the low-momentum behavior of $G(k)$ given in Eq. (16). Because of charge conservation, the monopoles and antimonopoles must be produced in pairs, and even if they move at the speed of light, the leading term in $G(k)$ grows as $G(k) \sim n_M k^2 t^2$. It will therefore take at least the time $t_{\rm eq} \approx (g^2T/n_M)^{1/2} \approx \xi_B$ to achieve the form (16). This conclusion can also be reached by considering the time it takes for the pairs to reach the equilibrium size $\sim\xi_B$.

This means that if the GUT symmetry is restored only very briefly, for a period shorter than $t_{\rm eq}\approx(g^2T_{\rm GUT})^{-1}$, the number density of monopoles will be suppressed. In reality, the equilibration process is probably significantly slower, and therefore $t_{\rm eq}$ can be much larger, perhaps even so large that the bounds on the reheat temperature disappear completely. In any case, a more careful analysis of the dynamics is needed to estimate how strong the suppression is in practice and whether it solves the monopole problem in the case of nonthermal symmetry restoration.

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