

Discriminating graviton exchange effects from other new physics scenarios in e^+e^- collisions

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We study the possibility of uniquely identifying the effects of graviton exchange from other new physics in high energy e^+e^- annihilation into fermion pairs. For this purpose, we use as the basic observable a specific asymmetry among integrated differential distributions that seems particularly suitable for direct testing for such gravitational effects in the data analysis.

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I. INTRODUCTION

All types of new physics (NP) scenarios are determined by nonstandard dynamics involving new building blocks and forces mediated by exchange corresponding to heavy states with mass scales Λ much greater than M_W . The unambiguous confirmation of such dynamics would require the experimental discovery of the envisaged new heavy objects and the measurement of their coupling constants to ordinary quarks and leptons. While there is substantial belief that the supersymmetric partners of the standard model (SM) particles should be directly produced, and identified at future proton-proton and electron-positron high energy colliders such as the CERN Large Hadron Collider (LHC) and the Linear Collider (LC), in other cases the current experimental limits on the new, heavy particles are so high, of the order of several (or tens of) TeV, that one cannot expect them to be directly produced at the energies foreseen for these machines. In this situation, the new interactions can manifest themselves only by indirect, virtual, effects represented by deviations of the measured observables from the SM numerical predictions. The problem, then, is to identify from the data analysis the possible new interactions, because different NP scenarios can in principle cause similar measurable deviations, and for this purpose suitable observables must be defined.

At “low” energies (compared to the above-mentioned large mass scales) the physical effects of the new interactions are conveniently accounted for, in reactions involving the familiar quarks and leptons, by effective *contact-interaction* (CI) Lagrangians that provide the expansion of the relevant transition amplitudes to leading order in the small ratio \sqrt{s}/Λ (\sqrt{s} being the c.m. energy).

Familiar classes of contact interactions are represented by composite models of quarks and leptons [1,2]; exchanges of very heavy Z' with a few TeV mass [3,4] and of scalar and vector heavy leptoquarks [5]; in the supersymmetry (SUSY)

context, R -parity breaking interactions mediated by sneutrino exchange [6,7]; bilepton boson exchanges [8]; anomalous gauge boson couplings (AGC) [9]; virtual Kaluza-Klein (KK) graviton exchange in the context of gravity propagating in large extra dimensions, exchange of gauge boson KK towers, or string excitations, etc. [10–15]. Of course, this list is not exhaustive, because other kinds of contact interaction may well exist.

In this note, we briefly discuss the deviations induced by contact interactions in the electron-positron annihilation into fermion pairs at the planned Linear Collider energies [16,17]. In particular, we propose a simple observable that can be used to unambiguously identify graviton KK tower exchange effects in the data, relying on its spin-2 character and “filtering” out contributions of other NP interactions.

If deviations from the SM predictions were effectively measured, the identification of the NP source could be attempted by Monte Carlo best fits of the observed effects, and this would apply also to graviton exchange [14]. Alternatively, moments of the differential cross section folded with Legendre polynomial weights appear to be a promising technique to pin down NP effects in the case of electron-positron reactions induced at the SM level by s -channel exchanges [18]. Here, we shall consider a suitably defined combination of integrated cross sections, the so-called “center-edge” asymmetry A_{CE} , that allows one to disentangle the graviton exchange in a very simple, and efficient, way. Specifically, in Sec. II we present the required kinematical details and discuss the properties of A_{CE} , in Sec. III we discuss beam polarization, in Sec. IV we evaluate the sensitivity of this observable to the characteristic mass parameter of the graviton KK tower exchange, in Sec. V we find the corresponding identification reaches and discuss an application to sneutrino exchange, differentiating it from KK graviton exchange, and finally Sec. VI is devoted to some comments and concluding remarks.

II. THE CENTER-EDGE ASYMMETRY A_{CE}

We consider the process (with $f \neq e, t$)

$$e^+ + e^- \rightarrow f + \bar{f}, \quad (1)$$

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TABLE I. Parametrization of the $\Delta_{\alpha\beta}$ functions in different models ($\alpha, \beta = L, R$).

Model	$\Delta_{\alpha\beta}$
Composite fermions [2]	$\pm \frac{s}{\alpha_{\text{e.m.}}} \frac{1}{\Lambda_{\alpha\beta}^2}$
Extra gauge boson Z' [3,4]	$g'_\alpha{}^e g'^f{}_\beta \chi_{Z'}$
AGC ($f = \ell$) [9]	$\Delta_{LL} = s \left(\frac{\tilde{f}_{DW}}{2s_W^2} + \frac{2\tilde{f}_{DB}}{c_W^2} \right),$
TeV-scale extra dimension [14,15]	$\frac{\Delta_{RR}}{2} = \Delta_{LR} = \Delta_{RL} = s \frac{4\tilde{f}_{DB}}{\pi^2 c_W^2}$ $(Q_e Q_f + g_\alpha^e g_\beta^f) \frac{\pi^2}{3M_C^2}$
ADD model [10,12]	$\Delta_{LL} = \Delta_{RR} = f_G(1-2z), \Delta_{LR} = \Delta_{RL} = -f_G(1+2z)$

and, neglecting all fermion masses with respect to \sqrt{s} , we can write the differential angular distribution for unpolarized e^+e^- beams in terms of s -channel γ and Z exchanges plus any contact-interaction terms in the following form [19]:

$$\frac{d\sigma}{dz} = \frac{1}{4} \left(\frac{d\sigma_{LL}}{dz} + \frac{d\sigma_{RR}}{dz} + \frac{d\sigma_{LR}}{dz} + \frac{d\sigma_{RL}}{dz} \right). \quad (2)$$

Here, $z \equiv \cos \theta$, with θ the angle between the incoming electron and the outgoing fermion in the c.m. frame, and $d\sigma_{\alpha\beta}/d\cos \theta$ ($\alpha, \beta = L, R$) are the helicity cross sections given by

$$\frac{d\sigma_{\alpha\beta}}{dz} = N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} |\mathcal{M}_{\alpha\beta}|^2 (1 \pm z)^2, \quad (3)$$

where the two signs \pm correspond to the LL , RR and LR , RL helicity configurations, respectively, and $N_C \approx 3(1 + \alpha_s/\pi)$ represents the number of colors of the final state, including the first-order QCD correction. The helicity amplitudes $\mathcal{M}_{\alpha\beta}$ can be written as

$$\mathcal{M}_{\alpha\beta} = \mathcal{M}_{\alpha\beta}^{\text{SM}} + \Delta_{\alpha\beta} = Q_e Q_f + g_\alpha^e g_\beta^f \chi_Z + \Delta_{\alpha\beta}, \quad (4)$$

where $\chi_Z = s/(s - M_Z^2 + iM_Z\Gamma_Z) \approx s/(s - M_Z^2)$ represents the Z propagator; $g_L^f = (I_{3L}^f - Q_f s_W^2)/s_W c_W$ and $g_R^f = -Q_f s_W/c_W$ are the SM left- and right-handed fermion couplings of the Z with $s_W^2 = 1 - c_W^2 \equiv \sin^2 \theta_W$; and Q_e and Q_f are the fermion electric charges. The $\Delta_{\alpha\beta}$ functions represent the contact interaction contributions coming from TeV-scale physics.

The structure of the differential cross section (2)–(4) is particularly interesting in that it is equally valid for a wide variety of new physics models listed in Table I. Note that only graviton exchange induces a modified angular dependence in the differential cross section via the z dependence of $\Delta_{\alpha\beta}$.

We define the generalized center-edge asymmetry A_{CE} as [20]

$$A_{\text{CE}} = \frac{\sigma_{\text{CE}}}{\sigma}, \quad (5)$$

in terms of the difference between the central and edge parts of the cross section

$$\sigma_{\text{CE}} = \left[\int_{-z^*}^{z^*} - \left(\int_{-1}^{-z^*} + \int_{z^*}^1 \right) \right] \frac{d\sigma}{dz} dz, \quad (6)$$

and the total cross section

$$\sigma = \int_{-1}^1 \frac{d\sigma}{dz} dz, \quad (7)$$

and $0 < z^* < 1$.¹

In Table I $\Lambda_{\alpha\beta}$ are compositeness scales; $\chi_{Z'}$ is the Z' propagator defined according to χ_Z ; \tilde{f}_{DW} and \tilde{f}_{DB} are related to f_{DW} and f_{DB} of Ref. [9] by $\tilde{f} = f/m_t^2$ [f_{DW} and f_{DB} parametrize new physics effects associated with the SU(2) and hypercharge currents, respectively]; M_C is the compactification scale; finally, $f_G = \lambda s^2/(4\pi\alpha_{\text{e.m.}}M_H^4)$ parametrizes the strength associated with massive graviton exchange with M_H the cutoff scale in the KK graviton tower sum. Note that, compared with, e.g., the composite fermion case, the KK graviton effect is suppressed by the (larger) power $(\sqrt{s}/M_H)^4$, so that a lower reach on M_H can be expected in comparison to the constraints obtainable, at the same c.m. energy, on Λ 's. The effect of the extra dimensional model [14] is s independent, and the sign of $\Delta_{\alpha\beta}$ is fixed.

First, let us consider graviton exchange effects. For definiteness we consider the Arkani-Hamed–Dimopoulos–Dvali (ADD) model [10]. From Eqs. (2)–(7) and Table I one can derive the asymmetry A_{CE} for the process (1) including graviton tower exchange:

$$A_{\text{CE}} = \frac{\sigma_{\text{CE}}^{\text{SM}} + \sigma_{\text{CE}}^{\text{INT}} + \sigma_{\text{CE}}^{\text{NP}}}{\sigma^{\text{SM}} + \sigma^{\text{INT}} + \sigma^{\text{NP}}}, \quad (8)$$

¹The center-edge asymmetry A_{CE} for W -pair production and fixed $z^* = 0.5$ was introduced in [21].

where ‘‘SM,’’ ‘‘INT,’’ and ‘‘NP’’ refer to ‘‘standard model,’’ ‘‘interference,’’ and (pure) ‘‘new physics’’ contributions. Explicitly, we have

$$\begin{aligned}\sigma_{\text{CE}}^{\text{SM}} &= N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} \frac{1}{4} [(\mathcal{M}_{\text{LL}}^{\text{SM}})^2 + (\mathcal{M}_{\text{RR}}^{\text{SM}})^2 + (\mathcal{M}_{\text{LR}}^{\text{SM}})^2 \\ &\quad + (\mathcal{M}_{\text{RL}}^{\text{SM}})^2] \frac{4}{3} [z^*(z^{*2} + 3) - 2], \\ \sigma_{\text{CE}}^{\text{INT}} &= N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} 2f_G \frac{1}{4} [\mathcal{M}_{\text{LL}}^{\text{SM}} + \mathcal{M}_{\text{RR}}^{\text{SM}} - \mathcal{M}_{\text{LR}}^{\text{SM}} \\ &\quad - \mathcal{M}_{\text{RL}}^{\text{SM}}] 4z^*(1 - z^{*2}), \\ \sigma_{\text{CE}}^{\text{NP}} &= N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} f_G^2 \frac{4}{5} [4z^{*5} + 5z^*(1 - z^{*2}) - 2],\end{aligned}\quad (9)$$

with

$$\begin{aligned}\sigma^{\text{SM}} &= N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} \frac{1}{4} [(\mathcal{M}_{\text{LL}}^{\text{SM}})^2 + (\mathcal{M}_{\text{RR}}^{\text{SM}})^2 + (\mathcal{M}_{\text{LR}}^{\text{SM}})^2 \\ &\quad + (\mathcal{M}_{\text{RL}}^{\text{SM}})^2] \frac{8}{3}, \\ \sigma^{\text{INT}} &= 0, \quad \sigma^{\text{NP}} = N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} f_G^2 \frac{8}{5}.\end{aligned}\quad (10)$$

Note that, at $z^* = 0$ and 1, $\sigma_{\text{CE}} = \mp \sigma$, respectively.

In the case of the SM the center-edge asymmetry $A_{\text{CE}}^{\text{SM}}$ can be obtained from Eqs. (8)–(10) taking $f_G = 0$:

$$A_{\text{CE}}^{\text{SM}} = \frac{\sigma_{\text{CE}}^{\text{SM}}}{\sigma^{\text{SM}}} = \frac{1}{2} z^*(z^{*2} + 3) - 1. \quad (11)$$

It is interesting to note that in Eq. (11) the helicity amplitudes in the numerator and denominator cancel and only a ratio of kinematical factors remains in the limit of neglecting external fermion masses. In addition, $A_{\text{CE}}^{\text{SM}}$ is independent of energy and of the flavor of the final-state fermions. It contains *only* the kinematical variable z^* . Figure 1 shows $A_{\text{CE}}^{\text{SM}}$ as a function of z^* . From Eq. (11) one can determine the value of z^* where $A_{\text{CE}}^{\text{SM}}$ vanishes [22]:

$$z_0^* = (\sqrt{2} + 1)^{1/3} - (\sqrt{2} - 1)^{1/3} = 0.596, \quad (12)$$

corresponding to $\theta = 53.4^\circ$ (see the solid curve in Fig. 1).

Graviton exchange in the ADD model affects A_{CE} , inducing a deviation from the SM prediction:

$$\Delta A_{\text{CE}} = A_{\text{CE}} - A_{\text{CE}}^{\text{SM}}. \quad (13)$$

For $(s/M_H^2)^2 \ll 1$, it will be $\sigma_{\text{CE}}^{\text{INT}}$ which will produce the largest deviation from the expectations of the SM, since this term is of order $(\sqrt{s}/M_H)^4$, whereas the pure NP contribution proportional to f_G^2 in Eqs. (9) and (10) is of the much higher order $(\sqrt{s}/M_H)^8$. Taking into account only SM-NP interference terms, one derives

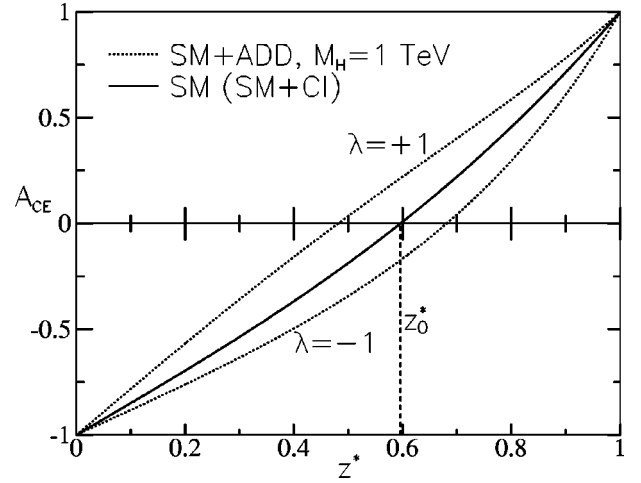


FIG. 1. Tree-diagram result for A_{CE} for the process $e^+e^- \rightarrow l^+l^-$ ($l = \mu, \tau$) as a function of z^* in the SM and in the ADD model with $M_H = 1$ TeV and $\lambda = \pm 1$.

$$\begin{aligned}\Delta A_{\text{CE}} &\approx f_G \frac{\mathcal{M}_{\text{LL}}^{\text{SM}} + \mathcal{M}_{\text{RR}}^{\text{SM}} - \mathcal{M}_{\text{LR}}^{\text{SM}} - \mathcal{M}_{\text{RL}}^{\text{SM}}}{[(\mathcal{M}_{\text{LL}}^{\text{SM}})^2 + (\mathcal{M}_{\text{RR}}^{\text{SM}})^2 + (\mathcal{M}_{\text{LR}}^{\text{SM}})^2 + (\mathcal{M}_{\text{RL}}^{\text{SM}})^2]} \\ &\quad \times 3z^*(1 - z^{*2}).\end{aligned}\quad (14)$$

For the lepton pair production process in the ADD model, the corresponding A_{CE} is shown in Fig. 1 for $M_H = 1$ TeV and $\lambda = \pm 1$. As one can see from Fig. 1, $\Delta A_{\text{CE}} = 0$ for $z^* = 0$ and 1. Clearly, in contrast to $A_{\text{CE}}^{\text{SM}}$, the ΔA_{CE} of Eq. (14) depends on the flavor of the final-state fermion f .

To illustrate the effect of graviton exchange on the center-edge asymmetry, we show in Fig. 2 the z^* distributions of the deviation ΔA_{CE} , taking as examples the values of M_H indicated in the caption. The deviation ΔA_{CE} [including also the pure NP term in addition to the simple result of Eq. (14)] is compared to the expected statistical uncertainties δA_{CE} represented by the vertical bars and given by

$$\delta A_{\text{CE}} = \sqrt{\frac{1 - (A_{\text{CE}}^{\text{SM}})^2}{\epsilon_f \mathcal{L}_{\text{int}} \sigma^{\text{SM}}}}. \quad (15)$$

Here, \mathcal{L}_{int} is the integrated luminosity, and ϵ_f the efficiency for reconstruction of $f\bar{f}$ pairs. We will assume that the efficiencies of identifying the final-state fermions are rather high: 100% for $l = \mu, \tau$, 80% for $f = b$, and 60% for $f = c$. Figure 2 qualitatively indicates that, for the chosen values of the c.m. energy \sqrt{s} and \mathcal{L}_{int} , the reach on M_H will be of the order of 2.5 TeV.

Now, let us consider the conventional contact-interaction-like effects parametrized by the z -independent $\Delta_{\alpha\beta}$ summarized in Table I. Application of Eq. (5) to compositelike contact interactions is straightforward; the result can be written as

$$A_{\text{CE}}^{\text{SM+CI}} = \frac{\sigma_{\text{CE}}^{\text{SM+CI}}}{\sigma^{\text{SM+CI}}}, \quad (16)$$

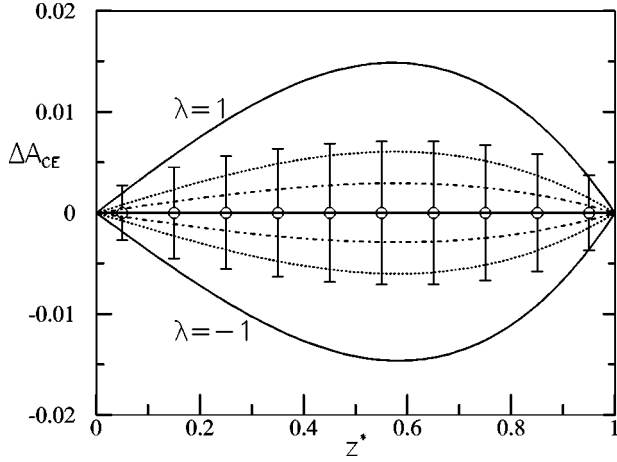


FIG. 2. The deviation of A_{CE} [cf. Eq. (13)] from the SM (or SM+CI) expectation (at the tree level) as a function of z^* for the process $e^+e^- \rightarrow l^+l^-$ for $M_H=2$ (solid), 2.5 (dotted), and 3 TeV (dash-dotted), $\lambda = \pm 1$, and $\sqrt{s}=0.5$ TeV. The expected statistical uncertainties at $\mathcal{L}_{\text{int}}=50 \text{ fb}^{-1}$ are shown as error bars.

where

$$\sigma_{CE}^{\text{SM+CI}} = N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} \frac{1}{4} [(\mathcal{M}_{LL})^2 + (\mathcal{M}_{RR})^2 + (\mathcal{M}_{LR})^2 + (\mathcal{M}_{RL})^2] \frac{4}{3} [z^*(z^{*2} + 3) - 2] \quad (17)$$

and

$$\sigma^{\text{SM+CI}} = N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} \frac{1}{4} [(\mathcal{M}_{LL})^2 + (\mathcal{M}_{RR})^2 + (\mathcal{M}_{LR})^2 + (\mathcal{M}_{RL})^2] \frac{8}{3}. \quad (18)$$

From Eqs. (16)–(18), one has

$$A_{CE}^{\text{SM+CI}} = \frac{1}{2} z^*(z^{*2} + 3) - 1. \quad (19)$$

This result is *identical* to the A_{CE}^{SM} defined by Eq. (11). In other words, A_{CE} has the form (19) in the SM and will remain so even if contact-interaction-like effects are present. Thus, conventional contact-interaction effects, being described by current-current interactions, yield the same center-edge asymmetry as the standard model. The reason is simply that both these interactions are described by vector currents, as opposed to the tensor couplings of gravity. The deviation of A_{CE} from the SM (and SM+CI) prediction is clearly a signal of the spin-2 particle exchange. Thus, it is clear that a nonzero value of ΔA_{CE} can provide a clean signature for graviton or more generally spin-2 exchange in the process $e^+e^- \rightarrow \bar{f}f$.

III. POLARIZED BEAMS

Let us now consider the case of longitudinally polarized beams, with P and \bar{P} the degrees of polarization of the electron and positron beams, respectively. The polarized differential cross section can then be written as

$$\frac{d\sigma}{dz} = \frac{D}{4} \left[(1 - P_{\text{eff}}) \left(\frac{d\sigma_{LL}}{dz} + \frac{d\sigma_{LR}}{dz} \right) + (1 + P_{\text{eff}}) \left(\frac{d\sigma_{RR}}{dz} + \frac{d\sigma_{RL}}{dz} \right) \right], \quad (20)$$

where $D=1-P\bar{P}$ and $P_{\text{eff}}=(P-\bar{P})/(1-P\bar{P})$ is the effective polarization [23]. For example, $P_{\text{eff}}=\pm 0.95$ and $D \approx 1.5$ for $P=\pm 0.8$ and $\bar{P}=\mp 0.6$.

In addition, in the case of a reduced kinematical region, with cuts around the beam pipe, $|z| \leq z_{\text{cut}}$ ($0 < z_{\text{cut}} < 1$), one can define the generalized center-edge asymmetry A_{CE} as above, with Eqs. (6) and (7) replaced by

$$\sigma_{CE} = \left[\int_{-z^*}^{z^*} - \left(\int_{-z_{\text{cut}}}^{-z^*} + \int_{z^*}^{z_{\text{cut}}} \right) \right] \frac{d\sigma}{dz} dz \quad (21)$$

and

$$\sigma = \int_{-z_{\text{cut}}}^{z_{\text{cut}}} \frac{d\sigma}{dz} dz, \quad (22)$$

with $0 < z^* < z_{\text{cut}}$.

Allowing for angular cuts, as discussed above, the asymmetry A_{CE} including graviton tower exchange for polarized beams can be expressed as given by Eq. (8), with

$$\begin{aligned} \sigma_{CE}^{\text{SM}}(z^*, z_{\text{cut}}) &= N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} \frac{D}{4} \{ (1 - P_{\text{eff}}) [(\mathcal{M}_{LL}^{\text{SM}})^2 + (\mathcal{M}_{LR}^{\text{SM}})^2] \\ &\quad + (1 + P_{\text{eff}}) [(\mathcal{M}_{RR}^{\text{SM}})^2 + (\mathcal{M}_{RL}^{\text{SM}})^2] \} \\ &\quad \times F^{\text{SM}}(z^*, z_{\text{cut}}), \\ \sigma_{CE}^{\text{INT}}(z^*, z_{\text{cut}}) &= N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} 2f_G \frac{D}{4} [(1 - P_{\text{eff}})(\mathcal{M}_{LL}^{\text{SM}} - \mathcal{M}_{LR}^{\text{SM}}) \\ &\quad + (1 + P_{\text{eff}})(\mathcal{M}_{RR}^{\text{SM}} - \mathcal{M}_{RL}^{\text{SM}})] F^{\text{INT}}(z^*, z_{\text{cut}}), \\ \sigma_{CE}^{\text{NP}}(z^*, z_{\text{cut}}) &= N_C \frac{\pi \alpha_{\text{e.m.}}^2}{2s} f_G^2 D F^{\text{NP}}(z^*, z_{\text{cut}}). \end{aligned} \quad (23)$$

Here, the dependences on the parameter z^* and on the angular cut z_{cut} are given by

$$F^{\text{SM}}(z^*, z_{\text{cut}}) = \frac{2}{3} [2z^*(z^{*2} + 3) - z_{\text{cut}}(z_{\text{cut}}^2 + 3)],$$

$$F^{\text{INT}}(z^*, z_{\text{cut}}) = 2[2z^*(1-z^{*2}) - z_{\text{cut}}(1-z_{\text{cut}}^2)],$$

$$F^{\text{NP}}(z^*, z_{\text{cut}}) = \frac{2}{5}[8z^{*5} + 10z^*(1-z^{*2}) - 4z_{\text{cut}}^5 - 5z_{\text{cut}}(1-z_{\text{cut}}^2)]. \quad (24)$$

The total cross sections in the denominator of Eq. (8) can be derived from Eqs. (23) and (24):

$$\sigma^{\text{SM}}(z_{\text{cut}}) = \sigma_{\text{CE}}^{\text{SM}}(z^* = z_{\text{cut}}), \quad \sigma^{\text{INT}}(z_{\text{cut}}) = \sigma_{\text{CE}}^{\text{INT}}(z^* = z_{\text{cut}}),$$

$$\sigma^{\text{NP}}(z_{\text{cut}}) = \sigma_{\text{CE}}^{\text{NP}}(z^* = z_{\text{cut}}). \quad (25)$$

From Eqs. (5), (17), (18), and (20)–(24) some immediate conclusions can be drawn. First, it is clear that in the case of longitudinally polarized beams and the chosen cut around the beam pipe, $|z| \leq z_{\text{cut}}$, the asymmetry A_{CE} within the SM and in any new physics scenario with Z' exchanges, and also in the four-fermion contact-interaction scenario, is given by

$$A_{\text{CE}}^{\text{SM}} = A_{\text{CE}}^{\text{SM+CI}} = 2 \frac{z^*(z^{*2} + 3)}{z_{\text{cut}}(z_{\text{cut}}^2 + 3)} - 1. \quad (26)$$

Secondly, the center-edge asymmetry (26) is identical to that for unpolarized beams [see Eqs. (11) and (19)] for $z_{\text{cut}} = 1$. Third, the asymmetry (26) is independent of the energy \sqrt{s} , the flavor of the final-state fermion f , and the SM and NP parameters. Moreover, there is a value z_0^* for which $A_{\text{CE}}^{\text{SM}}$ vanishes. One obtains $z_0^* = a - a^{-1}$, where $a = [(p + \sqrt{p^2 + 4})/2]^{1/3}$ and $p = (3z_{\text{cut}} + z_{\text{cut}}^3)/2$. These zeros of $A_{\text{CE}}^{\text{SM}}$ are important, since the graviton exchange will there give the only contribution. Finally, $\sigma^{\text{INT}}(z_{\text{cut}}) = 0$ at $z_{\text{cut}} = 1$, and in this limit for the angular cut the contribution to the total polarized cross section from the graviton exchange term would be of order f_G^2 , i.e., of order $(s/M_H^2)^4$, and hence negligible.

IV. SENSITIVITY

In order to get some feeling for the sensitivities of the processes $e^+e^- \rightarrow \mu^+\mu^-$, $b\bar{b}$, and $c\bar{c}$ to graviton exchange effects, let us consider the statistical significance defined as

$$S = \frac{|\Delta A_{\text{CE}}|}{\delta A_{\text{CE}}}, \quad (27)$$

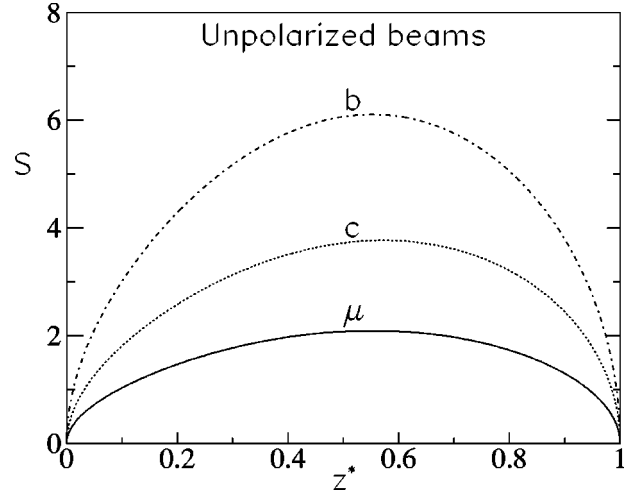


FIG. 3. Statistical significance S for unpolarized beams, $M_H = 2$ TeV, $\mathcal{L}_{\text{int}} = 50 \text{ fb}^{-1}$, $\lambda = 1$, and $\sqrt{s} = 500$ GeV. Different fermionic final states are considered: $\mu^+\mu^-$, $c\bar{c}$, and $b\bar{b}$. Here, no cut is imposed, $z_{\text{cut}} = 1$.

where ΔA_{CE} is defined by Eq. (13). Here, δA_{CE} is the expected statistical uncertainty defined by Eq. (15). Figure 3 shows the statistical significance S as a function of z^* for unpolarized beams for the process (1) at $M_H = 2$ TeV, $\mathcal{L}_{\text{int}} = 50 \text{ fb}^{-1}$, $\lambda = 1$, and $\sqrt{s} = 500$ GeV. In the following, we shall put $\lambda = 1$; our numerical results will turn out not to depend appreciably on the choice of the sign.

From Eqs. (27), (14), (15), and (11) one can derive the statistical significance for unpolarized initial beams, limited to the interference contribution (and for $z_{\text{cut}} = 1$):

$$S_f = f_G S_0 \frac{|(\mathcal{M}_{\text{LL}}^{\text{SM}} - \mathcal{M}_{\text{LR}}^{\text{SM}}) + (\mathcal{M}_{\text{RR}}^{\text{SM}} - \mathcal{M}_{\text{RL}}^{\text{SM}})|}{\sqrt{[(\mathcal{M}_{\text{LL}}^{\text{SM}})^2 + (\mathcal{M}_{\text{LR}}^{\text{SM}})^2] + [(\mathcal{M}_{\text{RR}}^{\text{SM}})^2 + (\mathcal{M}_{\text{RL}}^{\text{SM}})^2]}}, \quad (28)$$

$$S_0 = \sqrt{\frac{3\pi\alpha_{\text{e.m.}}^2 N_C \epsilon_f \mathcal{L}_{\text{int}}}{s} \frac{z^*(1-z^*)}{(z^{*2} + 3)(z^{*2} + z^* + 4)}} \times 2(1 + z^*). \quad (29)$$

The extension of Eq. (28) for polarized beams is straightforward:

$$S_f = f_G S_0 \sqrt{D} \frac{|(1 - P_{\text{eff}})(\mathcal{M}_{\text{LL}}^{\text{SM}} - \mathcal{M}_{\text{LR}}^{\text{SM}}) + (1 + P_{\text{eff}})(\mathcal{M}_{\text{RR}}^{\text{SM}} - \mathcal{M}_{\text{RL}}^{\text{SM}})|}{\sqrt{(1 - P_{\text{eff}})[(\mathcal{M}_{\text{LL}}^{\text{SM}})^2 + (\mathcal{M}_{\text{LR}}^{\text{SM}})^2] + (1 + P_{\text{eff}})[(\mathcal{M}_{\text{RR}}^{\text{SM}})^2 + (\mathcal{M}_{\text{RL}}^{\text{SM}})^2]}}. \quad (30)$$

Note that the maximum of \mathcal{S} occurs at $z_{\max}^* \approx 1/\sqrt{3} = 0.577$ ($\theta \approx 54.7^\circ$), which is very close to z_0^* where $A_{\text{CE}}^{\text{SM}} = 0$.² The dependence of \mathcal{S} in the vicinity of z_{\max}^* is quite smooth, as implied by the behavior of ΔA_{CE} and δA_{CE} shown in Fig. 2. In other words, variation of z^* around z_{\max}^* changes the sensitivity very little. Therefore, no stringent requirements on angular resolution are needed.

The statistical significance is expressed in terms of the SM amplitudes $\mathcal{M}_{\alpha\beta}^{\text{SM}} = Q_e Q_f [1 + (g_{\alpha\beta}^e g_{\alpha\beta}^f / Q_e Q_f) \chi_Z]$. The factor $Q_e Q_f$ is here extracted since it cancels in the ratios of Eqs. (28) and (30).

In order to clarify the dominant role of $q\bar{q}$ -pair production over $\mu^+ \mu^-$ production in searching for graviton exchange effects, as shown in Fig. 3, and also to reveal the role of polarization in this analysis, it is instructive to estimate the SM amplitudes in the limit where

$$z_W^2 = 0.25, \quad M_Z^2 \ll s \ll M_H^2. \quad (31)$$

With these approximations, the relations between the SM couplings can be written as

$$\frac{g_L^e}{Q_e} = \frac{1}{2} \frac{g_L^c}{Q_c} = \frac{1}{5} \frac{g_L^b}{Q_b} = \frac{1}{\sqrt{3}}, \quad \frac{g_R^e}{Q_e} = \frac{g_R^c}{Q_c} = \frac{g_R^b}{Q_b} = -\frac{1}{\sqrt{3}}, \quad (32)$$

and the SM amplitudes are related as ($\chi_Z \approx 1$)

$$\begin{aligned} \frac{1}{2} \mathcal{M}_{\text{LL}}^{e\mu} &= \frac{1}{2} \mathcal{M}_{\text{RR}}^{e\mu} = \mathcal{M}_{\text{LR}}^{e\mu} = \mathcal{M}_{\text{RL}}^{e\mu} = Q_e Q_\mu \frac{2}{3}, \\ \frac{1}{5} \mathcal{M}_{\text{LL}}^{ec} &= \frac{1}{4} \mathcal{M}_{\text{RR}}^{ec} = \frac{1}{2} \mathcal{M}_{\text{LR}}^{ec} = \mathcal{M}_{\text{RL}}^{ec} = Q_e Q_c \frac{1}{3}, \\ \frac{1}{4} \mathcal{M}_{\text{LL}}^{eb} &= \frac{1}{2} \mathcal{M}_{\text{RR}}^{eb} = \mathcal{M}_{\text{LR}}^{eb} = -\mathcal{M}_{\text{RL}}^{eb} = Q_e Q_b \frac{2}{3}. \end{aligned} \quad (33)$$

For unpolarized $e^+ e^-$ beams, we have

$$\mathcal{S}_\mu : \mathcal{S}_c : \mathcal{S}_b = 1 : \sqrt{\epsilon_c \frac{135}{23}} : \sqrt{\epsilon_b \frac{135}{11}} \approx 1 : 1.9 : 3.1. \quad (34)$$

Comparison of the ratios (34) obtained in the adopted approximation (31) with those presented in Fig. 3 and derived from the full expression of Eq. (27) shows that this approximation is quite reasonable. With fully polarized beams, $e_L^+ e_R^-$ ($P_{\text{eff}} = 1$) and $e_R^+ e_L^-$ ($P_{\text{eff}} = -1$), we find

$$\begin{aligned} \mathcal{S}_\mu(P_{\text{eff}}=0) : \mathcal{S}_\mu(P_{\text{eff}}=1) : \mathcal{S}_\mu(P_{\text{eff}}=-1) &= 1 : \sqrt{2} : \sqrt{2} \\ &= 1 : 1.4 : 1.4, \end{aligned} \quad (35)$$

²Strictly, $1/\sqrt{3}$ would be the value of z^* for which ΔA_{CE} in Eq. (14) is maximal. This represents to a very good approximation the location z_{\max}^* of the maximum of the statistical significance (27).

$$\begin{aligned} \mathcal{S}_c(P_{\text{eff}}=0) : \mathcal{S}_c(P_{\text{eff}}=1) : \mathcal{S}_c(P_{\text{eff}}=-1) \\ = 1 : \sqrt{\frac{46}{17}} : \sqrt{\frac{46}{29}} = 1 : 1.6 : 1.3, \end{aligned} \quad (36)$$

$$\begin{aligned} \mathcal{S}_b(P_{\text{eff}}=0) : \mathcal{S}_b(P_{\text{eff}}=1) : \mathcal{S}_b(P_{\text{eff}}=-1) \\ = 1 : \sqrt{\frac{22}{5}} : \sqrt{\frac{22}{17}} = 1 : 2.1 : 1.1. \end{aligned} \quad (37)$$

Note that the $b\bar{b}$ channel becomes more sensitive to graviton exchange effects for both unpolarized and polarized beams and would carry large statistical weight in the analysis. The advantage of polarization is lessened by the fact that the signal behaves as $(\sqrt{s}/M_H)^4$ compared to, e.g., the case of four-fermion contact interactions. This high power reduces the considerable gain in sensitivity to a less dramatic 20% gain in reach on M_H for the $b\bar{b}$ case [see Eq. (37)].

The sign of the SM-NP interference term in the A_{CE} asymmetry for the process $e^+ e^- \rightarrow c\bar{c}$ is opposite to those of $e^+ e^- \rightarrow \mu^+ \mu^-$ and $e^+ e^- \rightarrow b\bar{b}$. This sign correlation might yield additional information to identify graviton exchange effects.

V. IDENTIFICATION REACH

To assess a realistic reach on the mass scale M_H we can consider a χ^2 function made of the deviation of the asymmetry A_{CE} from its SM value. For a fixed integrated luminosity this can be done using the statistical errors as well as the systematic errors. We find that, to a very large extent, the systematic errors associated with the uncertainties expected on the luminosity measurements cancel out, and the same is true for the systematic errors induced by the uncertainty on beam polarizations. Accordingly, the errors on A_{CE} are largely dominated by statistics. In this estimation we assume the values $\delta\mathcal{L}_{\text{int}}/\mathcal{L}_{\text{int}} = \delta P/P = \delta\bar{P}/\bar{P} = 0.5\%$. We take the beam polarization to be 80% and 60% for electrons and positrons, respectively, and employ a 10° angular cut around the beam pipe, i.e., $z_{\text{cut}} = 0.98$. Since most of the error is statistical in origin, we expect the bound on M_H to scale as $\sim (\mathcal{L}_{\text{int}} s^3)^{1/8}$. The dependence of the reach on M_H on z_{cut} varying in a reasonable range close to 1 is, for the chosen values of energy, luminosity, and polarization, quite smooth. For example, in the range $z_{\text{cut}} = 0.96 - 1$, the bound on M_H is found to vary by only a few percent.

In the present analysis we also take into account the radiative corrections. Among the complete $\mathcal{O}(\alpha)$ corrections to the process (1), the numerically largest QED corrections are the effects of initial-state radiation, which in general are of major importance for new physics searches. The initial-state corrections have been calculated in the flux function approach (see, e.g., Ref. [4]). The structure of the corrected differential cross section in terms of $z_{\text{c.m.}} \equiv \cos \theta$ (where θ now refers to the final-state $f\bar{f}$ c.m. frame) is [24]

$$\frac{d\sigma}{dz_{\text{c.m.}}} \propto (1 + z_{\text{c.m.}}^2) \sigma_s + 2z_{\text{c.m.}} \sigma_a. \quad (38)$$

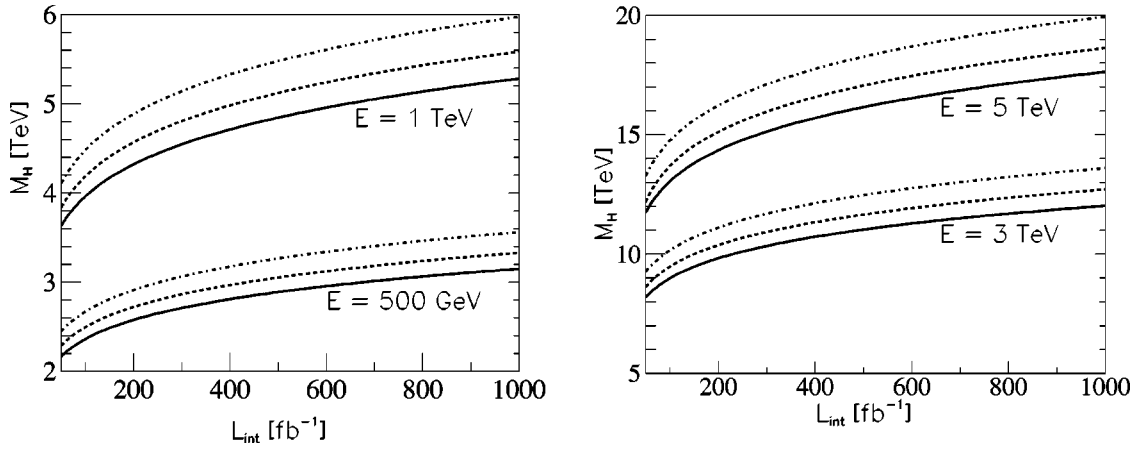


FIG. 4. 5σ reach on the mass scale M_H vs integrated luminosity from the process $e^+e^- \rightarrow f\bar{f}$, with f summed over μ, τ, b, c , and for a range of energies from 0.5 to 5 TeV. Solid: unpolarized; dashed: electrons polarized, $P=0.8$; dash-dotted: both beams polarized, $P=0.8$, $\bar{P} = -0.6$.

The symmetric and antisymmetric parts of the cross section are given by convolutions of the nonradiative cross section with the flux functions $H_A^e(v)$, with v the energy of the emitted photon in units of the beam energy. Because of the radiative return to the Z resonance for $\sqrt{s} > M_Z$, the energy spectrum of the radiated photons is peaked around $E_\gamma/E_{\text{beam}} \approx 1 - M_Z^2/s$. In order to increase a possible new physics signal, events with hard photons should be removed by a cut on the photon energy, $\Delta = E_\gamma/E_{\text{beam}} < 1 - M_Z^2/s$, with $\Delta = 0.9$. We also take into account electroweak corrections to the propagators and vertices amounting essentially to effective (momentum-dependent) coupling constants (effective Born approximation [25] with $m_{\text{top}} = 175$ GeV and $m_{\text{Higgs}} = 300$ GeV). Concerning the other QED corrections, the final-state ones and the initial-final state interference, they can be checked to be numerically unimportant for the chosen kinematical cuts, in particular that on Δ , using existing codes, e.g., ZFITTER [26]. In addition, z_0^* , the zero of $A_{\text{CE}}^{\text{SM}}$, is shifted by these corrections by a small amount from the effective Born approximation value. The box-diagram contributions, which introduce a different angular dependence, are found to be very small.

Since the form of the corrected cross section Eq. (38) is the same as that of Eq. (2), it follows that the radiatively corrected zero of $A_{\text{CE}}^{\text{SM}}$, z_0^* , can again be defined by

$$\left[\int_{-z_0^*}^{z_0^*} - \left(\int_{-1}^{-z_0^*} + \int_{z_0^*}^1 \right) \right] (1+z^2) dz = 0, \quad (39)$$

and one finds the same value for z_0^* as given by Eq. (12). Moreover, in both the SM and SM+CI cases the radiatively corrected asymmetry A_{CE} is still determined by Eq. (19).

Summing over $\mu^+\mu^-$, $\tau^+\tau^-$, $b\bar{b}$, and $c\bar{c}$ final states [the top quark is excluded as its mass effects would alter the angular distribution (38)] one can perform a conventional χ^2 analysis:

$$\chi^2 = \sum_{f=\mu, \tau, c, b} \frac{(\Delta A_{\text{CE}}^f)^2}{(\delta A_{\text{CE}}^f)^2}, \quad (40)$$

keeping $z^* = z_0^*$ fixed (recall from Fig. 3 that the sensitivities for the various final states are rather smooth in an interval around $z_0^* \approx z_{\text{max}}^*$). This leads to the 5σ identification reach as a function of integrated luminosity with energy $\sqrt{s} = 0.5, 1, 3$, and 5 TeV shown in Fig. 4. The chosen range of energy corresponds to the DESY TeV Energy Superconducting Linear Accelerator (TESLA), Next Linear Collider (NLC) [16], and CLIC [17]. Specifically, for $\sqrt{s} = 0.5-1$ TeV and 3-5 TeV machines with integrated luminosity 1 ab^{-1} , the identification reach with double beam polarization is found to be $(7-6) \times \sqrt{s}$ and $(4.5-4) \times \sqrt{s}$, respectively. The effects of spin-2 graviton exchange can be distinguished from the other forms of contact-interaction-like effects considered in Table I for $M_H \leq 3.5, 6, 13.6$, and 20 TeV at $\sqrt{s} = 0.5, 1, 3$, and 5 TeV, respectively.

It turns out that under the assumption of no observation of ΔA_{CE} within the expected experimental uncertainty, in which case only bounds on f_G can be derived, the 95% C.L. lower limits on M_H would be represented by the values shown in Fig. 4 essentially multiplied by a factor of the order of 1.3.

Finally, we consider a scenario that would most closely mimic massive graviton exchange, namely, the exchange of a scalar field in the s and t channels, limiting ourselves to the production of lepton pairs. To be specific, we can concentrate on the example of R -parity breaking SUSY interactions mediated by sneutrino exchange [6,7]. First, we consider the t -channel $\tilde{\nu}$ contribution to $e^+e^- \rightarrow \mu^+\mu^-$ or $\tau^+\tau^-$. In this case the helicity cross sections are given by Eq. (3) with an additional contribution to the helicity amplitudes caused by $\tilde{\nu}$ exchange:

$$\Delta_{\text{LL}} = \Delta_{\text{RR}} = 0, \quad \Delta_{\text{LR}} = \Delta_{\text{RL}} = \frac{1}{2} C_{\tilde{\nu}} P_{\tilde{\nu}}^t, \quad (41)$$

where $P_\nu^t = s/(t - m_\nu^2)$ and $t = -s(1 - z)/2$, $C_{\tilde{\nu}} = \lambda^2/4\pi\alpha_{\text{e.m.}}$, with λ in this case the Yukawa coupling [7]. It is clear that in the contact-interaction limit, i.e., $|t| \ll m_\nu^2$, these two new physics effects, graviton exchange and $\tilde{\nu}$ exchange, are easily separable by the previous analysis based on the asymmetry A_{CE} . If we are not in the contact-interaction limit, \mathcal{M}_{LR} and \mathcal{M}_{RL} pick up an additional z dependence resulting in a z^* dependence of ΔA_{CE} different from the one in Eq. (14) determined by graviton exchange. We find that polarization will also help to distinguish these two new physics effects. For this purpose one can define the polarized observable, the absolute center-edge left-right asymmetry:

$$\begin{aligned} \sigma A_{\text{CE,LR}} &\equiv \sigma_{\text{CE,LR}} \\ &= \left[\int_{-z_0^*}^{z_0^*} - \left(\int_{-1}^{-z_0^*} + \int_{z_0^*}^1 \right) \right] \left(\frac{d\sigma_{\text{L}}}{dz} - \frac{d\sigma_{\text{R}}}{dz} \right) dz. \end{aligned} \quad (42)$$

Here, z_0^* is the zero of $A_{\text{CE}}^{\text{SM}}$ [see Eq. (39)] and $d\sigma_{\text{L}}/dz$ and $d\sigma_{\text{R}}/dz$ are the differential cross sections defined by Eq. (20) with specific choices of electron and positron beam polarizations, for example, $(P, \bar{P}) = (-P_1, P_2)$ and $(P_1, -P_2)$, respectively, with P_1 and P_2 positive. The deviation from the SM prediction of the differential cross section difference involved in Eq. (42) and caused by $\tilde{\nu}$ exchange is given by

$$\begin{aligned} \Delta \frac{d\sigma_{\text{LR}}}{dz} &\equiv \left(\frac{d\sigma_{\text{L}}}{dz} - \frac{d\sigma_{\text{R}}}{dz} \right) - \left(\frac{d\sigma_{\text{L}}^{\text{SM}}}{dz} - \frac{d\sigma_{\text{R}}^{\text{SM}}}{dz} \right) \\ &\propto P_{\text{eff}} (\mathcal{M}_{\text{LR}}^{\text{SM}} - \mathcal{M}_{\text{RL}}^{\text{SM}}) C_{\tilde{\nu}} P_\nu^t = 0, \end{aligned} \quad (43)$$

because $\mathcal{M}_{\text{LR}}^{\text{SM}} = \mathcal{M}_{\text{RL}}^{\text{SM}}$ for the process (1) with $f = \mu, \tau$. Notice that this property, easily checked in the tree approximation of the SM, continues to hold also in the effective Born

approximation. Accordingly, $\sigma_{\text{CE,LR}}$ is unaltered by sneutrino exchange in the leptonic processes $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-$, i.e., $\Delta\sigma_{\text{CE,LR}}^{\tilde{\nu}} = 0$, whereas it is modified by graviton exchange, $\Delta\sigma_{\text{CE,LR}}^G \neq 0$. The choice of z_0^* as integration limits in Eq. (42) assures that the contribution of the SM as well as that of any conventional contact interaction vanishes, leaving room only for graviton and sneutrino exchanges. The role of polarization is that, in the combination (42), the sneutrino contributions cancel as explicitly seen in Eq. (43), so that only the signal of the graviton exchange term can survive. One can notice that this kind of analysis is allowed also in the case of only electron beam longitudinal polarization and an unpolarized positron, namely, $P_1 \neq 0$ and $P_2 = 0$. Also, the quadratic term in the differential cross sections, proportional to $(C_{\tilde{\nu}} P_\nu^t)^2$, cancels in Eq. (42), so that Eq. (43), linear in $(C_{\tilde{\nu}} P_\nu^t)$, is the exact representation of the deviation from the SM.

Concerning $\tilde{\nu}$ exchange in the s channel, the polarized differential cross section (20) picks up an additional, z -independent, term:

$$\frac{d\sigma^s}{dz} \propto (1 + P\bar{P})(C_{\tilde{\nu}} P_\nu^s)^2, \quad (44)$$

with $P_\nu^s \simeq s/(s - m_\nu^2)$. Indeed, the s -channel scalar exchange diagram does not interfere with the electroweak SM amplitudes mediated by the γ and Z boson and the resulting effects are of quadratic order, $(C_{\tilde{\nu}} P_\nu^s)^2$. As is easily seen from Eq. (44), either the electron beam polarization or both electron and positron polarizations allow one to remove the sneutrino s -channel exchange contribution to Eq. (42), i.e., $\Delta\sigma_{\text{CE,LR}}^{\tilde{\nu}} = 0$ in this case also.

Conversely, it is possible to define an observable ‘‘orthogonal’’ to $\sigma_{\text{CE,LR}}$ which is sensitive to $\tilde{\nu}$ exchange in the s channel and independent of the effects of graviton exchange, contact interactions, and Z' exchange. This is the double beam polarization asymmetry defined as [7]

$$A_{\text{double}} = \frac{\sigma(P_1, -P_2) + \sigma(-P_1, P_2) - \sigma(\bar{P}_1, P_2) - \sigma(-P_1, -P_2)}{\sigma(P_1, -P_2) + \sigma(-P_1, P_2) + \sigma(P_1, P_2) + \sigma(-P_1, -P_2)}. \quad (45)$$

Here, σ are the cross sections integrated over z in the indicated polarization configurations. One can see immediately that for the case of the SM, contact interactions, Z' exchange, $\tilde{\nu}$ exchange in the t channel, and graviton exchange one obtains $A_{\text{double}} = P_1 P_2$, since these exchanges contribute to the same amplitudes, whereas $\tilde{\nu}$ exchange in the s channel will force this observable to smaller values as $\Delta A_{\text{double}} \propto -P_1 P_2 (C_{\tilde{\nu}} P_\nu^s)^2 < 0$. A value of A_{double} smaller than $P_1 P_2$ can provide a signature of scalar exchange in the s channel.

In conclusion, we have seen that one can define a set of observables using cross sections integrated within appropri-

ate angular limits that can discriminate among deviations from the SM prediction related either to graviton or to scalar exchange in the s channel.

VI. SUMMARY AND OBSERVATIONS

We conclude with a summary of the main points and some observations. We have developed a specific approach based on an integrated observable, the center-edge asymmetry A_{CE} , to search for and identify spin-2 graviton exchange with a uniquely distinct signature. Indeed, the spin-2 graviton KK exchanges contribute to the asymmetry A_{CE} ,

whereas no deviation from the SM is induced by other kinds of new physics such as the compositelike contact interactions, a heavy vector boson Z' , or gauge boson KK excitations listed in Table I. Both in the SM and in any new physics scenario described by effective current-current interactions, the asymmetry A_{CE} is identical for any value of the parameter z^* .

Particularly convenient is the range of z^* values around the zero of $A_{CE}(z_0^*)$ for the SM. In this range, the sensitivity of A_{CE} to the graviton coupling f_G is maximal and rather smooth in z^* , so that one can not only discover but unambiguously identify this new physics effect. This kind of analysis based on A_{CE} can be applied also to the case where a cut is imposed on the full angular range covered by the experiment, and its nice distinctive features continue to hold to a very good approximation.

Initial electron and positron beam polarization appears to increase the sensitivity to graviton exchange, but its impact on the mass scale parameter M_H is not dramatic due to the large power $(\sqrt{s}/M_H)^4$ that parametrizes the graviton coupling. In particular, for an e^+e^- linear collider with energy $\sqrt{s}=0.5, 1, 3,$ and 5 TeV, with integrated luminosity 1 ab^{-1} , double beam polarization, and a 10° angular cut, the 5σ identification reach is found to be $M_H \leq 3.5, 6, 13.6,$ and 20 TeV, respectively.

Instead, initial polarization can play a key role in distin-

guishing graviton exchange from competing effects, such as those originating from exchange of scalar particles, for which appropriate polarization asymmetries can be defined.

An approach aiming to isolate graviton-exchange effects has recently been proposed in Ref. [18], based on the differential cross section convoluted with Legendre polynomials and integrated over the angular range. Alternatively, our method directly uses the integrated cross sections to construct the center-edge asymmetry A_{CE} . It has the main advantage of a mild dependence of A_{CE} on the kinematical cut, on systematics, and on the number of angular bins, and in particular it depends on the total luminosity and not on the statistics available in each bin. These features may lead to some improvement in the 5σ discovery reach on the mass scale M_H .

Finally, we note that an analysis based on asymmetries analogous to A_{CE} might be useful in the context of hadronic collisions, in the Drell-Yan process.

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