New physics contributions to the $B \to \phi K_S$ decay

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Recent measurements of the time-dependent *CP* asymmetry of the $B \to \phi K_S$ decay give results whose central values differ from standard model expectations. It is shown how such data can be used to identify new physics contributions in a model-independent manner. In general, a sizable new amplitude with nontrivial weak and strong phases would be required to explain current data. An improvement in the quality of data will allow one to form a more definite conclusion.

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I. INTRODUCTION

It has been suggested to look for discrepancies among the time-dependent *CP* asymmetries of different *B* decay modes as a means to detect new physics [1–6]. Since the $B \to J/\psi K_S$ decay is a tree-dominated process in the standard model (SM), its *CP* asymmetry $\mathcal{S}_{J/\psi K_S}$ is believed to be less affected by new physics and to give information on $\sin 2\beta$. Although the *CP* asymmetry of the $B \to \phi K_S$ mode is also expected to give the same $\sin 2\beta$ within the SM, this process is, however, particularly sensitive to new physics contributions because it is a purely penguin loop-mediated process in the SM. The SM pollution from a small *u*-penguin contribution with the weak phase γ has been studied in Ref. [7] and it is found that the deviation of $\mathcal{S}_{\phi K_S}$ from $\sin 2\beta$ is of $\mathcal{O}(\bar{\lambda}^2) \sim 5\%$, where $\bar{\lambda} \simeq \mathcal{O}(0.2)$ is a parameter close in magnitude to the Wolfenstein parameter $\lambda \simeq 0.22$ [8]. Therefore a large deviation of $\mathcal{S}_{\phi K_S}$ from its SM prediction would signal contributions from physics beyond the SM.

As argued by Fleischer and Mannel [4], even if one ignores rescattering effects, contributions from new physics at a TeV scale to $\Delta I = 0$ operators could be of the same order as the SM ones, while new $\Delta I = 1$ operators are suppressed by $\bar{\lambda}$. With rescattering effects taken into account, both the new $\Delta I = 1$ operators and the SM pollution will be enhanced by about $\bar{\lambda}$. In any case, both of the $\Delta I = 0,1$ operators from the new TeV-scale physics can be more significant in comparison with the above-mentioned SM pollution.

The world average of $\sin 2\beta$ as measured from the golden mode $B \to J/\psi K_S$, $\sin 2\beta = 0.734 \pm 0.054$ [9], agrees well with constraints obtained from other experiments. Recently both the BaBar and Belle groups have also reported measurements of time-dependent *CP* asymmetries in the $B \to \phi K_S$ decay. $\mathcal{S}_{\phi K_S}$ (the coefficient of $\sin \Delta mt$ in flavor-tagged decays) is found to be about 2.7σ away from $\mathcal{S}_{J/\psi K_S}$, while $\mathcal{A}_{\phi K_S}$ (the coefficient of $\cos \Delta mt$) is 1σ away from 0. If this situation continues as the data precision improves, it would be of interest to know the magnitude and phase of possible new physics contributions to the ϕK_S mode.

Instead of separating the new physics contributions into $\Delta I = 0$ and $\Delta I = 1$ parts as done in Ref. [4], we will simplify the discussion by considering the combined amplitude from such effects along with the smaller SM pollution amplitude. This enables us to obtain useful information from the three observables $\mathcal{S}_{\phi K_S}$, $\mathcal{A}_{\phi K_S}$, and the ratio *R* between the sum of squared amplitudes extracted from the measured $B^0(\overline{B^0}) \to \phi K^0(\overline{K^0})$ branching ratios and a "standard" squared amplitude, such as the SM predicted value or experimentally measured $B^\pm \to K^* \pi^\pm$ branching ratio. The algebraic structure of the problem then becomes very similar to that studied by several authors [10] for $B \to \pi\pi$. We try to find in a model-independent way the allowed magnitude and phases of the new amplitude and some generic properties associated with it. Such an analysis is useful in helping us narrow down new physics models [11] consistent with observed data.

The paper is organized as follows. Section II introduces a decomposition of decay amplitudes in terms of topological contributions. The formalism for time-dependent *CP* asymmetries is discussed in Sec. III. We present numerical analyses for two separate cases of new physics in Secs. IV and V. In Sec. VI, we summarize our results.

II. TOPOLOGICAL AMPLITUDE ANALYSIS

In the framework of the SM, both the ϕK^0 and $\phi K^\pm$ modes receive important contributions from QCD and electroweak (EW) penguin graphs, with the former having a dominant effect. Useful information about the QCD penguin contribution can be obtained from the $K^{*0}\pi^\pm$ decay mode using flavor-SU(3) symmetry [12]. It should be noted that a tiny annihilation diagram also exists in both the $\phi K^\pm$ and $K^{*0}\pi^\pm$ decay modes. From the arguments of both dynamical suppression and the fact that no asymmetry is observed between the $K^{*0}\pi^\pm$ modes, we shall ignore the annihilation amplitude in these charged decays. In this case, both the neutral and charged ϕK modes have the same decay amplitudes. We will then average over the branching ratios of these two sets of modes using their associated errors as the weights for our analysis.

Let us write down the amplitudes of the relevant modes in terms of independent topological components as [13,14]

$$\mathcal{A}(\phi K^0) = p e^{i(\phi_{SM}+\delta_p)} + s e^{i(\phi_{SM}+\delta_s)}, \quad (1)$$

$$\mathcal{A}(K^{*0}\pi^+) = p e^{i(\phi_{SM}+\delta_p)}, \quad (2)$$

in the SM. In the above two equations, the p part denotes the QCD penguin contribution which also contains a negligible color-suppressed EW penguin amplitude, and the s part denotes the EW penguin contribution along with a small flavor-SU(3)-singlet amplitude, as expected from the Okubo-Zweig-Iizuka (OZI) rule. The variables p and s are absolute values of the respective amplitudes and therefore are non-negative by definition. The weak phase ϕ_{SM} satisfying $e^{-2i\phi_{SM}} = V_{tb}V_{ts}^*/(V_{tb}^*V_{ts})$ is the same for both the p and s parts. Finally, δ_p and δ_s are the associated strong phases. Note that to simplify the notation given in Ref. [12], we omit from these amplitudes the subscript P indicating that the spectator quark ends up in the pseudoscalar meson in the final state and the prime that denotes $\Delta S = 1$ transitions.

It should be noted that we explicitly assume flavor SU(3) symmetry in Eqs. (1) and (2) in order to relate the amplitude for the $K^{*0}\pi^+$ mode to the penguin part in the ϕK process in later analysis. The $\mathrm{SU}(3)_F$ breaking effect will be characterized by the factor $[f_\phi F^{B\to K}(m_\phi^2)]/[f_{K^*}F^{B\to\pi}(m_{K^*}^2)] \sim 1.2$. We will simply treat this extra factor as 1 in our analysis.

New physics can give rise to new operators that contribute to the decays of the above processes. We will distinguish two cases in later discussions: (i) only the ϕK modes receive the new contributions while the $K^{*0}\pi^\pm$ modes are purely SM processes; and (ii) both types of decay modes receive the same contributions from new physics. Case (i) could happen, for example, when new physics enters the $b\to s$ EW penguin contribution only. In this case, we add an extra amplitude $n e^{i(\phi_n+\delta_n)}$ to Eq. (1). Case (ii) could happen when new physics modifies the $b\to s$ QCD penguin contribution. In that case we add the new amplitude to both Eqs. (1) and (2). In general, the new amplitude $n e^{i(\phi_n+\delta_n)}$ is a combination of $\Delta I = 0$ and $\Delta I = 1$ ones that may contribute at different strengths [4]. A more careful job can in principle be done by separating the new amplitude into those with different isospins and studying new physics contributions in each piece. However, one would find that there are not enough observables among the decay modes to solve for all the parameters in the amplitudes.

III. TIME-DEPENDENT *CP* ASYMMETRIES

In this section, we review the general analysis of time-dependent *CP* asymmetry of pure B^0 and $\overline{B}^0$ decays into a *CP* eigenstate f_{CP}. Let us define the asymmetry as

$$a_{f_{CP}}(t) \equiv \frac{\Gamma(\overline{B}^0_{\rm phys}(t)\to f_{CP}) - \Gamma(B^0_{\rm phys}(t)\to f_{CP})}{\Gamma(\overline{B}^0_{\rm phys}(t)\to f_{CP}) + \Gamma(B^0_{\rm phys}(t)\to f_{CP})}. \quad (3)$$

In our case, $f_{CP} = \phi K_S$. Denote

$$\lambda_{\phi K_S} = \eta_{\phi K_S}\left(\frac{q}{p}\right)_B \left(\frac{p}{q}\right)_K \frac{\overline{\mathcal{A}}(\phi\overline{K}^0)}{\mathcal{A}(\phi K^0)}, \quad (4)$$

where $\eta_{\phi K_S} = -1$ is the *CP* eigenvalue of the ϕK_S state,

$$\left(\frac{q}{p}\right)_B = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \quad \text{and} \quad \left(\frac{p}{q}\right)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} \quad (5)$$

are factors that account for the mixing effects in neutral B and K meson systems, respectively, and

$$\mathcal{A}(\phi K^0) \equiv \langle K^0|\mathcal{H}|B^0\rangle = a e^{i(\phi_a+\delta_a)} + b e^{i(\phi_b+\delta_b)}, \quad (6)$$

$$\overline{\mathcal{A}}(\phi\overline{K}^0) \equiv \langle \overline{K}^0|\mathcal{H}|\overline{B}^0\rangle = a e^{i(-\phi_a+\delta_a)} + b e^{i(-\phi_b+\delta_b)}, \quad (7)$$

where a,b are chosen to be positive, $\phi_{a,b} \in \{-\pi,\pi\}$ and $\delta_{a,b} \in \{0,2\pi\}$ are the associated weak and strong phases, respectively. The above amplitudes are invariant under the transformations $\phi_{a,b} \to \phi_{a,b} \pm m\pi, \delta_{a,b} \to \delta_{a,b} \mp m\pi$ and $\phi_{a,b} \to \phi_{a,b} \pm m\pi, \delta_{a,b} \to \delta_{a,b} \pm m\pi$ for $m \in \mathbf{Z}$. Here the separation of the total amplitude into two parts is done in accord with the nature of the problem. The ratio of the amplitudes in Eqs. (6) and (7) is then

$$\frac{\overline{\mathcal{A}}(\phi\overline{K}^0)}{\mathcal{A}(\phi K^0)} = e^{-2i\phi_a}\frac{1 + r e^{i(\phi-\delta)}}{1 + r e^{-i(\phi+\delta)}}, \quad (8)$$

where

$$r \equiv b/a \geqslant 0, \quad \phi \equiv \phi_a - \phi_b, \quad \text{and} \quad \delta \equiv \delta_a - \delta_b. \quad (9)$$

The *CP* asymmetry can then be written as

$$a_{\phi K_S}(t) = \mathcal{A}_{\phi K_S}\cos(\Delta M t) + \mathcal{S}_{\phi K_S}\sin(\Delta M t), \quad (10)$$

where ΔM is the mass difference between the two physical B meson states, and

$$\mathcal{A}_{\phi K_S} = \frac{|\lambda_{\phi K_S}|^2 - 1}{|\lambda_{\phi K_S}|^2 + 1}, \quad (11)$$

$$\mathcal{S}_{\phi K_S} = \frac{2\,\mathrm{Im}\,\lambda_{\phi K_S}}{|\lambda_{\phi K_S}|^2 + 1}. \quad (12)$$

New physics will affect the *CP* asymmetry observables through the parameter $\lambda_{\phi K_S}$. Therefore it may come in at two places: the mixing matrix and/or the decay amplitudes. As emphasized in Ref. [1], new physics effects on the mixing part will be universal and do not change the SM predicted pattern of *CP* asymmetries in different modes; their effects on the decay amplitudes, however, are nonuniversal so that the *CP* asymmetries can vary from channel to channel. Since current $\sin 2\beta$ measurements from other decay modes, such as $J/\psi K_S$, $\eta' K_S$, etc., seem to agree with one another and with the unitarity triangle constraints obtained from other processes pretty well, it is plausible to assume that any strange behavior in the ϕK_S mode is mostly due to new physics contributions in the amplitudes. In this case, we will use the SM mixing factors in Eq. (4) and obtain

$$\lambda_{\phi K_S} = -e^{-2i\beta_{\rm eff}}\frac{1+re^{i(\phi-\delta)}}{1+re^{-i(\phi+\delta)}}, \tag{13}$$

where

$$e^{-2i\beta_{\rm eff}} = \left(\frac{q}{p}\right)_B \left(\frac{p}{q}\right)_K e^{-2i\phi_a}. \tag{14}$$

Within the SM, $\phi_a \simeq \pi$ and one obtains the effective weak phase $\beta_{\rm eff}$ coinciding with β in the unitarity triangle. However, if new physics modifies the phase ϕ_a, then $\beta_{\rm eff}$ will in general differ from what the SM expects. If one writes $\phi_a = \phi_{SM} + \phi$ with ϕ_{SM} being the phase expected in the SM and ϕ being the deviation, then $\beta_{\rm eff} = \phi_{SM} + \phi$ (mod π).

We will be exclusively dealing with the decays of a B meson into a final state with one pesudoscalar meson (P) and one vector meson (V). The invariant amplitude $\mathcal{A}$ of such a process is conventionally related to its partial width in the following way:

$$\Gamma(B\to PV) = \frac{(p^*)^3}{8\pi m_B^2}|\mathcal{A}(B\to PV)|^2, \tag{15}$$

where p^* is the three-momentum of each final particle in the rest frame of the B meson, and m_B is the mass of the decaying B meson. Note that p^* is raised to its third power to appropriately account for the P-wave kinematic factor.

IV. NEW PHYSICS ONLY IN THE ϕK SYSTEM

In this section, we will discuss the case when new physics only enters the ϕK system but not the $K^*\pi$ system. Since the p and s parts of the ϕK decay amplitudes have the same weak phase ϕ_{SM}, we can combine them into a single v part and write, including the new physics part,

$$\begin{aligned}\mathcal{A}(\phi K^0) &= v e^{i(\phi_{SM}+\delta_v)} + n e^{i(\phi_n+\delta_n)}\\ &= v e^{i(\phi_{SM}+\delta_v)}[1+re^{-i(\phi+\delta)}],\end{aligned} \tag{16}$$

where $r=n/v$, $\phi=\phi_{SM}-\phi_n$, and $\delta=\delta_v-\delta_n$. Since here we assume that the $K^{*0}\pi^{\pm}$ modes are not affected by the new physics, we can use them to obtain reliable information on p, the magnitude of the QCD penguin contribution. The effective Hamiltonian approach indicates that there is a small relative strong phase between the p and s amplitudes in the SM [15,16]. The relative strong phase obtained in this approach comes purely from short-distance physics [17]. In general, there are nonperturbative strong phases from soft gluon exchanges in the final-state particles that may be different between the two types of penguin diagrams. For simplicity and definiteness, we will take $\delta_v = \delta_p \simeq \delta_s - \pi \simeq 0$ since the overall strong phase will not matter, and consider maximal destructive interference between the QCD and EW penguin contributions, in accord with the effective Hamiltonian analysis. The additional $-\pi$ in the above strong phase relation does not really have a strong interaction origin but simply comes from the charge coupling of the final-state s quark with the Z boson in the EW penguin contribution. Therefore the s part has a 180° phase from the p part within the SM. Under this assumption, $v=p-s$ can be computed once we know the prediction of the ratio s/p in the SM. We will also mention consequences of imperfect destructive interference between these two types of amplitudes.

A. Observables

We define for the ϕK system the ratio

$$\begin{aligned}R &\equiv \frac{|\mathcal{A}^{\rm exp}(B^0\to\phi K^0)|^2 + |\mathcal{A}^{\rm exp}(\overline{B^0}\to\phi\overline{K^0})|^2}{2|\mathcal{A}^{\rm SM}(B^0\to\phi K^0)|^2}\\ &= 1+2r\cos\phi\cos\delta + r^2,\end{aligned} \tag{17}$$

which is a combination of experimental observables and theoretical input from the SM. The numerator in the above definition is the sum of measured branching ratios of $B^0\to\phi K^0$ and $\overline{B^0}\to\phi\overline{K^0}$. As described before, we will actually take the weighted average of the neutral and charged modes. The denominator is the theoretical prediction for the same branching ratio sum within the SM. In terms of R and Eq. (13), we obtain

$$R\mathcal{S}_{\phi K_S} = \sin 2\beta + 2\,r\cos\delta\sin(2\beta-\phi) + r^2\sin 2(\beta-\phi), \tag{18}$$

$$R\mathcal{A}_{\phi K_S} = 2r\sin\phi\sin\delta. \tag{19}$$

Now we have three quantities R, $\mathcal{S}_{\phi K_S}$, and $\mathcal{A}_{\phi K_S}$ that allow us to solve for the three parameters r, ϕ, and δ. As the value of R may vary owing to the interference between p and s, we will estimate the SM contribution and also search the allowed parameter space by varying R over a reasonable range.

The self-tagging modes $B^{\pm}\to\phi K^{\pm}$ can provide additional statistical power to the determination of $\mathcal{A}_{\phi K_S}$ if we assume that $\mathcal{A}(B^+\to\phi K^+) = \mathcal{A}(B^0\to\phi K^0)$ as we have done above. In that case one finds just

$$A_{CP} \equiv \frac{|\mathcal{A}(\phi K^-)|^2 - |\mathcal{A}(\phi K^+)|^2}{|\mathcal{A}(\phi K^-)|^2 + |\mathcal{A}(\phi K^+)|^2} = \mathcal{A}_{\phi K_S} = \frac{2r\sin\phi\sin\delta}{R} \tag{20}$$

for the time-integrated CP rate asymmetry. The BaBar Collaboration [18] has recently reported $A_{CP}=0.039\pm0.086\pm0.011$. We shall not use this value in our averages but in principle it can greatly reduce the error on $\mathcal{A}_{\phi K_S}$.

B. Numerical studies

In this section, we will use the measured values of $\mathcal{A}_{\phi K_S}$, $\mathcal{S}_{\phi K_S}$, and R with some theoretical input from the SM to find the allowed ranges of r, ϕ, and δ. Solving for r in Eq. (17) in terms of R, ϕ, and δ, one obtains two solutions:

TABLE I. Experimental input of measured branching fractions.

$(\times 10^{-6})$	$\mathcal{B}(\phi K^0)$	$\mathcal{B}(\phi K^+)$	$\mathcal{B}(K^{*0}\pi^+)$
CLEO	$5.4^{+3.7}_{-2.7}\pm 0.7$ (<12.3) [19]	$5.5^{+2.1}_{-1.8}\pm 0.6$ [19]	$7.6^{+3.5}_{-3.0}\pm 1.6$ [20]
BaBar	$7.6^{+1.3}_{-1.2}\pm 0.5$ [18]	$10.0^{+0.9}_{-0.8}\pm 0.5$ [18]	$15.5\pm 3.4\pm 1.8$ [21]
Belle	$10.0^{+1.9+0.9}_{-1.7-1.3}$ [22]	$10.7\pm 1.0^{+0.9}_{-1.6}$ [22]	$19.4^{+4.2+2.1+3.5}_{-3.9-2.1-6.8}$ [23]
Average	7.98 ± 1.07	9.51 ± 0.78	12.3 ± 2.5

$$r_1 = -\cos\phi\cos\delta - \sqrt{\cos^2\phi\cos^2\delta + R - 1} \quad \text{(solution I)},$$
$$r_2 = -\cos\phi\cos\delta + \sqrt{\cos^2\phi\cos^2\delta + R - 1} \quad \text{(solution II)}. \tag{21}$$

First, it is seen that solution I is not allowed for $R>1$ because r_1 has to be positive. Therefore we see that $r_1<1$.

In the effective Hamiltonian approach, the EW penguin contribution is found to be of considerable importance, with the ratio $|s/p|$ predicted to be between 10% and 11% using the results given in Ref. [16]. On the other hand, the $B^+\to K^{*0}\pi^+$ decay mode involves only p, ignoring a small annihilation diagram that also contributes to the ϕK^+ mode. Using its branching ratio, we obtain $|p| = (1.42\pm 0.14)\times 10^{-8}$. Combining the above results and assuming maximal destructive interference between p and s, the SM predicts $|\mathcal{A}^{\rm SM}(\phi K)| = (1.27\pm 0.13)\times 10^{-8}$. To improve the statistics, we take the weighted average for the branching ratios of the neutral and charged ϕK modes as given in Table I and obtain $|\mathcal{A}^{\rm exp}(\phi K)| = (1.27\pm 0.04)\times 10^{-8}$ after removing the kinematic factors. Therefore we obtain an estimate of

$$R = \left|\frac{\mathcal{A}^{\rm exp}(\phi K)}{\mathcal{A}^{\rm SM}(\phi K)}\right|^2 \simeq 0.99\pm 0.21. \tag{22}$$

If a nontrivial relative strong phase exists between p and s, the central value of the resulting R will become smaller. In the case of maximal constructive interference between p and s, R could be as low as 0.5.

We will use the CP asymmetry observables measured by the BaBar and Belle groups [24,25] as given in Table II for our analysis. Replacing r in Eqs. (18) and (19) by one of the above solutions, it is then possible to find on the ϕ-δ plane regions that are consistent with the measured values $\mathcal{A}_{\phi K_S} = 0.19\pm 0.30$, $\mathcal{S}_{\phi K_S} = -0.38\pm 0.41$ [24,25], and the additional requirement that $r\geqslant 0$ by definition. The fact that $\mathcal{A}_{\phi K_S}$ is negative at the 1σ level gives the following possibilities: (i) $-\pi\leqslant\phi<0$ and $0\leqslant\delta<\pi$; and (ii) $0\leqslant\phi<\pi$ and $\pi\leqslant\delta<2\pi$.

In Fig. 1, we only show a set of representative solutions in the range $-\pi\leqslant\phi\leqslant 0$, $0\leqslant\delta\leqslant\pi$ for $R=0.8,1.0$, and 1.2. It is noticed that solution I does not exist when $R\gtrsim 0.8$. Therefore we only show those for solution II. As shown in Sec. III, solutions in other regions on the ϕ-δ plane can be obtained by the translations $\phi\to\phi\pm\pi$ and $\delta\to\delta\pm\pi$. As R increases, the allowed regions of solution II become larger.

TABLE II. Experimental input of measured CP asymmetries for the $B\to\phi K_S$ mode.

Quantity	BaBar [24]	Belle [25]	Average
$\mathcal{S}$	$-0.18\pm 0.51\pm 0.07$	$-0.73\pm 0.64\pm 0.22$	-0.38 ± 0.41
$\mathcal{A}$	$0.80\pm 0.38\pm 0.12$	$-0.56\pm 0.41\pm 0.16$	0.19 ± 0.30

In Fig. 2 we plot the allowed ranges of r for $0\leqslant R\leqslant 1.4$. The dark region in plot (a) corresponds to solution I, and that in plot (b) to solution II. It is seen from the plots that to satisfy the constraints of measured data, r has to be at least about 0.4 for either solution. This corresponds to a new physics amplitude with a magnitude of at least about 0.45×10^{-8}. It is also found that for solution I, R has to be less than about 0.8. Therefore the current value of R favors solution II. If we take the value $R=1$, solution II has a wide range for r: $0.40\lesssim r\lesssim 0.90$ and $1.05\lesssim r\lesssim 1.96$.

The fact that r has to be greater than a minimum can be readily understood. Should r be too small, then new physics [the n part in Eq. (16)] does not have enough weight to change $R\mathcal{S}_{\phi K_S}$ from that extracted from the $J/\psi K_S$ mode to the measured one as the modification is of $\mathcal{O}(r)$ according to Eq. (18). As mentioned in the beginning, $r\sim\mathcal{O}(1)$ means that the new amplitude has the same order of size as the SM contribution. This would point to the possibility of new physics at the TeV scale or below.

The sensitivity of $\mathcal{S}_{\phi K_S}$ and $\mathcal{A}_{\phi K_S}$ to the weak and strong phases ϕ and δ for a value of R close to the central one is illustrated in Fig. 3. Here each curve for a given ϕ intersects the axis $\mathcal{A}_{\phi K_S}=0$ at either $\delta=0$ or $\delta=\pi$, while curves with $\mathcal{A}_{\phi K_S}<0$ are related to those with $\mathcal{A}_{\phi K_S}>0$ and the same value of $\mathcal{S}_{\phi K_S}$ by the transformation $\phi\to\pi+\phi$, $\delta\to\pi-\delta$. The plotted cross shows the present status of the data summarized in Table II. The ranges of δ and ϕ are restricted in general by the requirement that the argument $\cos^2\phi\cos^2\delta + R - 1$ of the square roots in Eqs. (21) be non-negative. Because of the special value of R, we are able to draw curves for essentially any value of ϕ by varying δ. Therefore the constraints come merely from $\mathcal{S}_{\phi K_S}$ and $\mathcal{A}_{\phi K_S}$.

Assuming the central values of $\mathcal{S}_{\phi K_S}$ and $\mathcal{A}_{\phi K_S}$ stay the same in future experiments but the errors are improved by a factor of 3, we find that the allowed regions become smaller. This is demonstrated in Fig. 4.

We find that the variation of $\sin 2\beta$ within its experimental range makes little difference in the allowed solutions. The general behavior and regions presented in Figs. 1 and 2 remain the same.

V. NEW PHYSICS IN ϕK AND $K^*\pi$ SYSTEMS

In this section, we consider the situation of new physics entering both the ϕK and $K^*\pi$ systems. Since the new phys-

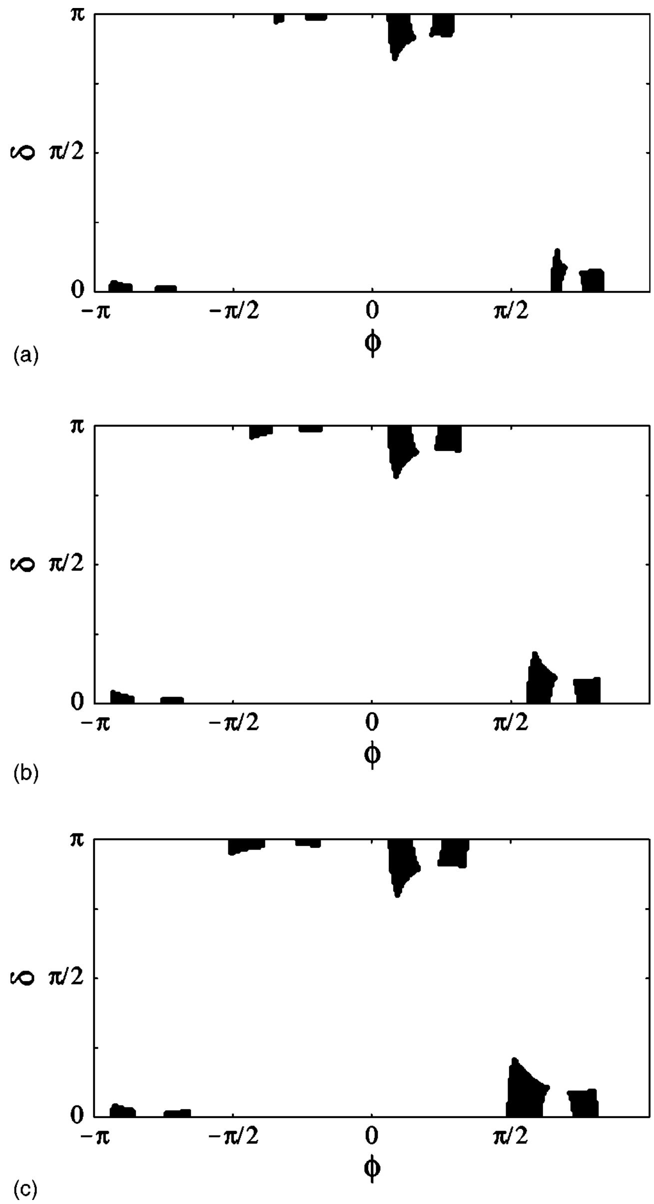

FIG. 1. The allowed regions in the ϕ-δ plane for solution II with (a) $R=0.8$, (b) $R=1.0$, and (c) $R=1.2$. Here we only show the allowed regions in the range $-\pi\leqslant\phi\leqslant\pi$, $0\leqslant\delta\leqslant\pi$. Other regions can be obtained by $\phi\to\phi\pm\pi$ and $\delta\to\delta\pm\pi$. Solution I is not allowed for $R\gtrsim 0.8$ and, therefore, no corresponding plots are shown here. For $R=1$ $\mathcal{S}_{\phi K_S}$ and $\mathcal{A}_{\phi K_S}$ are unchanged under $\delta\to\pi-\delta$ and $\phi\to\phi+\pi/2$ [Fig. 1(b)].

ics contribution $ne^{i(\phi_n+\delta_n)}$ is to be added to both Eqs. (1) and (2), we combine the p and n parts into a single q part as follows:

$$\mathcal{A}(\phi K^0)=qe^{i(\phi_q+\delta_q)}+se^{i(\phi_{SM}+\delta_s)}=qe^{i(\phi_q+\delta_q)}[1+r'e^{-i(\phi+\delta)}], \tag{23}$$

$$\mathcal{A}(K^{*0}\pi^+)=qe^{i(\phi_q+\delta_q)}, \tag{24}$$

where $qe^{i(\phi_q+\delta_q)}=pe^{i(\phi_{SM}+\delta_p)}+ne^{i(\phi_n+\delta_n)}$, $r'=s/q$, $\phi=\phi_q-\phi_{SM}$, and $\delta=\delta_q-\delta_s$. As mentioned in the previous section, $\phi_{SM}\simeq\pi$ and $\delta_s\simeq\pi$, one thus should use $(\phi_q,\delta_q)\simeq(\phi+\pi,\delta+\pi)$ to obtain the weak and strong phases associated with the q part when interpreting our following plots drawn on the ϕ-δ plane.

A. Observables

In this case, we use the observable

$$R'\equiv\frac{|\mathcal{A}^{\exp}(B^0\to\phi K^0)|^2+|\mathcal{A}^{\exp}(\overline{B^0}\to\phi\overline{K^0})|^2}{|\mathcal{A}^{\exp}(B^+\to K^{*0}\pi^+)|^2+|\mathcal{A}^{\exp}(B^-\to\overline{K}^{*0}\pi^-)|^2}=1+2r'\cos\phi\cos\delta+r'^2. \tag{25}$$

Note that in spite of the similarity in the forms between R' and R defined in the previous section, they are actually very different. Using Eq. (25), we have

$$\begin{aligned}R'\mathcal{S}_{\phi K_S}&=\sin 2\beta_{\rm eff}+2r'\cos\delta\sin(2\beta_{\rm eff}-\phi)+r'^2\sin 2(\beta_{\rm eff}-\phi)\\&=\sin 2(\beta+\phi)+2r'\cos\delta\sin(2\beta+\phi)+r'^2\sin 2\beta,\end{aligned} \tag{26}$$

$$R'\mathcal{A}_{\phi K_S}=2r'\sin\phi\sin\delta, \tag{27}$$

where $\beta_{\rm eff}=\beta+\phi$ is used. Here one quickly realizes that we also have only three parameters, r', ϕ, and δ for which to solve.

As in the previous case, the self-tagging rate asymmetry for $B^\pm\to\phi K^\pm$ provides additional statistical power for the measurement of $\mathcal{A}_{\phi K_S}$, since

$$A_{CP}=2r'\sin\phi\sin\delta/R'. \tag{28}$$

B. Numerical studies

Solving r' in Eq. (25) in terms of R', ϕ, and δ, one obtains two solutions:

$$\begin{aligned}r'_1&=-\cos\phi\cos\delta-\sqrt{\cos^2\phi\cos^2\delta+R'-1}\quad(\text{solution I}),\\r'_2&=-\cos\phi\cos\delta+\sqrt{\cos^2\phi\cos^2\delta+R'-1}\quad(\text{solution II}).\end{aligned} \tag{29}$$

First, as in the previous case, solution I is not allowed for $R'>1$ because r'_1 has to be positive. Therefore we see that $r'_1<1$.

We extract the amplitude for the $K^*\pi$ mode from Table I to be $|\mathcal{A}^{\exp}(K^*\pi)|=(1.42\pm0.14)\times10^{-8}$. Combined with the weighted average of the ϕK mode amplitude size given in the previous section, $|\mathcal{A}^{\exp}(\phi K)|=(1.25\pm0.05)\times10^{-8}$, we find $R'=0.79\pm0.17$. In Fig. 5 we show a set of representative solutions in the range $-\pi\leqslant\phi\leqslant0$, $0\leqslant\delta\leqslant\pi$. We take $R'=0.6,0.8$, and 1.0. It is seen that both solutions have two allowed regions except for solution I at $R'=1.0$. As R' increases, the allowed regions become larger for solution II.

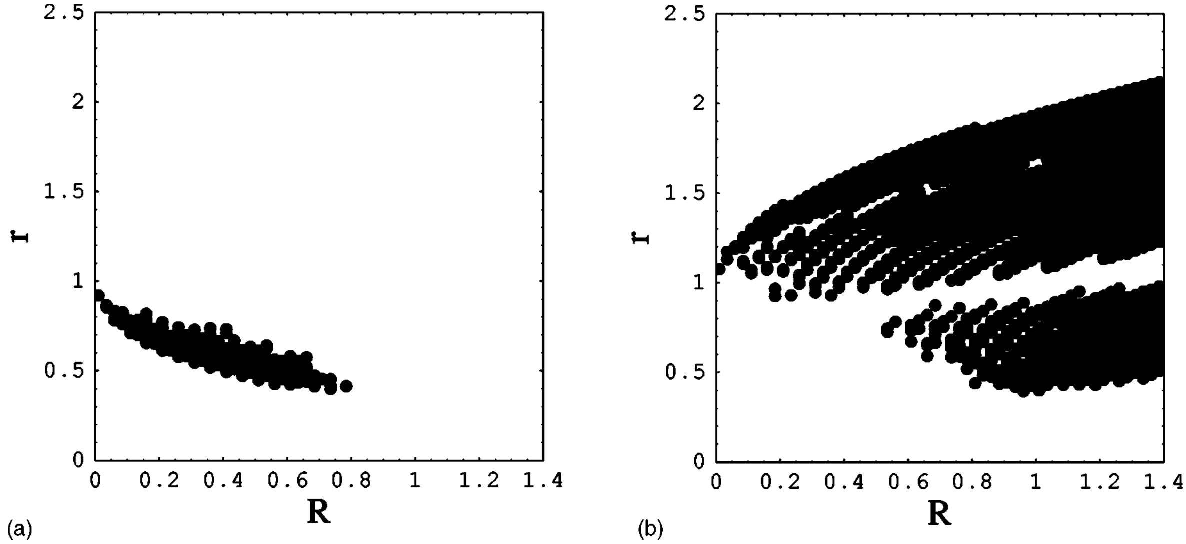

FIG. 2. The allowed range of r for specific values of R, using (a) solution I and (b) solution II.

In Fig. 6, we also draw the allowed ranges of r' for $0 \leqslant R' \leqslant 1.2$. The dark region in plot (a) corresponds to solution I, and that in plot (b) to solution II. It is seen from the plots that to satisfy the constraints of measured data, r' can go down to almost 0 for $R' \simeq 1$. If we take the value $R' = 0.8$, solution I has $0.16 \lesssim r' \lesssim 0.44$ while solution II has a wider range $0.47 \lesssim r' \lesssim 0.97$ and $1.20 \lesssim r' \lesssim 1.76$. In the standard model, one expects $r' \simeq 0.1$, $\phi = \pm\pi$, $\delta \simeq \pm\pi$ in accord with the expected contribution (mentioned previously) of the electroweak penguin amplitude.

If it turns out that $r' \ll 1$, that means the n part and the p part interfere to give an amplitude larger in size, agreeing with the fact that R' will be about 1. However, if $r' \sim \mathcal{O}(1)$, then there is a cancellation between n and p such that the combined amplitude of the two becomes comparable to the $SU(3)_F$-singlet amplitude. Either situation would tell us the new physics contribution is important.

The sensitivity of $\mathcal{S}_{\phi K_S}$ and $\mathcal{A}_{\phi K_S}$ to the weak and strong phases ϕ and δ in solutions I and II for the case in which new physics enters through the penguin amplitude into both $B \to \phi K$ and $B^+ \to K^{*0}\pi^+$ is illustrated in Fig. 7. As in Fig. 3, each curve for a given ϕ intersects the axis $\mathcal{A}_{\phi K_S} = 0$ at either $\delta = 0$ or $\delta = \pi$, while curves with $\mathcal{A}_{\phi K_S} < 0$ are related to those with $\mathcal{A}_{\phi K_S} > 0$ and the same value of $\mathcal{S}_{\phi K_S}$ by $\phi \to \pi + \phi$, $\delta \to \pi - \delta$. Contrary to Fig. 3, not all values of ϕ and δ are allowed for making a curve with a given value of $R' < 1$.

It is interesting to notice that for the large-r' solution (solution II), the curves for values of ϕ and $\pm\pi - \phi$ overlap, leading to the appearance of continuity. This is because both curves are part of a common ellipse, obtained by solving Eqs. (25), (26), and (27):

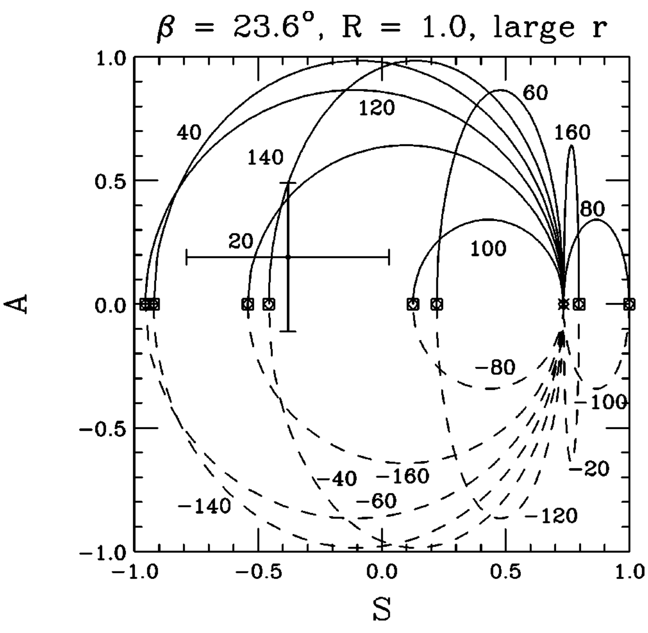

FIG. 3. The curves traced in the $\mathcal{S}_{\phi K_S}$-$\mathcal{A}_{\phi K_S}$ plane by varying the relative strong phase δ between 0 and π for fixed values of ϕ. The plot is for $\beta = 23.6°$ and $R = 1$ with r chosen according to solution II; no solution I exists for $R = 1$. Curves are labeled by values of ϕ (dashed: $\phi < 0$; solid: $\phi > 0$) in degrees. Squares and diamonds correspond to values of $\delta = 0$ or π. The point at $\mathcal{S}_{\phi K_S} = 0.734$, $\mathcal{A}_{\phi K_S} = 0$ corresponds to $\phi = 0, \pm\pi$ for all δ. The plotted data point is the average quoted in Table II.

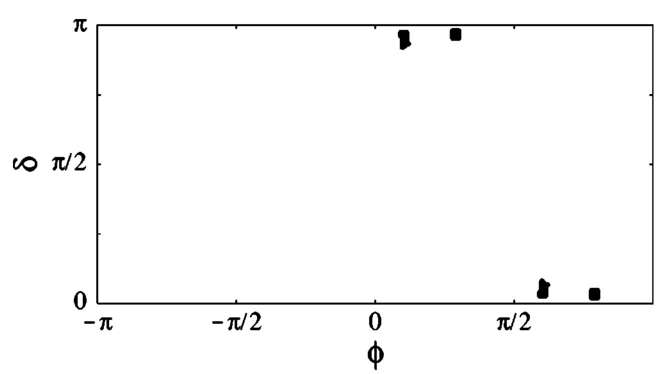

FIG. 4. The allowed regions on the ϕ-δ plane for $R = 1.0$ using solution II and with a factor of 3 improvement in $\mathcal{A}_{\phi K_S}$ and $\mathcal{S}_{\phi K_S}$.

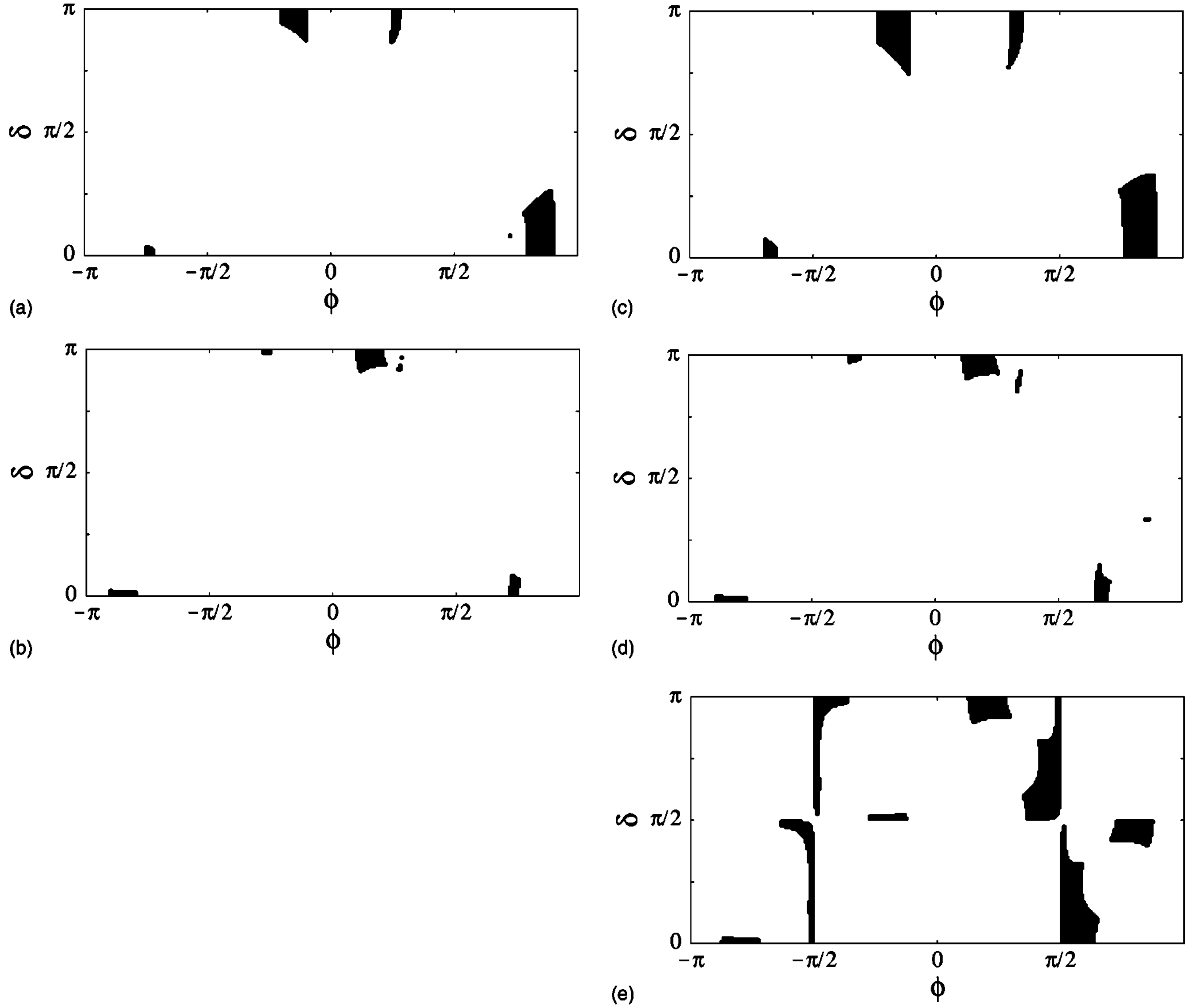

FIG. 5. The allowed regions on the ϕ-δ plane for $R'=0.6,0.8$, and 1.0, using solutions I [plots (a) and (b) in the left column] and II [plots (c), (d), and (e) in the right column]. Here we only show the allowed regions in the range $-\pi\leqslant\phi\leqslant 0,\ 0\leqslant\delta\leqslant\pi$. The other regions can be obtained by shifting: $\phi\to\phi+\pi$ and $\delta\to\delta+\pi$. Solution I is not allowed for $R'=1$. In this case (e) displays an additional symmetry of the solution under $\delta\to\delta+\pi/2,\ \phi\to\pm\pi-\phi$.

$$\left(\frac{R'\mathcal{S}_{\phi K_S}-(\cos 2\phi+R'-1)\sin 2\beta}{r_x}\right)^2+\left(\frac{R'\mathcal{A}_{\phi K_S}}{r_y}\right)^2 = \cos^2\phi+R'-1, \tag{30}$$

where $r_x=2\sin\phi\cos 2\beta$, and $r_y=2\sin\phi$. These ellipses have their centers at coordinates $([\cos 2\phi+R'-1]\sin 2\beta/R',0)$. One immediately sees that the above elliptic equation is invariant under the transformation $\phi\to\pi-\phi$. Since no explicit choice of solutions of r in Eq. (29) is made for deriving Eq. (30), it is valid for either solution. This is why each curve associated with ϕ in Fig. 7(a) is actually a portion of the corresponding curve associated with $\pi-\phi$ in Fig. 7(b). The curves for both solutions are truncated (although not seen in the plot for solution II because of the overlap) because of the conditions $\cos^2\phi\cos^2\delta+R'-1\geqslant 0$, $r_1'\geqslant 0$.

Again, we find that if the experimental precision of $\mathcal{S}_{\phi K_S}$ and $\mathcal{A}_{\phi K_S}$ can be improved by a factor of 3 with their current central values, then the allowed regions are considerably restricted. The variation of $\sin 2\beta$ within its experimental range also does not affect the general behavior and regions presented in this section.

VI. SUMMARY

We have shown how to estimate the magnitude and weak and strong phases of any new physics contribution which might account for the deviation of the *CP* asymmetry parameters in $B\to\phi K^0$ from their standard-model values. We find that it is useful to compare the overall rate for this process either with that predicted in the standard model (a ratio R) or with that for $B^+\to K^{*0}\pi^+$ (a ratio R'), which is expected to

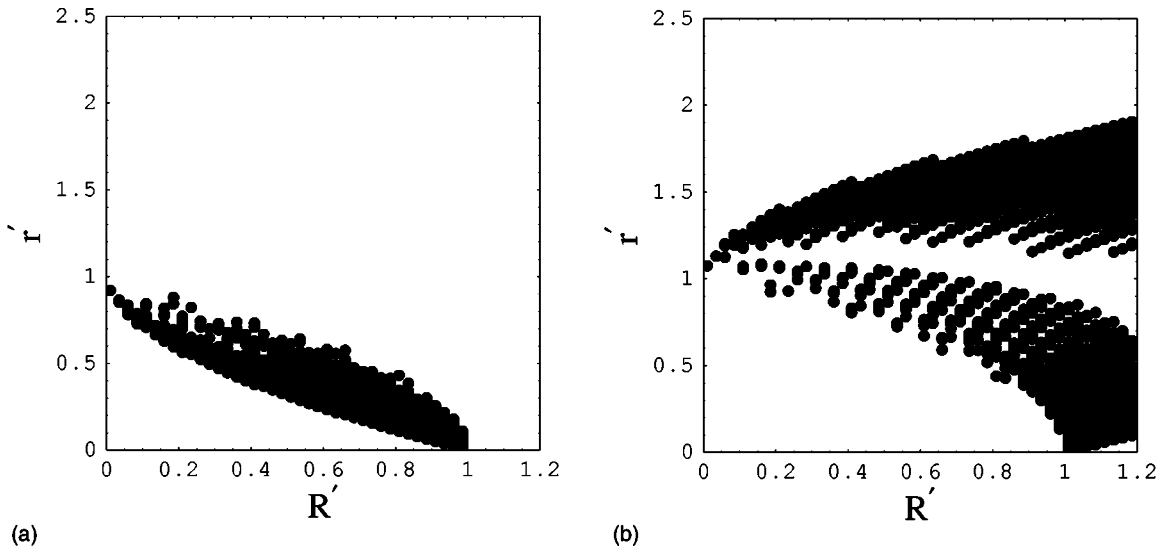

FIG. 6. The allowed range of r' for specific values of R', using (a) solution I and (b) solution II.

be dominated by the penguin amplitude in the standard model.

It is observed from our analysis that an amplitude associated with new physics, with considerable size and with nontrivial weak and/or strong phases, is required to fit the current experimental results. For example, in the case that new physics contributes to the ϕK modes but not the $K^*\pi$ mode, the ratio of the new amplitude to the SM contribution has to be $\gtrsim 0.4$, independent of the value of R.

Current experimental data indicate that the size r or r' of a new physics amplitude relative to that of the standard model could well be of $\mathcal{O}(1)$ for a wide range of R or R'. Such $\mathcal{O}(1)$ parameters could indicate new physics at about the TeV scale. Of course, these extra contributions would need the right strong and weak phases in order to explain the current data. Considerable refinement of the rate and asymmetry measurements in $B\to\phi K^0$ is necessary before the amplitude can be pinpointed satisfactorily, however.

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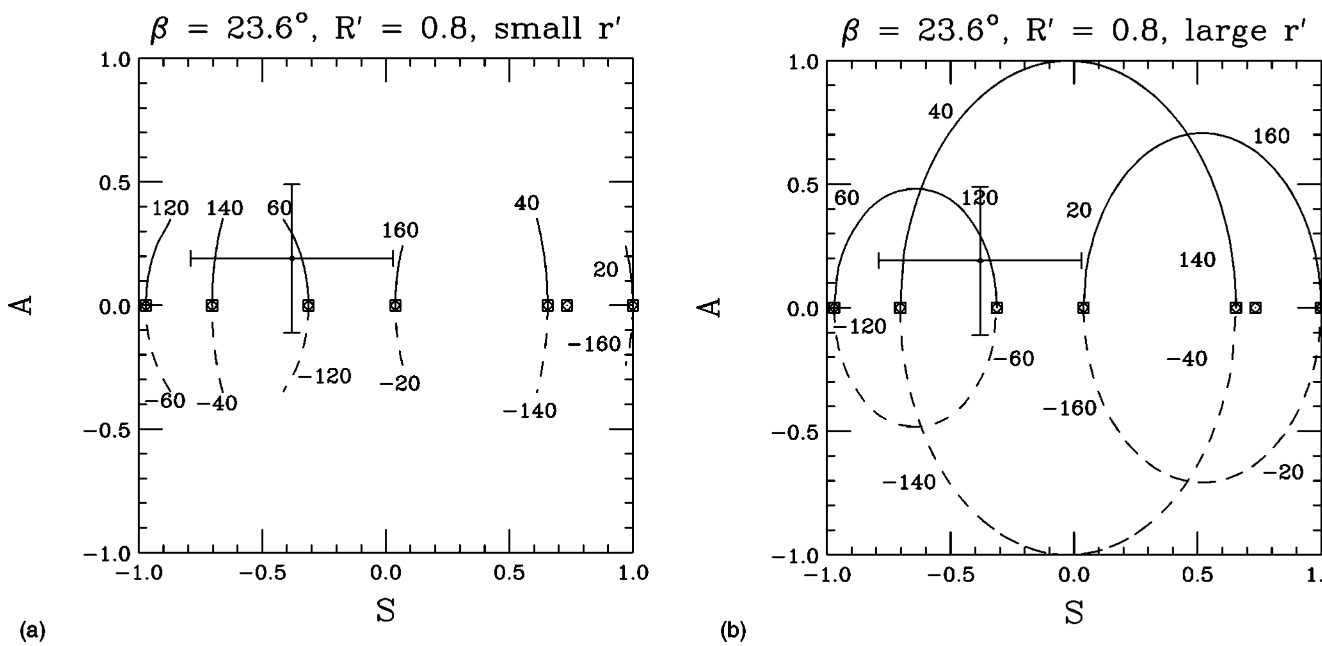

FIG. 7. Same as Fig. 3 but for new physics entering through the penguin amplitude into both $B\to\phi K$ and $B^+\to K^{*0}\pi^+$. The left plot (a) is for solution I and the right plot (b) for solution II. In both plots $\beta=23.6°$ and $R'=0.8$.

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