Lepton dipole moments and rare decays in the *CP*-violating MSSM with nonuniversal soft-supersymmetry breaking

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We investigate the muon anomalous magnetic dipole moment (MDM), the muon electric dipole moment (EDM) and the lepton-flavor-violating decays of the τ lepton, $\tau\to\mu\gamma$ and $\tau\to 3\mu$, in the *CP*-violating minimal supersymmetric standard model with nonuniversal soft-supersymmetry breaking. We evaluate numerically the muon EDM and the branching ratios $B(\tau\to\mu\gamma)$ and $B(\tau\to 3\mu)$, after taking into account the experimental constraints from the electron EDM and muon MDM. Upon imposition of the experimental limits on our theoretical predictions for the aforementioned branching ratios and the muon MDM, we obtain an upper bound of about $10^{-23}e$ cm on the muon EDM which lies well within the explorable reach of the proposed experiment at BNL.

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I. INTRODUCTION

Recently, the Muon-$g_\mu-2$ Collaboration has reported the world average experimental value on the muon anomalous magnetic dipole moment (MDM) [1]:

$$a_\mu\equiv\frac{1}{2}(g_\mu-2)=(11659203\pm 8)\times 10^{-10}\quad(0.7\ \text{ppm}).$$

In the framework of the standard model (SM), the value of a_μ(SM) is currently evaluated to be [2]

$$a_\mu(\text{SM})=(11659177\pm 7)\times 10^{-10}\quad(0.6\ \text{ppm}).$$

The experimental value differs from the SM prediction by 1.6 standard deviation (1.6σ). Even though the 1.6σ deviation is not very serious, this gap might be filled up by a contribution from new physics beyond the SM. It seems that a weak-scale new physics would fix the discrepancy [3,4]. In the framework of the SM, the contribution to a_μ is traditionally divided into several pieces:

$$a_\mu=a_\mu^{\text{QED}}+a_\mu^{\text{hadronic}}+a_\mu^{\text{EW}}.$$

The QED loop effects have already been computed to high orders [5,6]. A thorough analysis on hadronic contributions to the muon anomalous magnetic dipole moment is presented in Ref. [7]. At the one-loop level, the contribution of the standard model is formulated as [8–12]

$$a_\mu^{\text{EW}}=\frac{5}{3}\,\frac{G_F m_\mu^2}{8\sqrt{2}\pi^2}\left[1+\frac{1}{5}(1-4s_{\text{W}}^2)^2+\mathcal{O}\left(\frac{m_\mu^2}{m_{\text{W}}^2}\right)\right].$$

The two-loop electroweak- (EW-)sector contributions to a_μ are also discussed in Ref. [13]. Provided the recent measurement of a_μ is taken to be a signal of new physics beyond the SM, whose corrections to a_μ are extensively discussed in literature, some authors [14] have analyzed the muon anomalous magnetic dipole moment in the minimal supersymmetric standard model (MSSM). Considering the possible *CP*-violation phases, Ibrahim *et al.* made a similar analysis in the $N=1$ supergravity model [15]. A systematic analysis on lepton-flavor-violating processes and a_μ-value within the framework of the supersymmetry seesaw mechanism was given by Hisano *et al.* [16]. Involving the coupling of the second-generation with the third generation superparticles, the contributions of *R*-parity violation to a_μ have been evaluated [17] and discussions on a_μ in the supersymmetric grand unified theories (GUTs) are also made by some theorists [18]. A comparative study on a_μ in various supersymmetric models has also been presented in some recent works [19]. Provided one of the neutral Higgs bosons is of small mass, the authors of Ref. [20] calculated the anomalous magnetic dipole moment of a muon in the two-Higgs doublet model. An analysis of the muon anomalous magnetic dipole moment in other extensions of the standard model has been given [21]. Alternatively, we will apply the effective Lagrangian [22,23] to analyze the muon anomalous magnetic dipole moment in the *CP*-violating MSSM with nonuniversal soft-supersymmetry breaking, i.e., this interaction which violates the lepton flavor conservation is mediated by the nonuniversal soft-supersymmetry breaking parameters. It is well known that the lepton-flavor-violating decays are also ideal processes to detect possible ‘‘new’’ physics beyond the SM. So far, the experiments have not found any substantive evidence of such processes yet, instead, the experimental observation only sets upper bounds on those decay branching ratios, for example $B(\tau\to\mu\gamma)$ and $B(\tau\to 3\mu)$ [26]. Obviously, any new physics must be constrained by these bounds.

In this work, we investigate the lepton-flavor-violating decays $\tau\to\mu\gamma$, $\tau\to 3\mu$, and the muon MDM, electric dipole moment (EDM) in the framework of the MSSM with the nonuniversal soft-supersymmetry breaking. In the supersymmetric theories, there are many new physical *CP*-violating phases that are absent in the SM. Considering the renormalization condition on *CP*-odd Higgs boson [27], we can choose the μ-parameter[1] in the superpotential, and set the nondiagonal elements of the bilinear soft-supersymmetry breaking parameters m^2_{LIJ}, m^2_{RIJ}, m^2_{QIJ}, m^2_{UIJ}, m^2_{DIJ}, $(I\neq J)$ and soft trilinear couplings A^u, A^d, A^l with the physical *CP* phases after properly redefining the fields in the theory.[2] Up to one-loop order, those *CP*-violating phases induce the mixing among the *CP*-even and *CP*-odd Higgs boson [28,29] and modify the Higgs boson couplings to the up- and down-quarks, and to the gauge bosons drastically [29]. The current experimental lower bound on the mass of the lightest neutral Higgs can be reduced to 60 GeV [29]. At present, the experimental upper bound on the electron EDM is set [26]: $|d_e| < 0.5\times10^{-26}e$ cm. In order to rationally predict the muon EDM, we need to take the electron EDM as a rigorous constraint into account. It is well known that the two-loop Barr-Zee-type diagrams [24] may also give a large contribution to the electron EDM [25], thus in our discussion, we include the relevant two-loop Barr-Zee-type contributions to the EDM of charged leptons.

Here, we adopt the notation of Ref. [30], the relevant nonuniversal soft-supersymmetry breaking terms and Feynman rules can also be found in Ref. [30]. Our paper is organized as follows. In Sec. II, we introduce the *CP*-violating MSSM with nonuniversal soft-supersymmetry breaking. In Sec. III, we analyze the loop-corrections to the $\overline{e^J}e^I\gamma$ effective vertex. The muon anomalous magnetic dipole moment and the decay width of $\tau\to\mu\gamma$ in the supersymmetric models are eventually formulated. The $\tau\to3\mu$ is analyzed in Sec. IV. Within the experimentally allowed range for the concerned parameters, our numerical analysis is presented in Sec. V. Upon imposition of the experimental limits on the theoretical predictions of the branching ratios and the electron EDM and muon MDM, we obtain an upper bound on the muon EDM. Then we will make a brief summary about the method and model we employ in this work and discuss the obtained results in the last section. The tedious formulas are collected in Appendixes.

II. THE MSSM WITH NONUNIVERSAL SOFT-SUPERSYMMETRY BREAKING

The most general form of the superpotential which has the gauge invariance and retains all the conservation laws of the SM is written as

$$\mathcal{W}=\mu\epsilon_{ij}\hat{H}^1_i\hat{H}^2_j+\epsilon_{ij}h^l_{IJ}\hat{H}^1_i\hat{L}^I_j\hat{R}^J+\epsilon_{ij}h^d_{IJ}\hat{H}^1_i\hat{Q}^I_j\hat{D}^J+\epsilon_{ij}h^u_{IJ}\hat{H}^2_i\hat{Q}^I_j\hat{U}^J. \quad (1)$$

Here $\hat{H}^1$, $\hat{H}^2$ are the Higgs superfields; $\hat{Q}^I$ and $\hat{L}^I$ are quark and lepton superfields in doublets of the weak $SU(2)$ group, where I=1, 2, 3 are the indices of generations; the remaining superfields $\hat{U}^I$, $\hat{D}^I$ and $\hat{R}^I$ are the quark superfields of u- and d-types and charged leptons in singlets of the weak $SU(2)$ respectively. Indices i, j are contracted for the $SU(2)$ group, and h^l, $h^{u,d}$ are the Yukawa couplings. To break the supersymmetry, the nonuniversal soft-supersymmetry breaking terms are introduced as

$$\begin{aligned}\mathcal{L}_{soft}=&-m^2_{H^1}H^{1*}_iH^1_i-m^2_{H^2}H^{2*}_iH^2_i-m^2_{LIJ}\tilde{L}^{I*}_i\tilde{L}^J_i\\&-m^2_{RIJ}\tilde{R}^{I*}\tilde{R}^J-m^2_{QIJ}\tilde{Q}^{I*}_i\tilde{Q}^J_i-m^2_{UIJ}\tilde{U}^{I*}\tilde{U}^J\\&-m^2_{DIJ}\tilde{D}^{I*}\tilde{D}^J+(m_1\lambda_B\lambda_1+m_2\lambda^i_A\lambda^i_A+m_3\lambda^a_G\lambda^a_G\\&+\text{H.c.})+[\mu B\epsilon_{ij}H^1_iH^2_j+\epsilon_{ij}A^l_{IJ}H^1_i\tilde{L}^I_j\tilde{R}^J\\&+\epsilon_{ij}A^d_{IJ}H^1_i\tilde{Q}^I_j\tilde{D}^J+\epsilon_{ij}A^u_{IJ}H^2_i\tilde{Q}^I_j\tilde{U}^J+\text{H.c.}],\end{aligned} \quad (2)$$

where $m^2_{H^1}$, $m^2_{H^2}$, m^2_{LIJ}, m^2_{RIJ}, m^2_{QIJ}, m^2_{UIJ} and m^2_{DIJ} are the square masses of the superparticles, m_3, m_2, m_1 denote the masses of $\lambda^a_G(a=1,2,\cdots 8)$, $\lambda^i_A(i=1,2,3)$ and λ_B, which are the $SU(3)\times SU(2)\times U(1)$ gauginos. B is a free parameter in unit of mass. In $\mathcal{L}_{soft}$, the nonuniversal terms are (a) in the bilinear couplings: $m^2_{LIJ},m^2_{UIJ},m^2_{DIJ}$ with $I\neq J$; whereas for I=J, $m^2_{LII},m^2_{UII},m^2_{DII}$ are the universal soft terms; (b) in the trilinear couplings: $A^l_{IJ},A^u_{IJ},A^d_{IJ}$ with $I\neq J$ are the nonuniversal parts, whereas as I=J, $A^l_{II},A^u_{II},A^d_{II}$ are the universal parts. A^l_{IJ}, A^u_{IJ}, A^d_{IJ} $(I, J=1,2,3)$ are the soft-supersymmetry breaking parameters that result in mass splitting between leptons, quarks and their supersymmetric partners. With the soft-supersymmetry breaking terms in Eq. (2), we can study the phenomenology in the minimal supersymmetric extension of the standard model (MSSM). The resultant 6×6 square-mass matrix of the charged scalar leptons is written as

$$m^2_{\tilde{E}}=\begin{pmatrix} m^2_{LIJ}+\left[m^2_{e^I}+\left(\frac{1}{2}+s^2_{\rm W}\right)\cos2\beta m^2_{\rm Z}\right]\delta^{IJ} & -m_{e^I}\mu^*\tan\beta\delta^{IJ}+\frac{2m_{\rm W}s_{\rm W}}{e}c_\beta A^l_{IJ}\\ -m_{e^I}\mu\tan\beta\delta^{IJ}+\frac{2m_{\rm W}s_{\rm W}}{e}c_\beta A^{l*}_{JI} & m^2_{RIJ}+(m^2_{e^I}+s^2_{\rm W}\cos2\beta m^2_{\rm Z})\delta^{IJ}\end{pmatrix}, \quad (3)$$

[1]Please be noted, here we use the μ-parameter following the literature. We hope that it will not cause any confusion with the muon which is sometimes written as μ.

[2]At the Lagrangian level, all the couplings may be complex. However some phases are unphysical; for evaluating the physical processes, they are not necessary and we can remove those phases by redefining the wave function as $\Psi\to e^{i\phi}\Psi$. This step is the same as to define the physical *CP* phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements in the SM case.

while the 3×3 sneutrino square-mass matrix is expressed as

$$m_{\tilde{\nu}}^2=\left(-\frac{1}{2}\cos 2\beta m_Z^2\delta^{IJ}+m_{L^{IJ}}^2\right), \tag{4}$$

with m_{e^I} $(I=1, 2, 3)$ being the mass of the charged lepton of the I-th generation. Two mixing matrices $Z_{\tilde{\nu},\tilde{E}}$ which diagonalize the square-mass matrices of the sneutrino and charged slepton respectively are defined as

$$\mathcal{Z}_{\tilde{\nu}}^{\dagger}m_{\tilde{\nu}}^2\mathcal{Z}_{\tilde{\nu}}=\mathrm{diag}(m_{\tilde{\nu}_1}^2,\ \ m_{\tilde{\nu}_2}^2,m_{\tilde{\nu}_3}^2),$$

$$\mathcal{Z}_{\tilde{E}}^{\dagger}m_{\tilde{E}}^2\mathcal{Z}_{\tilde{E}}=\mathrm{diag}(m_{\tilde{E}_1}^2,\ \ m_{\tilde{E}_2}^2,\cdots,m_{\tilde{E}_6}^2). \tag{5}$$

As for the up- and down-type scalar quarks, we can define the mixing matrices similarly

$$\mathcal{Z}_{\tilde{U}}^{\dagger}m_{\tilde{U}}^2\mathcal{Z}_{\tilde{U}}=\mathrm{diag}(m_{\tilde{U}_1}^2,\ \ m_{\tilde{U}_2}^2,\cdots,m_{\tilde{U}_6}^2),$$

$$\mathcal{Z}_{\tilde{D}}^{\dagger}m_{\tilde{D}}^2\mathcal{Z}_{\tilde{D}}=\mathrm{diag}(m_{\tilde{D}_1}^2,\ \ m_{\tilde{D}_2}^2,\cdots,m_{\tilde{D}_6}^2), \tag{6}$$

the expressions for those mass matrices $m_{\tilde{U}}^2$, $m_{\tilde{D}}^2$ can be found in Ref. [30].

In order to suppress unexpectedly large effective FCNC interactions, it is natural to assume $m_{L^{IJ}}^2\ll m_{L^{II}}^2$, $m_{R^{IJ}}^2\ll m_{R^{II}}^2$ and $A_{IJ}^l\ll A_{II}^l$ with $J\neq I$. Accurate to order $\mathcal{O}([\Delta x_{\tilde{S}_{ij}}/(x_{S_i}^2-x_{S_j}^2)]^2)$, we write down the expression of the mixing matrices which diagonalize the scalar fermion square-mass matrices

$$\mathcal{Z}_{\tilde{\nu}}^{\dagger ij}=\mathbf{U}_{\tilde{\nu}_{ij}},\quad (i,\ j=1,2,3) \tag{7}$$

and

$$\mathcal{Z}_{\tilde{E}}^{\dagger\alpha i}=\mathbf{U}_{\tilde{E}_{\alpha i}}\cos\theta_{\tilde{E}_i}+\mathbf{U}_{\tilde{E}_{\alpha(3+i)}}\sin\theta_{\tilde{E}_i}e^{-i\varphi_{\tilde{E}_i}},$$

$$\mathcal{Z}_{\tilde{E}}^{\dagger\alpha(3+i)}=-\mathbf{U}_{\tilde{E}_{\alpha i}}\sin\theta_{\tilde{E}_i}e^{i\varphi_{\tilde{E}_i}}+\mathbf{U}_{\tilde{E}_{\alpha(3+i)}}\cos\theta_{\tilde{E}_i}$$

$$(\alpha=1,\cdots,6;\ \ i=1,2,3). \tag{8}$$

In general, the $N\times N$ transformation matrices $\mathbf{U}_{S_{\alpha i}}$ can be written as

$$\mathbf{U}_{S_{ii}}=1-\sum_{j\neq i}\frac{|\Delta x_{S_{ij}}|^2}{2(x_{S_i}-x_{S_j})^2},$$

$$\mathbf{U}_{S_{ij}}=\frac{\Delta x_{S_{ij}}}{x_{S_j}-x_{S_i}}+\sum_{k\neq i,\ j}\frac{\Delta x_{S_{ik}}\Delta x_{S_{kj}}}{(x_{S_j}-x_{S_i})(x_{S_j}-x_{S_k})}, \tag{9}$$

where $|\Delta m_{S_{ij}}^2|\ll|m_{S_i}^2-m_{S_j}^2|$ $(i,\ j=1,\ \cdots,\ N)$. The symbols are defined as $\Delta x_{S_{ij}}\equiv\Delta m_{S_{ij}}^2/m_W^2$, $x_{S_i}\equiv m_{S_i}^2/m_W^2$ with $S=\tilde{\nu},\ \tilde{E}$. For the sneutrinos, $\Delta m_{\tilde{\nu}_{ij}}^2(i\neq j)=m_{L^{ij}}^2$. The expressions for the off-diagonal elements of the charged slepton square-mass matrix are more complicated:

$$\begin{aligned}\Delta m_{\tilde{E}_{ij}}^2=&\cos\theta_{\tilde{E}_i}\cos\theta_{\tilde{E}_j}m_{L^{ij}}^2\\&+\sin\theta_{\tilde{E}_i}\sin\theta_{\tilde{E}_j}e^{i(\phi_{\tilde{E}_j}-\phi_{\tilde{E}_i})}m_{R^{ij}}^2\\&+\frac{2m_Ws_Wc_\beta}{e}[\cos\theta_{\tilde{E}_j}\sin\theta_{\tilde{E}_i}e^{-i\phi_{\tilde{E}_i}}A_{ji}^{l*}\\&+\cos\theta_{\tilde{E}_i}\sin\theta_{\tilde{E}_j}e^{i\phi_{\tilde{E}_j}}A_{ij}^l],\end{aligned}$$

$$\begin{aligned}\Delta m_{\tilde{E}_{i(3+j)}}^2=&\frac{2m_Ws_Wc_\beta}{e}[\cos\theta_{\tilde{E}_i}\cos\theta_{\tilde{E}_j}A_{ij}^l\\&-\sin\theta_{\tilde{E}_i}\sin\theta_{\tilde{E}_j}e^{-i(\phi_{\tilde{E}_i}+\phi_{\tilde{E}_j})}A_{ji}^{l*}]\\&-\cos\theta_{\tilde{E}_i}\sin\theta_{\tilde{E}_j}e^{-i\phi_{\tilde{E}_j}}m_{L^{ij}}^2\\&+\cos\theta_{\tilde{E}_j}\sin\theta_{\tilde{E}_i}e^{-i\phi_{\tilde{E}_i}}m_{R^{ij}}^2,\end{aligned}$$

$$\Delta m_{\tilde{E}_{(3+i)j}}^2=\Delta m_{\tilde{E}_{j(3+i)}}^{2*},$$

$$\begin{aligned}\Delta m_{\tilde{E}_{(3+i)(3+j)}}^2=&\cos\theta_{\tilde{E}_i}\cos\theta_{\tilde{E}_j}m_{R^{ij}}^2\\&+\sin\theta_{\tilde{E}_i}\sin\theta_{\tilde{E}_j}e^{i(\phi_{\tilde{E}_i}-\phi_{\tilde{E}_j})}m_{L^{ij}}^2\\&-\frac{2m_Ws_Wc_\beta}{e}[\cos\theta_{\tilde{E}_i}\sin\theta_{\tilde{E}_j}e^{-i\phi_{\tilde{E}_j}}A_{ji}^{l*}\\&+\cos\theta_{\tilde{E}_j}\sin\theta_{\tilde{E}_i}e^{i\phi_{\tilde{E}_i}}A_{ij}^l],\end{aligned}$$

$$\Delta m_{\tilde{E}_{(3+i)i}}^2=\Delta m_{\tilde{E}_{i(3+i)}}^2=0,\ (i,\ \ j=1,\ \ 2,\ \ 3;\quad i\neq j). \tag{10}$$

For a special case where there is degeneracy among the eigenvalues of the square-mass matrix, the explicit forms of the mixing matrices are given in Appendix A.

As we will find in the following sections, the effective Lagrangians of $\overline{e^J}e^I\gamma$ and $e^I\rightarrow 3e^J$ are mediated by a combination of the following couplings:

$$\mathbf{G}_{\nu(a)}^{\{ijI\beta J\alpha\}}=\mathcal{Z}_+^{1i*}\mathcal{Z}_+^{1j}\mathcal{Z}_\nu^{I\beta*}\mathcal{Z}_\nu^{J\alpha},$$

$$\mathbf{G}_{\nu(b)}^{\{ijI\beta J\alpha\}}=\mathcal{Z}_+^{1i}\mathcal{Z}_-^{2j}\mathcal{Z}_\nu^{I\beta*}\mathcal{Z}_\nu^{J\alpha},$$

$$\mathbf{G}_{\nu(c)}^{\{ijI\beta J\alpha\}}=\mathcal{Z}_+^{1i*}\mathcal{Z}_-^{2j*}\mathcal{Z}_\nu^{I\beta*}\mathcal{Z}_\nu^{J\alpha},$$

$$\mathbf{G}_{\nu(d)}^{\{ijI\beta J\alpha\}}=\mathcal{Z}_-^{2i*}\mathcal{Z}_-^{2j}\mathcal{Z}_\nu^{I\beta*}\mathcal{Z}_\nu^{J\alpha},$$

$$\mathbf{G}_{L(a)}^{\{ijI\beta J\alpha\}}=\Bigg[\mathcal{Z}_{\tilde{E}}^{J\alpha *}(\mathcal{Z}_N^{1i*}s_{\rm W}+\mathcal{Z}_N^{2i*}c_{\rm W})-\frac{m_{e^J}c_{\rm W}}{m_{\rm W}c_\beta}\mathcal{Z}_{\tilde{E}}^{(3+J)\alpha *}\mathcal{Z}_N^{3i*}\Bigg]\Bigg[\mathcal{Z}_{\tilde{E}}^{I\beta}(\mathcal{Z}_N^{1j}s_{\rm W}+\mathcal{Z}_N^{2j}c_{\rm W})-\frac{m_{e^I}c_{\rm W}}{m_{\rm W}c_\beta}\mathcal{Z}_{\tilde{E}}^{(3+I)\beta}\mathcal{Z}_N^{3j}\Bigg],$$

$$\mathbf{G}_{L(b)}^{\{ijI\beta J\alpha\}}=\Bigg[2s_{\rm W}\mathcal{Z}_{\tilde{E}}^{(3+J)\alpha *}\mathcal{Z}_N^{1i}+\frac{m_{e^J}c_{\rm W}}{m_{\rm W}c_\beta}\mathcal{Z}_{\tilde{E}}^{J\alpha *}\mathcal{Z}_N^{3i}\Bigg]\times\Bigg[\mathcal{Z}_{\tilde{E}}^{I\beta}(\mathcal{Z}_N^{1j}s_{\rm W}+\mathcal{Z}_N^{2j}c_{\rm W})-\frac{m_{e^I}c_{\rm W}}{m_{\rm W}c_\beta}\mathcal{Z}_{\tilde{E}}^{(3+I)\beta}\mathcal{Z}_N^{3j}\Bigg],$$

$$\mathbf{G}_{L(c)}^{\{ijI\beta J\alpha\}}=\Bigg[\mathcal{Z}_{\tilde{E}}^{J\alpha *}(\mathcal{Z}_N^{1i*}s_{\rm W}+\mathcal{Z}_N^{2i*}c_{\rm W})-\frac{m_{e^J}c_{\rm W}}{m_{\rm W}c_\beta}\mathcal{Z}_{\tilde{E}}^{(3+J)\alpha *}\mathcal{Z}_N^{3i*}\Bigg]\Bigg[2s_{\rm W}\mathcal{Z}_{\tilde{E}}^{(3+I)\beta}\mathcal{Z}_N^{1j*}+\frac{m_{e^I}c_{\rm W}}{m_{\rm W}c_\beta}\mathcal{Z}_{\tilde{E}}^{I\beta}\mathcal{Z}_N^{3j*}\Bigg],$$

$$\mathbf{G}_{L(d)}^{\{ijI\beta J\alpha\}}=\Bigg[2s_{\rm W}\mathcal{Z}_{\tilde{E}}^{(3+J)\alpha *}\mathcal{Z}_N^{1i}+\frac{m_{e^J}c_{\rm W}}{m_{\rm W}c_\beta}\mathcal{Z}_{\tilde{E}}^{J\alpha *}\mathcal{Z}_N^{3i}\Bigg]\times\Bigg[2s_{\rm W}\mathcal{Z}_{\tilde{E}}^{(3+I)\beta}\mathcal{Z}_N^{1j*}+\frac{m_{e^I}c_{\rm W}}{m_{\rm W}c_\beta}\mathcal{Z}_{\tilde{E}}^{I\beta}\mathcal{Z}_N^{3j*}\Bigg]. \quad (11)$$

Generally, we can recast the combined couplings in terms of Eq. (7) and Eq. (8). For example, we can write the coupling $\mathbf{G}_{\nu(a)}^{\{ijI\beta J\alpha\}}$ as

$$\mathbf{G}_{\nu(a)}^{\{ijI\beta J\alpha\}}=\mathcal{Z}_+^{1i*}\mathcal{Z}_+^{1j}\left[\delta_{\beta I}\delta_{\alpha J}+\delta_{\alpha J}\frac{\Delta x_{\tilde{\nu}_{\beta I}}}{x_{\tilde{\nu}_I}-x_{\tilde{\nu}_\beta}}\Bigg|_{I\neq\beta}+\delta_{\beta I}\frac{\Delta x^*_{\tilde{\nu}_{\alpha J}}}{x_{\tilde{\nu}_J}-x_{\tilde{\nu}_\alpha}}\Bigg|_{J\neq\alpha}\right], \quad (12)$$

where the first term just contributes to the muon MDM and EDM, while the other terms contribute to both the muon MDM, EDM and the flavor changing neutral current (FCNC) processes of leptons. The other couplings can also be written down in a similar way; for saving space, we omit those concrete expressions here.

It is well known that one can apply the mass insertion approximation (MIA) to simplify the expressions of supersymmetric contributions to FCNC processes which are induced via loop diagrams [31,32]. In that approach, a small off-diagonal mass is inserted into the mass matrix which is written in the basis of the weak interaction and an approximate degeneracy of the squark masses is assumed. Thus the drawback is obvious while evaluating some processes where degeneracy of masses does not exist. Instead, in this work, we carry out all the calculations rigorously in the mass basis and keep appropriate pole masses in the propagators and mixing entries between various flavors at the vertices. We only expand the mixing matrix $\mathcal{Z}_{\tilde{\nu}}^{\dagger ij}$ in the soft-supersymmetry breaking terms with respect to the mixing parameters $[\Delta x_{\tilde{S}_{ij}}/(x_{S_i}^2-x_{S_j}^2)]^2$ which are small as long as $i\neq j$.

To be more explicit, we would like to compare our approach with the MIA method, and point out the improvements of our scheme from the MIA.

(i) When the mixing between left- and right-handed sfermions is negligible, i.e. $\theta_S\sim 0$ ($S=\tilde{U}_i,\ \tilde{D}_i,\ \tilde{E}_i$) and $|\Delta m^2_{S_{ij}}|\ll|m^2_{S_i}-m^2_{S_j}|$ $(i,\ j=1,\dots,N)$, our results are in accord with the result of MIA at the lowest order of $\Delta x_{S_{ij}}$. When $\theta_S\neq 0$ ($S=\tilde{U}_i,\ \tilde{D}_i,\ \tilde{E}_i$), indeed, our approach is an improvement over the MIA.

(ii) When all the eigenvalues of the mass matrix are approximately degenerate, just as proved in [32], our results are the same as that of MIA at the lowest order of $\Delta x_{S_{ij}}$.

(iii) As some eigenvalues of the mass matrix are only approximately degenerate, simple applications of the MIA method are invalid [33]. Now, there is a large flavor mixing in the sfermion sector (certainly, such flavor changing effects will be suppressed by the mass of the heavy sfermion in our detectable processes).

In our numerical results, we will extensively discuss the first and third statements. In the following section, we give the formulas of the muon MDM, EDM, and the decay width of $\tau\to\mu\gamma$.

III. THE MUON ANOMALOUS MAGNETIC AND ELECTRIC DIPOLE MOMENTS

The effective Lagrangian is extensively applied to evaluating rare decay widths of b, c-quarks [34–37]. Derivation of the Lagrangian is carried out according to the following principle: if all the masses m_i's of the internal particles in the loops are much larger than the external momenta i.e. $m_i^2\gg p^2$, thus the heavy particles can be integrated out. In our case, all the SUSY particles are integrated out and their contributions are attributed into the Wilson coefficients in the effective Lagrangian.

For the W-boson propagator, we adopt the nonlinear R_ξ gauge whose gauge fixing term is [38]

$$\mathcal{L}_{gauge-fixing}=-\frac{1}{\xi}f^\dagger f \quad (13)$$

with $f=(\partial_\mu W^{+\mu}-ieA_\mu W^{+\mu}-i\xi m_{\rm W}\phi^+)$ and specifically we set $\xi=1$ in the later calculations. A thorough discussion about the gauge invariance in this situation has been given [39,40].

The Feynman diagrams for $\overline{e^J}e^I\gamma$ in the supersymmetric model are drawn in Fig. 1, the effective Lagrangian is written as

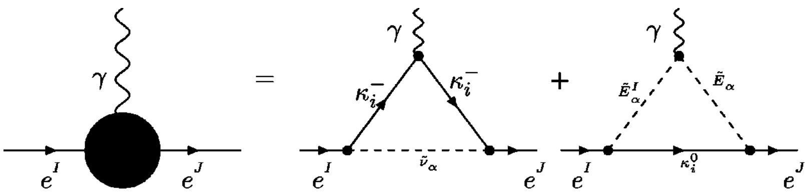


FIG. 1. The Feynman diagrams which contribute to the $\bar{e}^I e^J\gamma$ effective Lagrangian in MSSM.

$$\mathcal{L}_{\bar{e}^J e^I \gamma} = \frac{4G_F}{\sqrt{2}} \sum_{i=1}^{5} C_i^{\mp}(\mu_{\rm W}) \mathcal{O}_i^{\mp}, \tag{14}$$

and the operator basis consists of ten operators

$$\mathcal{O}_1^{\mp} = \frac{1}{(4\pi)^2} \overline{e^J} (i\not{D})^3 \omega_{\mp} e^I,$$

$$\mathcal{O}_2^{\mp} = \frac{1}{(4\pi)^2} \overline{e^J} \{ i\not{D}, eF\cdot\sigma \} \omega_{\mp} e^I,$$

$$\mathcal{O}_3^{\mp} = \frac{1}{(4\pi)^2} \overline{e^J} i D_\mu (ieF^{\mu\nu}) \gamma_\nu \omega_{\mp} e^I,$$

$$\mathcal{O}_4^{-} = \frac{1}{(4\pi)^2} m_{e^J} \overline{e^J} (i\not{D})^2 \omega_{-} e^I,$$

$$\mathcal{O}_5^{-} = \frac{1}{(4\pi)^2} m_{e^J} \overline{e^J} eF\cdot\sigma \omega_{-} e^I,$$

$$\mathcal{O}_4^{+} = \frac{1}{(4\pi)^2} m_{e^I} \overline{e^J} (i\not{D})^2 \omega_{+} e^I,$$

$$\mathcal{O}_5^{+} = \frac{1}{(4\pi)^2} m_{e^I} \overline{e^J} eF\cdot\sigma \omega_{+} e^I. \tag{15}$$

This basis also exists in the case of SM [35]. In these operators, $D_\mu \equiv \partial_\mu - ieQ_e A_\mu$, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ denoting the electromagnetic field strength tensor and $F\cdot\sigma \equiv F_{\mu\nu}\sigma^{\mu\nu}$. The terms of dimension-four which are related to the $\overline{e^J}\gamma_\rho \omega_{\pm} e^I$ vertex cancel each other as long as we let e^I and e^J leptons be on their mass shells [35], so that they do not exist in $\mathcal{L}^{\bar{e}^J e^I \gamma}$ at all. To shorten the text length, we present the one-loop contributions to the Wilson coefficients $C_i^{\mp}(\mu_{\rm W})$ in an Appendix. Here, we give the detailed expressions for the two-loop Barr-Zee-type diagrams (Fig. 2). After integrating out the heavy degrees of freedom, we obtain the two-loop Wilson coefficients:

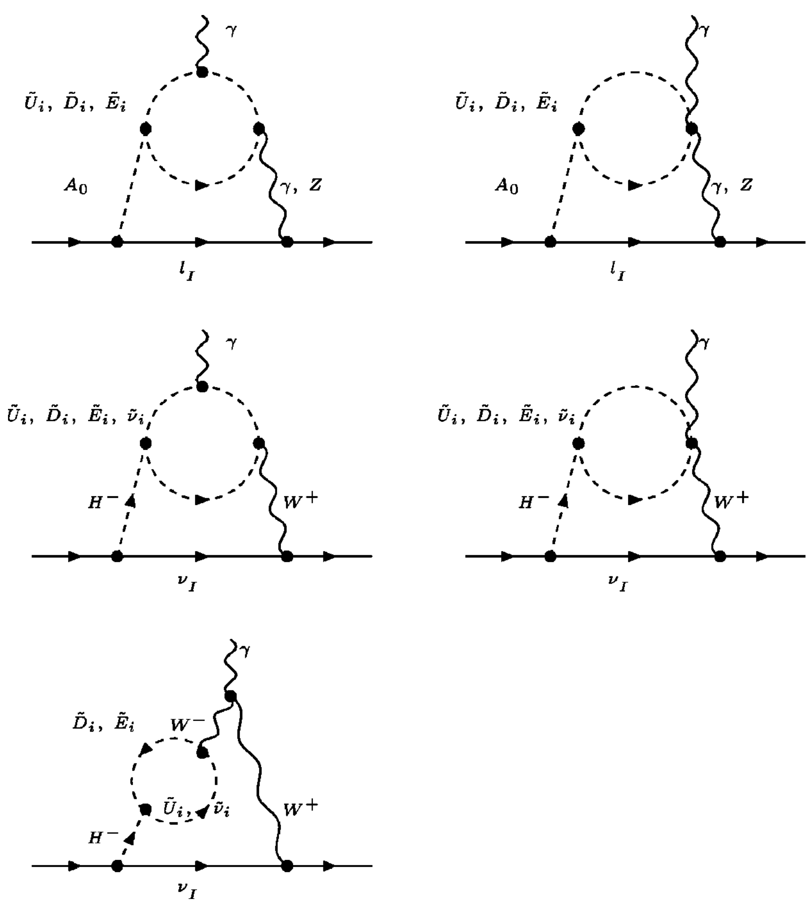


FIG. 2. The two-loop Barr-Zee-type Feynman diagrams which contribute to the $e^I e^J\gamma$ effective Lagrangian in MSSM.

$$
\begin{aligned}
C^{+}_{5(2-loop)}(\mu_{\rm W}) = \frac{e\tan\beta}{128\sqrt{2}\pi^2 m_{\rm W} s_{\rm W}} \Bigg\{ & N_c \sum_{K,L,i,j} \mathcal{Z}_{\tilde{U}}^{Ki*}\mathcal{Z}_{\tilde{D}}^{Lj*} V^{KL} (\Gamma^{H^+\tilde{U}_i\tilde{D}_j})^* [Q_u G(x_{H^-},x_{\tilde{U}_i},x_{\tilde{D}_j}) + Q_d G(x_{H^-},x_{\tilde{D}_j},x_{\tilde{U}_i})] \\
& - \sum_{K,i,j} \mathcal{Z}_{\tilde{\nu}}^{Ki*}\mathcal{Z}_{\tilde{E}}^{Kj*} (\Gamma^{H^+\tilde{\nu}_i\tilde{E}_j})^* G(x_{H^-},x_{\tilde{E}_j},x_{\tilde{\nu}_i}) + 4\sqrt{2} N_c \sum_{\tilde{U},\tilde{D}} Q_q G(c_{\rm W}^2 x_A^2, c_{\rm W}^2 x_{\tilde{Q}_i}^2, c_{\rm W}^2 x_{\tilde{Q}_j}) \\
& \times \Bigg[-\Gamma^{A\tilde{Q}_i^*\tilde{Q}_j}\Bigg(T_q \sum_K \mathcal{Z}_{\tilde{Q}}^{Ki*}\mathcal{Z}_{\tilde{Q}}^{Kj} - Q_q^2 s_{\rm W}^2 \delta^{ij}\Bigg)(T_l + s_{\rm W}^2) + (\Gamma^{A\tilde{Q}_i^*\tilde{Q}_j})^* \Bigg(T_q \sum_K \mathcal{Z}_{\tilde{Q}}^{Ki}\mathcal{Z}_{\tilde{Q}}^{Kj*} - Q_q^2 s_{\rm W}^2 \delta^{ij}\Bigg) s_{\rm W}^2 \Bigg] \\
& + 4\sqrt{2} \sum_{\tilde{E}} G(c_{\rm W}^2 x_A^2, c_{\rm W}^2 x_{\tilde{E}_i}^2, c_{\rm W}^2 x_{\tilde{E}_j}) \Bigg[\Gamma^{A\tilde{E}_i^*\tilde{E}_j}\Bigg(T_l \sum_K \mathcal{Z}_{\tilde{E}}^{Ki*}\mathcal{Z}_{\tilde{E}}^{Kj} - s_{\rm W}^2\delta^{ij}\Bigg)(T_l + s_{\rm W}^2) \\
& - (\Gamma^{A\tilde{E}_i^*\tilde{E}_j})^* \Bigg(T_l \sum_K \mathcal{Z}_{\tilde{E}}^{Ki}\mathcal{Z}_{\tilde{E}}^{Kj*} - s_{\rm W}^2\delta^{ij}\Bigg) s_{\rm W}^2 \Bigg] + 4\sqrt{2}\,\frac{s_{\rm W}^2}{x_A}\Bigg[N_c \sum_{\tilde{U},\tilde{D}} Q_q^2 \Gamma^{A\tilde{Q}_j^*\tilde{Q}_j} F\Bigg(\frac{x_{\tilde{Q}_i}}{x_A}\Bigg) \\
& + \sum_{\tilde{E}} \Gamma^{A\tilde{E}_j^*\tilde{E}_j} F\Bigg(\frac{x_{\tilde{E}_j}}{x_A}\Bigg)\Bigg]\Bigg]\Bigg\} \delta_{IJ}, \\
C^{-}_{5(2-loop)}(\mu_{\rm W}) = \frac{e\tan\beta}{128\sqrt{2}\pi^2 m_{\rm W} s_{\rm W}} \Bigg\{ & N_c \sum_{K,L,i,j} \mathcal{Z}_{\tilde{U}}^{Ki}\mathcal{Z}_{\tilde{D}}^{Lj} V^{KL*} (\Gamma^{H^+\tilde{U}_i\tilde{D}_j}) [Q_u G(x_{H^-},x_{\tilde{U}_i},x_{\tilde{D}_j}) + Q_d G(x_{H^-},x_{\tilde{D}_j},x_{\tilde{U}_i})] \\
& - \sum_{K,i,j} \mathcal{Z}_{\tilde{\nu}}^{Ki}\mathcal{Z}_{\tilde{E}}^{Kj} (\Gamma^{H^+\tilde{\nu}_i\tilde{E}_j}) G(x_{H^-},x_{\tilde{E}_j},x_{\tilde{\nu}_i}) + 4\sqrt{2} N_c \sum_{\tilde{U},\tilde{D}} Q_q G(c_{\rm W}^2 x_A^2, c_{\rm W}^2 x_{\tilde{Q}_i}^2, c_{\rm W}^2 x_{\tilde{Q}_j}) \\
& \times \Bigg[-\Gamma^{A\tilde{Q}_i^*\tilde{Q}_j}\Bigg(T_q \sum_K \mathcal{Z}_{\tilde{Q}}^{Ki*}\mathcal{Z}_{\tilde{Q}}^{Kj} - Q_q^2 s_{\rm W}^2 \delta^{ij}\Bigg) s_{\rm W}^2 + (\Gamma^{A\tilde{Q}_i^*\tilde{Q}_j})^* \Bigg(T_q \sum_K \mathcal{Z}_{\tilde{Q}}^{Ki}\mathcal{Z}_{\tilde{Q}}^{Kj*} - Q_q^2 s_{\rm W}^2 \delta^{ij}\Bigg)(T_l + s_{\rm W}^2) \Bigg] \\
& + 4\sqrt{2} \sum_{\tilde{E}} G(c_{\rm W}^2 x_A^2, c_{\rm W}^2 x_{\tilde{E}_i}^2, c_{\rm W}^2 x_{\tilde{E}_j}) \Bigg[\Gamma^{A\tilde{E}_i^*\tilde{E}_j}\Bigg(T_l \sum_K \mathcal{Z}_{\tilde{E}}^{Ki*}\mathcal{Z}_{\tilde{E}}^{Kj} - s_{\rm W}^2\delta^{ij}\Bigg) s_{\rm W}^2 \\
& - (\Gamma^{A\tilde{E}_i^*\tilde{E}_j})^* \Bigg(T_l \sum_K \mathcal{Z}_{\tilde{E}}^{Ki}\mathcal{Z}_{\tilde{E}}^{Kj*} - s_{\rm W}^2\delta^{ij}\Bigg)(T_l + s_{\rm W}^2) \Bigg] - 4\sqrt{2}\,\frac{s_{\rm W}^2}{x_A}\Bigg[N_c \sum_{\tilde{U},\tilde{D}} Q_q^2 \Gamma^{A\tilde{Q}_j^*\tilde{Q}_j} F\Bigg(\frac{x_{\tilde{Q}_i}}{x_A}\Bigg) \\
& + \sum_{\tilde{E}} \Gamma^{A\tilde{E}_j^*\tilde{E}_j} F\Bigg(\frac{x_{\tilde{E}_j}}{x_A}\Bigg)\Bigg]\Bigg]\Bigg\} \delta_{IJ}
\end{aligned}
\tag{16}
$$

with $q=u_i,\ d_i,\ Q_{u_i}=\frac{2}{3},\ Q_{d_i}=\frac{1}{3},\ T_{u_i}=-T_{d_i}=-T_{l_i}=\frac{1}{2}\ (i=1,2,3)$ and $N_c=3$ is the color factor. The two-loop functions $F(a)$, $G(a,b,c)$ are given by

$$
\begin{aligned}
F(a) &= \int_0^1 dx \frac{x(1-x)}{a-x(1-x)} \ln\Bigg(\frac{x(1-x)}{a}\Bigg), \\
G(a,b,c) &= \int_0^1 dx x \Bigg\{ \frac{a x(1-x)\ln a}{(a-1)[a x(1-x) - b x - c(1-x)]} \\
&\quad + \frac{x(1-x)[bx + c(1-x)]}{[a x(1-x) - b x - c(1-x)][x(1-x) - b x - c(1-x)]} \ln\Bigg(\frac{bx + c(1-x)}{x(1-x)}\Bigg)\Bigg\}.
\end{aligned}
\tag{17}
$$

The couplings between the scalar quarks and Higgs are presented in Appendix C. Integrating out the heavy degrees of freedom in the loops, effective vertices of γHV where H is a neutral or charged Higgs boson and V stands for the vector gauge bosons W or Z, are obtained [25]. The gauge invariant forms of the vertices should be

$$
\text{factor}\times[(k\cdot q)g_{\mu\nu} - k_\mu q_\nu]
$$

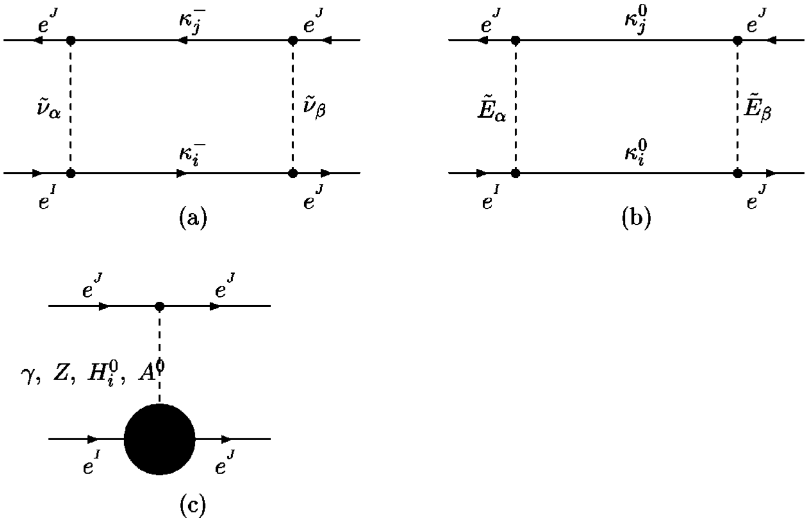


FIG. 3. The Feynman diagrams which contribute to the $e^I \to 3e^J$ in MSSM.

where q_μ and k_ν are the four-momenta of the photon and vector gauge boson (W or Z) and the factor depends on the heavy degrees of freedom, the two-loop Barr-Zee type diagrams only induce the nonzero contributions to the coefficient $C_5^{\pm}$.

Setting $I=J=2$ in Eq. (14), we obtain the muon anomalous magnetic dipole moment in the MSSM:

$$\delta a_\mu^{SUSY} = \frac{G_F m_\mu^2}{\sqrt{2}\pi^2}\left[C_2^+ + C_2^- + \frac{1}{2}(C_5^- + C_5^+)\right]_{I=J=2}. \quad (18)$$

Correspondingly, the muon electric dipole moment is

$$d_\mu^{SUSY} = e\frac{\sqrt{2}G_F m_\mu}{i4\pi^2}[C_5^- - C_5^+]_{I=J=2}. \quad (19)$$

For the FCNC process $\tau \to \mu\gamma$, the contributions from the SM and the Higgs sector are suppressed by the small ratio $x_{\nu_i} = m_{\nu_i}^2/m_W^2$. The supersymmetric contribution originates from the sneutrino-chargino loop and the amplitude reads

$$\mathcal{A}_{\tau\to\mu\gamma} = -\frac{eG_F}{4\sqrt{2}\pi^2}(m_\mu F^L_{\tau\to\mu\gamma}\bar{\mu}[/q,/\epsilon]\omega_-\tau + m_\tau F^R_{\tau\to\mu\gamma}\bar{\mu}[/q,/\epsilon]\omega_+\tau), \quad (20)$$

where ϵ is the polarization of the emitted photon. The form factors $F^L_{\tau\to\mu\gamma}, F^R_{\tau\to\mu\gamma}$ are formulated as

$$F^L_{\tau\to\mu\gamma} = C_2^- + C_5^- + \frac{m_\tau}{m_\mu}C_2^+,$$

$$F^R_{\tau\to\mu\gamma} = C_2^+ + C_5^+ + \frac{m_\mu}{m_\tau}C_2^-. \quad (21)$$

From Eq. (20), we have the decay width as

$$\Gamma_{\tau\to\mu\gamma} = \frac{e^2 G_F^2 m_\tau^3}{128\pi^5}[m_\mu^2|F^L_{\tau\to\mu\gamma}|^2 + m_\tau^2|F^R_{\tau\to\mu\gamma}|^2]. \quad (22)$$

IV. THE LEPTON-FLAVOR-VIOLATING DECAY $\tau\to 3\mu$

The effective Lagrangian for $\tau \to 3\mu$ is induced by the following four pieces: γ, Z, H-mediating penguin and box diagrams. Those Feynman diagrams are shown in Fig. 3 and Fig. 4. After integrating out these heavy degrees of freedom, the effective Lagrangian is written as

$$\mathcal{L}_{eff}^{\tau\to 3\mu} = \frac{G_F^2 m_W^2}{2\pi^2}\sum_i C_i Q_i \quad (23)$$

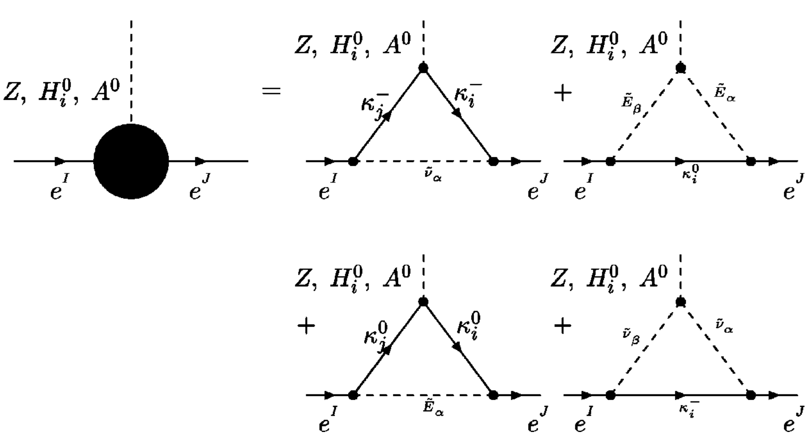


FIG. 4. The penguin diagrams in MSSM where Z, H_i^0, A^0 are involved respectively.

with those operators being

$$\mathcal{Q}_1=\overline{\mu}\gamma_\rho\omega_-\tau\overline{\mu}\gamma^\rho\omega_-\mu,$$

$$\mathcal{Q}_2=\overline{\mu}\gamma_\rho\omega_+\tau\overline{\mu}\gamma^\rho\omega_+\mu,$$

$$\mathcal{Q}_3=\overline{\mu}\omega_-\tau\overline{\mu}\omega_-\mu,$$

$$\mathcal{Q}_4=\overline{\mu}\omega_-\tau\overline{\mu}\omega_+\mu,$$

$$\mathcal{Q}_5=\overline{\mu}\omega_+\tau\overline{\mu}\omega_-\mu,$$

$$\mathcal{Q}_6=\overline{\mu}\omega_+\tau\overline{\mu}\omega_+\mu. \tag{24}$$

The differential width of $\tau\to3\mu$ is

$$\frac{d^2\Gamma}{dm_{12}^2dm_{23}^2}=\frac{G_F^4m_W^4}{(2\pi)^7}\frac{1}{32m_\tau^3}|\mathcal{M}|^2, \tag{25}$$

where $m_{ij}^2=(p_i+p_j)^2$, and $p_i(i=1, 2, 3)$ are the momenta of the outgoing muons in the rest frame of τ. With Eq. (23), one obtains the square of the transition matrix element $|\mathcal{M}|^2$ as

$$\begin{aligned}|\mathcal{M}|^2=\{&4(m_\tau^2+m_\mu^2-m_{12}^2)(m_{12}^2-2m_\mu^2)(|C_1|^2+|C_2|^2)+4(m_\tau^2+m_\mu^2-m_{23}^2)(m_{23}^2-2m_\mu^2)(|C_3|^2+|C_4|^2+|C_5|^2+|C_6|^2)\\&+8m_\tau m_\mu^3\mathbf{Re}(4C_1C_2^\dagger+C_3C_6^\dagger+C_4C_5^\dagger)+4m_\mu^2(m_\tau^2+m_\mu^2-m_{12}^2)\mathbf{Re}(C_1C_3^\dagger+C_2C_6^\dagger)+4m_\mu^2(m_\tau^2+m_\mu^2-m_{13}^2)\mathbf{Re}(C_1C_4^\dagger\\&+C_2C_5^\dagger)+4m_\tau m_\mu(m_\tau^2+m_\mu^2-m_{12}^2-m_{23}^2)\mathbf{Re}(C_1C_5^\dagger+C_2C_4^\dagger)+4m_\tau m_\mu(m_{12}^2-2m_\mu^2)\mathbf{Re}(C_1C_6^\dagger+C_2C_4^\dagger)\\&+4m_\mu^2(m_\tau^2+m_\mu^2-m_{23}^2)\mathbf{Re}(C_3C_4^\dagger+C_5C_6^\dagger)+4m_\tau m_\mu(m_{23}^2-2m_\mu^2)\mathbf{Re}(C_3C_5^\dagger+C_4C_6^\dagger)\}.\end{aligned} \tag{26}$$

V. NUMERICAL RESULT AND DISCUSSION

In this section, we present our numerical analysis on the muon MDM, EDM and the lepton-flavor-violating decay processes in the supersymmetric scenario with the nonuniversal soft-supersymmetry breaking. In the lepton sector of MSSM, there are $15+3G$ (G denotes the generation number) new free parameters besides the SM parameters g_1, g_2, m_{e^I} $(I=1, 2, 3)$. Too many parameters reduce the predictability of the model and before any direct evidence of the SUSY particles is found, determining the free parameters is the most subtle and tough task in the supersymmetric theory. To find a way out, the grand unified theory (GUT) assumption is frequently adopted where all new physics parameters are fully fixed from only five free parameters at the grand unification scale. In that scenario, all the input masses conserve flavors, namely there are no off-diagonal masses to be introduced. Thus when all the concerned parameters evolve from the GUT scale down to the electroweak scale by the renormalization group equations (RGE), the flavor changing parts in the effective Lagrangian emerge only through the CKM mechanism. The induced FCNC-related decay modes would have very small branching ratios which are much lower than the experimental bounds. Therefore, we would rather adopt an alternative parametrization which may imply a different physics picture from the GUT scenario, i.e. to choose a set of parameters at the electroweak scale as inputs. The numerical results depend on these parameters and by imposing the experimental bounds, one can obtain constraints on the parameters. Just as in Ref. [27], we require the phase $\arg(\mu)=0$ in order to suppress the one-loop contribution to the electron EDM. At the lowest order we can choose the parameter basis which is responsible for changing lepton flavors as following:

$$\mu,\ m_1,\ m_2,\ \tan\beta,\ m_{\tilde{E}_I}^2,\quad m_{\tilde{E}_{(3+I)}}^2,\ \theta_{\tilde{E}_I},\ \phi_{\tilde{E}_I},\ m_{L^{IJ}}^2,\ m_{R^{IJ}}^2,\ A_{IJ}^l,\ (I,\ J=1,\ 2,\ 3;\ I\neq J).$$

Correspondingly, the square mass of the sneutrino is given through the relation

$$m_{\tilde{\nu}_I}^2=\cos^2\theta_{\tilde{E}_I}m_{\tilde{E}_I}^2+\sin^2\theta_{\tilde{E}_I}m_{\tilde{E}_{(3+I)}}^2-m_{e^I}^2+m_W^2\cos2\beta.$$

There are several relevant *CP* phases: $\phi_{\tilde{\mu}}=\phi_{\tilde{L}^I}$ for $I=2$, $\phi_{\tilde{\tau}}=\phi_{\tilde{L}^I}$ for $I=3$, and $\phi_{A_{IJ}^l}$, $\phi_{m_{L_{IJ}}^2}$, $\phi_{m_{R_{IJ}}^2}$ $(I\neq J)$ are the *CP* phases of parameters A_{IJ}^l, $m_{L_{IJ}}^2$ and $m_{R_{IJ}}^2$ respectively. To simplify our discussion, we assume that the bilinear and trilinear couplings of scalar quarks are all universal, i.e. $A_{IJ}^u=A_{u^I}\delta_{IJ}$, $A_{IJ}^d=A_{d^I}\delta_{IJ}$ and $m_{\tilde{U}_{IJ}}^2=m_{\tilde{U}_I}^2\delta_{IJ}$, $m_{\tilde{D}_{IJ}}^2=m_{\tilde{D}_I}^2\delta_{IJ}$, $m_{\tilde{Q}_{IJ}}^2=m_{\tilde{Q}_I}^2\delta_{IJ}$. As aforementioned, the nontrivial *CP* phases lead to a large mixing among the neutral Higgs fields [29]. Considering the two-loop Yukawa and QCD corrections to the effective potential, the square mass matrix for the neutral Higgs boson is written as

$$m_{H^0}^2=\begin{pmatrix} \begin{pmatrix} m_a^2 s_\beta^2-\frac{8m_W^2 s_W^2}{e^2}[\lambda_1 c_\beta^2 \\ +\mathbf{Re}(\lambda_5)s_\beta^2+\mathbf{Re}(\lambda_6)s_\beta c_\beta] \end{pmatrix} & \begin{pmatrix} -m_a^2 s_\beta c_\beta-\frac{8m_W^2 s_W^2}{e^2}[(\lambda_3+\lambda_4)s_\beta c_\beta \\ +\mathbf{Re}(\lambda_6)c_\beta^2+\mathbf{Re}(\lambda_7)s_\beta^2] \end{pmatrix} & \begin{pmatrix} \mathbf{Im}(\lambda_5)s_\beta \\ +\mathbf{Im}(\lambda_6)c_\beta \end{pmatrix} \\ \begin{pmatrix} -m_a^2 s_\beta c_\beta-\frac{8m_W^2 s_W^2}{e^2}[(\lambda_3+\lambda_4)s_\beta c_\beta \\ +\mathbf{Re}(\lambda_6)c_\beta^2+\mathbf{Re}(\lambda_7)s_\beta^2 \end{pmatrix} & \begin{pmatrix} m_a^2 c_\beta^2-\frac{8m_W^2 s_W^2}{e^2}[\lambda_2 s_\beta^2 \\ +\mathbf{Re}(\lambda_5)c_\beta^2+\mathbf{Re}(\lambda_7)s_\beta c_\beta \end{pmatrix} & \begin{pmatrix} \mathbf{Im}(\lambda_5)s_\beta \\ +\mathbf{Im}(\lambda_6)c_\beta \end{pmatrix} \\ \mathbf{Im}(\lambda_5)s_\beta+\mathbf{Im}(\lambda_6)c_\beta & \mathbf{Im}(\lambda_5)c_\beta+\mathbf{Im}(\lambda_7)s_\beta & m_a^2 \end{pmatrix} \quad (27)$$

with the square mass m_a^2:

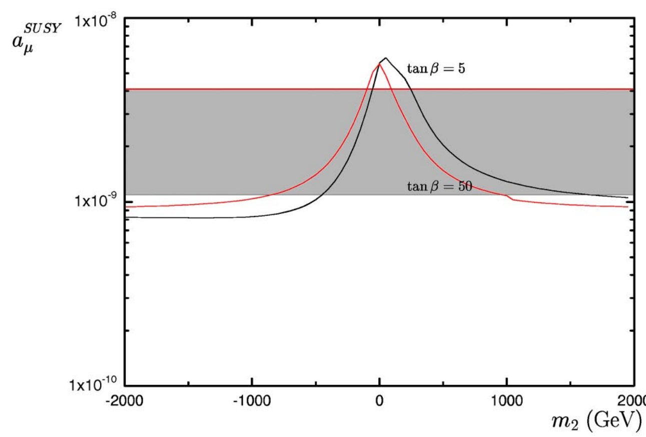
$$m_a^2=m_{H^\pm}^2-\frac{4m_W^2 s_W^2}{e^2}\left[\frac{1}{2}\lambda_4-\mathbf{Re}(\lambda_5)\right]. \quad (28)$$

Here, the parameter $m_{H^\pm}$ represents the mass of the physical charged Higgs-bosons, and the concrete expressions of the other parameters $\lambda_i (i=1,2,\dots,7)$ are presented in Appendix F. As pointed out by the authors of Ref. [29], the experimental lower bound on the mass of the lightest neutral Higgs boson mass can be reduced to 60 GeV in the *CP*-violating MSSM. In the numerical analysis of this work, we take this lower bound as an input. With above specification about the parameter space, we carry out our numerical computations. Without losing generality, we take $m_{H^\pm}=300$ GeV, $m_1=1$ TeV, $m_{\tilde{e}_1}=m_{\tilde{e}_2}=10$ TeV, $m_{\tilde{\mu}_1}=m_{\tilde{\tau}_1}=500$ GeV, $m_{\tilde{\mu}_2}=m_{\tilde{\tau}_2}=700$ GeV, $\theta_{\tilde{e}}=\phi_{\tilde{e}}=0$, $\theta_{\tilde{\mu}}=\theta_{\tilde{\tau}}=\pi/4$, $m_{L12}^2=m_{L13}^2=m_{R12}^2=m_{R13}^2=0$ GeV^2, $A_{12}^l=A_{13}^l=A_{21}^l=A_{31}^l=0$ GeV, $M_{\rm SUSY}=1$ TeV, $m_t=174$ GeV, $m_b=4.5$ GeV, $m_{\tilde{Q}^3}=m_{\tilde{U}^3}=m_{\tilde{D}^3}=500$ GeV, $A_t=A_b=e^{i(\pi/4)}$ TeV all through the paper. Indeed, the electron EDM is a very rigorous constraint to the parameters. Although we set the *CP* phase which is only related to the first generation of sleptons to be zero (in this case the one-loop contribution to the electron EDM is also zero), the nonzero *CP* phases which are related to the second and third generations can also lead to a large EDM of electron via the two-loop Baar-Zee diagrams. For some regions in the parameter space, the two-loop Barr-Zee diagrams would contribute an EDM of an electron which is larger than the experimental upper bound. Thus the electron EDM restricts the relevant parameters via the two-loop diagrams. Considering the constraint of the electron EDM through two-loop Barr-Zee diagrams involving sleptons, we set the *CP* phases $\phi_{\tilde{\mu}}=\phi_{\tilde{\tau}}=\pi/18$.

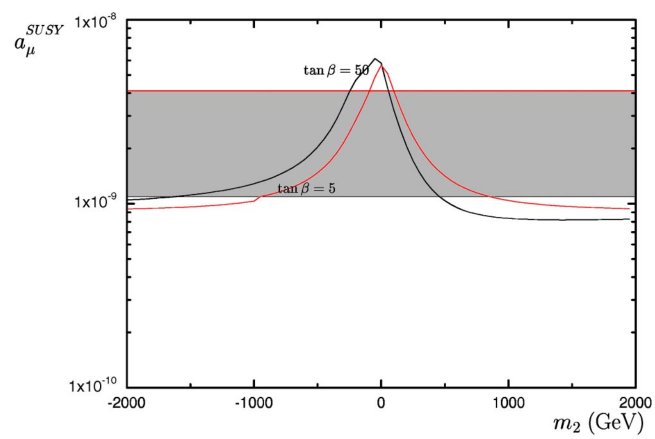
At present, the experimental upper bounds on the branching ratios of the two lepton-flavor-violating processes are

$$B(\tau\to\mu\gamma)<1.1\times10^{-6}, \quad B(\tau\to3\mu)<1.9\times10^{-6}.$$

Upon imposition of the experimental limits on our theoretical predictions for the branching ratios and the muon MDM, we find that the contribution to the muon MDM from slepton generation mixing parameters is much less than the contribution from the flavor conserving parameters through scanning the parameter space. Taking $\phi_{A_{23}^l}=\phi_{A_{32}^l}=\phi_{m_{L_{23}}^2}=\phi_{m_{R_{23}}^2}=\pi/2$, and $|m_{L_{23}}^2|=|m_{R_{23}}^2|=100$ GeV^2, $|A_{23}^l|=|A_{23}^l|=100$ GeV, we plot the muon MDM versus the parameter m_2 in Fig. 5. Within 1σ tolerance, we find that the theoretical prediction coincides with the experimental data rather well if a suitable parameter range is adopted. As for



(a)



(b)

FIG. 5. The muon anomalous magnetic dipole moment versus m_2 in MSSM with (a) $\mu=-200$ GeV, (b) $\mu=200$ GeV and $\tan\beta=5$, 50, the other parameters are set as in the text. The shaded region is allowed by 1σ tolerance from the most recent experimental observation.

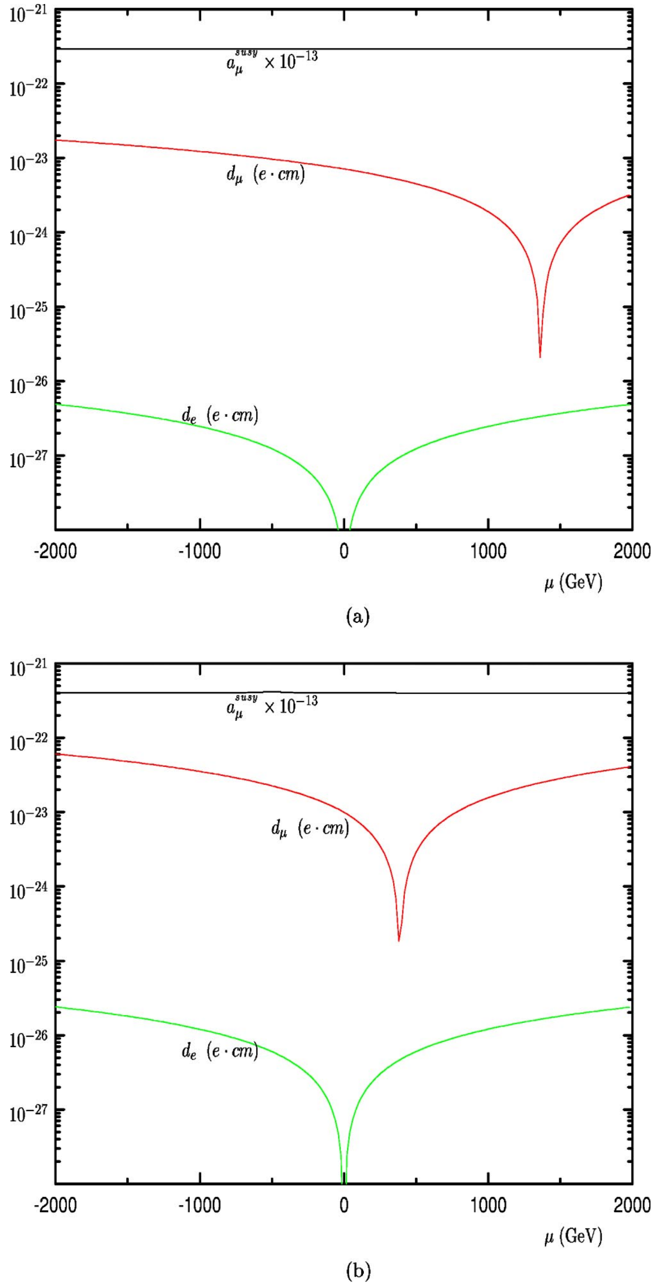


FIG. 6. The muon's MDM and EDM, the electron's EDM versus the parameter μ in MSSM with (a) tan $\beta = 5$, (b) tan $\beta = 50$; the other parameters are the same as in the text.

the calculation on the muon EDM, we must consider the constraint from the electron EDM. With 2σ tolerance, the experimental upper bound on the electron EDM is $|d_e| < 0.5 \times 10^{-26}$ e cm. With $m_2 = 500$ GeV, $\phi_{A^l_{23}} = \phi_{A^l_{32}} = \phi_{m^2_{L_{23}}} = \phi_{m^2_{R_{23}}} = \pi/2$, and $|m^2_{L_{23}}| = |m^2_{R_{23}}| = 100\ \mathrm{GeV}^2$, $|A^l_{23}| = |A^l_{23}| = 100$ GeV, and $\tan\beta = 5{,}50$ respectively, we plot the electron EDM, the muon EDM, and the muon MDM versus the parameter μ in Fig. 6. We find that the muon EDM can reach 10^{-23} $(e$ cm$)$, which is one order above the proposed sensitivity of the coming experiments [41]: 10^{-24} $(e$ cm$)$, while the muon MDM and the electron EDM are also consistent with the experimental data.

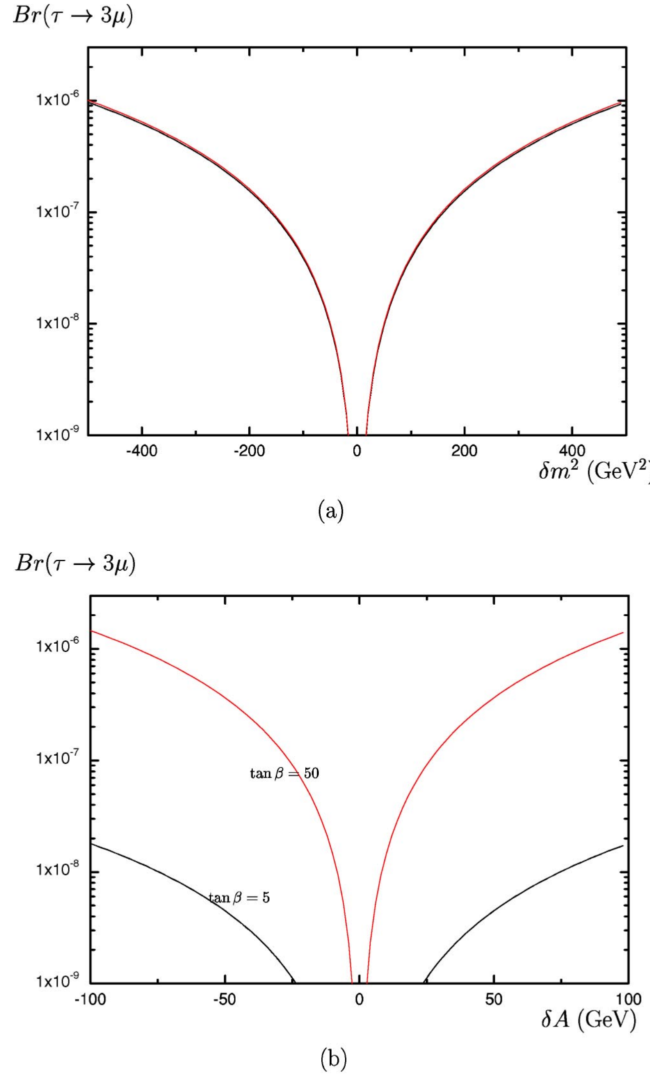


FIG. 7. With $\mu = 200$ GeV and all zero *CP* phases, the branching ratio of $\tau \to 3\mu$ versus (a) $m^2_{L_{23}} = m^2_{R_{23}} = \delta m^2$ or (b) $A^l_{23} = A^l_{23} = \delta A$, the other parameters are taken as in the text.

Now, we analyze the lepton-flavor-violating decays of heavy-lepton τ: $\tau \to \mu\gamma$, $\tau \to 3\mu$. With $\mu = 200$ GeV and setting all *CP* phases to be zero, we plot the branching ratio of $\tau \to 3\mu$ versus $m^2_{L_{23}} = m^2_{R_{23}} = \delta m^2$ [Fig. 7(a)] and $A^l_{23} = A^l_{23} = \delta A$ [Fig. 7(b)]. This result is slightly different from that given in previous literature. We not only consider the contributions from the γ-penguin, Z-penguin and the box diagrams, but also include the Higgs-penguin diagram. We notice that for larger $\tan\beta$ values, the contribution of the Higgs-penguin diagram is not negligible, It is in analogue to its role for the rare leptonic decays of B meson. Numerically, we find that the contribution of the γ-penguin is much less than that of the Z-penguin and box diagrams. For the case of nonzero A^l_{23}, A^l_{32} and large $\tan\beta$, the Higgs-penguin diagram is also non-negligible. When $A^l_{23} = A^l_{32} = 0$, but $m^2_{L_{23}}$, and $m^2_{R_{23}}$ are not zero, the main contribution to the decay width of $\tau \to 3\mu$ comes from the Z-penguin and box diagrams. By this we can explain why the difference of the curves corresponding to $\tan\beta=5$ and 50, is rather small, whereas in Fig. 7(b) the difference is so obvious. As to $\tau\to\mu\gamma$, when we take into account the constraint from $\tau\to 3\mu$, we find that its branching ratio is obviously lower than the experimental bound. If the lepton flavor violation originates from the term $m^2_{L_{ij}}$, $m^2_{R_{ij}}$ [Fig. 7(a)], the difference between $\tan\beta=5$ and $\tan\beta=50$ is very small. If the lepton flavor violation originates from the term A^l_{ij} [Fig. 7(b)], the difference between $\tan\beta=5$ and $\tan\beta=50$ is about two orders. We also investigate the $\tau\to\mu\gamma$ in the model. Imposing the constraint on the parameters from $\tau\to 3\mu$, we find that the branching ratio for $\tau\to\mu\gamma$ is smaller than 10^{-8}, beyond the present experimental detecting ability. Assuming that the flavor violation originates from m^2_{Lij}, m^2_{Rij}, we plot the $B(\tau\to\mu\gamma)$ within the possible parameter range in Fig. 8. The situation is similar to the case of $\tau\to 3\mu$, namely if the flavor violation originates from the term A^l_{ij}, the branching ratio $B(\tau\to\mu\gamma)$ is much less than that if the mixing m^2_{Lij}, m^2_{Rij} induces the flavor violation.

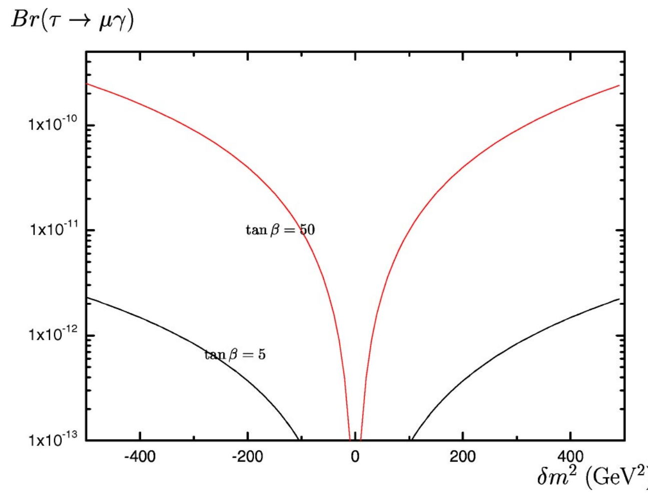


FIG. 8. The $Br(\tau\to\mu\gamma)$ versus $m^2_{L_{23}}=m^2_{R_{23}}=\delta m^2$, the other parameters are taken as in the text.

VI. CONCLUSION

In this work, we investigate the muon anomalous magnetic dipole moment, muon electric dipole moment, and the branching ratios of $\tau\to\mu\gamma$ and $\tau\to 3\mu$ in the framework of the *CP*-violating MSSM with the nonuniversal soft-supersymmetry breaking. For the muon anomalous magnetic dipole moment and electric dipole moment, the main contribution comes from the parameters which conserve the flavors. Process $\tau\to 3\mu$ occurs mainly through the Z-, H^0_i-, A^0-penguins and box diagrams. It can help in understanding why the branching ratio of $\tau\to\mu\gamma$ is so small as long as we consider the constraints from $\tau\to 3\mu$. From the methodology aspect, our method is equivalent to the MIA scheme when all mass eigenvalues of the mass matrix are almost degenerate, or the square mass difference among the eigenvalues is much larger than the corresponding flavor violation parameters. For the case where only several mass eigenvalues are degenerate, the MIA is invalid. Indeed, our method is an improvement over the MIA.

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APPENDIX A: THE MIXING MATRIX FOR THE SFERMION SECTOR

For the general case, the mixing matrices for the sfermions are given in the text. In this appendix, we present the mixing matrix for the case where there is degeneracy in the mass spectra. In fact, we can always perform a transformation on the sfermion mass matrix and turn it into a standard form:

$$\mathcal{Z}_{S_{LR}} m^2_S \mathcal{Z}^{-1}_{S_{LR}} = \begin{pmatrix} m^2_{S_1} & \Delta m^2_{S_{12}} & \Delta m^2_{S_{13}} & 0 & \Delta m^2_{S_{15}} & \Delta m^2_{S_{16}} \\ \Delta m^{2*}_{S_{12}} & m^2_{S_2} & \Delta m^2_{S_{23}} & \Delta m^2_{S_{24}} & 0 & \Delta m^2_{S_{26}} \\ \Delta m^{2*}_{S_{13}} & \Delta m^{2*}_{S_{23}} & m^2_{S_3} & \Delta m^2_{S_{34}} & \Delta m^2_{S_{35}} & 0 \\ 0 & \Delta m^{2*}_{S_{24}} & \Delta m^{2*}_{S_{34}} & m^2_{S_4} & \Delta m^2_{S_{45}} & \Delta m^2_{S_{46}} \\ \Delta m^{2*}_{S_{15}} & 0 & \Delta m^{2*}_{S_{35}} & \Delta m^{2*}_{S_{45}} & m^2_{S_5} & \Delta m^2_{S_{56}} \\ \Delta m^{2*}_{S_{16}} & \Delta m^{2*}_{S_{26}} & 0 & \Delta m^{2*}_{S_{46}} & \Delta m^{2*}_{S_{56}} & m^2_{S_6} \end{pmatrix}, \tag{A1}$$

with $\Delta m^2_{S_{ij}}$ being given in Eq. (10) and the matrix $\mathcal{Z}_{S_{LR}}$ is formulated as

$$\mathcal{Z}_{S_{LR}}=\begin{pmatrix} \cos\theta_{S_1} & 0 & 0 & \sin\theta_{S_1}e^{-i\phi_{S_1}} & 0 & 0 \\ 0 & \cos\theta_{S_2} & 0 & 0 & \sin\theta_{S_2}e^{-i\phi_{S_2}} & 0 \\ 0 & 0 & \cos\theta_{S_3} & 0 & 0 & \sin\theta_{S_3}e^{-i\phi_{S_3}} \\ -\sin\theta_{S_1}e^{i\phi_{S_1}} & 0 & 0 & \cos\theta_{S_1} & 0 & 0 \\ 0 & -\sin\theta_{S_2}e^{i\phi_{S_2}} & 0 & 0 & \cos\theta_{S_2} & 0 \\ 0 & 0 & -\sin\theta_{S_3}e^{i\phi_{S_2}} & 0 & 0 & \cos\theta_{S_3} \end{pmatrix}. \tag{A2}$$

Without losing generality, we assume that the first $n(2\leqslant n\leqslant 5)$ eigenvalues satisfy the condition $m^2_{\tilde{S}_i}-m^2_{\tilde{S}_j}\sim\Delta m^2_{\tilde{S}_{ij}}$ $(i,\ j=1,\dots,n)$, for the scalar lepton sector the mixing matrix is given as

$$\mathcal{Z}_S=\mathcal{Z}_{S_{LR}}\mathcal{Z}_{\mathbf{A}}\mathbf{V} \tag{A3}$$

with

$$\mathbf{V}_{S_{ii}}=1-\sum_{j\neq i}\sum_{\alpha}\frac{|\Delta x_{S_{j\alpha}}\Delta x_{S_{\alpha i}}|^2}{(x_{S_i}-x_{S_j})^2(x_{S_i}-x_{S_\alpha})^2}-\sum_{\alpha}\frac{|\Delta x_{S_{i\alpha}}|^2}{(x_{S_i}-x_{S_\alpha})^2},$$

$$\mathbf{V}_{S_{ji}}=\sum_{\alpha}\frac{\Delta x_{S_{j\alpha}}\Delta x_{S_{\alpha i}}}{(x_{S_i}-x_{S_j})(x_{S_i}-x_{S_\alpha})}-\frac{1}{(x_{S_i}-x_{S_j})^2}\sum_{\alpha,\beta}\frac{|\Delta x_{S_{\alpha i}}|^2\Delta x_{S_{j\beta}}\Delta x_{S_{\beta i}}}{(x_{S_i}-x_{S_\alpha})(x_{S_\beta}-x_{S_i})},$$

$$\mathbf{V}_{S_{\alpha i}}=\frac{\Delta x_{S_{\alpha i}}}{x_{S_i}-x_{S_\alpha}}+\sum_{\beta\neq\alpha}\frac{\Delta x_{S_{\alpha\beta}}\Delta x_{S_{\beta i}}}{(x_{S_i}-x_{S_\alpha})(x_{S_i}-x_{S_\beta})}+\sum_{j}\sum_{\beta}\frac{\Delta x_{S_{\alpha j}}\Delta x_{S_{j\beta}}\Delta x_{S_{\beta i}}}{(x_{S_i}-x_{S_j})(x_{S_i}-x_{S_\alpha})(x_{S_i}-x_{S_\beta})},$$

$$\mathbf{V}_{S_{i\alpha}}=\frac{\Delta x_{S_{i\alpha}}}{x_{S_\alpha}-x_{S_i}}+\sum_{\beta\neq\alpha}\frac{\Delta x_{S_{i\beta}}\Delta x_{S_{\beta\alpha}}}{(x_{S_\alpha}-x_{S_i})(x_{S_\alpha}-x_{S_\beta})},$$

$$\mathbf{V}_{S_{\alpha\alpha}}=1-\sum_{\beta}\frac{|\Delta x_{S_{\alpha\beta}}|^2}{(x_{S_\alpha}-x_{S_\beta})^2},$$

$$\mathbf{V}_{S_{\beta\alpha}}=\frac{\Delta x_{S_{\beta\alpha}}}{(x_{S_\alpha}-x_{S_\beta})}+\sum_{i}\frac{\Delta x_{S_{\beta i}}\Delta x_{S_{i\alpha}}}{(x_{S_\alpha}-x_{S_\beta})(x_{S_\beta}-x_{S_i})}+\sum_{\gamma\neq\alpha,\beta}\frac{\Delta x_{S_{\beta\gamma}}\Delta x_{S_{\gamma\alpha}}}{(x_{S_\alpha}-x_{S_\beta})(x_{S_\alpha}-x_{S_\gamma})}, \tag{A4}$$

and

$$\mathcal{Z}_{\mathbf{A}}=\begin{pmatrix} \mathbf{A}_{n\times n} & \mathbf{0}_{n\times(6-n)} \\ \mathbf{0}_{(6-n)\times n} & \mathbf{1}_{(6-n)\times(6-n)} \end{pmatrix}. \tag{A5}$$

Here, the matrix $\mathbf{A}$ is used to diagonalize the block matrix $m^2_{S_{ij}}$ $(i,\ j=1,\dots,n)$. Expanding Eq. (A3), we have

$$\mathcal{Z}_S^{\dagger\alpha i}=\mathbf{U}_{\tilde{E}_{\alpha i}}\cos\theta_{\tilde{E}_i}+\mathbf{U}_{\tilde{E}_{\alpha(3+i)}}\sin\theta_{\tilde{E}_i}e^{-i\varphi_{\tilde{E}_i}},$$

$$\mathcal{Z}_S^{\dagger\alpha(3+i)}=-\mathbf{U}_{\tilde{E}_{\alpha i}}\sin\theta_{\tilde{E}_i}e^{i\varphi_{\tilde{E}_i}}+\mathbf{U}_{\tilde{E}_{\alpha(3+i)}}\cos\theta_{\tilde{E}_i}$$

$$(\alpha=1,\dots,6;\ \ i=1,2,3), \tag{A6}$$

where

$$\mathbf{U}_{S_{ij}}=\sum_k \mathbf{V}_{S_{ik}}\mathbf{A}_{S_{kj}},$$

$$\mathbf{U}_{S_{i\alpha}}=\mathbf{V}_{S_{i\alpha}},$$

$$\mathbf{U}_{S_{\alpha i}}=\sum_k \mathbf{V}_{S_{\alpha k}}\mathbf{A}_{S_{ki}},$$

$$\mathbf{U}_{S_{\alpha\beta}}=\mathbf{V}_{S_{\alpha\beta}}. \tag{A7}$$

APPENDIX B: EXPRESSIONS OF THE COEFFICIENTS IN EQ. (14) FOR THE LEPTON ANOMALOUS MAGNETIC DIPOLE MOMENT AND THE AMPLITUDE OF DECAY MODE $e^I\to e^J\gamma$

The expressions of the coefficients in Eq. (14) for the vertex $e^I e^J\gamma$ are

$$C_1^-(\mu_{\rm W})=-2e\mathbf{G}_{\nu(a)}^{\{iiI\alpha J\alpha\}}x^2_{\kappa_i^-}\mathcal{B}^{(0)}_{[41]}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})+\frac{1}{c^2_{\rm W}}\mathbf{G}_{L(a)}^{\{iiI\alpha J\alpha\}}\mathcal{F}_1(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),$$

$$C_2^-(\mu_{\rm W})=-\frac{1}{2}\mathbf{G}_{\nu(a)}^{\{iiI\alpha J\alpha\}}\mathcal{F}_1(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})-\frac{1}{4c^2_{\rm W}}\mathbf{G}_{L(a)}^{\{iiI\alpha J\alpha\}}\mathcal{F}_1(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),$$

$$C_3^-(\mu_W) = -\frac{2}{3}\mathbf{G}_{\nu(a)}^{\{iiI\alpha J\alpha\}}\Big[\mathcal{B}_{[21]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha}) - \frac{5}{2}x_{\kappa_i^-}\mathcal{B}_{[31]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha}) - \frac{7}{2}x_{\kappa_i^-}^2\mathcal{B}_{[41]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})\Big] - \frac{1}{6c_W^2}\mathbf{G}_{L(a)}^{\{iiI\alpha J\alpha\}}\mathcal{B}_{[41]}^{(2)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),$$

$$C_4^-(\mu_W) = -\sqrt{\frac{2}{c_\beta}}\mathbf{G}_{\nu(b)}^{\{iiI\alpha J\alpha\}}x_{\kappa_i^-}^{3/2}x_{e^J}^{1/2}\mathcal{B}_{[31]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha}) - \frac{1}{c_W^2}\mathbf{G}_{L(b)}^{\{iiI\alpha J\alpha\}}x_{\tilde{E}^\alpha}x_{\kappa_i^0}^{1/2}\mathcal{B}_{[31]}^{(0)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),$$

$$C_5^-(\mu_W) = -\frac{1}{\sqrt{2}c_\beta}\mathbf{G}_{\nu(b)}^{\{iiI\alpha J\alpha\}}(x_{\kappa_i^-}x_{e^J})^{1/2}\mathcal{B}_{[31]}^{(1)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha}) + \frac{1}{2c_W^2}\mathbf{G}_{L(b)}^{\{iiI\alpha J\alpha\}}x_{\tilde{E}^\alpha}x_{\kappa_i^0}^{1/2}\mathcal{B}_{[31]}^{(0)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),$$

$$C_1^+(\mu_W) = -\frac{1}{c_\beta^2}\mathbf{G}_{\nu(d)}^{\{iiI\alpha J\alpha\}}(x_{e^I}x_{e^J})^{1/2}x_{\kappa_i^-}^2\mathcal{B}_{[41]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha}) + \frac{1}{c_W^2}\mathbf{G}_{L(d)}^{\{iiI\alpha J\alpha\}}\mathcal{F}_1(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),$$

$$C_2^+(\mu_W) = -\frac{1}{4c_\beta^2}\mathbf{G}_{\nu(d)}^{\{iiI\alpha J\alpha\}}(x_{e^I}x_{e^J})^{1/2}\mathcal{F}_1(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha}) - \frac{1}{4c_W^2}\mathbf{G}_{L(d)}^{\{iiI\alpha J\alpha\}}\mathcal{F}_1(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),$$

$$C_3^+(\mu_W) = -\frac{1}{3c_\beta^2}\mathbf{G}_{\nu(d)}^{\{iiI\alpha J\alpha\}}(x_{e^I}x_{e^J})^{1/2}\Big[\mathcal{B}_{[21]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha}) - \frac{5}{2}x_{\kappa_i^-}\mathcal{B}_{[31]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha}) - \frac{7}{2}x_{\kappa_i^-}^2\mathcal{B}_{[41]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})\Big] - \frac{1}{6c_W^2}\mathbf{G}_{L(d)}^{\{iiI\alpha J\alpha\}}\mathcal{B}_{[41]}^{(2)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),$$

$$C_4^+(\mu_W) = -\sqrt{\frac{2}{c_\beta}}\mathbf{G}_{\nu(c)}^{\{iiI\alpha J\alpha\}}x_{\kappa_i^-}^{3/2}x_{e^I}^{1/2}\mathcal{B}_{[31]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha}) - \frac{1}{c_W^2}\mathbf{G}_{L(c)}^{\{iiI\alpha J\alpha\}}x_{\tilde{E}^\alpha}x_{\kappa_i^0}^{1/2}\mathcal{B}_{[31]}^{(0)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),$$

$$C_5^+(\mu_W) = -\frac{1}{\sqrt{2}c_\beta}\mathbf{G}_{\nu(c)}^{\{iiI\alpha J\alpha\}}(x_{\kappa_i^-}x_{e^I})^{1/2}\mathcal{B}_{[31]}^{(1)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha}) + \frac{1}{2c_W^2}\mathbf{G}_{L(c)}^{\{iiI\alpha J\alpha\}}x_{\tilde{E}^\alpha}x_{\kappa_i^0}^{1/2}\mathcal{B}_{[31]}^{(0)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0}). \tag{B1}$$

APPENDIX C: THE COUPLINGS AMONG THE SFERMIONS AND HIGGS FIELDS

In this appendix, we give the couplings among the scalar fermions and Higgs fields that involve the two-loop Barr-Zee-type contributions to the Wilson coefficients:

$$\Gamma^{H^+\tilde{U}_i\tilde{D}_j} = \Bigg\{\frac{e}{\sqrt{2}s_W}V^{IJ*}\Bigg[\Bigg(-2m_W s_\beta c_\beta + \frac{m_{d^J}^2}{m_W}\tan\beta + \frac{m_{u^I}^2}{m_W\tan\beta}\Bigg)\mathcal{Z}_{\tilde{U}}^{Ii}\mathcal{Z}_{\tilde{D}}^{Jj} + \frac{m_{u^I}m_{d^J}}{m_W s_\beta c_\beta}\mathcal{Z}_{\tilde{U}}^{(3+I)i}\mathcal{Z}_{\tilde{D}}^{(3+J)j}\Bigg] + \Bigg(\frac{e\mu^* m_{u^I}}{\sqrt{2}m_W s_W}V^{IJ*} - V^{KJ*}A_{KI}^u c_\beta\Bigg)\mathcal{Z}_{\tilde{U}}^{(3+I)i}\mathcal{Z}_{\tilde{D}}^{Jj} + \Bigg(\frac{e\mu m_{d^J}}{\sqrt{2}m_W s_W}V^{IJ*} + V^{IK*}A_{KJ}^d s_\beta\Bigg)\mathcal{Z}_{\tilde{U}}^{(3+I)i}\mathcal{Z}_{\tilde{D}}^{Jj}\Bigg\},$$

$$\Gamma^{H^+\tilde{\nu}_i\tilde{E}_j} = \mathcal{Z}_\nu^{Ii}\Bigg\{\frac{e}{\sqrt{2}s_W}\Bigg(\frac{\mu m_{e^I}}{m_W}\mathcal{Z}_E^{(3+I)j} - 2m_W s_\beta c_\beta \mathcal{Z}_E^{Ij}\Bigg) + \Bigg(\frac{e m_{e^I}^2}{\sqrt{2}m_W s_W}\tan\beta\,\mathcal{Z}_E^{Ij} + s_\beta A_{IK}^l\mathcal{Z}_E^{(3+K)j}\Bigg)\Bigg\},$$

$$\Gamma^{A\tilde{U}_i\tilde{U}_j} = i\Bigg\{\frac{e m_{u^I}}{2m_W s_W}(\mu\mathcal{Z}_U^{Ii*}\mathcal{Z}_U^{(3+I)j} - \mu^*\mathcal{Z}_U^{Ij}\mathcal{Z}_U^{(3+I)i*}) + \frac{c_\beta}{\sqrt{2}}(A_{IJ}^u\mathcal{Z}_U^{Ij}\mathcal{Z}_U^{(3+J)i*} - A_{IJ}^{u*}\mathcal{Z}_U^{Ii*}\mathcal{Z}_U^{(3+J)j})\Bigg\},$$

$$\Gamma^{A\tilde{D}_i\tilde{D}_j} = i\Bigg\{\frac{s_\beta}{\sqrt{2}}(A_{IJ}^{d*}\mathcal{Z}_D^{Ij}\mathcal{Z}_D^{(3+J)i*} - A_{IJ}^d\mathcal{Z}_D^{Ii*}\mathcal{Z}_D^{(3+J)j}) - \frac{e m_{d^I}}{2m_W s_W}(\mu\mathcal{Z}_D^{Ii*}\mathcal{Z}_D^{(3+I)j} - \mu^*\mathcal{Z}_D^{Ij}\mathcal{Z}_D^{(3+I)i*})\Bigg\},$$

$$\Gamma^{A\tilde{E}_i\tilde{E}_j} = i\Bigg\{\frac{s_\beta}{\sqrt{2}}(A_{IJ}^{l*}\mathcal{Z}_E^{Ij}\mathcal{Z}_E^{(3+J)i*} - A_{IJ}^l\mathcal{Z}_E^{Ii*}\mathcal{Z}_E^{(3+J)j}) - \frac{e m_{e^I}}{2m_W s_W}(\mu\mathcal{Z}_E^{Ii*}\mathcal{Z}_E^{(3+I)j} - \mu^*\mathcal{Z}_E^{Ij}\mathcal{Z}_E^{(3+I)i*})\Bigg\}. \tag{C1}$$

APPENDIX D: EXPRESSIONS OF THE COEFFICIENTS IN EQ. (23) FOR DECAY MODE $e^I\to 3e^J$

The contributions from the gamma penguin diagrams are written as

$$
\begin{aligned}
C_1^\gamma(\mu_W)&=\frac{4s_W^2}{3}\mathbf{G}_{\nu(a)}^{\{iiI\alpha J\alpha\}}\left[\mathcal{B}_{[21]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})-\frac{5}{2}x_{\kappa_i^-}\mathcal{B}_{[31]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})-\frac{7}{2}x_{\kappa_i^-}^2\mathcal{B}_{[41]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})\right]+\frac{s_W^2}{3c_W^2}\mathbf{G}_{L(a)}^{\{iiI\alpha J\alpha\}}\mathcal{B}_{[41]}^{(2)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),\\
C_2^\gamma(\mu_W)&=\frac{2s_W^2}{3c_\beta^2}\mathbf{G}_{\nu(d)}^{\{iiI\alpha J\alpha\}}(x_{e^I}x_{e^J})^{\frac{1}{2}}\left[\mathcal{B}_{[21]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})-\frac{5}{2}x_{\kappa_i^-}\mathcal{B}_{[31]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})-\frac{7}{2}x_{\kappa_i^-}^2\mathcal{B}_{[41]}^{(0)}(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})\right]\\
&\quad+\frac{s_W^2}{3c_W^2}\mathbf{G}_{L(d)}^{\{iiI\alpha J\alpha\}}\mathcal{B}_{[41]}^{(2)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0}),\\
C_4^\gamma(\mu_W)&=-2C_1^\gamma(\mu_W),\\
C_5^\gamma(\mu_W)&=-2C_2^\gamma(\mu_W),\\
C_3^\gamma(\mu_W)&=C_6^\gamma(\mu_W)=0.
\end{aligned}\tag{D1}
$$

The pieces from the Z-penguin diagrams are

$$
\begin{aligned}
C_1^Z(\mu_W)&=\frac{\cos^2 2\theta_W}{c_W^2}\mathbf{G}_{L(a)}^{\{iiI\alpha J\alpha\}}\mathcal{F}_2(x_{\kappa_i^0},x_{\tilde{E}^\alpha})+\frac{\cos 2\theta_W}{4c_W^2}\mathbf{G}_{L(a)}^{\{ijI\alpha J\alpha\}}(\mathcal{Z}_N^{4i*}\mathcal{Z}_N^{4j}-\mathcal{Z}_N^{3i}\mathcal{Z}_N^{3j*})\left[\frac{1}{2}\mathcal{T}_{[111]}^{(1)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})\right.\\
&\quad\left.+(x_{\kappa_i^0}x_{\kappa_j^0})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})\right]+\frac{\cos 2\theta_W}{2c_W^2}\mathbf{G}_{L(a)}^{\{iiI\beta J\alpha\}}(2s_W^2\delta^{\alpha\beta}-\mathcal{Z}_{\tilde{E}}^{K\alpha}\mathcal{Z}_{\tilde{E}}^{K\beta*})\mathcal{T}_{[111]}^{(1)}(x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta},x_{\kappa_i^0})\\
&\quad+\frac{2\cos^2 2\theta_W}{c_W^2}\mathbf{G}_{\nu(a)}^{\{iiI\alpha J\alpha\}}\mathcal{F}_2(X_{\kappa_i^0},x_{\tilde{\nu}^\alpha})+\cos 2\theta_W\mathbf{G}_{\nu(a)}^{\{ijI\alpha J\alpha\}}\left[\frac{1}{2}\mathcal{T}_{[111]}^{(1)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})(2\delta^{ij}\cos 2\theta_W+\mathcal{Z}_-^{1i}\mathcal{Z}_-^{1j*})\right.\\
&\quad\left.-(x_{\kappa_i^-}x_{\kappa_j^-})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})(2\delta^{ij}\cos 2\theta_W+\mathcal{Z}_+^{1i*}\mathcal{Z}_+^{1j})\right]+\cos 2\theta_W\mathbf{G}_{L(a)}^{\{iiI\beta J\alpha\}}\mathcal{T}_{[111]}^{(1)}(x_{\tilde{\nu}^\alpha},x_{\tilde{\nu}^\beta},x_{\kappa_i^-}),\\
C_2^Z(\mu_W)&=\frac{4s_W^4}{c_W^2}\mathbf{G}_{L(d)}^{\{iiI\alpha J\alpha\}}\mathcal{F}_2(x_{\kappa_i^0},x_{\tilde{E}^\alpha})+\frac{s_W^2}{2c_W^2}(\mathcal{Z}_N^{4i*}\mathcal{Z}_N^{4j}-\mathcal{Z}_N^{3i}\mathcal{Z}_N^{3j*})\mathbf{G}_{L(d)}^{\{ijI\alpha J\alpha\}}\left[\frac{1}{2}\mathcal{T}_{[111]}^{(1)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})\right.\\
&\quad\left.-(x_{\kappa_i^0}x_{\kappa_j^0})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})\right]-\frac{s_W^2}{c_W^2}(2s_W^2\delta^{\alpha\beta}-\mathcal{Z}_{\tilde{E}}^{K\alpha}\mathcal{Z}_{\tilde{E}}^{K\beta*})\mathbf{G}_{L(d)}^{\{iiI\beta J\alpha\}}\mathcal{T}_{[111]}^{(1)}(x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta},x_{\kappa_i^0})\\
&\quad+\frac{4s_W^4}{c_\beta^2}(x_{e^I}x_{e^J})^{1/2}\mathbf{G}_{\nu(d)}^{\{iiI\alpha J\alpha\}}\mathcal{F}_2(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})-\frac{s_W^2}{c_\beta^2}(x_{e^I}x_{e^J})^{1/2}\mathbf{G}_{\nu(d)}^{\{ijI\alpha J\alpha\}}\left[\frac{1}{2}\mathcal{T}_{[111]}^{(1)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})(2\delta^{ij}\cos 2\theta_W\right.\\
&\quad\left.+\mathcal{Z}_+^{1i*}\mathcal{Z}_+^{1j})-(x_{\kappa_i^-}x_{\kappa_j^-})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})(2\delta^{ij}\cos 2\theta_W+\mathcal{Z}_-^{1i}\mathcal{Z}_-^{1j*})\right]\\
&\quad-\frac{2s_W^2}{c_\beta^2}\mathbf{G}_{\nu(d)}^{\{iiI\beta J\alpha\}}(x_{e^I}x_{e^J})^{1/2}\mathcal{T}_{[111]}^{(1)}(x_{\tilde{\nu}^\alpha},x_{\tilde{\nu}^\beta},x_{\kappa_i^-}),\\
C_4^Z(\mu_W)&=\frac{4s_W^2}{\cos 2\theta_W}C_1^Z(\mu_W),\\
C_5^Z(\mu_W)&=\frac{\cos 2\theta_W}{s_W^2}C_2^Z(\mu_W),
\end{aligned}
$$

$$C_3^Z(\mu_{\rm W})=C_6^Z(\mu_{\rm W})=0. \tag{D2}$$

The *CP*-even Higgs penguin-diagram contributions are formulated as

$$C_1^{H_k^0}(\mu_{\rm W})=C_2^{H_k^0}(\mu_{\rm W})=0,$$

$$\begin{aligned}
C_3^{H_k^0}(\mu_{\rm W})=&\frac{1}{2c_{\rm W}^2c_\beta^2x_{H_k^0}}\mathcal{F}_2(x_{\kappa_i^0},x_{\tilde{E}^\alpha})(\mathcal{Z}_R^{1k})^2[(x_{e^I}x_{e^J})^{1/2}\mathbf{G}_{L(d)}^{\{iiI\alpha J\alpha\}}+x_{e^J}\mathbf{G}_{L(a)}^{\{iiI\alpha J\alpha\}}]\\
&+\frac{1}{2c_{\rm W}^3c_\beta x_{H_k^0}}x_{e^I}^{1/2}\mathcal{Z}_R^{1k}\mathbf{G}_{L(b)}^{\{ijI\alpha J\alpha\}}[\mathcal{T}_{[111]}^{(1)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})(\mathcal{Z}_R^{1k}\mathcal{Z}_N^{3i*}-\mathcal{Z}_R^{2k}\mathcal{Z}_N^{4i*})(\mathcal{Z}_N^{1j*}s_{\rm W}-\mathcal{Z}_N^{2j*}c_{\rm W})\\
&+(x_{\kappa_i^0}x_{\kappa_j^0})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})(\mathcal{Z}_R^{1k}\mathcal{Z}_N^{3j}-\mathcal{Z}_R^{2k}\mathcal{Z}_N^{4j})(\mathcal{Z}_N^{1i}s_{\rm W}-\mathcal{Z}_N^{2i}c_{\rm W})]\\
&+\frac{s_{\rm W}}{em_{\rm W}c_{\rm W}^2c_\beta x_{H_k^0}}(x_{e^I}x_{\kappa_i^0})^{1/2}\mathcal{Z}_R^{1k}\mathbf{E}\mathbf{G}_{L(b)}^{\{iiI\beta J\alpha\}}\mathcal{T}_{[111]}^{(0)}(x_{\kappa_i^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})+\frac{1}{c_\beta^2x_{H_k^0}}\mathcal{F}_2(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})\\
&\times(\mathcal{Z}_R^{1k})^2\left[(x_{e^I}x_{e^J})^{1/2}\mathbf{G}_{\nu(a)}^{\{iiI\alpha J\alpha\}}+\frac{(x_{e^I}^3x_{e^J})^{1/2}}{2c_\beta^2}\mathbf{G}_{\nu(d)}^{\{iiI\alpha J\alpha\}}\right]-\frac{2}{c_\beta^2x_{H_k^0}}(x_{e^I}x_{e^J})^{1/2}\mathbf{G}_{\nu(b)}^{\{jiI\alpha J\alpha\}}[\mathcal{T}_{[111]}^{(1)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})\\
&\times(\mathcal{Z}_R^{1k}\mathcal{Z}_-^{2j*}\mathcal{Z}_+^{1i*}+\mathcal{Z}_R^{2k}\mathcal{Z}_-^{1j*}\mathcal{Z}_+^{2i*})+(x_{\kappa_i^-}x_{\kappa_j^-})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})(\mathcal{Z}_R^{1k}\mathcal{Z}_-^{2i}\mathcal{Z}_+^{1j}+\mathcal{Z}_R^{2k}\mathcal{Z}_-^{1i}\mathcal{Z}_+^{2j})]\mathcal{Z}_R^{1k}\\
&-\frac{e}{2\sqrt{2}m_{\rm W}s_{\rm W}c_{\rm W}^2c_\beta^2x_{H_k^0}}(x_{e^I}x_{e^J}x_{\kappa_i^-})^{1/2}\mathcal{Z}_R^{1k}B_R^k\mathbf{G}_{\nu(b)}^{\{iiI\alpha J\alpha\}}\mathcal{B}_{[21]}^{(0)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-}),
\end{aligned}$$

$$C_4^{H_k^0}(\mu_{\rm W})=C_3^{H_k^0}(\mu_{\rm W}),$$

$$\begin{aligned}
C_5^{H_k^0}(\mu_{\rm W})=&\frac{1}{2c_{\rm W}^2c_\beta^2x_{H_k^0}}\mathcal{F}_2(x_{\kappa_i^0},x_{\tilde{E}^\alpha})(\mathcal{Z}_R^{1k})^2[x_{e^J}\mathbf{G}_{\nu(d)}^{\{iiI\alpha J\alpha\}}+(x_{e^I}x_{e^J})^{1/2}\mathbf{G}_{\nu(a)}^{\{iiI\alpha J\alpha\}}]\\
&+\frac{2}{2c_{\rm W}^3c_\beta x_{H_k^0}}x_{e^I}^{1/2}\mathcal{Z}_R^{1k}\mathbf{G}_{\nu(c)}^{\{ijI\alpha J\alpha\}}[\mathcal{T}_{[111]}^{(1)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})(\mathcal{Z}_R^{1k}\mathcal{Z}_N^{3j}-\mathcal{Z}_R^{2k}\mathcal{Z}_N^{4j})(\mathcal{Z}_N^{1i}s_{\rm W}-\mathcal{Z}_N^{2i}c_{\rm W})\\
&+(x_{\kappa_i^0}x_{\kappa_j^0})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})(\mathcal{Z}_R^{1k}\mathcal{Z}_N^{3i*}-\mathcal{Z}_R^{2k}\mathcal{Z}_N^{4i*})(\mathcal{Z}_N^{1j*}s_{\rm W}-\mathcal{Z}_N^{2j*}c_{\rm W})]\\
&+\frac{s_{\rm W}}{em_{\rm W}c_{\rm W}^2c_\beta x_{H_k^0}}(x_{e^I}x_{\kappa_i^0})^{1/2}\mathcal{Z}_R^{1k}\mathbf{E}\mathbf{G}_{\nu(c)}^{\{iiI\beta J\alpha\}}\mathcal{T}_{[111]}^{(0)}(x_{\kappa_i^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})\\
&+\frac{1}{c_\beta^2x_{H_k^0}}\mathcal{F}_2(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})(\mathcal{Z}_R^{1k})^2\left[x_{e^I}\mathbf{G}_{\nu(a)}^{\{iiI\alpha J\alpha\}}+\frac{x_{e^I}x_{e^J}}{2c_\beta^2}\mathbf{G}_{\nu(d)}^{\{iiI\alpha J\alpha\}}\right]\\
&-\frac{2}{c_\beta^2x_{H_k^0}}x_{e^I}[\mathcal{T}_{[111]}^{(1)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})(\mathcal{Z}_R^{1k}\mathcal{Z}_-^{2i}\mathcal{Z}_+^{1j}+\mathcal{Z}_R^{2k}\mathcal{Z}_-^{1i}\mathcal{Z}_+^{2j})+(x_{\kappa_i^-}x_{\kappa_j^-})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})\\
&\times(\mathcal{Z}_R^{1k}\mathcal{Z}_-^{2j*}\mathcal{Z}_+^{1i*}+\mathcal{Z}_R^{2k}\mathcal{Z}_-^{1j*}\mathcal{Z}_+^{2i*})]\mathcal{Z}_R^{1k}\mathbf{G}_{\nu(c)}^{\{ijI\alpha J\alpha\}}\\
&-\frac{e}{2\sqrt{2}m_{\rm W}s_{\rm W}c_{\rm W}^2c_\beta^2x_{H_k^0}}x_{e^I}x_{\kappa_i^-}^{1/2}\mathbf{G}_{\nu(c)}^{\{iiI\alpha J\alpha\}}\mathcal{Z}_R^{1k}B_R^k\mathcal{B}_{[21]}^{(0)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-}),
\end{aligned}$$

$$C_6^{H_k^0}(\mu_{\rm W})=C_5^{H_k^0}(\mu_{\rm W}). \tag{D3}$$

The *CP*-odd Higgs contributions are

$$
\begin{aligned}
C_1^{A^0}(\mu_{\rm W}) &= C_2^{A^0}(\mu_{\rm W}) = 0,\\[2ex]
C_3^{A^0}(\mu_{\rm W}) &= \frac{1}{2c_{\rm W}^2 x_{A^0}}\tan^2\beta\mathcal{F}_2(x_{\kappa_i^0},x_{\tilde{E}^\alpha})[(x_{e^I}x_{e^J})^{1/2}\mathbf{G}_{L(d)}^{\{iiI\alpha J\alpha\}}+x_{e^J}\mathbf{G}_{L(a)}^{\{iiI\alpha J\alpha\}}]+\frac{1}{2c_{\rm W}^3 x_{A^0}}x_{e^I}^{1/2}\tan\beta[-\mathcal{T}_{[111]}^{(1)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})\\
&\quad\times(s_\beta\mathcal{Z}_N^{3i*}-c_\beta\mathcal{Z}_N^{4i*})(\mathcal{Z}_N^{1j*}s_{\rm W}-\mathcal{Z}_N^{2j*}c_{\rm W})+(x_{\kappa_i^0}x_{\kappa_j^0})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})(s_\beta\mathcal{Z}_N^{3j}-c_\beta\mathcal{Z}_N^{4j})(\mathcal{Z}_N^{1i}s_{\rm W}\\
&\quad-\mathcal{Z}_N^{2i}c_{\rm W})]\mathbf{G}_{L(b)}^{\{ijI\alpha J\alpha\}}+\frac{s_{\rm W}}{em_{\rm W}c_{\rm W}^2 x_{A^0}}(x_{e^I}x_{\kappa_i^0})^{1/2}\tan\beta\mathbf{F}\mathbf{G}_{L(b)}^{\{iiI\beta J\alpha\}}\mathcal{T}_{[111]}^{(0)}(x_{\kappa_i^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})+\frac{1}{x_{A^0}}\tan^2\beta\mathcal{F}_2(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})\\
&\quad\times\left[x_{e^J}\mathbf{G}_{\nu(a)}^{\{iiI\alpha J\alpha\}}+\frac{x_{e^I}x_{e^J}}{2c_\beta^2}\mathbf{G}_{\nu(d)}^{\{iiI\alpha J\alpha\}}\right]-\frac{2}{s_\beta x_{A^0}}(x_{e^I}x_{e^J})^{1/2}\tan^2\beta[\mathcal{T}_{[111]}^{(1)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})(s_\beta\mathcal{Z}_-^{2j*}\mathcal{Z}_+^{1i*}\\
&\quad+c_\beta\mathcal{Z}_-^{1j*}\mathcal{Z}_+^{2i*})-(x_{\kappa_i^-}x_{\kappa_j^-})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-}x_{\kappa_j^-})(s_\beta\mathcal{Z}_-^{2i}\mathcal{Z}_+^{1j}+c_\beta\mathcal{Z}_-^{1i}\mathcal{Z}_+^{2j})]\mathbf{G}_{L(b)}^{\{jiI\alpha J\alpha\}},\\[2ex]
C_4^{A^0}(\mu_{\rm W}) &= -C_3^{A^0}(\mu_{\rm W}),\\[2ex]
C_5^{A^0}(\mu_{\rm W}) &= -\frac{1}{2c_{\rm W}^2 x_{A^0}}\tan^2\beta\mathcal{F}_2(x_{\kappa_i^0},x_{\tilde{E}^\alpha})[x_{e^J}\mathbf{G}_{L(d)}^{\{iiI\alpha J\alpha\}}+(x_{e^I}x_{e^J})^{1/2}\mathbf{G}_{L(a)}^{\{iiI\alpha J\alpha\}}]\\
&\quad-\frac{1}{2c_{\rm W}^3 x_{A^0}}x_{e^I}^{1/2}\tan\beta\mathbf{G}_{L(c)}^{\{ijI\alpha J\alpha\}}[\mathcal{T}_{[111]}^{(1)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})(s_\beta\mathcal{Z}_N^{3j}-c_\beta\mathcal{Z}_N^{4j})(\mathcal{Z}_N^{1i}s_{\rm W}-\mathcal{Z}_N^{2i}c_{\rm W})\\
&\quad-(x_{\kappa_i^0}x_{\kappa_j^0})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{E}^\alpha},x_{\kappa_i^0},x_{\kappa_j^0})(s_\beta\mathcal{Z}_N^{3i*}-c_\beta\mathcal{Z}_N^{4i*})(\mathcal{Z}_N^{1j*}s_{\rm W}-\mathcal{Z}_N^{2j*}c_{\rm W})]\\
&\quad-\frac{s_{\rm W}}{em_{\rm W}c_{\rm W}^2 x_{A^0}}(x_{e^I}x_{\kappa_i^0})^{1/2}\tan\beta\mathbf{F}\mathbf{G}_{L(c)}^{\{iiI\beta J\alpha\}}\mathcal{T}_{[111]}^{(0)}(x_{\kappa_i^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})-\frac{1}{x_{A^0}}\tan^2\beta\mathcal{F}_2(x_{\kappa_i^-},x_{\tilde{\nu}^\alpha})\left[(x_{e^I}x_{e^J})^{1/2}\mathbf{G}_{\nu(a)}^{\{iiI\alpha J\alpha\}}\right.\\
&\quad\left.+\frac{(x_{e^I}x_{e^J}^3)^{1/2}}{2c_\beta^2}\mathbf{G}_{\nu(d)}^{\{iiI\alpha J\alpha\}}\right]+\frac{1}{s_\beta x_{A^0}}x_{e^I}\tan^2\beta\mathbf{G}_{\nu(c)}^{\{ijI\alpha J\alpha\}}[\mathcal{T}_{[111]}^{(1)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})(s_\beta\mathcal{Z}_-^{2i}\mathcal{Z}_+^{1j}+c_\beta\mathcal{Z}_-^{1i}\mathcal{Z}_+^{2j})\\
&\quad-(x_{\kappa_i^-}x_{\kappa_j^-})^{1/2}\mathcal{T}_{[111]}^{(0)}(x_{\tilde{\nu}^\alpha},x_{\kappa_i^-},x_{\kappa_j^-})(s_\beta\mathcal{Z}_-^{2j*}\mathcal{Z}_+^{1i*}+c_\beta\mathcal{Z}_-^{1j*}\mathcal{Z}_+^{2i*})],\\[2ex]
C_6^{A^0}(\mu_{\rm W}) &= -C_5^{A^0}(\mu_{\rm W}).
\end{aligned}
\tag{D4}
$$

The box diagram contributions to the coefficients are

$$
\begin{aligned}
C_1^{box}(\mu_{\rm W}) &= \frac{1}{4c_{\rm W}^4}\mathbf{G}_{L(a)}^{\{iiI\beta J\alpha\}}\mathbf{G}_{L(a)}^{\{jjJ\alpha J\beta\}}\mathcal{D}_{[111]}^{(1)}(x_{\kappa_i^0},x_{\kappa_j^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})+\mathbf{G}_{\nu(a)}^{\{iiI\beta J\beta\}}\mathbf{G}_{\nu(a)}^{\{jjJ\alpha J\alpha\}}\mathcal{D}_{[111]}^{(1)}(x_{\kappa_i^-},x_{\kappa_j^-},x_{\tilde{\nu}^\alpha},x_{\tilde{\nu}^\beta}),\\[2ex]
C_2^{box}(\mu_{\rm W}) &= \frac{1}{4c_{\rm W}^4}\mathbf{G}_{L(d)}^{\{iiI\beta J\alpha\}}\mathbf{G}_{L(d)}^{\{jjJ\alpha J\beta\}}\mathcal{D}_{[111]}^{(1)}(x_{\kappa_i^0},x_{\kappa_j^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})+\frac{1}{4c_\beta^4}\mathbf{G}_{\nu(d)}^{\{iiI\beta J\beta\}}\mathbf{G}_{\nu(a)}^{\{jjJ\alpha J\alpha\}}x_{e^I}^{1/2}x_{e^J}^{1/3}\mathcal{D}_{[111]}^{(1)}(x_{\kappa_i^-},x_{\kappa_j^-},x_{\tilde{\nu}^\alpha},x_{\tilde{\nu}^\beta}),\\[2ex]
C_3^{box}(\mu_{\rm W}) &= \frac{1}{c_{\rm W}^4}\mathbf{G}_{L(b)}^{\{iiI\beta J\alpha\}}\mathbf{G}_{L(b)}^{\{jjJ\alpha J\beta\}}(x_{\kappa_i^0}x_{\kappa_j^0})^{1/2}\mathcal{D}_{[111]}^{(0)}(x_{\kappa_i^0},x_{\kappa_j^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})\\
&\quad+\frac{2}{c_\beta^2}\mathbf{G}_{\nu(b)}^{\{iiI\beta J\beta\}}\mathbf{G}_{\nu(b)}^{\{jjJ\alpha J\alpha\}}x_{e^J}(x_{\kappa_i^-}x_{\kappa_j^-})^{1/2}\mathcal{D}_{[111]}^{(0)}(x_{\kappa_i^-},x_{\kappa_j^-},x_{\tilde{\nu}^\alpha},x_{\tilde{\nu}^\beta}),
\end{aligned}
$$

$$C_4^{box}(\mu_W)=\frac{1}{c_W^4}\left\{\mathbf{G}_{L^{(b)}}^{\{iiI\beta J\alpha\}}\mathbf{G}_{L^{(c)}}^{\{iiJ\alpha J\beta\}}(x_{\kappa_i^0}x_{\kappa_j^0})^{1/2}\mathcal{D}_{[111]}^{(0)}(x_{\kappa_i^0},x_{\kappa_j^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})-\frac{1}{2}\mathbf{G}_{L^{(a)}}^{\{iiI\beta J\alpha\}}\mathbf{G}_{L^{(d)}}^{\{jjJ\alpha J\beta\}}\mathcal{D}_{[111]}^{(1)}(x_{\kappa_i^0},x_{\kappa_j^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})\right\}$$
$$+\frac{2}{c_\beta^2}\left\{\mathbf{G}_{\nu^{(a)}}^{\{jiI\beta J\beta\}}\mathbf{G}_{\nu^{(d)}}^{\{jiJ\alpha J\alpha\}}x_{e^J}(x_{\kappa_i^-}x_{\kappa_j^-})^{1/2}\mathcal{D}_{[111]}^{(0)}(x_{\kappa_i^-},x_{\kappa_j^-},x_{\tilde{\nu}^\alpha},x_{\tilde{\nu}^\beta})\right.$$
$$\left.-\mathbf{G}_{\nu^{(a)}}^{\{iiI\beta J\beta\}}\mathbf{G}_{\nu^{(d)}}^{\{jjJ\alpha J\alpha\}}\frac{x_{e^J}}{2}\mathcal{D}_{[111]}^{(1)}(x_{\kappa_i^-},x_{\kappa_j^-},x_{\tilde{\nu}^\alpha},x_{\tilde{\nu}^\beta})\right\},$$

$$C_5^{box}(\mu_W)=\frac{1}{c_W^4}\left\{\mathbf{G}_{L^{(a)}}^{\{ijJ\alpha J\alpha\}}\mathbf{G}_{L^{(d)}}^{\{jiI\beta J\beta\}}(x_{\kappa_i^0}x_{\kappa_j^0})^{1/2}\mathcal{D}_{[111]}^{(0)}(x_{\kappa_i^0},x_{\kappa_j^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})\right.$$
$$\left.-\frac{1}{2}\mathbf{G}_{L^{(d)}}^{\{iiI\beta J\alpha\}}\mathbf{G}_{L^{(a)}}^{\{jjJ\alpha J\beta\}}\mathcal{D}_{[111]}^{(1)}(x_{\kappa_i^0},x_{\kappa_j^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})\right\}$$
$$+\frac{2}{c_\beta^2}\left\{\mathbf{G}_{\nu^{(a)}}^{\{ijI\beta J\beta\}}\mathbf{G}_{\nu^{(d)}}^{\{ijJ\alpha J\alpha\}}(x_{e^I}x_{e^J}x_{\kappa_i^-}x_{\kappa_j^-})^{1/2}\mathcal{D}_{[111]}^{(0)}(x_{\kappa_i^-},x_{\kappa_j^-},x_{\tilde{\nu}^\alpha},x_{\tilde{\nu}^\beta})\right.$$
$$\left.-\mathbf{G}_{\nu^{(a)}}^{\{jjI\beta J\beta\}}\mathbf{G}_{\nu^{(d)}}^{\{iiJ\alpha J\alpha\}}\frac{(x_{e^I}x_{e^J})^{1/2}}{2}\mathcal{D}_{[111]}^{(1)}(x_{\kappa_i^-},x_{\kappa_j^-},x_{\tilde{\nu}^\alpha},x_{\tilde{\nu}^\beta})\right\},$$

$$C_6^{box}(\mu_W)=\frac{1}{c_W^4}\mathbf{G}_{L^{(c)}}^{\{iiI\beta J\alpha\}}\mathbf{G}_{L^{(c)}}^{\{jjJ\alpha J\beta\}}(x_{\kappa_i^0}x_{\kappa_j^0})^{1/2}\mathcal{D}_{[111]}^{(0)}(x_{\kappa_i^0},x_{\kappa_j^0},x_{\tilde{E}^\alpha},x_{\tilde{E}^\beta})$$
$$+\frac{2}{c_\beta^2}\mathbf{G}_{\nu^{(a)}}^{\{ijI\beta J\beta\}}\mathbf{G}_{\nu^{(d)}}^{\{jiJ\alpha J\alpha\}}(x_{e^I}x_{e^J}x_{\kappa_i^-}x_{\kappa_j^-})^{1/2}\mathcal{D}_{[111]}^{(0)}(x_{\kappa_i^-},x_{\kappa_j^-},x_{\tilde{\nu}^\alpha},x_{\tilde{\nu}^\beta}). \quad \text{(D5)}$$

APPENDIX E: THE LOOP INTEGRAL FUNCTIONS

In this appendix, we present the expressions of various loop integral functions which appear in the text.

$$f_1(x_1,x_2)=\begin{cases}\dfrac{x_1x_2^2(\ln x_1-\ln x_2)}{(x_2-x_1)^4}-\dfrac{x_1^2-5x_1x_2-2x_2^2}{6(x_2-x_1)^3} & (\text{for } x_1\neq x_2),\\[2ex] \dfrac{1}{12x_1} & (\text{for } x_1=x_2);\end{cases}$$

$$f_2(x_1,x_2)=\begin{cases}\dfrac{(2x_1x_2-x_2^2)\ln x_2-x_1^2\ln x_1}{2(x_2-x_1)^2}-\dfrac{x_1}{2(x_2-x_1)}-\dfrac{x_1}{2(x_2-x_1)} & (\text{for } x_1\neq x_2),\\[2ex] -\dfrac{1}{4}-\dfrac{1}{2}\ln x_1 & (\text{for } x_1=x_2);\end{cases}$$

$$B_{[21]}^{(0)}(x_1,x_2)=\begin{cases}\dfrac{x_2(\ln x_1-\ln x_2)}{(x_2-x_1)^2}+\dfrac{1}{x_2-x_1} & (\text{for } x_1\neq x_2),\\[2ex] -\dfrac{1}{2x_1} & (\text{for } x_1=x_2);\end{cases}$$

$$B_{[31]}^{(0)}(x_1,x_2)=\begin{cases}\dfrac{x_2(\ln x_1-\ln x_2)}{(x_2-x_1)^3}+\dfrac{x_1+x_2}{2x_1(x_2-x_1)^2} & (\text{for } x_1\neq x_2),\\[2ex] \dfrac{1}{6x_1^2} & (\text{for } x_1=x_2);\end{cases}$$

$$B^{(0)}_{[41]}(x_1,x_2)=\begin{cases} \dfrac{x_2(\ln x_1-\ln x_2)}{(x_2-x_1)^4}+\dfrac{2x_1^2+5x_1x_2-x_2^2}{6x_1^2(x_2-x_1)^3} & (\text{for } x_1\neq x_2),\\ -\dfrac{1}{12x_1^3} & (\text{for } x_1=x_2); \end{cases}$$

$$B^{(1)}_{[31]}(x_1,x_2)=B^{(0)}_{[21]}(x_1,x_2)+x_1B^{(0)}_{[31]}(x_1,x_2),$$

$$B^{(2)}_{[41]}(x_1,x_2)=B^{(0)}_{[21]}(x_1,x_2)+2x_1B^{(0)}_{[31]}(x_1,x_2)+x_1^2B^{(0)}_{[41]}(x_1,x_2),$$

$$\mathcal{T}^{(0)}_{[111]}(x_1,x_2,x_3)=\begin{cases} -\dfrac{x_1\ln x_1}{(x_2-x_1)(x_3-x_1)}-\dfrac{x_2\ln x_2}{(x_1-x_2)(x_3-x_2)}-\dfrac{x_3\ln x_3}{(x_1-x_3)(x_2-x_3)} & (\text{for } x_1\neq x_2\neq x_3),\\ \dfrac{x_3(\ln x_1-\ln x_3)}{(x_3-x_1)^2}+\dfrac{1}{x_3-x_1} & (\text{for } x_1=x_2\neq x_3),\\ -\dfrac{1}{2x_1} & (\text{for } x_1=x_2=x_3); \end{cases}$$

$$\mathcal{T}^{(1)}_{[111]}(x_1,x_2,x_3)=\begin{cases} -\dfrac{x_1^2\ln x_1}{(x_2-x_1)(x_3-x_1)}-\dfrac{x_2^2\ln x_2}{(x_1-x_2)(x_3-x_2)}-\dfrac{x_3^2\ln x_3}{(x_1-x_3)(x_2-x_3)} & (\text{for } x_1\neq x_2\neq x_3),\\ \dfrac{(2x_1x_3-x_1^2)\ln x_1-x_3^2\ln x_3}{(x_3-x_1)^2}+\dfrac{x_1}{x_3-x_1} & (\text{for } x_1=x_2\neq x_3),\\ -\dfrac{3}{2}-\ln x_1 & (\text{for } x_1=x_2=x_3); \end{cases}$$

$$\mathcal{D}^{(0)}_{[1111]}(x_1,x_2,x_3,x_4)=\begin{cases} -\dfrac{x_1\ln x_1}{(x_2-x_1)(x_3-x_1)(x_4-x_1)}-\dfrac{x_2\ln x_2}{(x_1-x_2)(x_3-x_2)(x_4-x_2)} & \\ -\dfrac{x_3\ln x_3}{(x_1-x_3)(x_2-x_3)(x_4-x_3)}-\dfrac{x_4\ln x_4}{(x_1-x_4)(x_2-x_4)(x_3-x_4)} & (\text{for } x_1\neq x_2\neq x_3\neq x_4),\\ \dfrac{x_3\ln x_3}{(x_1-x_3)^2(x_4-x_3)}+\dfrac{x_4\ln x_4}{(x_1-x_4)^2(x_3-x_4)} & \\ -\dfrac{(x_3x_4-x_1^2)\ln x_1}{(x_3-x_1)^2(x_4-x_1)^2}-\dfrac{1}{(x_3-x_1)(x_4-x_1)} & (\text{for } x_1=x_2\neq x_3\neq x_4),\\ -\dfrac{2x_1\ln x_1-2x_3\ln x_3}{(x_3-x_1)^3}-\dfrac{2+\ln x_1+\ln x_3}{(x_3-x_1)^2} & (\text{for } x_1=x_2\neq x_3=x_4);\\ \dfrac{x_4(\ln x_1-\ln x_4)}{(x_4-x_1)^3}+\dfrac{x_4}{2x_1(x_4-x_1)^2} & (\text{for } x_1=x_2=x_3\neq x_4),\\ \dfrac{1}{6x_1^2} & (\text{for } x_1=x_2=x_3=x_4); \end{cases}$$

$$\mathcal{D}^{(1)}_{[1111]}(x_1,x_2,x_3,x_4)=\begin{cases} -\frac{x_1^2\ln x_1}{(x_2-x_1)(x_3-x_1)(x_4-x_1)}-\frac{x_2^2\ln x_2}{(x_1-x_2)(x_3-x_2)(x_4-x_2)} \\ -\frac{x_3^2\ln x_3}{(x_1-x_3)(x_2-x_3)(x_4-x_3)}-\frac{x_4^2\ln x_4}{(x_1-x_4)(x_2-x_4)(x_3-x_4)} \quad (\text{for} \quad x_1\neq x_2\neq x_3\neq x_4), \\ \frac{x_3^2\ln x_3}{(x_1-x_3)^2(x_4-x_3)}+\frac{x_4^2\ln x_4}{(x_1-x_4)^2(x_3-x_4)} \\ -\frac{(2x_1x_3x_4-x_1^2x_3-x_1^2x_4)\ln x_1}{(x_3-x_1)^2(x_4-x_1)^2}-\frac{x_1}{(x_3-x_1)(x_4-x_1)} \quad (\text{for} \ x_1=x_2\neq x_3\neq x_4), \\ -\frac{2x_1^2\ln x_1-2x_3^2\ln x_3}{(x_3-x_1)^3}-\frac{(x_1+x_3+2x_1\ln x_1+2x_3\ln x_3)}{(x_3-x_1)^2} \quad (\text{for} \ x_1=x_2\neq x_3=x_4); \\ \frac{x_4^2(\ln x_1-\ln x_4)}{(x_4-x_1)^3}+\frac{3x_4-x_1}{2(x_4-x_1)^2} \quad (\text{for} \ x_1=x_2=x_3\neq x_4), \\ -\frac{1}{3x_1} \quad (\text{for} \ x_1=x_2=x_3=x_4). \end{cases} \tag{E1}$$

APPENDIX F: THE EXPRESSION FOR THE λ_i PARAMETERS

The λ parameters that involve the neutral Higgs mass mixing are given as

$$\lambda_1=-\frac{g_1^2+g_2^2}{8}\left(1-\frac{3}{8\pi^2}h_b^2t\right)-\frac{3}{16\pi^2}h_b^4\left[t+\frac{1}{2}X_b+\frac{1}{16\pi^2}\left(\frac{3}{2}h_b^2+\frac{1}{2}h_t^2-8g_s^2\right)(X_bt+t^2)\right]$$
$$+\frac{3}{192\pi^2}h_t^4\frac{|\mu|^4}{M_{\rm SUSY}^4}\left[1+\frac{1}{16\pi^2}(9h_t^2-5h_b^2-16g_s^2)t\right],$$

$$\lambda_2=-\frac{g_1^2+g_2^2}{8}\left(1-\frac{3}{8\pi^2}h_t^2t\right)-\frac{3}{16\pi^2}h_t^4\left[t+\frac{1}{2}X_t+\frac{1}{16\pi^2}\left(\frac{3}{2}h_t^2+\frac{1}{2}h_b^2-8g_s^2\right)(X_tt+t^2)\right]$$
$$+\frac{3}{192\pi^2}h_b^4\frac{|\mu|^4}{M_{\rm SUSY}^4}\left[1+\frac{1}{16\pi^2}(9h_b^2-5h_t^2-16g_s^2)t\right],$$

$$\lambda_3=-\frac{g_2^2-g_1^2}{4}\left[1-\frac{3}{16\pi^2}(h_t^2+h_b^2)t\right]-\frac{3}{8\pi^2}h_t^2h_b^2\left[t+\frac{1}{2}X_{tb}+\frac{1}{16\pi^2}(h_t^2+h_b^2-8g_s^2)(X_{tb}t+t^2)\right]-\frac{3}{96\pi^2}h_t^4\left(\frac{3|\mu|^2}{M_{\rm SUSY}^2}\right.$$
$$\left.-\frac{|\mu|^2|A_t|^2}{M_{\rm SUSY}^4}\right)\left[1+\frac{1}{16\pi^2}(6h_t^2-2h_b^2-16g_s^2)t\right]-\frac{3}{96\pi^2}h_b^4\left(\frac{3|\mu|^2}{M_{\rm SUSY}^2}-\frac{|\mu|^2|A_b|^2}{M_{\rm SUSY}^4}\right)\left[1+\frac{1}{16\pi^2}(6h_b^2-2h_t^2-16g_s^2)t\right],$$

$$\lambda_4=\frac{g_2^2}{2}\left[1-\frac{3}{16\pi^2}(h_t^2+h_b^2)t\right]+\frac{3}{8\pi^2}h_t^2h_b^2\left[t+\frac{1}{2}X_{tb}+\frac{1}{16\pi^2}(h_t^2+h_b^2-8g_s^2)(X_{tb}t+t^2)\right]-\frac{3}{96\pi^2}h_t^4\left(\frac{3|\mu|^2}{M_{\rm SUSY}^2}-\frac{|\mu|^2|A_t|^2}{M_{\rm SUSY}^4}\right)$$
$$\times\left[1+\frac{1}{16\pi^2}(6h_t^2-2h_b^2-16g_s^2)t\right]-\frac{3}{96\pi^2}h_b^4\left(\frac{3|\mu|^2}{M_{\rm SUSY}^2}-\frac{|\mu|^2|A_b|^2}{M_{\rm SUSY}^4}\right)\left[1+\frac{1}{16\pi^2}(6h_b^2-2h_t^2-16g_s^2)t\right],$$

$$\lambda_5=\frac{3}{192\pi^2}h_t^4\frac{\mu^2A_t^2}{M_{\rm SUSY}^4}\left[1-\frac{1}{16\pi^2}(2h_b^2-6h_t^2+16g_s^2)t\right]+\frac{3}{192\pi^2}h_b^4\frac{\mu^2A_b^2}{M_{\rm SUSY}^4}\left[1-\frac{1}{16\pi^2}(2h_t^2-6h_b^2+16g_s^2)t\right],$$

$$\lambda_6 = -\frac{3}{96\pi^2} h_t^4 \frac{|\mu|^2 \mu A_t}{M_{\rm SUSY}^4} \left[1 - \frac{1}{16\pi^2} \left(\frac{7}{2} h_b^2 - \frac{15}{2} h_t^2 + 16 g_s^2 \right) t \right] + \frac{3}{96\pi^2} h_b^4 \frac{\mu}{M_{\rm SUSY}} \left(\frac{6A_b}{M_{\rm SUSY}} - \frac{|A_b|^2 A_b}{M_{\rm SUSY}^3} \right)$$
$$\times \left[1 - \frac{1}{16\pi^2} \left(\frac{1}{2} h_t^2 - \frac{9}{2} h_b^2 + 16 g_s^2 \right) t \right],$$

$$\lambda_7 = -\frac{3}{96\pi^2} h_b^4 \frac{|\mu|^2 \mu A_b}{M_{\rm SUSY}^4} \left[1 - \frac{1}{16\pi^2} \left(\frac{7}{2} h_t^2 - \frac{15}{2} h_b^2 + 16 g_s^2 \right) t \right] + \frac{3}{96\pi^2} h_t^4 \frac{\mu}{M_{\rm SUSY}} \left(\frac{6A_t}{M_{\rm SUSY}} - \frac{|A_t|^2 A_t}{M_{\rm SUSY}^3} \right)$$
$$\times \left[1 - \frac{1}{16\pi^2} \left(\frac{1}{2} h_b^2 - \frac{9}{2} h_t^2 + 16 g_s^2 \right) t \right], \tag{F1}$$

where $t = \ln(M_{\rm SUSY}^2/\overline{m}_t^2)$, $g_1 = e/c_{\rm W}$, $g_2 = e/s_{\rm W}$ and

$$h_t = \frac{\sqrt{2} m_t(\overline{m}_t)}{v \sin\beta}, \quad h_b = \frac{\sqrt{2} m_b(\overline{m}_t)}{v \cos\beta},$$

$$X_t = \frac{2|A_t|^2}{M_{\rm SUSY}^2} \left(1 - \frac{|A_t|^2}{12 M_{\rm SUSY}^2} \right),$$

$$X_b = \frac{2|A_b|^2}{M_{\rm SUSY}^2} \left(1 - \frac{|A_b|^2}{12 M_{\rm SUSY}^2} \right),$$

$$X_{tb} = \frac{|A_t|^2 + |A_b|^2 + 2\,{\rm Re}(A_b^* A_t)}{2 M_{\rm SUSY}^2} - \frac{|\mu|^2}{M_{\rm SUSY}^2} - \frac{||\mu|^2 - A_b^* A_t|^2}{6 M_{\rm SUSY}^4}. \tag{F2}$$

In Eq. (F2), $\overline{m}_t$ is the top-quark pole mass, which is related to the on-shell running mass m_t through

$$m_t(\overline{m}_t) = \frac{\overline{m}_t}{1 + \frac{4}{3\pi} \alpha_s(\overline{m}_t)}. \tag{F3}$$

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