Hadronic charmed meson decays involving tensor mesons

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Charmed meson decays into a pseudoscalar meson P and a tensor meson T are studied. The charm to tensor meson transition form factors are evaluated in the Isgur-Scora-Grinstein-Wise (ISGW) quark model. It is shown that the Cabibbo-allowed decay $D_s^+ \to f_2(1270)\pi^+$ is dominated by the W-annihilation contribution and has the largest branching ratio in $D \to TP$ decays. We argue that the Cabibbo-suppressed mode $D^+ \to f_2(1270)\pi^+$ should be suppressed by one order of magnitude relative to $D_s^+ \to f_2(1270)\pi^+$. When the finite width effect of the tensor resonances is taken into account, the decay rate of $D \to TP$ is generally enhanced by a factor of 2–3. Except for $D_s^+ \to f_2(1270)\pi^+$, the predicted branching ratios of $D \to TP$ decays are in general too small by one to two orders of magnitude compared to experiment. However, it is very unlikely that the $D \to T$ transition form factors can be enhanced by a factor of 3–5 within the ISGW quark model to account for the discrepancy between theory and experiment. As many of the current data are still preliminary and lack sufficient statistic significance, more accurate measurements are needed to pin down the issue.

I. INTRODUCTION

Cabibbo-allowed and Cabibbo-suppressed two-body hadronic D decays into a pseudoscalar meson P and a tensor meson T have been studied in [1] and [2], respectively. In both studies, the charm to tensor meson transition form factors are calculated using the Isgur-Scora-Grinstein-Wise (ISGW) quark model [3]. The calculated branching ratios are of the order of 10^{-5}–10^{-7}. Recently, the Cabibbo-allowed mode $D_s^+ \to f_2(1270)\pi^+$ and the Cabibbo-suppressed one $D^+ \to f_2(1270)\pi^+$ both have been measured by E791 at the level of 10^{-3} [4]. More recently, FOCUS [5] and BaBar [6] have also reported some new measurements of $D \to TP$ decays. Though their results are still preliminary and many of them do not have enough statistic significance (see Table I), the branching ratios are typically of the order of 10^{-3}. Therefore, it appears that there exists a large discrepancy between theory and experiment. It is thus important to understand the origin of the discrepancy.

In the present work, several improvements over the previous work [1,2] are made. First, the charm to tensor meson transition form factors will be calculated in the improved version of the ISGW model [7]. The updated version of this quark model gives a more realistic description of the form-factor momentum dependence, especially at small q^2. Second, the tensor meson has a width typically of order 100–200 MeV [8]. The finite width effect, which is very crucial to account for the decays such as $D \to K_2^*(1430)\bar{K}$ and $D \to f_2'(1525)\bar{K}$ that appear to be prohibited by kinematics at first sight, is carefully examined. Third, it is known that weak annihilation (W exchange or W annihilation) in charm decays can receive sizable contributions from nearby resonances through inelastic final-state interactions (see, e.g. [9]). Hence, it is important to take into account weak annihilation contributions.

This work is organized as follows. In Sec. II we summarize the current experimental measurements of $D \to TP$ decays. We discuss the various physical properties of the tensor mesons in Sec. III, such as the decay constants and the form factors, and then we analyze the $D \to TP$ decays in Sec. IV based on the generalized factorization approach in conjunction with final-state interactions. Conclusions are presented in Sec. V.

II. EXPERIMENTAL STATUS

It is known that three-body decays of heavy mesons provide a rich laboratory for studying the intermediate state resonances. The Dalitz plot analysis is a very useful technique for this purpose. We are interested in $D \to TP$ decays extracted from the three-body decays of charmed mesons. In addition to the earlier measurements by ARGUS [10] and E687 [11], some recent results are available from E791 [4], CLEO [12], FOCUS [5] and BaBar [6]. The $J^P = 2^+$ tensor mesons that have been studied in hadronic charm decays include $f_2(1270)$, $a_2(1320)$ and $K_2^*(1430)$. The results of various experiments are summarized in Table I where the product of $\mathcal{B}(D \to TP)$ and $\mathcal{B}(T \to P_1 P_2)$ is shown. In order to extract the branching ratios for the two-body decays $D \to TP$, we need to know the branching fractions of the strong decays of the tensor mesons [8]:

$$\mathcal{B}(f_2(1270) \to \pi\pi) = (84.7^{+2.4}_{-1.3})\%,$$

$$\mathcal{B}(f_2(1270) \to K\bar{K}) = (4.6 \pm 0.5)\%,$$

$$\mathcal{B}(a_2(1320) \to K\bar{K}) = (4.9 \pm 0.8)\%,$$

$$\mathcal{B}(K_2^*(1430) \to K\pi) = (49.9 \pm 1.2)\%. \qquad (2.1)$$

It is evident that most of the listed $D \to TP$ decays in Table I have branching ratios of order 10^{-3}, even though some of them are Cabibbo suppressed. Note that the results from FO-CUS and BaBar are still preliminary. Indeed, many of them have not yet sufficient statistical significance.

Note that at first sight it appears that the decay $D \to \bar{K}_2^*(1430)K$ is kinematically not allowed as the $K_2^*(1430)$

TABLE I. Experimental branching ratios of various $D \to TP$ decays measured by ARGUS, E687, E791, CLEO, FOCUS and BaBar. For simplicity and convenience, we have dropped the mass identification for $f_2(1270)$, $a_2(1320)$ and $K_2^*(1430)$.

Collaboration	$\mathcal{B}(D \to TP) \times \mathcal{B}(T \to P_1 P_2)$	$\mathcal{B}(D \to TP)$
E791	$\mathcal{B}(D^+ \to f_2 \pi^+)\mathcal{B}(f_2 \to \pi^+ \pi^-) = (6.0 \pm 1.1) \times 10^{-4}$	$\mathcal{B}(D^+ \to f_2 \pi^+) = (1.1 \pm 0.2) \times 10^{-3}$
FOCUS	$\mathcal{B}(D^+ \to f_2 \pi^+)\mathcal{B}(f_2 \to \pi^+ \pi^-) = (3.8 \pm 0.8) \times 10^{-5}$	$\mathcal{B}(D^+ \to f_2 \pi^+) = (6.8 \pm 1.4) \times 10^{-4}$
FOCUS	$\mathcal{B}(D^+ \to f_2 \pi^+)\mathcal{B}(f_2 \to K^+ K^-) = (7.0 \pm 1.9) \times 10^{-5}$	$\mathcal{B}(D^+ \to f_2 \pi^+) = (3.1 \pm 0.9) \times 10^{-3}$
E791	$\mathcal{B}(D_s^+ \to f_2 \pi^+)\mathcal{B}(f_2 \to \pi^+ \pi^-) = (2.0 \pm 0.7) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to f_2 \pi^+) = (3.5 \pm 1.2) \times 10^{-3}$
FOCUS	$\mathcal{B}(D_s^+ \to f_2 \pi^+)\mathcal{B}(f_2 \to \pi^+ \pi^-) = (1.0 \pm 0.3) \times 10^{-3}$	$\mathcal{B}(D_s^+ \to f_2 \pi^+) = (1.8 \pm 0.5) \times 10^{-3}$
ARGUS,E687	$\mathcal{B}(D^0 \to f_2 \bar{K}^0)\mathcal{B}(f_2 \to \pi^+ \pi^-) = (3.2 \pm 0.9) \times 10^{-3}$	$\mathcal{B}(D^0 \to f_2 \bar{K}^0) = (4.5 \pm 1.7) \times 10^{-3}$
CLEO	$\mathcal{B}(D^0 \to f_2 \bar{K}^0)\mathcal{B}(f_2 \to \pi^+ \pi^-) = (1.6^{+2.4}_{-1.3}) \times 10^{-3}$	$\mathcal{B}(D^0 \to f_2 \bar{K}^0) = (2.8^{+4.3}_{-2.3}) \times 10^{-3}$
FOCUS	$\mathcal{B}(D_s^+ \to f_2 K^+)\mathcal{B}(f_2 \to \pi^+ \pi^-) = (2.0 \pm 1.3) \times 10^{-4}$	$\mathcal{B}(D_s^+ \to f_2 K^+) = (3.5 \pm 2.3) \times 10^{-4}$
BaBar	$\mathcal{B}(D^0 \to a_2^- \pi^+)\mathcal{B}(a_2^- \to K^0 K^-) = (3.5 \pm 2.1) \times 10^{-5}$	$\mathcal{B}(D^0 \to a_2^- \pi^+) = (7.0 \pm 4.3) \times 10^{-4}$
E791	$\mathcal{B}(D^+ \to \bar{K}_2^{*0} \pi^+)\mathcal{B}(\bar{K}_2^{*0} \to K^- \pi^+) = (4.6 \pm 2.0) \times 10^{-4}$	$\mathcal{B}(D^+ \to \bar{K}_2^{*0} \pi^+) = (1.4 \pm 0.6) \times 10^{-3}$
CLEO	$\mathcal{B}(D^0 \to K_2^{*-} \pi^+)\mathcal{B}(K_2^{*-} \to \bar{K}^0 \pi^-) = (6.5^{+4.2}_{-2.2}) \times 10^{-4}$	$\mathcal{B}(D^0 \to K_2^{*-} \pi^+) = (2.0^{+1.3}_{-0.7}) \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \to K_2^{*+} K^-)\mathcal{B}(K_2^{*+} \to K^0 \pi^+) = (6.8 \pm 4.2) \times 10^{-4}$	$\mathcal{B}(D^0 \to K_2^{*+} K^-) = (2.0 \pm 1.3) \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \to \bar{K}_2^{*0} K^0)\mathcal{B}(\bar{K}_2^{*0} \to K^- \pi^+) = (6.6 \pm 2.7) \times 10^{-4}$	$\mathcal{B}(D^0 \to \bar{K}_2^{*0} K^0) = (2.0 \pm 0.8) \times 10^{-3}$

mass lies outside of the phase space for the decay. Nevertheless, it is physically allowed as $K_2^*(1430)$ has a decay width of order 100 MeV [8]. Likewise, the decay $D^0 \to f_2'(1525)\bar{K}^0$ is also allowed.

III. PHYSICAL PROPERTIES OF SCALAR MESONS

The observed $J^P = 2^+$ tensor mesons $f_2(1270)$, $f_2'(1525)$, $a_2(1320)$ and $K_2^*(1430)$ form an SU(3) $1\,^3P_2$ nonet. The $q\bar{q}$ content for isodoublet and isovector tensor resonances are obvious. Just as the η-η' mixing in the pseudoscalar case, the isoscalar tensor states $f_2(1270)$ and $f_2'(1525)$ also have a mixing and their wave functions are defined by

$$f_2(1270) = \frac{1}{\sqrt{2}}(f_2^u + f_2^d)\cos\theta + f_2^s \sin\theta,$$

$$f_2'(1525) = \frac{1}{\sqrt{2}}(f_2^u + f_2^d)\sin\theta - f_2^s \cos\theta, \qquad (3.1)$$

with $f_2^q \equiv q\bar{q}$. Since $\pi\pi$ is the dominant decay mode of $f_2(1270)$, whereas $f_2'(1525)$ decays predominantly into $K\bar{K}$ (see Particle Data Group [8]), it is obvious that this mixing angle should be small. More precisely, it is found $\theta = 7.8°$ [8,13]. Therefore, $f_2(1270)$ is primarily an $(u\bar{u}+d\bar{d})/\sqrt{2}$ state, while $f_2'(1525)$ is dominantly $s\bar{s}$.

The polarization tensor $\varepsilon_{\mu\nu}$ of a $^3P_2$ tensor meson with $J^{PC} = 2^{++}$ satisfies the relations

$$\varepsilon_{\mu\nu} = \varepsilon_{\nu\mu}, \quad \varepsilon^\mu_\mu = 0, \quad p_\mu \varepsilon^{\mu\nu} = p_\nu \varepsilon^{\mu\nu} = 0. \qquad (3.2)$$

Therefore,

$$\langle 0|(V-A)_\mu|T(\varepsilon,p)\rangle = a\varepsilon_{\mu\nu}p^\nu + b\varepsilon^\nu_\nu p_\mu = 0, \qquad (3.3)$$

and hence the decay constant of the tensor meson vanishes; that is, the tensor meson cannot be produced from the $V - A$ current.

As for the form factors, the $D \to P$ transition is defined by [14]

$$\langle P(p)|V_\mu|D(p_D)\rangle = \left(p_{D\mu} + p_\mu - \frac{m_D^2 - m_P^2}{q^2}q_\mu\right)F_1^{DP}(q^2)$$
$$+ \frac{m_D^2 - m_P^2}{q^2}q_\mu F_0^{DP}(q^2), \qquad (3.4)$$

where $q_\mu = (p_D - p)_\mu$, while the general expression for the $D \to T$ transition has the form [3]

$$\langle T(\varepsilon,p_T)|(V-A)_\mu|D(p_D)\rangle$$
$$= ih(q^2)\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu\alpha}p_{D\alpha}(p_D+p_T)^\rho(p_D-p_T)^\sigma$$
$$+ k(q^2)\varepsilon^*_{\mu\nu}p_D^\nu + b_+(q^2)\varepsilon^*_{\alpha\beta}p_D^\alpha p_D^\beta(p_D+p_T)_\mu$$
$$+ b_-(q^2)\varepsilon^*_{\alpha\beta}p_D^\alpha p_D^\beta(p_D-p_T)_\mu. \qquad (3.5)$$

The form factors h, k, b_+ and b_- can be calculated in the ISGW quark model [3] and its improved version, the ISGW2 model [7]. In general, the form factors evaluated in the ISGW model are reliable only at $q^2 = q_m^2 \equiv (m_D - m_T)^2$, the maximum momentum transfer. The reason is that the form-factor q^2 dependence in the ISGW model is proportional to $\exp[-(q_m^2 - q^2)]$ and hence the form factor decreases exponentially as a function of $(q_m^2 - q^2)$. This has been improved in the ISGW2 model in which the form factor has a more realistic behavior at large $(q_m^2 - q^2)$ which is expressed in terms of a certain polynomial term.

The calculated $D \to T$ form factors are listed in Table II. The form factor $h(q^2)$ is not shown there as it does not contribute to the factorizable $D \to TP$ amplitudes. It is convenient to express the form factors for $(D, D_s^+) \to f_2(1270)$

TABLE II. The form factors at $q^2 = m_\pi^2$ calculated in the ISGW2 model, where k is dimensionless and b_+ and b_- are in units of GeV^{-2}. Shown in parentheses are the results obtained in the ISGW model.

Transition	k	b_+	b_-
$D \to f_2^n$	0.61 (0.51)	$-0.052\ (-0.083)$	0.064
$D_s^+ \to f_2^s$	1.15 (1.02)	$-0.079\ (-0.120)$	0.101
$D \to a_2(1320)$	0.61 (0.51)	$-0.052\ (-0.083)$	0.064
$D \to K_2^*(1430)$	0.71 (0.58)	$-0.060\ (-0.098)$	0.069

and $(D, D_s^+) \to f_2'(1525)$ in terms of $D \to f_2^n$ with n standing for the light non-strange quark (i.e. $D^0 \to f_2^u$ for $n = u$ and $D^+ \to f_2^d$ for $n = d$) and $D_s^+ \to f_2^s$ transition form factors. Note that $D \to f_2^s$ and $D_s^+ \to f_2^n$ are prohibited. In the calculations of $D \to T$ form factors we follow [13] to use the masses: $m_{f_2^q} = 1.32$ GeV and $m_{f_2^s} = 1.55$ GeV.

Two remarks are in order. (i) The magnitude of the form factors for the $D_s^+ \to f_2^s$ transition is larger than that for $D \to f_2^n$ owing to the larger constituent s quark mass than the u and d quarks. That is, SU(3) symmetry breaking in $D \to f_2^n$ and $D_s^+ \to f_2^s$ is sizable. (ii) The difference between ISGW and ISGW2 model predictions for form factors at $q^2 = 0$ is not significant for the charm case, though form factors in the

ISGW model fall more rapidly at small q^2. However, the difference will be dramatic for the $B \to T$ case as noticed in [15]. For example, the $B \to a_2$ and $B \to f_2(1370)$ form factors at $q^2 = m_D^2$ obtained in the ISGW2 model are about 2–6 times larger than that in the ISGW model. This is because the region covered from zero recoil to small q^2 in B decays is much bigger than that in D decays.

IV. $D \to TP$ DECAYS AND FACTORIZATION

We will study the $D \to TP$ decays (T: tensor meson; P: pseudoscalar meson) within the framework of generalized factorization in which the hadronic decay amplitude is expressed in terms of factorizable contributions multiplied by the *universal* (i.e. process independent) effective parameters a_i that are renormalization scale and scheme independent. More precisely, the weak Hamiltonian has the form

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cq_1} V_{uq_2}^* [a_1 (\bar{u} q_2)(\bar{q}_1 c) + a_2 (\bar{q}_1 q_2)(\bar{u} c)] + \text{H.c.},$$

(4.1)

with $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$. For hadronic charm decays, we shall use $a_1 = 1.15$ and $a_2 = -0.55$. Since the decay constant of tensor mesons vanishes, the factorizable amplitude of $D \to TP$ always has the expression

TABLE III. Quark-diagram amplitudes and branching ratios for various $D \to TP$ decays with and without the long-distance weak annihilation terms induced from final-state interactions. The W-annihilation amplitude A is fixed by fitting to the data of $D_s^+ \to f_2(1270)\pi^+$ [see Eq. (4.6)]. The W-exchange amplitude E is assumed to have the expression of Eq. (4.8) for the purpose of illustration. Experimental results are taken from Table I and from [8]. The finite width effect of the tensor resonances has been taken into account in theoretical calculations.

Decay	Amplitude	$\mathcal{B}_{\text{naive}}$	$\mathcal{B}_{\text{FSI}}$	$\mathcal{B}_{\text{expt}}$
$D^+ \to f_2(1270)\pi^+$	$V_{cd}V_{ud}^*(T + C + 2A)\cos\theta/\sqrt{2}$	2.9×10^{-5}	2.2×10^{-4}	$(0.9 \pm 0.1) \times 10^{-3}$
$D^0 \to f_2(1270)\bar{K}^0$	$V_{cs}V_{ud}^*(C + E)\cos\theta/\sqrt{2}$	1.0×10^{-4}	2.5×10^{-4}	$(4.5 \pm 1.7) \times 10^{-3}$
$D_s^+ \to f_2(1270)\pi^+$	$V_{cs}V_{ud}^*(T\sin\theta + 2A\cos\theta/\sqrt{2})$	6.6×10^{-5}	2.1×10^{-3}	$(2.1 \pm 0.5) \times 10^{-3}$
$\to f_2(1270)K^+$	$V_{cs}V_{us}^*[T\sin\theta + C'\sin\theta + A(\sin\theta + \cos\theta/\sqrt{2})]$	5.2×10^{-6}	4.9×10^{-5}	$(3.5 \pm 2.3) \times 10^{-4}$
$D^+ \to f_2'(1525)\pi^+$	$V_{cd}V_{ud}^*(T + C + 2A)\sin\theta/\sqrt{2}$	1.4×10^{-6}	3.7×10^{-6}	
$D^0 \to f_2'(1525)\bar{K}^0$	$V_{cs}V_{ud}^*(C + E)\sin\theta/\sqrt{2}$	2.5×10^{-7}	6.0×10^{-7}	
$D_s^+ \to f_2'(1525)\pi^+$	$V_{cs}V_{ud}^*(T\cos\theta - 2A\sin\theta/\sqrt{2})$	1.6×10^{-4}	1.5×10^{-4}	
$\to f_2'(1525)K^+$	$V_{cs}V_{us}^*[T\cos\theta + C'\cos\theta + A(\cos\theta - \sin\theta/\sqrt{2})]$	4.9×10^{-6}	7.5×10^{-6}	
$D^+ \to a_2^+(1320)\bar{K}^0$	$V_{cs}V_{ud}^*(T' + C)$	1.3×10^{-6}	1.3×10^{-6}	$< 3 \times 10^{-3}$
$D^0 \to a_2^+(1320)K^-$	$V_{cs}V_{ud}^*(T' + E)$	0	8.9×10^{-8}	$< 2 \times 10^{-3}$
$\to a_2^-(1320)\pi^+$	$V_{cd}V_{ud}^*(T + E)$	5.7×10^{-6}	6.1×10^{-6}	$(7.0 \pm 4.3) \times 10^{-4}$
$D^+ \to \bar{K}_2^{*0}(1430)\pi^+$	$V_{cs}V_{ud}^*(T + C')$	2.6×10^{-4}	2.6×10^{-4}	$(1.4 \pm 0.6) \times 10^{-3}$
$D^0 \to K_2^{*-}(1430)\pi^+$	$V_{cs}V_{ud}^*(T + E)$	1.0×10^{-4}	1.1×10^{-4}	$(2.0^{+1.3}_{-0.7}) \times 10^{-3}$
$\to \bar{K}_2^{*0}(1430)\pi^0$	$\frac{1}{\sqrt{2}} V_{cs}V_{ud}^*(C' + E)$	0	1.3×10^{-5}	$< 3.4 \times 10^{-3}$
$\to K_2^{*+}(1430)K^-$	$V_{cs}V_{us}^*(T' + E)$	0	1.3×10^{-6}	$(2.0 \pm 1.3) \times 10^{-3}$
$\to \bar{K}_2^{*0}(1430)K^0$	$V_{cs}V_{us}^*(E_d) + V_{cd}V_{ud}^*(E_s)$	0	$\sim 10^{-8}$	$(2.0 \pm 0.8) \times 10^{-3}$

$$A(D \to TP) = i \frac{G_F}{\sqrt{2}} V_{cq_1} V_{uq_2}^* f_P \varepsilon_{\mu\nu}^* p_D^\mu p_D^\nu [k(m_P^2) + b_+(m_P^2)$$

$$\times (m_D^2 - m_T^2) + b_-(m_P^2) m_P^2]$$

$$\equiv \varepsilon_{\mu\nu}^* p_D^\mu p_D^\nu M(D \to TP), \qquad (4.2)$$

where use has been made of Eq. (3.5). The decay rate is given by

$$\Gamma(D \to TP) = \frac{k_T^5}{12\pi m_T^2} \left(\frac{m_D}{m_T}\right)^2 |M(D \to TP)|^2, \quad (4.3)$$

where k_T is the c.m. momentum of the tensor meson in the rest frame of the charmed meson.

In terms of the topological amplitudes [16]: T, the color-allowed external W-emission tree diagram; C, the color-suppressed internal W-emission diagram; E, the W-exchange diagram; A, the W-annihilation diagram, the topological quark-diagram amplitudes of various $D \to TP$ decays are shown in Table III. There exist also penguin diagrams. However, the penguin contributions are negligible owing to the good approximation $V_{ud} V_{cd}^* \approx -V_{us} V_{cs}^*$ and the smallness of $V_{ub} V_{cb}^*$. For $D \to TP$ and $D \to PT$ decays, one can have two different external W-emission and internal W-emission dia-

grams, depending on whether the emission particle is a tensor meson or a pseudoscalar one. We thus denote the prime amplitudes T' and C' for the case when the tensor meson is an emitted particle [17]. Under the factorization approximation, $T' = C' = 0$.

In general, TP final states are suppressed relative to PP states due to the less phase space available. More precisely,

$$\frac{\Gamma(D \to TP)}{\Gamma(D \to P_1 P_2)} = \frac{2}{3} \frac{k_T^5}{k_P} \left(\frac{m_D}{m_T}\right)^4 \left|\frac{M(D \to TP)}{M(D \to P_1 P_2)}\right|^2, \quad (4.4)$$

where k_P is the c.m. momentum of the pseudoscalar meson P_1 or P_2 in the charm rest frame. The kinematic factor $h = \frac{2}{3}(k_T^5/k_P)(m_D/m_T)^4$ is typically of order $(1-4) \times 10^{-2}$ GeV^{-4}. An inspection of Table III indicates that, in the absence of weak annihilation contributions, the Cabibbo-allowed decays $D^+ \to \bar{K}_2^{*0} \pi^+$ and $D^0 \to K_2^{*-} \pi^+$ will have the largest decay rates as they proceed through the color-allowed tree diagram T. It is easily seen that all other W-emission amplitudes in $D \to a_2 \bar{K}$, $D \to f_2 \pi$ and $D \to f_2 \bar{K}$ are suppressed for various reasons. For example, it is suppressed by the vanishing decay constant of the tensor meson, or by the small f_2-f_2' mixing angle or by the parameter a_2 or by the Cabibbo mixing angle. Let us compare $D^+ \to \bar{K}_2^{*0} \pi^+$ with $D^+ \to \bar{K}^0 \pi^+$

$$\frac{\Gamma(D^+ \to \bar{K}_2^{*0} \pi^+)}{\Gamma(D^+ \to \bar{K}^0 \pi^+)} = 1.3 \times 10^{-2} \left(\frac{k(m_\pi^2) + b_+(m_\pi^2)(m_D^2 - m_{K_2^*}^2) + b_-(m_\pi^2) m_\pi^2}{(m_D^2 - m_K^2) F_0^{DK}(m_\pi^2) + \frac{a_2}{a_1}(m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2)}\right)^2. \qquad (4.5)$$

Note that $D^+ \to \bar{K}_2^{*0} \pi^+$ does not receive the internal W-emission contribution owing to the vanishing K_2^* decay constant. The form factors $F_0^{DK}(0)$ and $F_0^{D\pi}(0)$ are of order 0.70 [14,18]. Hence, the expression in the parentheses of the above equation is of order 0.5. As a consequence, the predicted branching ratio of $D^+ \to \bar{K}_2^{*0} \pi^+$ is of order 10^{-4}, which is one order of magnitude smaller than experiment (see Table III). As for the decay $D^0 \to K_2^{*-} \pi^+$, its branching ratio is similar to that of $D^+ \to \bar{K}^{*0} \pi^+$ but it receives an additional W-exchange contribution. A fit of this mode to experiment will require $|E| > |T|$, namely, the W exchange dominates over the external W emission, which is very unlikely. If we demand that $|E| < |T|$, then the color-suppressed decay $D^0 \to \bar{K}_2^{*0} \pi^0$, which receives contributions only from the W-exchange diagram, will be at most of order 10^{-5} (see Table III).

For $D \to f_2(1270)\pi(K)$ decays, let us first consider $D_s^+ \to f_2 \pi^+$. Its external W-emission amplitude is suppressed owing to the small $s\bar{s}$ component in $f_2(1270)$. However, W annihilation is not subject to the f_2-f_2' mixing angle suppression. Moreover, the D_s^+ decay constant is much larger than

that of the pion. The magnitude of W annihilation obtained by fitting $D_s^+ \to f_2 \pi^+$ to the data reads

$$A/T|_{D \to TP} \approx 0.5 e^{-i75°}, \qquad (4.6)$$

where a relative phase of $-75°$ has been assigned in analog to $D \to PP$ [see Eq. (4.7)] and the tree amplitude T is referred to the one in $D_s^+ \to f_2(1270)\pi^+$.

The importance of the weak annihilation contribution (W exchange or W annihilation) in charm decays has been noticed long before (see, e.g. [16,9]). Even if the short-distance weak annihilation amplitude is helicity suppressed, it does receive long-distance contributions from nearby resonance via inelastic final-state interactions from the leading tree or color-suppressed amplitude. As a consequence, weak annihilation has a sizable magnitude comparable to the color-suppressed internal W emission with a large phase relative to the tree amplitude. A quark-diagram analysis of the Cabibbo-allowed $D \to PP$ decays yield [19]

$$A/T|_{D \to PP} \approx 0.39 e^{-i65°}, \quad E/T|_{D \to PP} \approx 0.63 e^{i115°}. \qquad (4.7)$$

We see that the ratio of $|A/T|$ in $D \to TP$ ad $D \to PP$ decays is similar.

Using the W-annihilation term inferred from $D_s^+ \to f_2 \pi^+$, we can fix the decay rates of $D^+ \to f_2 \pi^+$ and $D_s^+ \to f_2 K^+$. Note that the predicted branching ratio for $D^+ \to f_2 \pi^+$ is smaller than experiment by a factor of 4. Indeed, it is difficult to understand why the measured branching ratio of this mode is of the same order as $D_s^+ \to f_2(1270) \pi^+$ even when the former is Cabibbo suppressed.

$D \to f_2'(1525) \pi(K)$ decays are suppressed relative to $f_2(1270) \pi(K)$ due to the phase space suppression. Contrary to $D_s^+ \to f_2(1270) \pi^+$, the decay $D_s^+ \to f_2'(1525) \pi^+$ is dominated by the external W emission and hence it has the largest rate among $D \to f_2' \pi(K)$ decays.

For $D \to a_2(1320) \pi(K)$ decays, both $a_2^+ \bar{K}^0$ and $a_2^+ K^-$ are small since the factorizable external W emission vanishes owing to the vanishing a_2 decay constant. The decay $D^0 \to a_2^-(1320) \pi^+$ is of order 10^{-5} at most.

For $D \to \bar{K}_2^* \pi$ decays, it is found that the decay $D^+ \to \bar{K}_2^{*0} \pi^+$ is at most of order 10^{-4} as noted in passing and it does not receive any weak annihilation contributions. Furthermore, the unknown W-exchange amplitude cannot be extracted from $D^0 \to K_2^{*-}(1430) \pi^+$ or $D^0 \to f_2(1270) \bar{K}^0$ or $D^0 \to a_2^-(1320) \pi^+$ by fitting them to the data. It will require the unreasonable condition $|E| > |T|$. For the purpose of illustration of the W-exchange effect, we shall assume

$$E/T|_{D \to TP} = 0.5 e^{i100^\circ}. \tag{4.8}$$

Finite width effects

The decay $D \to K_2^*(1430) \bar{K}$ is physically allowed even though $K_2^*(1430)$ mass lies outside of the phase space for the decay. The point is that $K_2^*(1430)$ has a decay width of order 100 MeV [8] and hence it is necessary to take into account the finite width effect. Likewise, the decay $D^0 \to f_2'(1525) \bar{K}^0$ which is outside of phase space also can occur.

The measured decay widths of various tensor mesons are given by [8]

$$\Gamma_{f_2(1270)} = 185.1_{-2.6}^{+3.4} \text{ MeV},$$

$$\Gamma_{f_2'(1525)} = 76 \pm 10 \text{ MeV},$$

$$\Gamma_{a_2(1320)} = 107 \pm 5 \text{ MeV},$$

$$\Gamma_{K_2^{*\pm}(1430)} = 98.5 \pm 2.7 \text{ MeV},$$

$$\Gamma_{K_2^{*0}(1430)} = 109 \pm 5 \text{ MeV}. \tag{4.9}$$

To take into account the finite width effect of the tensor resonances, we employ the factorization relation to "define" the $D \to TP$ decay rate

$$\Gamma(D \to TP \to P_1 P_2 P) = \Gamma(D \to TP) \mathcal{B}(T \to P_1 P_2), \tag{4.10}$$

with

$$\Gamma(D \to TP \to P_1 P_2 P)$$

$$= \frac{1}{2m_D} \int_{(m_1+m_2)^2}^{(m_D-m_P)^2} \frac{dq^2}{2\pi} |\langle TP|\mathcal{H}_W|D\rangle|^2 \frac{\lambda^{1/2}(m_D^2, q^2, m_P^2)}{8\pi m_D^2}$$

$$\times \frac{1}{(q^2 - m_T^2)^2 + (\Gamma_{12}(q^2)m_T)^2} g_{TP_1P_2}^2 \frac{\lambda^{1/2}(q^2, m_1^2, m_2^2)}{8\pi q^2}, \tag{4.11}$$

where λ is the usual triangluar function $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, m_1 (m_2) is the mass of P_1 (P_2), $g_{TP_1P_2}$ is the strong coupling to be defined below, and the "running" or "comoving" width $\Gamma_{12}(q^2)$ is a function of the invariant mass $m_{12} = \sqrt{q^2}$ of the $P_1 P_2$ system and it has the expression [20]

$$\Gamma_{12}(q^2) = \Gamma_T \frac{m_T}{m_{12}} \left(\frac{p'(q^2)}{p'(m_T^2)} \right)^5 \frac{9 + 3R^2 p'^2(m_T^2) + R^4 p'^4(m_T^2)}{9 + 3R^2 p'^2(q^2) + R^4 p'^4(q^2)}, \tag{4.12}$$

with $p'(q^2) = \lambda^{1/2}(q^2, m_1^2, m_2^2)/(2\sqrt{q^2})$. We shall follow [12] to take R, the "radius" of the meson, to be 1.5 GeV^{-1}. From the measured decay width of the tensor meson, one can determine the strong coupling $g_{TP_1P_2}$ via

$$\Gamma(T \to P_1 P_2) = \frac{g_{TP_1P_2}^2 m_T}{15\pi} \left(\frac{p_c}{m_T} \right)^5, \tag{4.13}$$

where p_c is the c.m. momentum of P_1 and P_2 in the rest frame of the tensor meson.

Note that in the narrow width approximation, one can show that the factorization relation (4.10) holds. When the decay width is not negligible we will use Eq. (4.11) to evaluate the three-body decay $\Gamma(D \to TP \to P_1 P_2 P)$ and employ Eq. (4.10) to define the decay rate of $D \to TP$. To evaluate the decay rate of $D \to TP \to P_1 P_2 P$, we will assume that $g_{TP_1P_2}$ is insensitive to the q^2 dependence when the resonance is off its mass shell. Numerically it is found that when the finite decay width of the tensor meson is taken into account, the decay rate of $D \to TP$ is generally enhanced by a factor of 2–3. The results of the calculated branching ratios shown in Table III have included finite width effects.

V. DISCUSSION AND CONCLUSION

Charmed meson decays into a pseudoscalar meson and a tensor meson are studied. The charm to tensor meson transition form factors are evaluated in the Isgur-Scora-Grinstein-Wise quark model. The main conclusions are as follows:

The external W-emission contribution to the decay $D_s^+ \to f_2(1270) \pi^+$ is suppressed by the fact that $f_2(1270)$ is predominately $n\bar{n}$. Hence, this decay is dominated by the W-annihilation contribution. We argue that the Cabibbo-suppressed mode $D^+ \to f_2 \pi^+$ should be suppressed by one

order of magnitude relative to $D_s^+ \to f_2(1270)\pi^+$, contrary to the E791 measured results.

The long-distance W-annihilation contributions induced from nearby resonances via inelastic final-state interactions gives the dominant contributions to $(D^+, D_s^+) \to f_2(1270)\pi^+$, $D_s^+ \to f_2(1270)K^+$. Under the factorization approximation, the decays $D^0 \to a_2^+(1320)K^-$, $\bar{K}_2^{*0}(1430)\pi^0$, $K_2^{*+}(1430)K^-$ receive contributions solely from the W-exchange diagram.

Among the $D \to TP$ decays, $D_s^+ \to f_2(1270)\pi^+$ has the largest branching ratio of order 10^{-3}. The modes $D^+ \to f_2(1270)\pi^+$, $D^0 \to f_2(1270)\bar{K}^0$, $D_s^+ \to f_2'(1525)\pi^+$, $D^+ \to \bar{K}^{*0}\pi^+$ and $D^0 \to K_2^{*-}\pi^+$ are of order 10^{-4}.

The decay rate of $D \to TP$ is generally enhanced by a factor of 2–3 when the finite width effect of the tensor resonances is taken into account. In particular, it is necessary to include the finite width effect to explain the decays $D \to K_2^*(1430)\bar{K}$ and $D \to f_2'(1525)K$.

Except for the Cabibbo-allowed decay $D_s^+ \to f_2(1270)\pi^+$, the predicted branching ratios of $D \to TP$ decays are in general too small by one to two orders of magnitude compared to experiment. However, it is very unlikely that one can enhance the $D \to T$ transition form factors within the ISGW quark model by a factor of 3–5 to account for the discrepancy between theory and experiment.

There are two possible sources which may account for the discrepancy between theory and experiment. First, as pointed out in [21], the tensor meson, for example a_2^+, can be produced from the tensor operator $(\bar{u}_R\gamma^\mu\overleftrightarrow{\partial}^\nu d_R) + (\bar{u}_L\gamma^\mu\overleftrightarrow{\partial}^\nu d_L)$. However, this operator must be generated by gluon corrections and is suppressed by factors of $\alpha_s(m_c)/\pi$ and $1/m_c$. Therefore, both of them could be potentially large, being of order 30–40 %. Second, the $D \to T$ form factors are so far evaluated using the ISGW quark model. These form factors are accessible in semileptonic $D \to T\ell\nu_\ell$ decays and will surely be measured at CLEO-c. Such a measurement would remove one possible source of model dependence and isolate the source of the disagreement. Finally, it should be remarked that as many of the current data have not yet enough significance, it is important to have more accurate measurements in the near future to pin down the issue.

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