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Rare top quark decays $t \to cV$ in the top-color-assisted technicolor model

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We consider the rare top quark decays in the framework of the top-color-assisted technicolor (TC2) model. We find that the contributions of top-pions and top-Higgs-bosons predicted by the TC2 model can enhance the SM branching ratios by as much as 6–9 orders of magnitude; i.e., in the extreme case, the orders of magnitude of branching ratios are $Br(t \to cg) \sim 10^{-5}$, $Br(t \to cZ) \sim 10^{-5}$, $Br(t \to c\gamma) \sim 10^{-7}$. With reasonable values of the parameters in the TC2 model, such rare top quark decays may be testable in future experiments. Therefore, rare top quark decays provide a unique way to test the TC2 model.

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I. INTRODUCTION

It is widely believed that the top quark, with a mass of the order of the electroweak scale, plays an important role in particle physics. Its unusually large mass makes it more sensitive to certain types of flavor-changing (FC) interactions.

In the standard model (SM), because of the Glashow-Iliopoulos-Maiani (GIM) mechanism, the rare top quark decays $t \to cV$ $(V=Z,\gamma,g)$ are very small [1], far below the feasible experimental possibilities at the future colliders Large Hadron Collider (LHC) or Linear Collider (LC) [2]. In some new physics models beyond the SM, the decay widths of the rare top quark decays $t \to cV$ may be significantly enhanced because of the appearance of large flavor changing couplings at the tree level. Various rare top quark decays have been extensively studied in the SM [1], multi-Higgs-doublet model (MHDM) [3–5], technicolor model [6,11], minimal supersymmetric standard model (MSSM) [7–9], and other new physics models. They have shown that, with reasonable values for the parameters, the branching ratios $Br(t \to cV)$ could be within the observable threshold of future experiments.

The top-color-assisted technicolor (TC2) model [10] connects the top quark with electroweak symmetry breaking (EWSB). In this model, the top-color interactions make small contributions to the EWSB and give rise to the main part of the top quark mass $(1-\epsilon)m_t$ with a model dependent parameter $0.03 \leqslant \epsilon \leqslant 0.1$. The TC interactions play a main role in the breaking of the electroweak gauge symmetry. The extend TC (ETC) interactions give rise to the masses of the ordinary fermions including a very small portion of the top quark mass ϵm_t. This kind of model predicts three top-pions $(\Pi_t^0,\Pi_t^\pm)$ and one top-Higgs-boson (h_t) with large Yukawa couplings to the third generation. These new particles can be regarded as a typical feature of the TC2 model. Thus, studying the possible signature of these particles and their contributions to some processes at high energy colliders is a good method of testing the TC2 model. There have been many publications related to this field [11–13]. Another feature of the TC2 model is the existence of large flavor-changing couplings. For TC2 models, top-color interactions are non-universal and therefore do not possess a GIM mechanism, which results in a new flavor-changing coupling vertices when one writes the interactions in the quark mass eigenbasis. Thus, the top-pions and top-Higgs-bosons predicted by this kind of models have large Yukawa couplings to the third generation and can induce the new flavor-changing couplings. Such flavor-changing couplings would give contributions to the rare decays $t \to cV$. Because the rare top quark decays $t \to cV$ can hardly be detected in the SM, any observation of rare top quark decays would be an unambiguous signal of new physics. So the study of rare top quark decays within the framework of the TC2 model would be a feasible method to test the TC2 model. Reference [11] has considered the contributions of these particles to the rare top quark decay $t \to cg$. However, Ref. [11] only considered the contributions of neutral top-pions Π_t^0 and did not consider the contributions of the charged top-pions $\Pi_t^\pm$. In this paper, we systematically calculate the contributions of the top-pions $(\Pi_t^0,\Pi_t^\pm)$ and top-Higgs-bosons (h_t) to the rare top quark decays $t \to cV$ in the TC2 model, and find that the TC2 model can significantly enhanced the rare top quark decays $t \to cV$, and may approach the detectability threshold of future experiments.

II. RARE TOP QUARK DECAYS $t \to cV$ IN THE TC2 MODEL

The TC2 model predicts that the existence of the top-pions $\Pi_t^0,\Pi_t^\pm$, top-pions would give new flavor changing couplings at the tree level. The relevant interactions of these top-pions with the b, t and c quarks can be written as [10,12]

$$\frac{m_t}{\sqrt{2}F_t}\frac{\sqrt{v_\omega^2-F_t^2}}{v_\omega}[iK_{UR}^{tt}K_{UL}^{tt*}\overline{t_L}t_R\Pi_t^0+\sqrt{2}K_{UR}^{tt}K_{DL}^{bb*}\overline{b_L}t_R\Pi_t^- + iK_{UR}^{tc}K_{UL}^{tt*}\overline{t_L}c_R\Pi_t^0+\sqrt{2}K_{UR}^{tc}K_{DL}^{bb*}\overline{b_L}c_R\Pi_t^- + \text{H.c.}], \tag{1}$$

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where $v_\omega = v/\sqrt{2} \approx 174$ GeV, and F_t is the decay constant of the top-pions. K_{UL}^{ij} are the matrix elements of the unitary matrix K_{UL} from which the Cabibbo-Kobayashi-Maskawa (CKM) matrix can be derived as $V = K_{UL}^{-1} K_{DL}$, and K_{UR}^{ij} are the matrix elements of the right-handed rotation matrix K_{UR}. Their values can be written as

$$K_{UL}^{tt} = K_{DL}^{bb} = 1, \quad K_{UR}^{tt} = 1 - \epsilon, \quad K_{UR}^{tc} \leqslant \sqrt{2\epsilon - \epsilon^2}. \tag{2}$$

In the following calculation, we take $K_{UR}^{tc} = \sqrt{2\epsilon - \epsilon^2}$ and take ϵ as a free parameter.

The TC2 model also predicts a CP-even scalar h_t, called the top-Higgs-boson [12], which is a $\bar{t}t$ bound and analogous to the σ particle in low energy QCD. Its couplings to quarks are similar to that of the neutral top-pion except that the neutral top-pion is CP odd. All the Feynman rules of top-pions and top-Higgs-bosons relevant to $t \to cV$ are shown in Appendix A.

The above large Yukawa couplings will effect the rare top quark decays $t \to cV$. The relevant Feynman diagrams for the contributions of the top-pions and top-Higgs-bosons to the rare top quark decays $t \to cV$ are shown in Fig. 1. Using Eq. (1) and other relevant Feynman rules, we obtain the relative amplitudes of the rare top quark decays $t \to cV$:

$$M_V = \overline{u_c} L (F_{V1} \gamma^\mu + F_{V2} p_t^\mu + F_{V3} p_c^\mu) u_t \varepsilon_\mu(\lambda), \tag{3}$$

where $L = (1 - \gamma_5)/2$ is the left-handed projector. The expressions of F_{Vi} $(V = Z, \gamma, g,\ i = 1,2,3)$ in Eq. (3) can be obtained by straightforward calculations of the diagrams shown in Fig. 1. Because of $m_t \gg m_c(m_b)$, for the sake of simplicity, we have neglected the terms proportional to m_c, m_b in Eq. (3). It can be seen that each diagram actually contains ultraviolet divergences. Because there are no corresponding tree-level terms to absorb these divergences, all the ultraviolet divergences cancel in the effective vertex. Then, the widths of the rare top quark decays contributed by top-pions and top-Higgs-bosons can be written as

$$\Gamma(t \to cZ) = \frac{1}{16\pi m_t} \left(1 - \frac{M_Z^2}{m_t^2}\right) \frac{1}{8 M_Z^2} [F_{Z1}^2 (4 m_t^2 M_Z^2 - 8 M_Z^4 + 4 m_t^4) + F_{Z2}^2 (-3 m_t^4 M_Z^2 + m_t^6 + 3 m_t^2 M_Z^4 - M_Z^6) + F_{Z3}^2 (m_t^2 - M_Z^2)^3$$
$$+ (F_{Z1} \cdot F_{Z2}^* + F_{Z2} \cdot F_{Z1}^*)(-4 m_t^3 M_Z^2 + 2 m_t^5 + 2 M_Z^4 m_t) + (F_{Z2} \cdot F_{Z3}^* + F_{Z3} \cdot F_{Z2}^*)$$
$$\times (-3 m_t^4 M_Z^2 + m_t^6 + 3 m_t^2 M_Z^4 - M_Z^6) + 2 (F_{Z1} \cdot F_{Z3}^* + F_{Z3} \cdot F_{Z1}^*)(m_t^2 - M_Z^2)^2 m_t], \tag{4}$$

$$\Gamma(t \to c\gamma) = \frac{1}{16\pi m_t} \left[F_{\gamma 1}^2 m_t^2 - \frac{1}{2} F_{\gamma 2}^2 m_t^4 - \frac{1}{2} (F_{\gamma 1} F_{\gamma 2}^* + F_{\gamma 2} F_{\gamma 1}^*) m_t^3 - \frac{1}{4} (F_{\gamma 2} F_{\gamma 3}^* + F_{\gamma 3} F_{\gamma 2}^*) m_t^4 \right], \tag{5}$$

$$\Gamma(t \to cg) = \frac{1}{16\pi m_t} \left[F_{g1}^2 m_t^2 - \frac{1}{2} F_{g2}^2 m_t^4 - \frac{1}{2} (F_{g1} F_{g2}^* + F_{g2} F_{g1}^*) m_t^3 - \frac{1}{4} (F_{g2} F_{g3}^* + F_{g3} F_{g2}^*) m_t^4 \right], \tag{6}$$

where m_t and M_Z denote the masses of top quark and Z boson, respectively. The explicit expressions of the form factors $F_{\gamma i}, F_{Zi}, F_{gi}$ are given in Appendix B.

III. THE NUMERICAL RESULTS AND CONCLUSIONS

According to the above calculations, we can give the numerical results of the branching ratio of $t \to cV$ contributed by Π_t and h_t^0. In this paper, we adopt the branching ratios $Br(t \to cV)$ defined as [1]

$$Br(t \to cV) = \frac{\Gamma(t \to cV)}{\Gamma(t \to W^+ b)}. \tag{7}$$

Before numerical calculations, we need to specify the parameters involved. We take $m_t = 175$ GeV, $M_Z = 91.18$ GeV, $s_W^2 = \sin^2\theta_W = 0.23$, $\alpha_e = 1/128.9$, and $\alpha_s = 0.118$. Now, there are still four free parameters ϵ, $m_{\Pi_t^0}$, $m_{\Pi_t^\pm}, m_{h_t}$. Here ϵ is a model dependent parameter and we take it in the range of 0.03–0.1; $m_{\Pi_t^0}$, $m_{\Pi_t^\pm}, m_{h_t}$ denote the masses of neural top-pion Π_t^0, charged top-pion $\Pi_t^\pm$, top-Higgs-boson (h_t^0), respectively. As a result of the split of the $m_{\Pi_t^0}$ and $m_{\Pi_t^\pm}$ only coming from the electroweak interactions, the difference of $m_{\Pi_t^0}$ and $m_{\Pi_t^\pm}$ is very small and can be ignored. Here, we take $m_{\Pi_t^0} = m_{\Pi_t^\pm} = m_{\Pi_t}$. The mass of top-pions is estimated in [10]; the results show that the m_{Π_t} is allowed to be in the region of a few hundred GeV depending on the models. Estimating the contributions of top-pions to the rare top quark decays $t \to cV$, we take the mass of the top-pion to vary in the range of 200–500 GeV in this paper. The mass of h_t can be estimated in the Nambu–Jona-Lasinio (NJL) model in the large N_c approximation and is found to be of the order of $m_{h_t} \approx 2 m_t$ [12,14]. This estimation is rather crude and the masses well below the $\bar{t}t$ threshold are quite possible and occur in a variety of cases [15]. As the branching ratios are proportional to $(2\epsilon - \epsilon^2)(1 - \epsilon)^2$, to cancel the influence of ϵ on the branching ratio, we summarized the

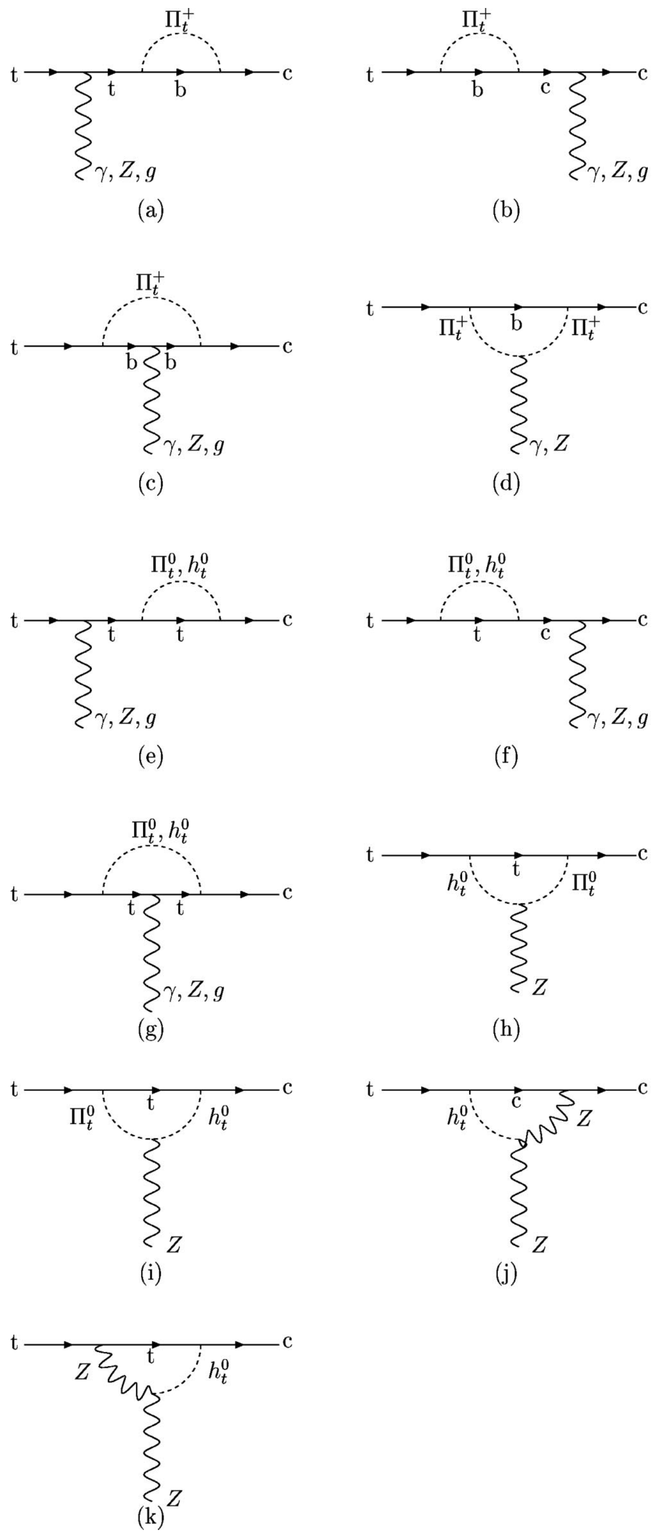

FIG. 1. The Feynman diagrams for the contributions of the top-pions ($\Pi_t^0, \Pi_t^{\pm}$) and top-Higgs-bosons (h_t) to the rare top quark decays $t \to cV$.

final numerical results of $Br(t \to cV)/(2\epsilon - \epsilon^2)(1-\epsilon)^2$ in Figs. 2–4.

Figures 2–4 are plots of $Br(t \to cV)/(2\epsilon - \epsilon^2)(1-\epsilon)^2$ versus m_{Π_t}(200–500 GeV) for $m_{h_t} = 200$ GeV, 250 GeV, 300 GeV, respectively. We can see that the branching ratio of

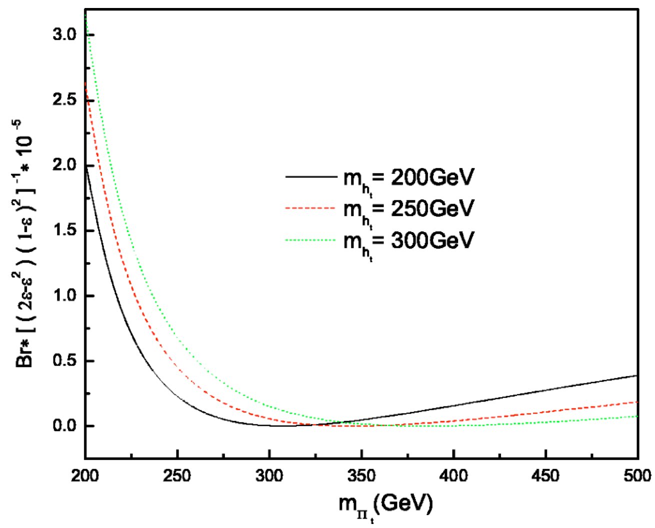

FIG. 2. The branching ratio $Br(t \to c\gamma)/(2\epsilon - \epsilon^2)(1-\epsilon)^2$ as a function of top-pion mass m_{Π_t} for the mass of top-Higgs-bosons $m_{h_t} = 200$ GeV (solid line), $m_{h_t} = 250$ GeV (dashed line), $m_{h_t} = 300$ GeV (dotted line), respectively.

$t \to c\gamma$ is two orders of magnitude smaller than that of $t \to cZ$ and $t \to cg$. The $Br(t \to c\gamma)$ decreases as m_{Π_t} increase and m_{h_t} decreases for small m_{Π_t}, but for large m_{Π_t}, it increases with m_{Π_t} increasing and m_{h_t} decreasing. The $Br(t \to cZ)$ is very sensitive to the top-pion mass and decreases with m_{Π_t} and m_{h_t} increasing. But for very large m_{Π_t}, the branching ratio of $t \to cZ$ hardly changes with the m_{h_t}. As for $t \to cg$, the branching ratio decreases very sharply as m_{Π_t} increases for small m_{Π_t}. In most cases, the orders of magnitude of the branching ratios are $Br(t \to cg) \sim 10^{-5}$, $Br(t \to cZ) \sim 10^{-5}$, $Br(t \to c\gamma) \sim 10^{-7}$.

Comparing with the theoretical predictions in the other models, we list the maximum levels of $Br(t \to cV)$ predicted by the SM [1], the MSSM [5] and the TC2 model in Table I.

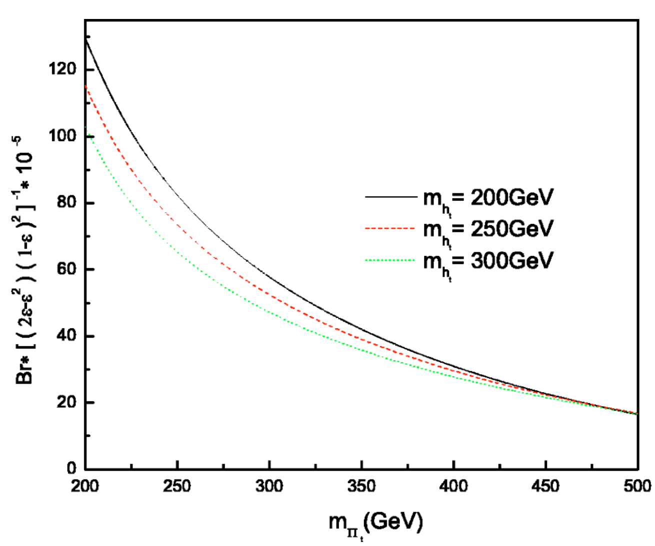

FIG. 3. The same as Fig. 2 but for the process of $t \to cZ$.

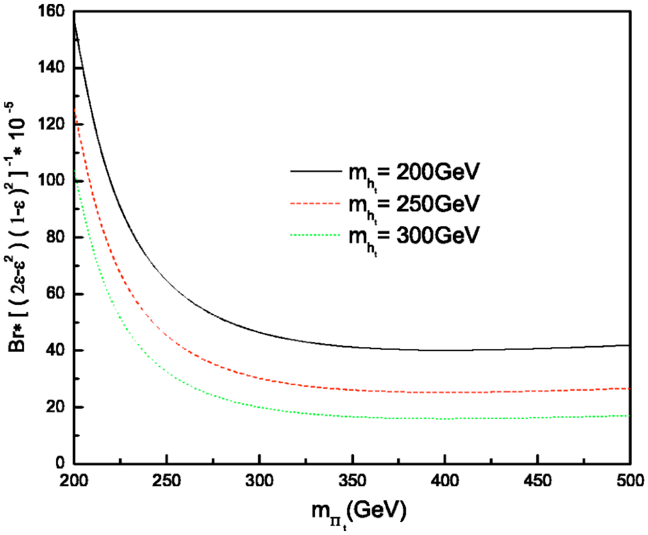

FIG. 4. The same as Fig. 2 but for the process of $t\rightarrow cg$.

It is shown that the branching ratios of $t\rightarrow cV$ in the TC2 model are significantly larger than that in the SM and MSSM. The contributions of Π_t and h_t^0 can enhance the SM branching ratios by as much as 6–9 orders of magnitude. On the other hand, $Br(t\rightarrow cZ)$ predicted by the TC2 model is about 3 orders of magnitude larger than that predicted by MSSM. So the mode of $t\rightarrow cZ$ is especially important for us to distinguish the TC2 model from the MSSM.

To assess the discovery reach of the rare top quark decays in future high energy colliders, Ref. [16] has roughly estimated the following sensitivities:

Run II (for 100 fb^{-1} of integrated luminosity):

$$Br(t\rightarrow q(u,c)\gamma)\geqslant 8.4\times 10^{-5},$$

$$Br(t\rightarrow q(u,c)Z)\geqslant 6.3\times 10^{-4};$$

Run II (for 10 fb^{-1} of integrated luminosity):

$$Br(t\rightarrow q(u,c)\gamma)\geqslant 4\times 10^{-4},$$

$$Br(t\rightarrow q(u,c)Z)\geqslant 3.8\times 10^{-3};$$

LHC (for 100 fb^{-1} of integrated luminosity):

$$Br(t\rightarrow q(u,c)\gamma)\geqslant 10^{-4}(\mathrm{ATLAS}),$$

$$Br(t\rightarrow q(u,c)\gamma)\geqslant 3.4\times 10^{-5}(\mathrm{MS}),$$

$$Br(t\rightarrow q(u,c)Z)\geqslant 2\times 10^{-4};$$

LC (for 100 fb^{-1} of integrated luminosity):

$$Br(t\rightarrow cV)\geqslant 5\times 10^{-4}. \tag{8}$$

Comparing the theoretical predictions in the TC2 model with the sensitivities of future high luminosity colliders (LHC, LC, TEV33), we can conclude that the TC2 model can enhance the branching ratios $Br(t\rightarrow cV)$ to be within the observable threshold of future experiments, especially for $t\rightarrow cZ$. LHC seems to be the most suitable collider where to test rare top quark decays. The LC is limited by statistics but in compensation every collected event is clear-cut. So this machine could eventually be of much help, especially for high luminosity.

In conclusion, we have calculated the rare top quark decays $t\rightarrow cV$ in the TC2 model. We find that the contributions arising from Π_t and h_t predicted by the TC2 model indeed significantly enhance the branching ratios of the rare top quark decays. The channels $t\rightarrow cZ$ and $t\rightarrow cg$ are found to have the larger branching ratios, which can reach 10^{-4} for the favorable parameter values and may be detectable in future high energy colliders. Therefore, the rare top quark decays provide us a unique way to test the TC2 model. Otherwise, $Br(t\rightarrow cZ)$ predicted by the TC2 model is much larger than that predicted by the MSSM. So we can distinguish the TC2 model from the MSSM via the $t\rightarrow cZ$ mode.

TABLE I. Theoretical predictions for branching ratios of the rare top quark decays $t\rightarrow cV$.

	SM	MSSM	TC2
$Br(t\rightarrow cZ)$	$O(10^{-13})$	$O(10^{-7})$	$O(10^{-4})$
$Br(t\rightarrow c\gamma)$	$O(10^{-13})$	$O(10^{-7})$	$O(10^{-6})$
$Br(t\rightarrow cg)$	$O(10^{-11})$	$O(10^{-4})$	$O(10^{-4})$

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APPENDIX A: THE FEYNMAN RULES NEEDED IN THE CALCULATIONS

Based on the effective Yukawa couplings to ordinary fermions of the top-pions and top-Higgs-bosons in the TC2 model, we can write down the relevant Feynman rules used in this paper:

$$\Pi_t^0 \bar{t}t:-\frac{m_t}{\sqrt{2}F_t}\frac{\sqrt{v_\omega^2-F_t^2}}{v_\omega}(1-\epsilon)\gamma_5, \tag{A1}$$

$$\Pi_t^0 \bar{t}c:\frac{m_t}{\sqrt{2}F_t}\frac{\sqrt{v_\omega^2-F_t^2}}{v_\omega}\frac{1-\gamma_5}{2}\sqrt{2\epsilon-\epsilon^2}, \tag{A2}$$

$$\Pi_t^+ \bar{t}b:i\sqrt{2}\frac{m_t}{\sqrt{2}F_t}\frac{\sqrt{v_\omega^2-F_t^2}}{v_\omega}\frac{1+\gamma_5}{2}(1-\epsilon), \tag{A3}$$

$$\Pi_t^0\overline{b}c: i\sqrt{2}\frac{m_t}{\sqrt{2}F_t}\frac{\sqrt{v_\omega^2-F_t^2}}{v_\omega}\frac{1-\gamma_5}{2}\sqrt{2\epsilon-\epsilon^2}, \tag{A4}$$

$$h_t^0\overline{t}t: \frac{im_t}{\sqrt{2}F_t}\frac{\sqrt{v_\omega^2-F_t^2}}{v_\omega}(1-\epsilon), \tag{A5}$$

$$h_t^0\overline{t}c: \frac{im_t}{\sqrt{2}F_t}\frac{\sqrt{v_\omega^2-F_t^2}}{v_\omega}\frac{1-\gamma_5}{2}\sqrt{2\epsilon-\epsilon^2}, \tag{A6}$$

$$Zh_t^0\Pi_t^0: \frac{g}{2c_W}(p_1-p_2)_\mu, \tag{A7}$$

$$ZZh_t^0: i\frac{F_t}{v_\omega}\frac{gM_Z}{c_W}g_{\mu\nu}. \tag{A8}$$

Here $g=e/2c_W$, and $c_W=\cos\theta_W$ is the Weinberg angle.

APPENDIX B: THE EXPLICIT EXPRESSIONS OF THE FORM FACTORS F_{Vi}

The explicit expressions of the form factors F_{Vi} used in Eqs. (3)–(6) can be written as

$$F_{Zi}=k_Z\sum_{\alpha=a}^{i}F_{Zi}^{\alpha}+k_Z'\sum_{\beta=j}^{k}F_{Zi}^{\beta}, \quad F_{\gamma i}=k_\gamma\sum_{\alpha=a}^{i}F_{\gamma i}^{\alpha},$$

$$F_{gi}=k_g\sum_{\alpha=a}^{i}F_{gi}^{\alpha}. \tag{B1}$$

Here, $\alpha=a,b,c,d,e,f,g,h,i$ and $\beta=j,k$ denote each Feynman diagrams in Fig. 1. $F_{Vi}^{\alpha}(V=\gamma,Z,gi=1,2,3)$ are the contributions arising from corresponding Feynman diagrams:

$$F_{Z1}^{b}=\frac{8}{3}s_W^2(B_0^b+B_1^b), \tag{B2}$$

$$F_{Z1}^{c}=2\left(1-\frac{2}{3}s_W^2\right)\Big[-m_t^2(C_{11}^c-C_{12}^c+C_{21}^c+C_{22}^c-2C_{23}^c)$$
$$+(m_t^2-M_Z^2)(C_{22}^c-C_{23}^c)-2C_{24}^c+\frac{1}{2}\Big], \tag{B3}$$

$$F_{Z2}^{c}=-4\left(1-\frac{2}{3}s_W^2\right)m_t(C_{22}^c-C_{23}^c), \tag{B4}$$

$$F_{Z3}^{c}=4\left(1-\frac{2}{3}s_W^2\right)m_t(C_{11}^c-C_{12}^c+C_{21}^c+C_{22}^c-2C_{23}^c), \tag{B5}$$

$$F_{Z1}^{d}=4(1-2s_W^2)C_{24}^d, \tag{B6}$$

$$F_{Z2}^{d}=-2(1-2s_W^2)m_t(4C_{23}^d-2C_{22}^d-2C_{21}^d+C_{12}^d-C_{11}^d), \tag{B7}$$

$$F_{Z3}^{d}=-2(1-2s_W^2)m_t(2C_{22}^d-2C_{23}^d+C_{12}^d-C_{11}^d), \tag{B8}$$

$$F_{Z1}^{e}=\frac{4}{3}s_W^2(B_0^e-B_0^{*e}), \tag{B9}$$

$$F_{Z1}^{f}=\frac{4}{3}s_W^2(B_1^f+B_1^{*f}+2B_0^{*f}), \tag{B10}$$

$$F_{Z1}^{g}=\left(1-\frac{4}{3}s_W^2\right)[m_t^2(C_{22}^g-2C_{23}^g+C_{21}^g$$
$$+C_{22}^{*g}-2C_{23}^{*g}+C_{21}^{*g})+(2C_{24}^g+2C_{24}^{*g}-1)]$$
$$-\frac{4}{3}s_W^2m_t^2(C_{11}^g-C_{12}^g+C_{11}^{*g}-C_{12}^{*g})$$
$$-\left(1-\frac{4}{3}s_W^2\right)(m_t^2-M_Z^2)(C_{22}^g-C_{23}^g+C_{22}^{*g}-C_{23}^{*g})$$
$$+\frac{8}{3}s_W^2m_t^2C_0^{*g}, \tag{B11}$$

$$F_{Z2}^{g}=-2\left(1-\frac{4}{3}s_W^2\right)m_t(C_{21}^g+C_{22}^g-2C_{23}^g+C_{21}^{*g}+C_{22}^{*g}$$
$$-2C_{23}^{*g}-2C_{12}^{*g}-2C_{11}^{*g}), \tag{B12}$$

$$F_{Z3}^{g}=-2\left(1-\frac{4}{3}s_W^2\right)m_t(C_{23}^g-C_{22}^g+C_{23}^{*g}-C_{22}^{*g})$$
$$-\frac{8}{3}s_W^2m_t(C_{12}^g-C_{12}^{*g}), \tag{B13}$$

$$F_{Z1}^{h}=-2C_{24}^h, \tag{B14}$$

$$F_{Z2}^{h}=m_t(4C_{23}^h-2C_{22}^h-2C_{21}^h+C_0^h-C_{12}^h+C_{11}^h), \tag{B15}$$

$$F_{Z3}^{h}=m_t(2C_{22}^h-2C_{23}^h+3C_{12}^h-C_{11}^h+C_0^h), \tag{B16}$$

$$F_{Z1}^{i}=-2C_{24}^i, \tag{B17}$$

$$F_{Z2}^{i}=m_t(4C_{23}^i-2C_{22}^i-2C_{21}^i-C_0^i+3C_{12}^i-3C_{11}^i), \tag{B18}$$

$$F_{Z3}^{i}=m_t(2C_{22}^i-2C_{23}^i-C_{12}^i-C_{11}^i-C_0^i), \tag{B19}$$

$$F_{Z1}^{j}=\frac{4}{3}s_W^2m_t(C_{12}^j-C_{11}^j), \tag{B20}$$

$$F_{Z3}^{j}=-\frac{8}{3}s_W^2C_{12}^j, \tag{B21}$$

$$F_{Z1}^{k}=m_t\left[\left(1-\frac{4}{3}s_W^2\right)(C_{12}^k-C_{11}^k)+\frac{4}{3}s_W^2C_0^k\right], \tag{B22}$$

$$F_{Z2}^{k}=-2\left(1-\frac{4}{3}s_W^2\right)(C_{12}^k-C_{11}^k), \tag{B23}$$

$$F^b_{\gamma 1}=-\frac{4}{3}(B^b_0+B^b_1), \tag{B24}$$

$$F^c_{\gamma 1}=\frac{2}{3}\left[-m_t^2(C^c_{11}-C^c_{12}+C^c_{21}-C^c_{23})-2C^c_{24}+\frac{1}{2}\right], \tag{B25}$$

$$F^c_{\gamma 2}=\frac{4}{3}m_t(C^c_{11}-C^c_{12}+C^c_{21}+C^c_{22}-2C^c_{23}), \tag{B26}$$

$$F^c_{\gamma 3}=-\frac{4}{3}m_t(C^c_{22}-C^c_{23}), \tag{B27}$$

$$F^d_{\gamma 1}=4C^d_{24}, \tag{B28}$$

$$F^d_{\gamma 2}=-2m_t(4C^d_{23}-2C^d_{22}-2C^d_{21}+C^d_{12}-C^d_{11}), \tag{B29}$$

$$F^d_{\gamma 3}=-2m_t(2C^d_{22}-2C^d_{23}+C^d_{12}-C^d_{11}), \tag{B30}$$

$$F^e_{\gamma 1}=-\frac{2}{3}(B^e_0-B^{*e}_0), \tag{B31}$$

$$F^f_{\gamma 1}=-\frac{2}{3}(B^f_1+B^{*f}_1+2B^{*f}_0), \tag{B32}$$

$$F^g_{\gamma 1}=-\frac{2}{3}[-m_t^2(C^g_{21}-C^g_{23}+C^g_{11}-C^g_{12}+C^{*g}_{21}-C^{*g}_{23}+C^{*g}_{11} -C^{*g}_{12})+(-2C^g_{24}-2C^{*g}_{24}+1)]-\frac{4}{3}m_t^2C^{*g}_0, \tag{B33}$$

$$F^g_{\gamma 2}=-\frac{4}{3}m_t(C^g_{21}+C^g_{22}-2C^g_{23}+2C^{*g}_{11}+C^{*g}_{22}-2C^{*g}_{23} +C^{*g}_{21}-2C^{*g}_{12}), \tag{B34}$$

$$F^g_{\gamma 3}=-\frac{4}{3}m_t[C^g_{23}-C^g_{22}-C^g_{12}+C^{*g}_{23}-C^{*g}_{22}+C^{*g}_{12}], \tag{B35}$$

$$F^b_{g1}=-2(B^b_0+B^b_1), \tag{B36}$$

$$F^c_{g1}=-2\left[-m_t^2(C^c_{11}-C^c_{12}+C^c_{21}-C^c_{23})-2C^c_{24}+\frac{1}{2}\right], \tag{B37}$$

$$F^c_{g2}=-4m_t(C^c_{11}-C^c_{12}+C^c_{21}+C^c_{22}-2C^c_{23}), \tag{B38}$$

$$F^c_{g3}=4m_t(C^c_{22}-C^c_{23}), \tag{B39}$$

$$F^e_{g1}=-(B^e_0-B^{*e}_0), \tag{B40}$$

$$F^f_{g1}=-(B^f_1+B^{*f}_1+2B^{*f}_0), \tag{B41}$$

$$F^g_{g1}=m_t^2(C^g_{21}-C^g_{23}+C^g_{11}-C^g_{12}+C^{*g}_{21}-C^{*g}_{23}+C^{*g}_{11}-C^{*g}_{12}) -2C^g_{24}-2C^{*g}_{24}+1-2m_t^2C^{*g}_0, \tag{B42}$$

$$F^g_{g2}=-2m_t(C^g_{21}+C^g_{22}-2C^g_{23}+2C^{*g}_{11}+C^{*g}_{22}-2C^{*g}_{23}+C^{*g}_{21} -2C^{*g}_{12}), \tag{B43}$$

$$F^g_{g3}=-2m_t[C^g_{23}-C^g_{22}+C^{*g}_{23}-C^{*g}_{22}+C^g_{12}-C^{*g}_{12}], \tag{B44}$$

$$k_Z=-\frac{i}{16\pi^2}\frac{m_t^2}{2F_t^2}\frac{\upsilon_\omega^2-F_t^2}{\upsilon_\omega^2}\sqrt{2\epsilon-\epsilon^2}(1-\epsilon)\frac{g}{2c_W},$$

$$k'_Z=-\frac{i}{32\pi^2}\frac{m_tM_Z}{\sqrt{2}\upsilon_\omega}\frac{\sqrt{\upsilon_\omega^2-F_t^2}}{\upsilon_\omega}\frac{g^2}{c_W^2},$$

$$k_\gamma=-\frac{i}{16\pi^2}\frac{m_t^2}{2F_t^2}\frac{\upsilon_\omega^2-F_t^2}{\upsilon_\omega^2}\sqrt{2\epsilon-\epsilon^2}(1-\epsilon)e,$$

$$k_g=-\frac{i}{16\pi^2}\frac{m_t^2}{2F_t^2}\frac{\upsilon_\omega^2-F_t^2}{\upsilon_\omega^2}\sqrt{2\epsilon-\epsilon^2}(1-\epsilon)g_sT^\alpha, \tag{B45}$$

$$B^b_i=B_i(-p_t,m_{\Pi_t^\pm},m_b),\quad B^e_i=B_n(-p_c,m_{\Pi_t^0},m_t),$$

$$B^{*e}_i=B_i(-p_c,m_{h_t},m_t),\quad B^f_i=B_i(-p_t,m_{\Pi_t^0},m_t),$$

$$B^{*f}_i=B_i(-p_t,m_{h_t},m_t),$$

$$C^c_{ij}=C_{ij}(-p_t,p_V,m_{\Pi_t^\pm},m_b,m_b),$$

$$C^d_{ij}=C_{ij}(-p_t,p_V,m_b,m_{\Pi_t^\pm},m_{\Pi_t^\pm}),$$

$$C^g_{ij}=C_{ij}(-p_t,p_V,m_{\Pi_t^0},m_t,m_t),$$

$$C^{*g}_{ij}=C_{ij}(-p_t,p_V,m_{h_t},m_t,m_t),$$

$$C^h_{ij}=C_{ij}(-p_t,p_V,m_t,m_{h_t},m_{\Pi_t^0}),$$

$$C^i_{ij}=C_{ij}(-p_t,p_V,m_t,m_{\Pi_t^0},m_{h_t}),$$

$$C^j_{ij}=C_{ij}(-p_t,p_V,m_t,m_{h_t},M_Z),$$

$$C^k_{ij}=C_{ij}(-p_t,p_V,m_t,M_Z,m_{h_t}). \tag{B46}$$

Here $g_s=\sqrt{4\pi\alpha_s}$, T^α are the Gell-Mann $SU(3)_c$ matrices. B_i, C_{ij} are two-point and there-point scalar integrals. p_V represents the momenta of Z, γ, g, respectively.

[1] G. Eilam, J.L. Hewett, and A. Soni, Phys. Rev. D **44**, 1473 (1991).

[2] R. Frey *et al.*, Report No. FERMIAB-CONF-97-085, hep-ph/9704243; J.A. Aguilar-Saavedra and G.C. Branco, Phys. Lett. B **495**, 347 (2000); J.A. Aguilar-Saavedra, *ibid.* **502**, 115 (2001).

[3] J.L. Diáz-Gruz *et al.*, Phys. Rev. D **41**, 891 (1990).

[4] B. Grzadfkowski, J.F. Gunion, and P. Krawcyzk, Phys. Lett. B **268**, 106 (1991); W.S. Hou, *ibid.* **296**, 179 (1992).

[5] R.A. Diaz, R. Martine, and J. Alexis Rodriguez, hep-ph/0103307.

[6] X.L. Wang *et al.*, Phys. Rev. D **50**, 5781 (1994).

[7] C.S. Li, R.J. Oakes, and J.M. Yang, Phys. Rev. D **49**, 293 (1994).

[8] C.S. Li, X. Zhang, and S.H. Zhu, Phys. Rev. D **60**, 077702 (1999).

[9] J.J. Cao, Z. Xiong, and J.M. Yang, Phys. Rev. D **67**, 035002 (2003).

[10] C.T. Hill, Phys. Lett. B **345**, 483 (1995); K. Lane and E. Eichten, *ibid.* **352**, 382 (1995); K. Lane, *ibid.* **433**, 96 (1998).

[11] C.X. Yue, G.R. Lu, G.L. Liu, and Q.J. Xu, Phys. Rev. D **64**, 095004 (2001).

[12] Hong-Jian He and C.P. Yuan, Phys. Rev. Lett. **83**, 28 (1999); G. Burdman, *ibid.* **83**, 2888 (1999).

[13] X.L. Wang, Y.L. Yang, B.Z. Li, C.X. Yue, and J.Y. Zhang, Phys. Rev. D **66**, 075009 (2002); X.L. Wang, Y.L. Yang, B.Z. Li, and L.D. Wan, *ibid.* **66**, 075013 (2002); J.J. Cao, Z. Xiong, and J.M. Yang, *ibid.* **67**, 071701(R) (2003).

[14] C.X. Yue, Y.P. Kuang, X.L. Wang, and W.B. Li, Phys. Rev. D **62**, 055005 (2000); C.T. Hill and E.H. Simmons, hep-ph/0203079.

[15] R.S. Chivukula, B. Dobrescu, H. Georgi, and C.T. Hill, Phys. Rev. D **59**, 075003 (1999).

[16] R.A. Diaz, R. Martinez, and J.A. Rodriguez, hep-ph/010330; E.H. Simmons, hep-ph/0211335; S. Bejar, J. Guasch, and J. Sola, hep-ph/0101294.