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# Symmetric textures in $SO(10)$ and large mixing angle solution for solar neutrinos

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We analyze a model based on supersymmetric $SO(10)$ combined with $SU(2)$ family symmetry and symmetric mass matrices constructed by the authors recently. Previously, only the parameter space for the low mass, low probability and vacuum oscillation solutions was investigated. We indicate in this Brief Report the parameter space that leads to a large mixing angle solution to the solar neutrino problem with a slightly modified effective neutrino mass matrix. The symmetric mass textures arising from the left-right symmetry breaking and the $SU(2)$ symmetry breaking give rise to very good predictions for the quark and lepton masses and mixing angles. The prediction of our model for the $|U_{e\nu_3}|$ element in the Maki-Nakagawa-Sakata matrix is close to the sensitivity of current experiments; thus the validity of our model can be tested in the near future. We also investigate the correlation between the $|U_{e\nu_3}|$ element and $\tan^2\theta_\odot$ in a general two-zero neutrino mass texture.

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The recently reported measurements from the KamLAND reactor experiment [1] confirmed the large mixing angle (LMA) solution to be the unique oscillation solution to the solar neutrino problem at the $4.7\sigma$ level [2–4]. The global analysis including solar+KamLAND+CHOOZ data indicates the following allowed region at $3\sigma$ [2]:

$$5.1\times10^{-5}<\Delta m_{21}^2<9.7\times10^{-5}\ \text{eV}^2, \tag{1}$$

$$0.29\leqslant\tan^2\theta_{12}\leqslant0.86, \tag{2}$$

$$(0.70\leqslant\sin^2 2\theta_{12}\leqslant0.994). \tag{3}$$

The allowed regions at the $3\sigma$ level based on a global fit including Super-Kamiokande (SK)+solar+CHOOZ data for the atmospheric parameters and the CHOOZ angle are [5]

$$1.4\times10^{-3}<\Delta m_{32}^2<6.0\times10^{-3}\ \text{eV}^2, \tag{4}$$

$$0.4\leqslant\tan^2\theta_{23}\leqslant3.0, \tag{5}$$

$$(0.82<\sin^2 2\theta_{23}), \tag{6}$$

$$\sin^2\theta_{13}<0.06. \tag{7}$$

There have been a few $SO(10)$ models constructed aiming to accommodate the observed neutrino masses and mixing angles (see, for example, [6–8]). By far, the LMA solution is the most difficult to obtain. Most of the models in the literature assume the mass matrices to be "lopsided." In our model based on supersymmetric (SUSY) $SO(10)\times SU(2)$ [6,7] (referred as CM hereafter), we consider *symmetric* mass matrices which result from the left-right symmetric breaking of $SO(10)$ and the breaking of the family symmetry $SU(2)$. Previously, we studied the parameter space for the low mass, low probability (LOW) and vacuum oscillation (VO) solutions to the solar neutrino problem in our model. In view of the KamLAND result, we reanalyze our model and find the parameter space for the LMA solution.

The details of our model based on $SO(10)\times SU(2)_F$ are contained in CM. The following is an outline of its salient features. In order to specify the superpotential uniquely, we invoke $Z_2\times Z_2\times Z_2$ discrete symmetry. The matter fields are

$$\psi_a\sim(16,2)^{-++}\ \ (a=1,2),\qquad \psi_3\sim(16,1)^{+++},$$

where the subscripts refer to family indices; the superscripts $+/-$ refer to $(Z_2)^3$ charges. The Higgs fields that break $SO(10)$ and give rise to mass matrices upon acquiring vacuum expectation values (VEV's) are

$$\begin{aligned}(10,1):&\quad T_1^{+++},\quad T_2^{-+-},\quad T_3^{--+},\quad T_4^{---},\quad T_5^{+--},\\ (\overline{126},1):&\quad \overline{C}^{---},\quad \overline{C}_1^{+++},\quad \overline{C}_2^{++-}.\end{aligned} \tag{8}$$

The Higgs representations 10 and $\overline{126}$ give rise to Yukawa couplings to the matter fields which are symmetric under the interchange of family indices. $SO(10)$ is broken through the left-right symmetry breaking chain

$$\begin{aligned}SO(10)\ &\rightarrow\ SU(4)\times SU(2)_L\times SU(2)_R\\ &\rightarrow\ SU(3)\times SU(2)_L\times SU(2)_R\times U(1)_{B\text{-}L}\\ &\rightarrow\ SU(3)\times SU(2)_L\times U(1)_Y\\ &\rightarrow\ SU(3)\times U(1)_{EM}.\end{aligned} \tag{9}$$

The $SU(2)$ family symmetry [9] is broken in two steps and the mass hierarchy is produced using the Froggatt-Nielsen mechanism:

$$SU(2)\overset{\epsilon M}{\rightarrow}U(1)\overset{\epsilon' M}{\rightarrow}\text{nothing}, \tag{10}$$

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where $M$ is the UV cutoff of the effective theory above which the family symmetry is exact, and $\epsilon M$ and $\epsilon' M$ are the VEV's accompanying the flavon fields given by

$$(1,2):\quad \phi_{(1)}^{++-},\quad \phi_{(2)}^{+-+},\quad \Phi^{-+-},$$

$$(1,3):\quad S_{(1)}^{+--},\quad S_{(2)}^{---},\quad \Sigma^{++-}. \tag{11}$$

The various aspects of VEV's of Higgs and flavon fields are given in CM.

The superpotential of our model is

$$W = W_{Dirac} + W_{\nu_{RR}}, \tag{12}$$

$$W_{Dirac} = \psi_3\psi_3 T_1 + \frac{1}{M}\psi_3\psi_a(T_2\phi_{(1)} + T_3\phi_{(2)}) + \frac{1}{M}\psi_a\psi_b(T_4 + \overline{C})S_{(2)} + \frac{1}{M}\psi_a\psi_b T_5 S_{(1)},$$

$$W_{\nu_{RR}} = \psi_3\psi_3\overline{C}_1 + \frac{1}{M}\psi_3\psi_a\Phi\overline{C}_2 + \frac{1}{M}\psi_a\psi_b\Sigma\overline{C}_2. \tag{13}$$

The mass matrices then can be read from the superpotential to be

$$M_{u,\nu_{LR}} = \begin{pmatrix} 0 & 0 & \langle 10_2^+\rangle\epsilon' \\ 0 & \langle 10_4^+\rangle\epsilon & \langle 10_3^+\rangle\epsilon \\ \langle 10_2^+\rangle\epsilon' & \langle 10_3^+\rangle\epsilon & \langle 10_1^+\rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 & r_2\epsilon' \\ 0 & r_4\epsilon & \epsilon \\ r_2\epsilon' & \epsilon & 1 \end{pmatrix} M_U, \tag{14}$$

$$M_{d,e} = \begin{pmatrix} 0 & \langle 10_5^-\rangle\epsilon' & 0 \\ \langle 10_5^-\rangle\epsilon' & (1,-3)\langle\overline{126}^-\rangle\epsilon & 0 \\ 0 & 0 & \langle 10_1^-\rangle \end{pmatrix} = \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & (1,-3)p\epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} M_D, \tag{15}$$

where $M_U \equiv \langle 10_1^+\rangle$, $M_D \equiv \langle 10_1^-\rangle$, $r_2 \equiv \langle 10_2^+\rangle/\langle 10_1^+\rangle$, $r_4 \equiv \langle 10_4^+\rangle/\langle 10_1^+\rangle$, and $p \equiv \langle\overline{126}^-\rangle/\langle 10_1^-\rangle$. The right-handed neutrino mass matrix is

$$M_{\nu_{RR}} = \begin{pmatrix} 0 & 0 & \langle\overline{126}_2'^0\rangle\delta_1 \\ 0 & \langle\overline{126}_2'^0\rangle\delta_2 & \langle\overline{126}_2'^0\rangle\delta_3 \\ \langle\overline{126}_2'^0\rangle\delta_1 & \langle\overline{126}_2'^0\rangle\delta_3 & \langle\overline{126}_1'^0\rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} M_R \tag{16}$$

with $M_R \equiv \langle\overline{126}_1'^0\rangle$. Here the superscripts $+/-/0$ refer to the sign of the hypercharge. It is to be noted that there is a factor of $-3$ difference between the (22) elements of mass matrices $M_d$ and $M_e$. This is due to the Clebsch-Gordan coefficients associated with $\overline{126}$; as a consequence, we obtain the phenomenologically viable Georgi-Jarlskog relation. We then parametrize the Yukawa matrices as follows, after removing all the nonphysical phases by rephasing various matter fields:

$$Y_{u,\nu_{LR}} = \begin{pmatrix} 0 & 0 & a \\ 0 & be^{i\theta} & c \\ a & c & 1 \end{pmatrix} d, \tag{17}$$

$$Y_{d,e} = \begin{pmatrix} 0 & ee^{-i\xi} & 0 \\ ee^{i\xi} & (1,-3)f & 0 \\ 0 & 0 & 1 \end{pmatrix} h. \tag{18}$$

This is one of the five sets of symmetric texture combinations [labeled set (v)] proposed by Ramond, Roberts, and Ross [10].

We use the following inputs at $M_Z = 91.187$ GeV [11,12]:

$$m_u = 2.32\ \text{MeV}\ (2.33^{+0.42}_{-0.45}),$$

$$m_c = 677\ \text{MeV}\ (677^{+56}_{-61}),$$

$$m_t = 182\ \text{GeV}\ (181 \pm 13),$$

$$m_e = 0.485\ \text{MeV}\ (0.486847),$$

$$m_\mu = 103\ \text{MeV}\ (102.75),$$

$$m_\tau = 1.744\ \text{GeV}\ (1.7467),$$

$$|V_{us}| = 0.222\ (0.219\text{–}0.224),$$

$$|V_{ub}| = 0.0039\ (0.002\text{–}0.005),$$

$$|V_{cb}| = 0.036\ (0.036\text{–}0.046),$$

where the values extrapolated from experimental data are given inside the parentheses. These values correspond to the following set of input parameters at the grand unified theory scale $M_{GUT} = 1.03 \times 10^{16}$ GeV:

$$a = 0.00246,\quad b = 3.50 \times 10^{-3},$$

$$c = 0.0320,\quad d = 0.650,$$

$$\theta = 0.110,$$

$$e = 4.03 \times 10^{-3},\quad f = 0.0195,$$

$$h = 0.0686,\quad \xi = -0.720,$$

$$g_1 = g_2 = g_3 = 0.746, \tag{19}$$

the one-loop renormalization group equations for the minimal supersymmetric standard model (MSSM) spectrum with three right-handed neutrinos are solved numerically down to the effective right-handed neutrino mass scale $M_R$. At $M_R$, the seesaw mechanism is implemented. With the constraints $|m_{\nu_3}| \gg |m_{\nu_2}|, |m_{\nu_1}|$ and maximal mixing in the atmospheric sector, the up-type mass texture leads us to choose the following effective neutrino mass matrix:

$$M_{\nu_{LL}} = \begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1+t^{3/2} \\ t & 1+t^{3/2} & 1 \end{pmatrix} \frac{d^2 v_u^2}{M_R} \tag{20}$$

and from the seesaw formula we obtain

$$\delta_1 = \frac{a^2}{c^2 t + a^2(2t^{1/2}+t^2) + 2a[1-c(1+t^{3/2})]},$$

$$\delta_2 = \frac{b^2 t e^{2i\theta}}{c^2 t + a^2(2t^{1/2}+t^2) + 2a[1-c(1+t^{3/2})]},$$

$$\delta_3 = \frac{-a[be^{i\theta}(1+t^{3/2})-c] + bcte^{i\theta}}{c^2 t + a^2(2t^{1/2}+t^2) + 2a[1-c(1+t^{3/2})]}. \tag{21}$$

We then solve the two-loop renormalization group equations (RGE's) for the MSSM spectrum down to the SUSY breaking scale, taken to be $m_t(m_t) = 176.4$ GeV, and then the SM RGE's from $m_t(m_t)$ to the weak scale $M_Z$. We assume that $\tan\beta \equiv v_u/v_d = 10$, with $v_u^2 + v_d^2 = (246/\sqrt{2}\ \text{GeV})^2$. At the weak scale $M_Z$, the predictions for $\alpha_i \equiv g_i^2/4\pi$ are

$$\alpha_1 = 0.01663, \quad \alpha_2 = 0.03374, \quad \alpha_3 = 0.1242.$$

These values compare very well with the values extrapolated to $M_Z$ from the experimental data, $(\alpha_1, \alpha_2, \alpha_3) = (0.01696, 0.03371, 0.1214 \pm 0.0031)$. The predictions at the weak scale $M_Z$ for the charged fermion masses, Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and strengths of *CP* violation are summarized in Table I of Ref. [7]. Using the mass squared difference in the atmospheric sector $\Delta m_{atm}^2 = 2.78 \times 10^{-3}$ eV$^2$ and the mass squared difference for the LMA solution $\Delta m_\odot^2 = 7.25 \times 10^{-5}$ eV$^2$ as input parameters, we determine $t = 0.35$ and $M_R = 5.94 \times 10^{12}$ GeV, and correspondingly

$$(\delta_1, \delta_2, \delta_3) = (0.00119, 0.000841 e^{i(0.220)}, 0.0211 e^{-i(0.029)}).$$

We obtain the following predictions in the neutrino sector. The three mass eigenvalues are give by

$$(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = (0.00363, 0.00926, 0.0535)\ \text{eV}. \tag{22}$$

The prediction for the Maki-Nakagawa-Sakata (MNS) matrix is

$$|U_{MNS}| = \begin{pmatrix} 0.787 & 0.599 & 0.149 \\ 0.508 & 0.496 & 0.705 \\ 0.350 & 0.629 & 0.694 \end{pmatrix}, \tag{23}$$

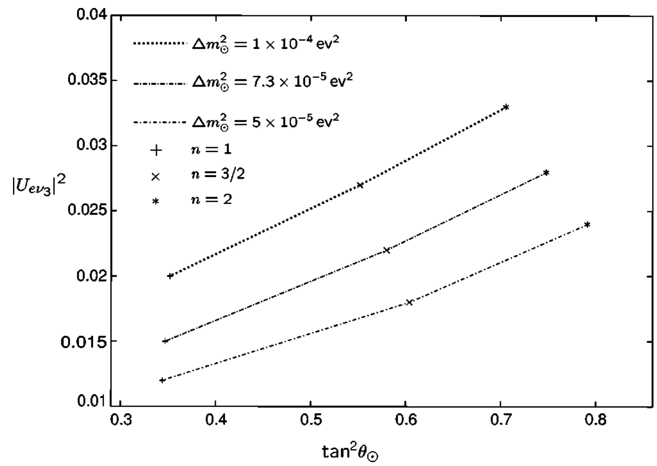


FIG. 1. Correlation between $|U_{e\nu_3}|^2$ and $\tan^2\theta_\odot$ for different values of *n*. The value of $\Delta m_{atm}^2$ is $2.8 \times 10^{-3}$ eV$^3$. The dotted line corresponds to the upper bound $\Delta m_\odot^2 = 10^{-4}$ eV$^2$; the dotted-long-dashed line corresponds to the best fit value $\Delta m_\odot^2 = 7.3 \times 10^{-5}$ eV$^2$; the dotted-short-dashed line corresponds to the lower bound $\Delta m_\odot^2 = 5 \times 10^{-5}$ eV$^2$. So a generic viable prediction of the texture given in Eq. (26) is in the region bounded by the dotted line and the dotted-short-dashed line.

which translates into the mixing angles in the atmospheric, solar, and reactor sectors,

$$\sin^2 2\theta_{atm} \equiv \frac{4|U_{\mu\nu_3}|^2 |U_{\tau\nu_3}|^2}{(1-|U_{e\nu_3}|^2)^2} = 1,$$

$$\tan^2\theta_{atm} \equiv \frac{|U_{\mu\nu_3}|^2}{|U_{\tau\nu_3}|^2} = 1.03,$$

$$\sin^2 2\theta_\odot \equiv \frac{4|U_{e\nu_1}|^2 |U_{e\nu_2}|^2}{(1-|U_{e\nu_3}|^2)^2} = 0.93,$$

$$\tan^2\theta_\odot \equiv \frac{|U_{e\nu_2}|^2}{|U_{e\nu_1}|^2} = 0.58,$$

$$\sin^2\theta_{13} = |U_{e\nu_3}|^2 = 0.022. \tag{24}$$

To our precision, the atmospheric mixing angle is maximal, while the solar angle is within the allowed region at the $1\sigma$ level $(0.37 \leq \tan^2\theta_\odot \leq 0.60$ [3]). The MNS matrix given in Eq. (23) agrees with the recently obtained one from a global analysis [13] at 1 sigma level. We comment that $M_{\nu_{LL}}$ given in Eq. (20) is a special case of a two-zero texture

$$\begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix} \tag{25}$$

first proposed in [6] in which the elements in the (23) block are taken to have equal strengths to accommodate near bimaximal mixing. Here we consider a slightly different case,

$$\begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1+t^n \\ t & 1+t^n & 1 \end{pmatrix}. \tag{26}$$

This modification is needed in order to accommodate a large, but nonmaximal solar angle in the so-called "light side" region ($0<\theta<\pi/4$) [14]. We find that it is possible to obtain the LMA solution at the $3\sigma$ level with $n$ ranging from 1 to 2. To obtain the LMA solution within the allowed region at the $1\sigma$ level, we have considered above $n=3/2$. The correlation between $|U_{e\nu_3}|^2$ and $\tan^2\theta_\odot$ for different values of $n$ is plotted in Fig. 1.

The predictions of our model for the strengths of *CP* violation in the lepton sector are

$$J_{CP}^l \equiv \mathrm{Im}\{U_{11}U_{12}^* U_{21}^* U_{22}\} = -0.00690,$$

$$(\alpha_{31},\alpha_{21}) = (0.490,-2.29). \tag{27}$$

Using the predictions for the neutrino masses, the mixing angles, and the two Majorana phases $\alpha_{31}$ and $\alpha_{21}$, the matrix element for the neutrinoless double $\beta$ decay can be calculated and is given by $|\langle m\rangle| = 2.22\times10^{-3}$ eV. The masses of the heavy right-handed neutrinos are $(M_1,M_2,M_3)=(1.72\times10^7, 2.44\times10^9, 5.94\times10^{12})$ GeV. As in the case of the LOW and VO solutions in our model [7], the amount of baryogenesis due to the decay of heavy right-handed neutrinos is too small to account for the observed amount. Thus another mechanism for baryogenesis is needed in our model. The prediction for the $\sin^2\theta_{13}$ value is 0.022, in agreement with the current bound 0.06. Because our prediction for $\sin^2\theta_{13}$ is very close to the present sensitivity of the experiment, the validity of our model can be tested in the foreseeable future.

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[1] KamLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).
[2] M. Maltoni, T. Schwetz, and J.W.F. Valle, hep-ph/0212129.
[3] J.N. Bahcall, M.C. Gonzalez-Garcia, and C. Pena-Garay, J. High Energy Phys. **02**, 009 (2003).
[4] G.L. Fogli *et al.*, Phys. Rev. D **67**, 073002 (2003); V. Barger and D. Marfatia, Phys. Lett. B **555**, 144 (2003).
[5] M.C. Gonzalez-Garcia and Y. Nir, hep-ph/0202058.
[6] M.-C. Chen and K.T. Mahanthappa, Phys. Rev. D **62**, 113007 (2000).
[7] M.-C. Chen and K.T. Mahanthappa, Phys. Rev. D **65**, 053010 (2002).
[8] T. Blazek, S. Raby, and K. Tobe, Phys. Rev. D **60**, 113001 (1999); C.H. Albright and S.M. Barr, *ibid.* **62**, 093008 (2000); Z. Berezhiani and A. Rossi, Nucl. Phys. **B594**, 113 (2001); references to earlier papers on $SO(10)$ models can be found in Ref. [6].
[9] R. Barbieri, L.J. Hall, S. Raby, and A. Romanino, Nucl. Phys. **B493**, 3 (1997).
[10] P. Ramond, R.G. Roberts, and G.G. Ross, Nucl. Phys. **B406**, 19 (1993).
[11] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).
[12] A. Hocker, H. Lacker, S. Laplace, and F. Le Diberder, Eur. Phys. J. C **21**, 225 (2001).
[13] M.C. Gonzalez-Garcia and C. Pena-Garay, hep-ph/0306001.
[14] G.L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, Phys. Rev. D **62**, 013002 (2000); A. de Gouvea, A. Friedland, and H. Murayama, Phys. Lett. B **490**, 125 (2000).