SU(3) relations and the *CP* asymmetries in *B* decays to $\eta' K_S$, ϕK_S, and $K^+K^-K_S$

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We consider *CP* asymmetries in neutral *B* meson decays to $\eta' K_S$, ϕK_S, and $K^+K^-K_S$. We use SU(3) relations to estimate or bound the contributions to these amplitudes proportional to $V_{ub}^* V_{us}$. Such contributions induce a deviation of the S_f terms measured in these time dependent *CP* asymmetries from that measured for ψK_S. For the $K^+K^-K_S$ mode, we estimate the deviation to be of order 0.1. For the $\eta' K_S$ mode, we obtain an upper bound on this deviation of order 0.3. For the ϕK_S mode, we have to add a mild dynamical assumption to the SU(3) analysis due to insufficient available data, yielding an upper bound of order 0.25. These bounds may improve significantly with future data. While they are large at present compared to the usually assumed standard model contribution, they are obtained with minimal assumptions and hence provide more rigorous tests for new physics. If measurements yield $|S_f - S_{\psi K}|$ that are much larger than our bounds, it would make a convincing case for new physics.

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I. INTRODUCTION

Recent measurements of *CP* asymmetries in neutral *B* meson decays into final *CP* eigenstates test the Kobayashi-Maskawa mechanism and probe new sources of *CP* violation. The time dependent asymmetries depend on two parameters S_f and C_f (f denotes here a final *CP* eigenstate):

$$\mathcal{A}_f(t) \equiv \frac{\Gamma(\overline{B}^0_{\rm phys}(t)\to f) - \Gamma(B^0_{\rm phys}(t)\to f)}{\Gamma(\overline{B}^0_{\rm phys}(t)\to f) + \Gamma(B^0_{\rm phys}(t)\to f)} = -C_f \cos(\Delta m_B t) + S_f \sin(\Delta m_B t). \quad (1)$$

CP violation in decay induces C_f, while *CP* violation in the interference of decays with and without mixing induces S_f. (The contribution from *CP* violation in mixing is at or below the percent level and can be safely neglected with the present experimental accuracy.)

If the decay is dominated by a single weak phase, $C_f \approx 0$ and the value of S_f can be cleanly interpreted in terms of *CP* violating parameters of the Lagrangian. This is the case for decays which are dominated by the tree $b \to c\bar{c}s$ transition or by the gluonic penguin $b \to s\bar{s}s$ transition. If one neglects the subdominant amplitudes with a different weak phase, the *CP* asymmetries in these two classes of decays are given by $S_f = -\eta_f \sin 2\beta$, where $\eta_f = +1(-1)$ for final *CP*-even (-odd) states and β is one of the angles of the unitarity triangle. In particular, in this approximation, the *CP* asymmetries in the two classes are equal to each other, for example, $S_{\psi K} = S_{\phi K}$. A strong violation of such a relation would indicate new physics [1].

Our aim in this paper is to quantify this statement with minimal assumptions for three modes of interest: ϕK_S, $\eta' K_S$ and $K^+K^-K_S$. We would like to estimate or to find bounds on the deviations of the corresponding asymmetries from $S_{\psi K_S}$ that are (hadronic-)model independent. The ingredients of our analysis are SU(3) relations and experimental information on related modes. We will be able to carry out this program to the end for $S_{\eta' K_S}$. As concerns $S_{\phi K_S}$, we derive SU(3) relations that can, in principle, lead to model independent bounds. In practice, however, some experimental information is still missing. Nevertheless, by using a mild dynamical assumption, we obtain a bound for this mode too. The situation is more complicated for S_{KKK}, where we point out some subtleties in the interpretation of the experimental results. For this mode, however, we are able to estimate (rather than just bound) the deviation of the extracted asymmetry from $\sin 2\beta$ in the standard model by using *U*-spin relations and experimental data.

II. EXPERIMENTAL AND THEORETICAL BACKGROUND

The *CP* asymmetry in $B \to \psi K_S$ decays (and other, related, modes that proceed via $b \to c\bar{c}s$) has been measured, with a world average [2] of

$$S_{\psi K_S} = +0.734 \pm 0.054 \quad [3,4],$$

$$C_{\psi K_S} = +0.05 \pm 0.04 \quad [3,4]. \quad (2)$$

The value of $S_{\psi K_S}$ is consistent with predictions made on the basis of other measurements of the Cabibbo-Kobayashi-Maskawa (CKM) parameters (Δm_B, Δm_{B_s}, ε_K and tree level decays).

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CP asymmetries have also been searched for in three modes that are dominated by $b \to s\bar{s}s$ gluonic penguin transitions:

$$S_{\eta' K_S} = +0.33 \pm 0.34 \quad [5,6],$$

$$C_{\eta' K_S} = -0.08 \pm 0.18 \quad [5,6], \tag{3}$$

$$S_{\phi K_S} = -0.39 \pm 0.41 \quad [5,7],$$

$$C_{\phi K_S} = +0.56 \pm 0.44 \quad [5], \tag{4}$$

$$-S_{K^+K^-K_S} = +0.49 \pm 0.44^{+0.33}_{-0.00} \quad [5],$$

$$C_{K^+K^-K_S} = +0.40 \pm 0.34^{+0.26}_{-0.00} \quad [5]. \tag{5}$$

The standard model predicts that in these modes $-\eta_f S_f = S_{\psi K_S}$ and $C_f = 0$ to a good approximation. The statistical errors in Eqs. (3)–(5) are too large to make any firm conclusions. It is clear, however, that there is still much room left for deviations from the standard model generated by possible new physics in $b \to s$ transitions.

The standard model amplitude for these three decay modes can be written as follows:

$$A_f \equiv A(B^0 \to f) = V_{cb}^* V_{cs} a_f^c + V_{ub}^* V_{us} a_f^u. \tag{6}$$

The second term is CKM-suppressed compared to the first one since

$$\mathcal{I}m\left(\frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}}\right) = \left|\frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}}\right| \sin\gamma = \mathcal{O}(\lambda^2), \tag{7}$$

where $\lambda = 0.22$ is the Wolfenstein parameter. It is convenient to define

$$\xi_f \equiv \frac{V_{ub}^* V_{us} a_f^u}{V_{cb}^* V_{cs} a_f^c}, \tag{8}$$

and thus rewrite the amplitude of Eq. (6),

$$A_f = V_{cb}^* V_{cs} a_f^c (1 + \xi_f). \tag{9}$$

The SU(3) analysis we carry out allows us to bound $|\xi_f|$. To first order in this quantity, the deviation of the asymmetry from $\sin 2\beta$ is given by [8,9]

$$-\eta_f S_f - \sin 2\beta = 2\cos 2\beta \sin\gamma \cos\delta_f |\xi_f|, \tag{10}$$

where $\delta_f = \arg(a_f^u / a_f^c)$. The ξ_f parameter characterizes also the size of C_f:

$$C_f = -2\sin\gamma \sin\delta_f |\xi_f|. \tag{11}$$

Note that δ_f can be determined from $\tan\delta_f = (\eta_f S_f + \sin 2\beta)/(C_f \cos 2\beta)$, while the following ($\delta_f$-independent) relation between S_f, C_f and $|\xi_f|$ may become useful in the future:

$$C_f^2 + [(\eta_f S_f + \sin 2\beta)/\cos 2\beta]^2 = 4\sin^2\gamma |\xi_f|^2. \tag{12}$$

The crucial question, when thinking of the deviation of $-\eta_f S_f$ from $\sin 2\beta$, is the size of a_f^u / a_f^c. While a_f^c is dominated by the contribution of $b \to s\bar{s}s$ gluonic penguin diagrams, a_f^u gets contributions from both penguin diagrams and $b \to u\bar{u}s$ tree diagrams. For the penguin contributions, it is clear that $|a_f^u / a_f^c| \sim 1$. (The a_f^c term comes from the charm penguin minus the top penguin, while the up penguin minus the top penguin contributes to a_f^u.) Thus our main concern is the possibility that the tree contributions might yield $|a_f^u / a_f^c|$ significantly larger than one.

For final states with zero strangeness, f', we write the amplitudes as

$$A_{f'} \equiv A(B^0 \to f') = V_{cb}^* V_{cd} b_{f'}^c + V_{ub}^* V_{ud} b_{f'}^u. \tag{13}$$

Here neither term is CKM suppressed compared to the other. We use SU(3) flavor symmetry to relate the $a_f^{u,c}$ amplitudes to sums of $b_{f'}^{u,c}$. While similar SU(3) relationships have been explored elsewhere [10–14], most of our results and applications are new.

The SU(3) relations, together with the measurements or upper bounds on the rates for the nonstrange channels plus the measured rate for the channel of interest yield an upper bound on $|\xi_f|$. Let us first provide an intuitive explanation of this. The decays to final strange states, *f*, are dominated by the a_f^c terms. Those to final states with zero strangeness, f', are dominated by the $b_{f'}^u$ terms. Thus we can estimate $|a_f^c|$ and $|b_{f'}^u|$ from the measured branching ratios (or the upper bounds on them). Then the SU(3) relations give upper bounds on certain sums of the $b_{f'}^c$ and a_f^u amplitudes from the extracted values of a_f^c and $b_{f'}^u$, respectively. This then gives a bound on $|a_f^u / a_f^c|$, and consequently on $|\xi_f|$. We can also check the self-consistency of the analysis, namely that $|a_f^u| < |A_f| / |V_{ub} V_{us}|$ and $|b_{f'}^c| < |A_{f'}| / |V_{cb} V_{cd}|$. However, as we show below, the assumptions made in this paragraph can be avoided entirely.

The SU(3) relations actually provide an upper bound on $|V_{cb}^* V_{cd} a_f^c + V_{ub}^* V_{ud} a_f^u|$, in terms of the measured branching ratios of some zero strangeness final states (or limits on them). Therefore, without any approximations, we can bound

$$\hat{\xi}_f \equiv \left|\frac{V_{us}}{V_{ud}} \times \frac{V_{cb}^* V_{cd} a_f^c + V_{ub}^* V_{ud} a_f^u}{V_{cb}^* V_{cs} a_f^c + V_{ub}^* V_{us} a_f^u}\right|$$
$$= \left|\frac{\xi_f + (V_{us} V_{cd})/(V_{ud} V_{cs})}{1 + \xi_f}\right|. \tag{14}$$

If the bound on $\hat{\xi}_f$ is less than unity, then it gives a bound on $|\xi_f|$. We work to first order in ξ_f, since the naive expectation is $\xi_f = \mathcal{O}(\lambda^2)$. At the present state of the data the bounds we obtain on $\hat{\xi}_f$ are significantly larger than λ^2, so we also work in the approximation $\lambda^2 \ll \hat{\xi}_f < 1$. This is appropriate because we want to constrain the possibility $|a_f^u / a_f^c| \gg 1$. Therefore, we take $|\xi_f| \approx \hat{\xi}_f$ in what follows (although this approximation should not be made once the bounds on $\hat{\xi}_f$ are of order λ^2).

The SU(3) decomposition of a_f^u and $b_{f'}^u$ is identical with that of a_f^c and $b_{f'}^c$, although the values of the reduced matrix elements are independent for the u- and the c-terms. The SU(3) decomposition is given in Appendix A for the channels discussed in this paper. We use the notation $a(f) \equiv a_f^{u,c}$ and $b(f') \equiv b_{f'}^{u,c}$ for equations that apply for both cases, with either all u or all c upper indices. Our normalization of the various amplitudes is the same as that of Ref. [11]. It corresponds to $\Gamma = |A|^2$ independent of whether the final particles are identical or not.

The contributions to a_f^c and $b_{f'}^c$ come from penguin diagrams or the tree $b \to c\bar{c}q$ transition plus some form of rescattering (such as D-exchange) to replace the $c\bar{c}$ with lighter quark flavors. Aside from small electroweak penguin contributions, there is only an SU(3) triplet term in the Hamiltonian for these amplitudes. Neglecting electroweak penguins would result in additional SU(3) relations between the a_f^c and $b_{f'}^c$ terms. We do not make such an approximation in our analysis, but it might be useful for other purposes.

III. THE *CP* ASYMMETRY IN $B \to \eta' K_S$

A. SU(3) relations

The *CP* asymmetry in $B \to \eta' K_S$ is expected to yield a less accurate measurement of $\sin 2\beta$ than the ψK_S mode. The reason is that, while this decay is dominated by gluonic penguins, there are CKM-suppressed tree contributions that induce a deviation from the leading result. Nevertheless, it was argued in Ref. [15] that this deviation is below the two percent level. The argument was based on relating the tree contributions in $B \to \eta' K$ and $B \to \pi\pi$ decays. While this may be a reasonable hypothesis, it is based on neither approximate symmetry nor obvious dynamical assumptions. In this section we derive a more rigorous (though weaker) bound on the "problematic" subleading contribution.

The results of Appendix A imply the following amplitude relations:

$$a(\eta_1 K^0) = -\frac{1}{\sqrt{2}} b(\eta_1 \pi^0) + \sqrt{\frac{3}{2}} b(\eta_1 \eta_8),$$

$$a(\eta_8 K^0) = \frac{1}{2\sqrt{2}} b(\eta_8 \pi^0) - \sqrt{\frac{3}{4}} [b(\pi^0\pi^0) - b(\eta_8\eta_8)]. \tag{15}$$

The states η_1 and η_8 transform as a singlet and an octet of SU(3), respectively. They are related to the physical η' and η states through an orthogonal rotation:

$$\eta' = c\,\eta_1 - s\,\eta_8, \quad \eta = s\,\eta_1 + c\,\eta_8, \tag{16}$$

where $s \equiv \sin\theta_{\eta\eta'}$ and $c \equiv \cos\theta_{\eta\eta'}$. Most extractions of the mixing angle, $\theta_{\eta\eta'}$, vary in the 10–20° range, and we will use $\theta_{\eta\eta'} = 20°$ in our numerical calculations [16].

In terms of physical states, we obtain from Eq. (15) the relation

$$\begin{aligned} a(\eta' K^0) = {} & \frac{s^2 - 2c^2}{2\sqrt{2}} b(\eta'\pi^0) - \frac{3cs}{2\sqrt{2}} b(\eta\pi^0) \\ & + \frac{\sqrt{3}s}{4} b(\pi^0\pi^0) - \frac{\sqrt{3}s(s^2+4c^2)}{4} b(\eta'\eta') \\ & + \frac{3\sqrt{3}sc^2}{4} b(\eta\eta) + \frac{\sqrt{3}c(2c^2-s^2)}{2\sqrt{2}} b(\eta\eta'). \end{aligned} \tag{17}$$

The SU(3) analysis gives many more relations, involving both charged and neutral B decay amplitudes. The most general such relation, involving up to thirteen amplitudes on the right hand side, is given in Appendix B. With current data, Eq. (17) gives the strongest bound:

$$\begin{aligned} |\xi_{\eta' K_S}| < \left|\frac{V_{us}}{V_{ud}}\right| \Bigg[& 0.59\sqrt{\frac{\mathcal{B}(\eta'\pi^0)}{\mathcal{B}(\eta' K^0)}} + 0.33\sqrt{\frac{\mathcal{B}(\eta\pi^0)}{\mathcal{B}(\eta' K^0)}} \\ & + 0.14\sqrt{\frac{\mathcal{B}(\pi^0\pi^0)}{\mathcal{B}(\eta' K^0)}} + 0.53\sqrt{\frac{\mathcal{B}(\eta'\eta')}{\mathcal{B}(\eta' K^0)}} \\ & + 0.38\sqrt{\frac{\mathcal{B}(\eta\eta)}{\mathcal{B}(\eta' K^0)}} + 0.96\sqrt{\frac{\mathcal{B}(\eta\eta')}{\mathcal{B}(\eta' K^0)}} \Bigg]. \end{aligned} \tag{18}$$

This bound is obtained from Eq. (17) by taking all amplitudes to interfere constructively, and using $\theta_{\eta\eta'} = 20°$. The experimental upper bounds on the relevant branching ratios are collected in Appendix C. Using these values, we obtain

$$|\xi_{\eta' K_S}| < 0.36. \tag{19}$$

So far only upper limits are available for many of the rates that enter in Eq. (18). Hence this bound is probably a significant overestimate and will improve with further data. At the present state of the data, we do not consider it necessary to be concerned about SU(3) breaking corrections. Eventually, there may be sufficient data to fix all the amplitudes $a_f^{u,c}$, including their relative phases. At that point a much stronger bound can be expected, and allowance for SU(3) breaking corrections will need to be made.

Using the known CKM dependence, the bound of Eq. (19) can be translated into a bound on the hadronic parameters,

$$\left|\frac{a^u_{\eta' K^0}}{a^c_{\eta' K^0}}\right| < 18. \tag{20}$$

This bound is much weaker than most theoretical estimates. Since the amplitudes involved in Eq. (17) carry different strong phases, we do not expect that they all add up coherently, as assumed in Eq. (18). A more plausible (though less rigorous) estimate would be that the left hand side of Eq. (18) is unlikely to be larger than the largest term on the right hand side. This estimate would give $|\xi_{\eta' K_S}|<0.14$ instead of 0.36. Clearly, more data could significantly improve these bounds.

The same set of SU(3) relationships can be used to carry out a similar analysis for a number of other modes. The relevant general relationship for ηK_S is given in Appendix B. Once experimental data on the asymmetry in this mode are available, it will be interesting to use this relationship to obtain a similar constraint on $\xi_{\eta K_S}$.

B. Using charged modes and a dynamical assumption

One can obtain a similar bound on the ratio a_f^u/a_f^c for the charged mode, $f=\eta' K^+$. The experimental situation is such that this bound is significantly stronger than the one in the neutral mode. The SU(3) relations for the decompositions of a_f^u and $b_{f'}^u$ in $B^+\to PP$ decays are also given in Appendix A. They lead to the following relations:

$$a(\eta_1 K^+)=b(\eta_1\pi^+),$$

$$a(\eta_8 K^+)=\frac{1}{\sqrt{3}}b(\pi^+\pi^0)-\frac{1}{\sqrt{6}}b(\overline{K^0}K^+), \tag{21}$$

and

$$\sqrt{6}b(\eta_8\pi^+)=\sqrt{2}b(\pi^+\pi^0)+2b(\overline{K^0}K^+). \tag{22}$$

This last equation allows us to bound $\xi_{\eta' K^+}$ by many different combinations of decay modes. The most general relationship that involves only B^+ decay modes is

$$\begin{aligned}a(\eta' K^+)=&\frac{(3-x)cs}{2}b(\eta\pi^+)\\&+\frac{(x-1)s^2+2c^2}{2}b(\eta'\pi^+)\\&+\frac{(x-3)s}{2\sqrt{3}}b(\pi^+\pi^0)+\frac{xs}{\sqrt{6}}b(\overline{K^0}K^+),\end{aligned} \tag{23}$$

where, as before, c and s parametrize $\eta-\eta'$ mixing. The parameter x is free: it allows us to choose, based on the state of the data, the optimal (that is, the most constraining) combination of amplitudes. With the branching ratios collected in Appendix C, we find that at present $x=3$ in Eq. (23) gives the strongest constraint,

$$|\xi_{\eta' K^+}|<0.09. \tag{24}$$

While the a_f^c amplitudes for the charged and neutral $\eta' K$ modes are the same, the a_f^u are not. The nontriplet contributions coming from the tree $b\to u\bar{u}\bar{d}$ terms in the Hamiltonian cause the differences. [This can be seen in the SU(3) relations in Table II of Appendix A.] If we examine the quark diagrams for these two channels we find that $a^u_{\eta' K^+}$ has a color-allowed tree diagram contribution, while $a^u_{\eta' K_S}$ only arises from a color-suppressed tree diagram or penguins. Our dynamical assumption is that the color-suppressed $a^u_{\eta' K_S}$ is not bigger than the color-allowed $a^u_{\eta' K^+}$. This could only be violated by an accidental cancellation between two terms that are formally different orders in $1/N_c$. By making this mild assumption, we improve the bound on $\xi_{\eta' K_S}$ by more than a factor of three over that given by the pure SU(3) analysis.

There are experimental tests that could indicate that $a^u_{\eta' K^+}$ is small compared to $a^u_{\eta' K_S}$. First, if direct *CP* violation in the neutral mode were established to be large, $|C_{\eta' K_S}|\not\ll 1$, it would place a *lower* bound on $|a^u_{\eta' K_S}/a^c_{\eta' K_S}|$. Second, if the difference of the neutral and charged $B\to\eta' K$ rates were sizable, given our strong bound on $|a^u_{\eta' K^+}/a^c_{\eta' K^+}|$, it would imply a large $|a^u_{\eta' K_S}/a^c_{\eta' K_S}|$ with a relative strong phase that is not close to $\pi/2$. If either of these measurements violate the *upper* bound on $|a^u_{\eta' K^+}/a^c_{\eta' K^+}|$, it would suggest either an accidental cancellation in the charged mode, or, more interestingly, possible new physics. This would make it important to improve the direct neutral mode test for new physics.

Equation (3) shows that $C_{\eta' K_S}$ is consistent with zero and the data in Appendix C show that the ratio of charged and neutral rates is not necessarily much different from one. However, this does not validate our assumption. For example, a small strong phase, $\delta_{\eta' K}\approx 0$, and a large weak phase, $\gamma\approx\pi/2$, would make direct *CP* violation small and induce approximately equal rates, independently of the size of $a^u_{\eta' K^+}/a^u_{\eta' K_S}$.

IV. THE *CP* ASYMMETRY IN $B\to\phi K_S$

A. SU(3) relations

A similar analysis can also be applied to $B\to\phi K_S$. Again, the existence of CKM-suppressed contributions induces a deviation from the leading result, and our goal is to constrain that effect using SU(3) related modes. Here it is usually assumed that these corrections are not large, since the $b\to u\bar{u}s$ tree diagram can only contribute via rescattering to the ϕ final state which is pure $s\bar{s}$. Thus it was generally argued that the deviation of $S_{\phi K_S}$ from $\sin 2\beta$ is likely to be no larger than $\mathcal{O}(\lambda^2)$. Reference [12] proposed an SU(3)-based relation that can potentially bound this deviation, however, it involves an implicit dynamical assumption. In this subsection we present exact SU(3) relations that can, in principle, give a model independent bound. In the next subsection, we explain the dynamical assumption that leads to the bound of Ref. [12] and update it with current data.

The SU(3) decomposition of a_f^u and $b_{f'}^u$ for final states composed of a vector and a pseudoscalar meson is given in

Appendix A. These results imply the following relations:

$$a(\phi_1 K^0) = -\frac{1}{\sqrt{2}} b(\phi_1 \pi^0) + \sqrt{\frac{3}{2}} b(\phi_1 \eta_8),$$

$$\begin{aligned} a(\phi_8 K^0) = & \frac{1}{4\sqrt{2}}[3b(\rho^0\eta_8) - b(\phi_8\pi^0)] \\ & + \frac{1}{4}\sqrt{\frac{3}{2}}[b(\phi_8\eta_8) - b(\rho^0\pi^0)] \\ & + \frac{1}{2}\sqrt{\frac{3}{2}}[b(K^{*0}\overline{K^0}) - b(\overline{K^{*0}}K^0)]. \end{aligned} \quad (25)$$

The states ϕ_1 and ϕ_8 transform as a singlet and an octet of SU(3), respectively. They are related to the physical ϕ and ω through an orthogonal rotation:

$$\phi = \sqrt{\frac{1}{3}}\phi_1 - \sqrt{\frac{2}{3}}\phi_8, \quad \omega = \sqrt{\frac{2}{3}}\phi_1 + \sqrt{\frac{1}{3}}\phi_8, \quad (26)$$

which defines the ϕ as a pure $s\bar{s}$ state. Thus, in terms of physical states, we obtain

$$\begin{aligned} a(\phi K^0) = & \frac{1}{2}[b(\overline{K^{*0}}K^0) - b(K^{*0}\overline{K^0})] \\ & + \frac{1}{2}\sqrt{\frac{3}{2}}[c b(\phi\eta) - s b(\phi\eta')] \\ & + \frac{\sqrt{3}}{4}[c b(\omega\eta) - s b(\omega\eta')] \\ & - \frac{\sqrt{3}}{4}[c b(\rho^0\eta) - s b(\rho^0\eta')] \\ & + \frac{1}{4} b(\rho^0\pi^0) - \frac{1}{4} b(\omega\pi^0) - \frac{1}{2\sqrt{2}} b(\phi\pi^0). \end{aligned} \quad (27)$$

This relation could give a bound on $\xi_{\phi K_S}$ in a similar fashion as Eq. (18) in Sec. III A. However, a survey of the experimental data shows that currently no useful bound can be obtained from Eq. (27). While it is possible, using SU(3) relations, to replace some modes that occur on the right-hand side of Eq. (27) with a combination of others, there is no relation that yields a bound on $\hat{\xi}_{\phi K_S}$ below unity at present.

We conclude that, while in the future it will be possible to use relations such as Eq. (27) to constrain $a^u_{\phi K^0}/a^c_{\phi K^0}$ in a model independent way, it is impossible to do so with current data.

B. Using charged modes and a dynamical assumption

For the charged mode $f = \phi K^+$, one can similarly obtain a bound on the ratio $\xi_{\phi K^+}$ based purely on SU(3). The SU(3) relations for $B^+ \to VP$ decays are given in Appendix A. They lead to the following relations:

$$a(\phi_1 K^+) = b(\phi_1 \pi^+),$$

$$a(\phi_8 K^+) = b(\phi_8 \pi^+) - \sqrt{\frac{3}{2}} b(\overline{K^{*0}}K^+). \quad (28)$$

In terms of physical states we thus obtain

$$a(\phi K^+) = b(\phi\pi^+) + b(\overline{K^{*0}}K^+). \quad (29)$$

Using the experimental upper limits on rates collected in Appendix C, we obtain

$$|\xi_{\phi K^+}| < 0.25. \quad (30)$$

Once again there is no immediate relationship between the $a^u_{\phi K^+}$ and $a^u_{\phi K_S}$. They differ by nontriplet Hamiltonian contributions arising from the tree-type $b \to u\bar{u}s$ transition (and a small but similar effect from electroweak penguins). To use the above result as a bound on $\xi_{\phi K_S}$ requires an additional assumption that $a^u_{\phi K_S}$ is not much larger than $a^u_{\phi K^+}$. Here we cannot readily justify this assumption, although we know of no reason why it should not hold. Because the ϕ is a pure $s\bar{s}$ state, there is no order N_c^2 tree contribution to $a^u_{\phi K^+}$, as there was in the case of $a^u_{\eta K^+}$. The tree $b \to u\bar{u}s$ contribution must undergo a rescattering in order to contribute. This brings it to be an order N_c term, at the same level as the penguin contributions. Traditional analyses of this channel assume that the rescattering contribution is negligible compared to the $b \to s\bar{s}s$ penguin terms, in which case the charged and neutral $a^u_{\phi K}$ would be approximately equal. Indeed, only an accidental cancellation could make $|a^u_{\phi K^+}|$ much smaller than $|a^u_{\phi K_S}|$.

With this assumption, the bound for the charged mode also applies for the neutral mode. In this case, there is presently no pure SU(3)-based bound, and so this assumption is necessary to obtain any result. The bound in Eq. (30) was applied to the neutral mode in Ref. [12]. We have shown here that implicit in this bound is the assumption that there is no accidental cancellation of the two contributions to the charged mode.

As in the $\eta' K$ modes, here too there is no evidence either for large $C_{\phi K_S}$ or for a large difference between $\mathcal{B}(\phi K^0)$ and $\mathcal{B}(\phi K^+)$. If such evidence existed, it would indicate that $|a^u_{\phi K^+}|$ is small compared to $|a^u_{\phi K_S}|$. This could indicate that there is indeed a cancellation between two independent contributions to $a^u_{\phi K^+}$, or a harbinger of new physics effects. Data on the nonstrange neutral modes would then be desirable, as they could distinguish these two possibilities.

V. THE *CP* ASYMMETRY IN $B \to K^+K^-K_S$

The $B \to K^+K^-K_S$ decay is different from the other decay modes discussed in one, very important, aspect: since the final state is three-body, it does not have a definite *CP*. Thus, the *CP* asymmetry is diluted with respect to $\sin 2\beta$. To extract the value of $\sin 2\beta$ from this measurement, one has to know the relative fractions of *CP*-even and *CP*-odd final states. The BELLE Collaboration employed a beautiful isospin analysis for this purpose [17]. The accuracy of the isospin analysis affects the accuracy with which the true S_{KKK} (that is, S_{KKK} for a final KKK state with a definite *CP*) is determined. Consequently, the relation between the experimental value, $S_{KKK}^{\rm exp}$, and $\sin 2\beta$ is more complicated. We will first analyze the accuracy of determining S_{KKK} from $S_{KKK}^{\rm exp}$ and then the deviation of S_{KKK} from $\sin 2\beta$.[1]

A. Isospin analysis

The $B\to K^I K^J K^L$ decays (where $I,J,L=\{+,-,0,S\}$ specify the kaon states) involve an initial $I=1/2$ state and final $I=1/2$ and 3/2 states. There are five independent isospin amplitudes, A_1^2, $A_1^{2'}$, A_3^2, $A_3^{2'}$ and A_3^4, where the lower index denotes the isospin representation of the Hamiltonian and the upper index gives that of the final state. (We follow the notation of the previous sections, instead of using isospin labels.) For the isospin-doublet final states, the representation 2 denotes where the two $S=-1$ mesons are in an isospin-singlet state, while $2'$ denotes where they are in an isospin-triplet. Defining

$$A_{IJL}(p_1,p_2,p_3)\equiv A[B\to K^I(p_1)\overline{K}^J(p_2)K^L(p_3)], \quad (31)$$

we obtain the isospin decompositions given in Appendix D.

Let us start by neglecting the tree contributions, as was done in the BELLE analysis. This corresponds to $A_3^2=A_3^{2'}=A_3^4=0$. Then, the following amplitude relations arise:

$$A_{00+}=A_{+-0}, \quad A_{+00}=A_{0-+}, \quad A_{000}=A_{+-+}. \quad (32)$$

When integrating over phase space, the contribution from the interference between A_1^2 and $A_1^{2'}$ vanishes. (Thus, although A_{IJL} is not invariant under $I\leftrightarrow L$, the rates Γ_{IJL} and the branching ratios $\mathcal{B}_{IJL}$ are.) Consequently, the equalities of the following rates are predicted:

$$\Gamma_{+-0}=\Gamma_{+00}, \quad \Gamma_{+-+}=\Gamma_{000}. \quad (33)$$

Branching ratios of four $B\to KKK$ decays have been measured [17]:

$$\mathcal{B}_{+-+}=(3.30\pm 0.18\pm 0.32)\times 10^{-5},$$

$$\mathcal{B}_{+-0}=(2.93\pm 0.34\pm 0.41)\times 10^{-5},$$

$$\mathcal{B}_{+SS}=(1.34\pm 0.19\pm 0.15)\times 10^{-5},$$

$$\mathcal{B}_{SSS}=(0.43^{+0.16}_{-0.14}\pm 0.75)\times 10^{-5}. \quad (34)$$

Thus the approximation in Eq. (32) that led to Eq. (33) has not yet been tested (in particular, isospin symmetry implies no relation between $\mathcal{B}_{+-+}$ and $\mathcal{B}_{+-0}$). Because Γ_{I00} include both *CP*-odd and even states for the pair of neutral kaons, the measured rates in Eq. (34) are not sufficient to test the relations in Eq. (33). For example, the first relation in Eq. (33) becomes

$$\Gamma_{+-S}=\frac{1}{2}\Gamma_{+SL}+\Gamma_{+SS}. \quad (35)$$

We now focus on the $K^+K^-K^0$ and $K^0\overline{K^0}K^+$ modes. One can write down the effective Hamiltonian in terms of the meson fields. The two $I=0$ terms are of the form $(B^iK_i)(K^jK_j)$ where i and j are isospin indices. We can decompose the Hamiltonian into components where the K^jK_j pair is either in an $l=$even or in an $l=$odd angular momentum state:

$$H_{\rm eff}\propto(B^iK_i)[x(K^jK_j)_{l=\rm even}+\sqrt{1-x^2}(K^jK_j)_{l=\rm odd}]. \quad (36)$$

The equality of the amplitudes in Eq. (32) guarantees that x is equal for the two decay modes, and allows the extraction of the *CP* even/odd fractions in the $B^0\to K_SK^+K^-$ decay from measurements of the $B^+\to K^+K_SK_S$ decay as follows [17].

Consider the K^+K^- subsystem in the $B^0\to K^+K^-K^0$ decay. Charge conjugation exchanges K^+ and K^-, and parity exchanges them again (in the center of mass frame). Thus the K^+K^- system has $CP=+1$. Then, the $K^+K^-K_S$ system has $CP=(-1)^l$, where l is the relative angular momentum between K_S and (K^+K^-). (It also equals the relative angular momentum between K^+ and K^-.) There is then a one-to-one correspondence between angular momentum and *CP*. In particular, x^2 in Eq. (36) gives the *CP*-even fraction in the $B\to K^+K^-K_S$ decay.

Next consider the $\overline{K^0}K^0$ subsystem in the $B^+\to K^+\overline{K^0}K^0$ decay. Bose symmetry implies that $l=$even corresponds to a final $K_SK_S+K_LK_L$ state, while $l=$odd corresponds to a K_SK_L state. Thus, $x^2=2\Gamma_{+SS}/\Gamma_{+00}$. Since $\mathcal{B}_{+00}$ has not been measured, one can use again Eq. (33) to arrive at the following relation [17]:

$$x^2=2\frac{\Gamma_{+SS}}{\Gamma_{+-0}}=0.97\pm 0.15\pm 0.07. \quad (37)$$

We learn that the $K^+K^-K_S$ final state is dominantly *CP*-even.

From a measured value of the *CP* asymmetry, $S_{KKK}^{\rm exp}$, we can deduce the value of the *CP* asymmetry for the *CP*-even component, S_{KKK}, according to $S_{KKK}=S_{KKK}^{\rm exp}/(2x^2-1)$. We learn that in the limit that $I=1$ contributions to the Hamiltonian are neglected, we have

$$S_{KKK}=\frac{S_{KKK}^{\rm exp}}{4\Gamma_{+SS}/\Gamma_{+-0}-1}. \quad (38)$$

[1]In the original version of this section, there were errors in the isospin and *U*-spin decompositions of the relevant amplitudes, which were pointed out in Ref. [18]. We agree with their results.

When the higher isospin contributions are taken into account, the three amplitude equalities of Eq. (32) and the two rate equalities of Eq. (33) no longer hold. There remains a single amplitude relation:

$$A_{000}+A_{+-+}+A_{+00}+A_{00+}+A_{+-0}+A_{0-+}=0. \quad (39)$$

The angular momentum analysis is modified by the three A_3 terms. In particular, both the relation $\Gamma_{+-S}(l=\text{even})/\Gamma_{+-S}=\Gamma_{+00}(l=\text{even})/\Gamma_{+00}$ and the relation $\Gamma_{+00}=\Gamma_{+-0}$ are corrected by terms of $\mathcal{O}[(A_3^2+\sqrt{2}A_3^4)/(\sqrt{3}A_1^2)]$ and of $\mathcal{O}[A_3^{2'}/(\sqrt{3}A_1^{2'})]$. At present there is no experimental information on the size of these corrections.

One might worry about isospin violation in the $\phi\to KK$ decays, since $\mathcal{B}(\phi\to K^+K^-)\approx 49\%$ and $\mathcal{B}(\phi\to K_SK_L)\approx 34\%$ should be equal in the isospin limit. (This large violation can be understood as arising chiefly from the phase space difference for the two channels.) Since $\mathcal{B}(B\to\phi K)\times\mathcal{B}(\phi\to K^+K^-)$ is between 10–15 % of $\mathcal{B}_{+-0}$, this could give an additional error of up to $\sim 4\%$ on x^2, not a very large effect.

Note that even if the A_3 amplitudes were negligibly small and thus the isospin analysis to find the *CP*-even fraction in the *KKK* state very precise, it would not imply that the extracted *CP* asymmetry is equal to $\sin 2\beta$ to the same precision. The $b\to u\overline{u}s$ tree contribution also has an isosinglet component, which would not affect the isospin analysis but would shift S_{KKK} from $\sin 2\beta$. In the next subsection we estimate the overall effect of that contribution.

B. *U*-spin analysis

In the previous subsection, we used isospin symmetry to estimate the *CP*-even fraction in the $K^+K^-K_S$ final state. Isospin symmetry relates the $B^0\to K^+K^-K^0$ mode to the $B^+\to K^+K^0\overline{K^0}$ mode. In this subsection we use *U*-spin symmetry to estimate the overall effect of contributions to $B\to K^+K^-K^+$ that are proportional to $V_{ub}^*V_{us}$. *U*-spin relates certain $B^+\to h_i^+h_j^-h_k^+$ modes to each other, where $h_{i,j,k}=K$ or π. [Since *U*-spin is a subgroup of SU(3), this analysis is just a simplified form of the analysis that we have given for the other channels in this paper.]

Under *U*-spin, B^+ is a singlet, while $M_i=(K^+,\pi^+)$ is a doublet. A crucial point in our discussion is the *U*-spin transformation properties of the Hamiltonian. Both the penguin amplitudes, $b\to(\overline{u}u+\overline{d}d+\overline{s}s)q$, and the tree contributions $b\to u\overline{u}q$ (with $q=d,s$) are $\Delta U=1/2$. Consequently, there are two *U*-spin amplitudes for the charged *B* decays into three charged kaons or pions. The decomposition of the various decay amplitudes in terms of these two *U*-spin amplitudes is given in Table IV. We find the following relation:

$$a(K^+K^-K^+)=b(\pi^+\pi^-\pi^+). \quad (40)$$

The experimental data are [17,19]

$$\mathcal{B}_{KKK}\equiv\mathcal{B}(B^+\to K^+K^-K^+)=(3.1\pm 0.2)\times 10^{-5},$$

$$\mathcal{B}_{\pi KK}\equiv\mathcal{B}(B^+\to\pi^+K^-K^+)=(6.6\pm 3.4)\times 10^{-6},$$

$$\mathcal{B}_{K\pi\pi}\equiv\mathcal{B}(B^+\to K^+\pi^-\pi^+)=(5.7\pm 0.4)\times 10^{-5},$$

$$\mathcal{B}_{\pi\pi\pi}\equiv\mathcal{B}(B^+\to\pi^+\pi^-\pi^+)=(1.1\pm 0.4)\times 10^{-5}. \quad (41)$$

To relate the two pairs of rates in a useful way, we make our usual approximation: we take the *KKK* rate to be dominated by the a_{KKK}^c term, and the $\pi\pi\pi$ rate to be dominated by the $a_{\pi\pi\pi}^u$ term. Then we obtain

$$|\xi_{KKK}|=\left|\frac{V_{us}}{V_{ud}}\right|\sqrt{\frac{\mathcal{B}(B^+\to\pi^+\pi^-\pi^+)}{\mathcal{B}(B^+\to K^+K^-K^+)}}\approx 0.13. \quad (42)$$

Given the size of *U*-spin breaking effects and the crudeness of our approximations, we estimate that the corrections to $-S_{KKK}=\sin 2\beta$ is of the following size:

$$\xi_{K^+K^-K_S}=0.13\pm 0.06. \quad (43)$$

Additional constraint on $|\xi_{KKK}|$ can be derived from $\mathcal{B}_{\pi KK}$. Our amplitude relations imply that, in the *U*-spin limit, $\mathcal{B}_{\pi KK}\geqslant\mathcal{B}_{\pi\pi\pi}/2$. Consequently,

$$|\xi_{KKK}|\leqslant\left|\frac{V_{us}}{V_{ud}}\right|\sqrt{\frac{2\mathcal{B}(B^+\to\pi^+K^-K^+)}{\mathcal{B}(B^+\to K^+K^-K^+)}}\approx 0.14. \quad (44)$$

The above analysis does not distinguish between quasi-two-body and true three-body contributions to the eventual three-body rate. Indeed it includes all three-body final states, whether or not reached by a resonant contribution. We used here the SU(3) relationship only for the total rates, integrated over the entire Dalitz plots. Comparisons of more restricted regions of the Dalitz plots would be much more subject to SU(3) breaking corrections.

VI. CONCLUSIONS

Within the standard model, the *CP* asymmetries S_f in neutral *B* decays to the final *CP* eigenstates ϕK_S, $\eta' K_S$ and $(K^+K^-K_S)_{CP=-1}$ are equal to the CKM parameter $\sin 2\beta$ measured in $B\to\psi K_S$, to a good approximation. Furthermore, the direct *CP* asymmetries C_f in these modes are expected to be small. The goodness of this approximation is different between the various modes and its estimate suffers, in general, from hadronic uncertainties. We used SU(3) relations and experimental data (and, in some cases, a mild dynamical assumption) to estimate or to derive upper bounds on the deviation of the S_f from $\sin 2\beta$ and on the size of C_f. We obtained

$$|\xi_{\eta' K_S}| < \begin{cases} 0.36 & \text{SU(3)}, \\ 0.09 & \text{SU(3)+leading } N_c \text{ assumption}, \end{cases}$$

$$|\xi_{\phi K_S}| < 0.25 \quad \text{SU(3)+noncancellation assumption},$$

$$|\xi_{K^+K^-K_S}| \sim 0.13 \quad U\text{-spin}, \tag{45}$$

where ξ_f is defined in Eq. (8). The approximations and assumptions that lead to these results are spelled out in the corresponding sections. While our bounds for the first two modes are considerably weaker than estimates based on explicit calculations of the hadronic amplitudes, they have the advantage that they are model independent. Although SU(3) breaking effects could be significant, our bounds for the two-body modes are probably still conservative because they arise from a sum over several complex amplitudes that we assumed to interfere constructively. Furthermore only experimental upper bounds are available for many of the rates that enter these bounds. As data improve, these bounds could become significantly stronger. Certainly, if deviations from $\sin 2\beta$ are established that are larger than the SU(3) bounds, the case for new physics would be convincing. Since our bounds apply more generally to minimal flavor violation models, the new physics would have to be beyond this framework. Even where our results require additional assumptions, the situation here is better than the usual, in that we are making assumptions about nonleading corrections to the a_f^u amplitudes, rather than about the full a_f^u terms.

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APPENDIX A: SU(3) DECOMPOSITION FOR $\langle f^{(\prime)}|(\bar{b}u)(\bar{u}q)|B\rangle$ AMPLITUDES

In this appendix we give the SU(3) decomposition of matrix elements that are relevant to our analysis. The operator that creates a B meson containing a $\bar{b}$ quark transforms as a $\bar{3}$ of SU(3). The $\Delta B=+1$ Hamiltonian, which has the flavor structure $(\bar{b}q_i)(\bar{q}_j q_k)$, transforms as $3\times 3\times \bar{3}=15+\bar{6}+3+3$. Our calculations follow closely that in Ref. [20], the only difference being the decomposition of the Hamiltonian that can be read off from Ref. [10].

For final states composed of an SU(3) singlet and an octet, there are three reduced matrix elements. The reason is that there is a unique way of making a singlet from an octet plus any one of the three representations of the Hamiltonian and the B operator. In Table I we give the decomposition of $a(f)\equiv a_f^{u,c}$ and $b(f')\equiv b_{f'}^{u,c}$ for $f^{(\prime)}=\eta_1 P_8$, where η_1 is the SU(3)-singlet pseudoscalar, and P_8 is the SU(3)-octet pseudoscalar. The matrix elements S_α^β that occur in the decomposition of a_f^u and $b_{f'}^u$ are independent of those for a_f^c and $b_{f'}^c$. In our notation, the lower index of the reduced matrix elements denotes the SU(3) representation of the Hamiltonian and the upper index is that of the final state. (If electroweak penguin contributions were neglected, the decomposition of a_f^c and $b_{f'}^c$ is given by the last column, corresponding to H in a triplet.) The decomposition for $f^{(\prime)}=\phi_1 P_8$, where ϕ_1 is the SU(3)-singlet vector-meson, is the same as that for $f^{(\prime)}=\eta_1 P_8$, with different values of the reduced matrix ele-

TABLE I. SU(3) decomposition of $a(f)$ and $b(f')$ for $f^{(\prime)}=\eta_1 P_8$.

$f^{(\prime)}$	S_{15}^8	$S_{\bar{6}}^8$	S_3^8
$\eta_1 K^0$	-1	-1	1
$\eta_1 K^+$	3	1	1
$\eta_1 \pi^0$	$5/\sqrt{2}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$
$\eta_1 \eta_8$	$\sqrt{3/2}$	$-\sqrt{3/2}$	$1/\sqrt{6}$
$\eta_1 \pi^+$	3	1	1

TABLE II. SU(3) decomposition of $a(f)$ and $b(f')$ for $f^{(\prime)}=P_8P_8$.

$f^{(\prime)}$	A_{15}^{27}	A_{15}^8	$A_{\bar{6}}^8$	A_3^8	A_3^1
$\eta_8 K^0$	$4\sqrt{6}/5$	$1/\sqrt{6}$	$-1/\sqrt{6}$	$-1/\sqrt{6}$	0
$K^0\pi^0$	$12\sqrt{2}/5$	$1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0
$K^+\pi^-$	$16/5$	-1	1	1	0
$\eta_8 K^+$	$8\sqrt{6}/5$	$-\sqrt{3/2}$	$1/\sqrt{6}$	$-1/\sqrt{6}$	0
$K^+\pi^0$	$16\sqrt{2}/5$	$3/\sqrt{2}$	$-1/\sqrt{2}$	$1/\sqrt{2}$	0
$K^0\pi^+$	$-8/5$	3	-1	1	0
$\eta_8\pi^0$	0	$5/\sqrt{3}$	$1/\sqrt{3}$	$-1/\sqrt{3}$	0
$\pi^0\pi^0$	$-13\sqrt{2}/5$	$1/\sqrt{2}$	$1/\sqrt{2}$	$1/(3\sqrt{2})$	$\sqrt{2}$
$\eta_8\eta_8$	$3\sqrt{2}/5$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/(3\sqrt{2})$	$\sqrt{2}$
$\pi^-\pi^+$	$14/5$	1	1	$1/3$	2
K^-K^+	$-2/5$	2	0	$-2/3$	2
$K^0\bar{K}^0$	$-2/5$	-3	-1	$1/3$	2
$\eta_8\pi^+$	$4\sqrt{6}/5$	$\sqrt{6}$	$-\sqrt{2/3}$	$\sqrt{2/3}$	0
$\pi^+\pi^0$	$4\sqrt{2}$	0	0	0	0
$K^+\bar{K}^0$	$-8/5$	3	-1	1	0

TABLE III. SU(3) decomposition of $a(f)$ and $b(f')$ for $f^{(\prime)}=V_8P_8$.

$f^{(\prime)}$	B^{27}_{15}	$B^{8_S}_{15}$	$B^{8_S}_{\bar{6}}$	$B^{8_S}_{3}$	B^{1}_{3}	B^{10}_{15}	$B^{\overline{10}}_{\bar{6}}$	$B^{8_A}_{15}$	$B^{8_A}_{\bar{6}}$	$B^{8_A}_{3}$
$\phi_8 K^0$	$2\sqrt{6}/5$	$1/(2\sqrt{6})$	$-1/(2\sqrt{6})$	$-1/(2\sqrt{6})$	0	$-4\sqrt{2/3}$	0	$\sqrt{3/2}/2$	$-\sqrt{3/2}/2$	$-\sqrt{3/2}/2$
$K^{*0}\eta_8$	$2\sqrt{6}/5$	$1/(2\sqrt{6})$	$-1/(2\sqrt{6})$	$-1/(2\sqrt{6})$	0	$4\sqrt{2/3}$	0	$-\sqrt{3/2}/2$	$\sqrt{3/2}/2$	$\sqrt{3/2}/2$
$K^{*0}\pi^0$	$6\sqrt{2}/5$	$1/(2\sqrt{2})$	$-1/(2\sqrt{2})$	$-1/(2\sqrt{2})$	0	$4\sqrt{2}/3$	$4\sqrt{2}/3$	$1/(2\sqrt{2})$	$-1/(2\sqrt{2})$	$-1/(2\sqrt{2})$
$\rho^0 K^0$	$6\sqrt{2}/5$	$1/(2\sqrt{2})$	$-1/(2\sqrt{2})$	$-1/(2\sqrt{2})$	0	$-4\sqrt{2}/3$	$-4\sqrt{2}/3$	$-1/(2\sqrt{2})$	$1/(2\sqrt{2})$	$1/(2\sqrt{2})$
$K^{*+}\pi^-$	$8/5$	$-1/2$	$1/2$	$1/2$	0	$-8/3$	$4/3$	$-1/2$	$1/2$	$1/2$
$\rho^- K^+$	$8/5$	$-1/2$	$1/2$	$1/2$	0	$8/3$	$-4/3$	$1/2$	$-1/2$	$-1/2$
$\phi_8 K^+$	$4\sqrt{6}/5$	$-\sqrt{3/2}/2$	$1/(2\sqrt{6})$	$-1/(2\sqrt{6})$	0	0	0	$-3\sqrt{3/2}/2$	$\sqrt{3/2}/2$	$-\sqrt{3/2}/2$
$K^{*+}\eta_8$	$4\sqrt{6}/5$	$-\sqrt{3/2}/2$	$1/(2\sqrt{6})$	$-1/(2\sqrt{6})$	0	0	0	$3\sqrt{3/2}/2$	$-\sqrt{3/2}/2$	$\sqrt{3/2}/2$
$K^{*+}\pi^0$	$8\sqrt{2}/5$	$3/(2\sqrt{2})$	$-1/(2\sqrt{2})$	$1/(2\sqrt{2})$	0	0	$4\sqrt{2}/3$	$3/(2\sqrt{2})$	$-1/(2\sqrt{2})$	$1/(2\sqrt{2})$
$\rho^0 K^+$	$8\sqrt{2}/5$	$3/(2\sqrt{2})$	$-1/(2\sqrt{2})$	$1/(2\sqrt{2})$	0	0	$-4\sqrt{2}/3$	$-3/(2\sqrt{2})$	$1/(2\sqrt{2})$	$-1/(2\sqrt{2})$
$K^{*0}\pi^+$	$-4/5$	$3/2$	$-1/2$	$1/2$	0	0	$-4/3$	$3/2$	$-1/2$	$1/2$
$\rho^+ K^0$	$-4/5$	$3/2$	$-1/2$	$1/2$	0	0	$4/3$	$-3/2$	$1/2$	$-1/2$
$\phi_8\pi^0$	0	$5/(2\sqrt{3})$	$1/(2\sqrt{3})$	$-1/(2\sqrt{3})$	0	$4/\sqrt{3}$	$2/\sqrt{3}$	0	0	0
$\rho^0\eta_8$	0	$5/(2\sqrt{3})$	$1/(2\sqrt{3})$	$-1/(2\sqrt{3})$	0	$-4/\sqrt{3}$	$-2/\sqrt{3}$	0	0	0
$\rho^0\pi^0$	$-13/5$	$1/2$	$1/2$	$1/6$	1	0	0	0	0	0
$\phi_8\eta_8$	$3/5$	$-1/2$	$-1/2$	$-1/6$	1	0	0	0	0	0
$\rho^-\pi^+$	$7/5$	$1/2$	$1/2$	$1/6$	1	$4/3$	$-2/3$	$5/2$	$1/2$	$-1/2$
$\rho^+\pi^-$	$7/5$	$1/2$	$1/2$	$1/6$	1	$-4/3$	$2/3$	$-5/2$	$-1/2$	$1/2$
$K^{*-}K^+$	$-1/5$	1	0	$-1/3$	1	$-4/3$	$2/3$	2	1	0
$K^{*+}K^-$	$-1/5$	1	0	$-1/3$	1	$4/3$	$-2/3$	-2	-1	0
$K^{*0}\overline{K^0}$	$-1/5$	$-3/2$	$-1/2$	$1/6$	1	$-4/3$	$2/3$	$1/2$	$-1/2$	$-1/2$
$\overline{K^{*0}}K^0$	$-1/5$	$-3/2$	$-1/2$	$1/6$	1	$4/3$	$-2/3$	$-1/2$	$1/2$	$1/2$
$\phi_8\pi^+$	$2\sqrt{6}/5$	$\sqrt{3/2}$	$-1/\sqrt{6}$	$1/\sqrt{6}$	0	0	$-2\sqrt{2/3}$	0	0	0
$\rho^+\eta_8$	$2\sqrt{6}/5$	$\sqrt{3/2}$	$-1/\sqrt{6}$	$1/\sqrt{6}$	0	0	$2\sqrt{2/3}$	0	0	0
$\rho^+\pi^0$	$2\sqrt{2}$	0	0	0	0	0	$2\sqrt{2}/3$	$3/\sqrt{2}$	$-1/\sqrt{2}$	$1/\sqrt{2}$
$\rho^0\pi^+$	$2\sqrt{2}$	0	0	0	0	0	$-2\sqrt{2}/3$	$-3/\sqrt{2}$	$1/\sqrt{2}$	$-1/\sqrt{2}$
$\overline{K^{*0}}K^+$	$-4/5$	$3/2$	$-1/2$	$1/2$	0	0	$-4/3$	$3/2$	$-1/2$	$1/2$
$K^{*+}\overline{K^0}$	$-4/5$	$3/2$	$-1/2$	$1/2$	0	0	$4/3$	$-3/2$	$1/2$	$-1/2$

ments, S^{β}_{α}, and the replacement $\eta_1\to\phi_1$.

Final states containing two SU(3) octets can be decomposed as $8\times 8=27+10+\overline{10}+8_S+8_A+1$. The final state composed of P_8P_8 is symmetric, and so it transforms as an element of the symmetric part, $(8\times 8)_S=27+8+1$. In Table II we give the decomposition of $a(f)$ and $b(f)$ for $f^{(\prime)}=P_8P_8$; it contains five reduced matrix elements. When the final mesons are different, such as $f^{(\prime)}=P_8V_8$, where V_8 is the SU(3)-octet vector-meson, all six representations appear. In Table III we give the decomposition of $a(f)$ and $b(f')$ for $f^{(\prime)}=P_8V_8$, which contains ten reduced matrix elements. Again, the matrix elements A^{β}_{α} and B^{β}_{α} that occur in the decomposition of a^u_f and $b^u_{f'}$ are independent of those for a^c_f and $b^c_{f'}$. (If electroweak penguins are neglected, the decomposition of a^c_f and $b^c_{f'}$ is given by the columns corresponding to H in a triplet, A^{8}_{3}, A^{1}_{3}, $B^{8_S}_{3}$, $B^{8_A}_{3}$, and B^{1}_{3}.)

Related tables have been presented in Ref. [10], however, the $(\bar{b}u)(\bar{u}s)$ contributions to strangeness changing decays were neglected. Furthermore, in that paper, when there are several independent amplitudes with the Hamiltonian in a given SU(3) representation, these contributions are not decomposed according to the SU(3) representation of the final state. Reference [11] gives the SU(3) decomposition for $f^{(\prime)}=P_8P_8$ in a somewhat different notation from ours. They do not discuss the applications we investigate here. In Ref. [14] a nonet [U(3)] symmetry is assumed (which can be justified in the large N_c limit). It relates matrix elements involving ϕ_1 and ϕ_8 (and, similarly, η_1 and η_8) that are independent of one another based only on SU(3). Thus, in Ref. [14], eleven amplitudes describe $B\to VP$ decays (with V in a singlet or octet and P in an octet), while we need thirteen. The use of the fewer number of matrix elements amounts to a dynamical assumption beyond SU(3).

APPENDIX B: SU(3) RELATIONS FOR $B^0 \to \eta' K^0$ AND $B^0 \to \eta K^0$

The most general SU(3) relation between the $a(\eta' K^0)$ and $b(f')$'s of charged and neutral B decays can be written as follows:

$$\begin{aligned} a(\eta' K^0) = {} & \left[\frac{s^2 - 2c^2}{2\sqrt{2}} - \frac{\sqrt{3}s^2(x_1 - x_2)}{2} \right] b(\eta'\pi^0) - \left[\frac{3sc}{2\sqrt{2}} - \frac{\sqrt{3}sc(x_1 - x_2)}{2} \right] b(\eta\pi^0) + \left[\frac{\sqrt{3}s}{4} + \frac{s(x_1 + x_2 + 4x_3)}{2\sqrt{2}} \right] b(\pi^0\pi^0) \\ & - \left[\frac{\sqrt{3}s(s^2 + 4c^2)}{4} - \frac{3s^3(x_1 + x_2)}{2\sqrt{2}} \right] b(\eta'\eta') + \left[\frac{3\sqrt{3}sc^2}{4} + \frac{3sc^2(x_1 + x_2)}{2\sqrt{2}} \right] b(\eta\eta) \\ & + \left[\frac{\sqrt{6}c(2c^2 - s^2)}{4} - \frac{3cs^2(x_1 + x_2)}{2} \right] b(\eta\eta') - s x_3 b(\pi^+\pi^-) - s x_1 b(K^+K^-) - s x_2 b(K^0\overline{K^0}) - s x_4 b(\overline{K^0}K^+) \\ & + \sqrt{\frac{3}{2}} s c x_4 b(\eta\pi^+) - \sqrt{\frac{3}{2}} s^2 x_4 b(\eta'\pi^+) + \left(\sqrt{2} s x_3 - \frac{s x_4}{\sqrt{2}} \right) b(\pi^0\pi^+). \end{aligned} \quad \text{(B1)}$$

Here, x_1, x_2, x_3 and x_4 are free parameters that allow us to choose between various combinations of amplitudes on the right hand side of this relation. In particular, given a set of experimental measurements of (or bounds on) the corresponding branching ratios, we can vary the x_i parameters so that we get the strongest constraint. With current data, the optimal choice is $x_1 = x_2 = x_3 = x_4 = 0$, which yields Eq. (17).

The analogous relation that will allow to bound the deviation of the *CP* asymmetry in $B \to \eta K_S$ decay (once it is measured) from $\sin 2\beta$ is

$$\begin{aligned} a(\eta K^0) = {} & -\frac{sc}{2}\left(\sqrt{3}x_1 + \frac{3}{\sqrt{2}} \right) b(\eta'\pi^0) - \left[\frac{s^2}{\sqrt{2}} - \frac{c^2}{2}\left(\sqrt{3}x_1 + \frac{1}{\sqrt{2}} \right) \right] b(\eta\pi^0) + \sqrt{\frac{3}{2}} s c x_2 b(\eta'\pi^+) - \sqrt{\frac{3}{2}} c^2 x_2 b(\eta\pi^+) \\ & + \frac{s}{2}\left(c^2\left(\sqrt{\frac{3}{2}} + 3x_3 \right) - s^2\sqrt{6} \right) b(\eta'\eta) - \frac{3s^2c}{4}(\sqrt{3} + \sqrt{2}x_3) b(\eta'\eta') \\ & + c\left[\frac{c^2}{4}(\sqrt{3} - 3\sqrt{2}x_3) + s^2\sqrt{3} \right] b(\eta\eta) - \frac{c}{4}(\sqrt{3} + \sqrt{2}(4x_4 + x_3))\, b(\pi^0\pi^0) + c x_4 b(\pi^+\pi^-) \\ & + \frac{c}{\sqrt{2}}(x_2 - 2x_4) b(\pi^0\pi^+) + \frac{c}{2}(x_3 - x_1) b(K^+K^-) + \frac{c}{2}(x_3 + x_1) b(K^0\overline{K^0}) + c x_2 b(\overline{K^0}K^+). \end{aligned} \quad \text{(B2)}$$

We have also derived the most general SU(3) relation between the $a(\phi_8 K^0)$ and the (sixteen) $b^u_{f'}$'s of charged and neutral B decays. The relation is quite complicated and it does not seem likely that it will become useful in the near future, so we do not present it explicitly here.

APPENDIX C: RELEVANT BRANCHING RATIOS

The values below are collected from Ref. [16].

$$\mathcal{B}(\eta' K^0) = (5.8^{+1.4}_{-1.3}) \times 10^{-5},$$

$$\mathcal{B}(\eta' K^+) = (7.5 \pm 0.7) \times 10^{-5},$$

$$\mathcal{B}(\pi^+\pi^-) = (4.4 \pm 0.9) \times 10^{-6},$$

$$\mathcal{B}(K^+K^-) < 1.9 \times 10^{-6},$$

$$\mathcal{B}(\overline{K^0}K^+) < 2.4 \times 10^{-6},$$

$$\mathcal{B}(\eta\pi^0) < 2.9 \times 10^{-6},$$

$$\mathcal{B}(\pi^0\pi^0) < 5.7 \times 10^{-6},$$

$$\mathcal{B}(\eta\pi^+) < 5.7 \times 10^{-6},$$

$$\mathcal{B}(\eta'\pi^0) < 5.7 \times 10^{-6},$$

$$\mathcal{B}(\eta'\pi^+) < 7.0 \times 10^{-6},$$

$$\mathcal{B}(\pi^0\pi^+)<9.6\times10^{-6},$$
$$\mathcal{B}(K^0\overline{K^0})<1.7\times10^{-5},$$
$$\mathcal{B}(\eta\eta)<1.8\times10^{-5},$$
$$\mathcal{B}(\eta\eta')<2.7\times10^{-5},$$
$$\mathcal{B}(\eta'\eta')<4.7\times10^{-5}, \quad \text{(C1)}$$

while for vector-pseudoscalar modes:

$$\mathcal{B}(\phi K^0)=(8.1^{+3.2}_{-2.6})\times10^{-6},$$
$$\mathcal{B}(\phi K^+)=(7.9^{+2.0}_{-1.8})\times10^{-6},$$
$$\mathcal{B}(\eta\omega)<1.2\times10^{-5},$$
$$\mathcal{B}(\eta'\omega)<6.0\times10^{-5},$$
$$\mathcal{B}(\eta\phi)<0.9\times10^{-5},$$
$$\mathcal{B}(\eta'\phi)<3.1\times10^{-5},$$
$$\mathcal{B}(\eta\rho^0)<1.0\times10^{-5},$$
$$\mathcal{B}(\eta'\rho^0)<1.2\times10^{-5},$$
$$\mathcal{B}(\rho^0\pi^0)<5.5\times10^{-6},$$
$$\mathcal{B}(\omega\pi^0)<3.0\times10^{-6},$$
$$\mathcal{B}(K^+\overline{K^{*0}})<5.3\times10^{-6},$$
$$\mathcal{B}(\phi\pi^+)<1.4\times10^{-6}. \quad \text{(C2)}$$

TABLE IV. Top half: Isospin decomposition of $A_{IJL}=A[B\to K^I(p_1)\overline{K}^J(p_2)K^L(p_3)]$. Bottom half: U-spin decomposition of $A_{h_ih_jh_l}=A[B\to h_i^+(p_1)h_j^-(p_2)h_l^+(p_3)]$.

A_{IJL}	A_1^2	$A_1^{2'}$	A_3^2	$A_3^{2'}$	A_3^4
A_{+00}	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3\sqrt{2}}$
A_{00+}	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{3\sqrt{2}}$
A_{+-0}	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{3\sqrt{2}}$
A_{0-+}	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3\sqrt{2}}$
A_{000}	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{3}$	0	$-\frac{1}{3\sqrt{2}}$
A_{+-+}	$-\frac{1}{\sqrt{3}}$	0	$-\frac{1}{3}$	0	$\frac{1}{3\sqrt{2}}$

$A_{h_ih_lh_j}$	X_1^2	$X_1^{2'}$
$A_{K\pi\pi}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}$
$A_{\pi\pi K}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$
$A_{KK\pi}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$
$A_{\pi KK}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}$
$A_{\pi\pi\pi}$	$-\frac{1}{\sqrt{3}}$	0
A_{KKK}	$-\frac{1}{\sqrt{3}}$	0

APPENDIX D: SU(2) DECOMPOSITION FOR $\langle KKK|(\overline{b}u)(\overline{u}s)|B\rangle$ AMPLITUDES

In this appendix we give the isospin decomposition of matrix elements that are relevant to our analysis. The operator that creates a B meson containing a $\overline{b}$ quark transforms as a 2 of isospin-SU(2). The $\Delta B=+1$ Hamiltonian, which has the flavor structure $(\overline{b}s)(\overline{q_i}q_i)$, transforms as either $2\times 2=1+3$ for $q_i=u,d$ or 1 for $q_i=s$. For final isospin-doublet states, the nonprimed (primed) isospin amplitudes correspond to the $K^I K^L$ subsystem being in an isospin-singlet (-triplet) state. A similar form of decomposition holds for the U-spin amplitudes in $B^+\to h_i^+(p_1)h_j^-(p_2)h_l^+(p_3)$ decays (where $h_{i,l}^+=K^+,\pi^+$ and $h_j^-=\pi^-,K^-$). They are given in Table IV.

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