Chiral QCD, general QCD parametrization, and constituent quark models

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Several recent papers, using effective QCD chiral Lagrangians, reproduced results obtained with the general QCD parametrization (GP). These include the baryon **8**+**10** mass formula, the octet magnetic moments and the coincidental nature of the "perfect" $(\mu_p/\mu_n)=-(3/2)$ ratio. Although we anticipated that the GP covers the case of chiral treatments, the above results explicitly exemplify this fact. Also we show by the GP that, in any model or theory (chiral or nonchiral) reproducing the results of QCD, the Franklin (Coleman-Glashow) sum rule for the octet magnetic moments must be violated.

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I. INTRODUCTION: CHIRAL RESULTS AND GP RESULTS

(1) In a recent publication Durand *et al.* [1] (see also Ref. [2]) derived, by an effective chiral Lagrangian, the same **8** +**10** baryon mass formula that, using the general QCD parametrization (GP) [3,4], had been obtained in Ref. [5] and rediscussed in Ref. [6]. Also they derived in Ref. [1] an expression for the baryon octet magnetic moments similar to that obtained with the GP in Refs. [3,4,6,9]. They kindly acknowledged all this in Ref. [1].

(2) These interesting results of Durand *et al.* add to that of Leinweber *et al.* In Ref. [7] (see also Ref. [8]) they show that, in a chiral description, the fact that the ratio $\mu(p)/\mu(n)$ is so near to $-(3/2)$ [the nonrelativistic (NR) quark model result] is coincidental. We agree: In fact we reached the same conclusion by the GP method [4,6,9] independently of a specific chiral description.

(3) Another point discussed recently (e.g., Refs. [10,11]) is the Franklin sum rule [12] for the octet baryon magnetic moments. The revived interest in the rule is due to the fact [10] that according to a chiral quark model, the Franklin rule should be exact, while experimentally it is not. The GP shows that, in fact, the rule is violated by two specific first-order flavor breaking terms, present in the QCD but not in the chiral quark model (χQM).

(4) The GP explains the Pondrom fit [13] of the magnetic moments.

II. THE GENERAL PARAMETRIZATION OF QCD

The method (GP) [3,6] is derived exactly from the QCD Lagrangian exploiting only few general properties.[1] Although noncovariant, the GP is relativistic. Also [6] the renormalization point for the quark masses can be selected at will in the QCD Lagrangian. The GP is compatible, in particular, with a quasichiral Lagrangian (with light u,d quarks) if the latter does not violate the properties of the QCD Lagrangian. By integrating over the virtual $q\overline{q}$ and gluon variables, the method parametrizes exactly the results of the QCD calculations of various hadron properties, expressing them in a few body language. It allows to write, almost at first sight, the most general expression for the spin-flavor structure of quantities relevant to the lowest baryons (**8** +**10**) and mesons. Unexpectedly one finds that, for the lowest hadrons, the GP is characterized, usually, by a rather small number of terms.

A consequence of the GP is that it allows to know if a constituent quark model is consistent with QCD. For any given property under study (masses, magnetic moments, etc.), it displays the result of QCD as a parametrized spin-flavor expression. The terms (and only the terms) present in the GP are compatible with QCD.

Of course, solving QCD (if one could do that) would express all parameters in the GP, in terms of $\Lambda_{\rm QCD}$ and the masses of the quarks. But much can be understood even if one is unable to calculate by QCD the values of the parameters. This goes as follows: Consider, e.g., the masses of the lowest baryons (**8**+**10**). Neglecting the electromagnetic (e.m.) corrections and the u,d mass difference, these are 8. But there are only eight GP parameters in this case; so they can be empirically determined. After doing this, a hierarchy in the parameters emerges: The parameters multiplying spin-flavor structures of increasing complexity are smaller and smaller. This is true for any quantity (not only the masses). Often this hierarchy allows us to neglect some terms in the GP; in particular, it explains why the nonrelativistic quark model (NRQM) [14] works.

III. THE 8+10 MASS FORMULA: A COMPARISON WITH THE CHIRAL RESULTS

The parametrization of the masses M_B's of the **8** and **10** baryons is [for the notation and use of Eq. (1) see Refs. [5,6]; only the combination $a+b$ enters in the masses]

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[1] These are (1) flavor breaking is due only to the mass term in the Lagrangian, (2) only quarks carry electric charge, (3) exact QCD eigenstates can be put in correspondence (for baryons) to a set of three quark-no gluon states or (for mesons) to a set of quark-antiquark-no gluon states, and (4) the flavor matrices in the electromagnetic (e.m.) interaction and in the flavor breaking term in the QCD Lagrangian commute.

"parametrized mass"

$$=A+B\sum_i P_i^s+C\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)+D\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)\times(P_i^s+P_k^s)+E\sum_{\substack{i\neq k\neq j\\(i>k)}}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)P_j^s+a\sum_{i>k}P_i^sP_k^s+b\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)P_i^sP_k^s+c\sum_{\substack{i\neq k\neq j\\(i>k)}}(\sigma_i\cdot\boldsymbol{\sigma}_k)(P_i^s+P_k^s)P_j^s+dP_1^sP_2^sP_3^s. \quad (1)$$

In Eq. (1) P^s is the projector on the strange quark and the flavor breaking term $\Delta m\bar{\psi}P^s\psi$ in the QCD Lagrangian is taken into account to all orders in $\Delta m=m_s-m$, no matter how large is Δm; thus Eq. (1) includes all orders in flavor breaking. The values (in MeV) of the parameters in Eq.(1), obtained fitting the baryon masses, are [5,6]

$$A=1076,\quad B=192,\quad C=45.6,\quad D=-13.8\pm0.3,$$
$$(a+b)=-16\pm1.4,\quad E=5.1\pm0.3,\quad c=-1.1\pm0.7,$$
$$d=4\pm3. \quad (2)$$

The hierarchy is evident. The values (2) decrease rather strongly with increasing complexity of the accompanying spin-flavor structure, so that one can neglect c and d in Eq. (1) and obtain the following mass formula [5], a generalization of the Gell-Mann–Okubo formula including octet and decuplet:

$$\tfrac{1}{2}(p+\Xi^0)+T=\tfrac{1}{4}(3\Lambda+2\Sigma^+-\Sigma^0). \quad (3)$$

The symbols stay for the masses and T is the following combination of decuplet masses:

$$T=\Xi^{*-}-\tfrac{1}{2}(\Omega+\Sigma^{*-}). \quad (4)$$

Because of the accuracy reached, we wrote Eq. (3) so as to be free of electromagnetic effects before comparing it to the data. [The combinations in Eq. (3) are independent of electromagnetic and of m_d-m_u effects, to zero order in flavor breaking (m_s-m).] The data satisfy Eq. (3) as follows (using the pole values, in MeV, of the masses):

$$\text{lhs}=1133.86\pm1.25,\quad \text{rhs}=1133.93\pm0.04 \quad (5)$$

an agreement confirming the smallness of the c,d terms neglected in Eq. (1) (with the conventional values of the masses the agreement is similar).

Of course (Ref. [6]) a full QCD calculation, if feasible, would express each $(A,B,\ldots,c,d)$ in Eq. (1) in terms of the quantities in the QCD Lagrangian, the running quark masses and the dimensional (mass) parameter $\Lambda\equiv\Lambda_{QCD}$; for instance, $A\equiv\Lambda\hat{A}(m/\Lambda,m_s/\Lambda)$ where $\hat{A}$ is some function. Similarly for B,C,D,E,a,b,c,d.

The results of Durand et al. [1] (see Sec. I) are obtained using an effective chiral QCD Lagrangian and heavy baryon chiral perturbation theory. They so reobtain our mass formula [Eq. (3)] (our T is their $\hat{\alpha}_{MM'}$ in their Eq. (3.38); they did not, however, extract the e.m. corrections from their sum rule).[2] Of course this result of Ref. [1] was expected. As stated in Sec. II, the GP is compatible with any relativistic chiral description, satisfying the listed general properties of QCD (for the pion field in the chiral Lagrangian, compare the end of Sec. IV). But it is interesting to see this in practice, especially in view of the heavy calculations in the chiral treatment of [1,2].[3]

Clearly the work of Durand *et al.* also confirms indirectly the existence of a hierarchy, as shown by the fact that they reobtain our mass formula [Eq. (3)] which is due to the smallness of c,d in Eq. (2).

As to the hierarchy in the GP, its field theoretical basis is discussed in Ref. [4]. The terms in the GP can be related to classes of the Feynman diagrams in QCD (Fig. 1 of Ref. [4]). The decrease of the coefficients that multiply terms with more quark indices is due: (1) to the increase of the number of gluons exchanged, (2) to the fact that each flavor breaking P_i^s also carries a reduction factor. Above we saw this for the masses. Indeed the values in Eq. (2) show that in Eq. (1) an additional pair of indices (corresponding to at least an additional gluon exchange between quark lines) implies a reduction factor in the range from 0.22 to 0.37 [4,6]. (One gets 0.37 using the pole values of the decuplet masses as we did in Eq. (2) and 0.22 using the conventional values.) The range of values 0.20–0.37 covers all the hadron properties examined so far. We will adopt usually 0.3 for the reduction factor due to "one gluon exchange more". The flavor reduction factor is in the range 0.3–0.33. Our reduction factor ≈ 0.3 is just an *empirical* number derivable in principle from QCD. Some papers relate this $\approx 1/3$ to the $1/N_c$ expansion. We do not see similarities between the basis of the GP, that refers *only* to the $N_c=3$ sector, and the $1/N_c$ expansion (see Ref. [15]).

IV. THE MAGNETIC MOMENTS OF THE BARYON OCTET

To first order in flavor breaking the parametrized magnetic moments $M_z(B)$ of the octet baryons B derived from QCD are *necessarily* (notation in Ref. [6]):

[2]It is only after doing this that the agreement becomes striking, as in Eq. (5) above.

[3]In Ref. [1] (E) "the parameter T" should be read as "the quantity T" [in fact T is defined by Eq. (4)]. Also the statement from "so is not to be used" to "Our approaches differ in that respect" is not too clear to us, because our T is identical to their $\hat{\alpha}_{MM'}$ (except that we included the e.m. corrections). If the above statement means that, with the chiral effective Lagrangian, the *individual* baryon masses can be calculated in terms of the couplings introduced in the Lagrangian, this may be true in principle; but, as in many chiral treatments, uncertainties often arise in practice.

$$M_z(B)=\sum_{\nu=1}^{7}\tilde{g}_\nu(\mathbf{G}_\nu)_z, \tag{6}$$

where

$$\mathbf{G}_1=\sum_i Q_i\boldsymbol{\sigma}_i,\quad \mathbf{G}_2=\sum_i Q_iP_i^s\boldsymbol{\sigma}_i,\quad \mathbf{G}_3=\sum_{i\neq k} Q_i\boldsymbol{\sigma}_k,$$

$$\mathbf{G}_4=\sum_{i\neq k} Q_iP_i^s\boldsymbol{\sigma}_k$$

$$\mathbf{G}_5=\sum_{i\neq k} Q_kP_i^s\boldsymbol{\sigma}_i,\quad \mathbf{G}_6=\sum_{i\neq k} Q_iP_k^s\boldsymbol{\sigma}_i,$$

$$\mathbf{G}_7=\sum_{i\neq j\neq k} Q_iP_j^s\boldsymbol{\sigma}_k. \tag{7}$$

In Eq. (7) Q_i is the quark charge. It is understood that the expectation value of the rhs of Eq. (6) on the octet spin-flavor states W_B (compare Ref. [6]) must be taken. Eight $\mathbf{G}_\nu$'s appear in Eq. (23) of Ref. [6]; but due to the following Eq. (8), holding for the expectation values of the $\mathbf{G}_\nu$'s in W_B's (see Ref. [9]), $\mathbf{G}_0=\mathrm{Tr}[QP^s]\Sigma_i\boldsymbol{\sigma}_i$ is expressed in terms of the $\mathbf{G}_\nu$'s with $\nu=1,\ldots,7$; thus the sum in Eq. (6) contains seven terms; their coefficients $\tilde{g}_\nu$ differ inappreciably from the g_ν multiplying the $\mathbf{G}_\nu$'s. This is a very general consequence of QCD (compare the evaluation of the quark loop effect in Ref. [17]):

$$\mathbf{G}_0=-\tfrac{1}{3}\mathbf{G}_1+\tfrac{2}{3}\mathbf{G}_2-\tfrac{5}{6}\mathbf{G}_3+\tfrac{5}{3}\mathbf{G}_4+\tfrac{1}{6}\mathbf{G}_5+\tfrac{1}{6}\mathbf{G}_6+\tfrac{2}{3}\mathbf{G}_7. \tag{8}$$

Though the $\mathbf{G}_\nu$'s look nonrelativistic, Eq. (6) is an exact consequence of full QCD (to first order in flavor breaking). We repeat this to avoid misinterpreting Eq. (6) as a sort of generalized NRQM. Note the relative dominance of $\tilde{g}_1$ and $\tilde{g}_2$ in the sum in Eq. (6) [see Eq. (12) below]; this explains the fairly good two-parameter fit of the naive NRQM:

$$M_z(B)=\tilde{g}_1(\mathbf{G_1})_z+\tilde{g}_2(\mathbf{G_2})_z\quad (NRQM). \tag{9}$$

We now compare the above results with the chiral treatment of Durand et al. [1]. It is notable that the latter produces precisely seven terms for the octet baryon magnetic moments, to first order flavor breaking [their Eqs. (4.6)–(4.12)] written in terms of the Pauli spin matrices, similarly to our Eqs. (14) (their symbols M are our P^s). Their seven $\mathbf{m}$'s are linear combinations of our $\mathbf{G}$'s. In their footnote 14 a relation appears similar to our Eq. (8).

In the following we will need the magnetic moments in terms of the $\tilde{g}_\nu$'s. From Eq. (6) we get (the baryon symbol indicates the magnetic moment)

$$p=\tilde{g}_1,$$

$$n=-(2/3)(\tilde{g}_1-\tilde{g}_3), \tag{10}$$

$$\Lambda=-(1/3)(\tilde{g}_1-\tilde{g}_3+\tilde{g}_2-\tilde{g}_5),$$

$$\Sigma^+=\tilde{g}_1+(1/9)(\tilde{g}_2-4\tilde{g}_4-4\tilde{g}_5+8\tilde{g}_6+8\tilde{g}_7),$$

$$\Sigma^-=-(1/3)(\tilde{g}_1+2\tilde{g}_3)+(1/9)(\tilde{g}_2-4\tilde{g}_4+2\tilde{g}_5-4\tilde{g}_6-4\tilde{g}_7),$$

$$\Xi^0=-(2/3)(\tilde{g}_1-\tilde{g}_3)+(1/9)(-4\tilde{g}_2-2\tilde{g}_4+4\tilde{g}_5-8\tilde{g}_6+10\tilde{g}_7),$$

$$\Xi^-=-(1/3)(\tilde{g}_1+2\tilde{g}_3)+(1/9)(-4\tilde{g}_2-2\tilde{g}_4-8\tilde{g}_5-2\tilde{g}_6-2\tilde{g}_7),$$

and

$$\mu(\Sigma\Lambda)=-(1/\sqrt{3})(\tilde{g}_1-\tilde{g}_3+\tilde{g}_6-\tilde{g}_7). \tag{11}$$

From the Particle Data Group (PDG) values of the magnetic moments [16] we obtain

$$\tilde{g}_1=2.793,\quad \tilde{g}_2=-0.934,\quad \tilde{g}_3=-0.076,\quad \tilde{g}_4=0.438,$$

$$\tilde{g}_5=0.097,\quad \tilde{g}_6=-0.147,\quad \tilde{g}_7=0.154. \tag{12}$$

In Eq. (12) the hierarchy is apparent. The average value of the one gluon exchange reduction factor derived from the values of $|\tilde{g}_6|,|\tilde{g}_5|,|\tilde{g}_4|$ is 0.25, having adopted 0.3 for the flavor reduction factor (this is 0.33 from the ratio of $|\tilde{g}_2|$ and $|\tilde{g}_1|$). Here we go on using 0.3 for both reduction factors; doing so, the maximum discrepancy between estimated and empirical values is 2.5 for each $|\tilde{g}_\nu|$ with $\nu=4,5,6$.

An exception is $|\tilde{g}_3|\simeq 0.08$. This is much too small: One expects from the hierarchy $2.79\times 0.3\simeq 0.84$, a value ten times larger. We discuss this in Secs. V and VI.

A comment to *the Pondrom's four-parameter fit* [13] of the baryon moments is appropriate here. Pondrom's fit is based on the conjecture of assigning to the quarks different magnetic moments in different baryons and assume additivity. But these assumptions lead to the following approximate empirical relations (holding to ± 0.1):

$$-(2/3)p\simeq n,\ \ (1/2)(\Sigma^- - \Sigma^+)\simeq n,\ \ \Xi^-+(1/2)\Xi^0\simeq 2\Lambda. \tag{13}$$

The Eqs. (13), of course, reduce from 7 to 4, the number of $\tilde{g}_\nu$'s. One thus finds that the GP plus the smallness of $\tilde{g}_3$ explain why a four-parameter fit is rather good, independently of any symmetry (a question raised in Ref. [13]). Finally, the fit [13] gives $\mu(\Sigma\Lambda)=-1.61$. The GP formula (11) gives instead $\mu(\Sigma\Lambda)=-1.48\pm 0.04$. Experimentally, it is $|\mu(\Sigma\Lambda)|=1.61\pm 0.08$; errors are still large.

To go on with the comparison to the work of Durand *et al.* [1], a remark on the pion exchange terms in a class of calculations of the baryon moments is necessary. For instance, for the p and n moments $M_z(p,n)$, a typical such term is

$$M_z(p,n)=\cdots+\alpha\sum_{i\neq k}(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k)_z(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_k)_3, \tag{14}$$

where the dots in Eq. (14) refer to the contributions other than pion exchange and α is some coefficient. Because in the QCD Lagrangian only the quark and gluon fields intervene (not those of pions), the question arises of the meaning of such pion exchange terms. The answer (see Ref. [18]) is that they simply duplicate terms already present in the GP; they can be always incorporated into them. It is

$$\sum_{i\neq k} (\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k)(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_k)_3=-8\mathbf{G}_1+4\mathbf{G}_3. \tag{15}$$

Durand et al. [1] *showed* that all pion exchange magnetic moments could be rewritten in terms of their seven quantities **m**'s (their Eqs. (4.6)–(4.12) in Ref. [1]), which are simply certain linear combinations of our **G**'s. We found this result [from their Eq. (4.29) to their Eq. (4.36)] interesting also because it confirms the GP on this rather subtle point; the pion loops in the chiral treatment of Ref. [1] are eliminated by a mechanism that must be equivalent to that of Eq. (15).

V. THE COINCIDENTAL NATURE OF THE "PERFECT" 3/2 PREDICTION FOR $|\mu(p)/\mu(n)|$

Equation (6) of the GP for the magnetic moments, applied to p and n, gives

$$\mathbf{M}(n,p)=\tilde{g}_1\mathbf{G}_1+\tilde{g}_3\mathbf{G}_3=\tilde{g}_1\sum_i Q_i\boldsymbol{\sigma}_i+\tilde{g}_3\sum_{i\neq k} Q_i\boldsymbol{\sigma}_k. \tag{16}$$

In Sec. IV we noted that $\tilde{g}_3$ is ten times smaller than the value $2.79\times0.3\simeq0.84$ expected from the hierarchy; the other $\tilde{g}_\nu$'s (with $\nu=4,5,6$) differ by no more than 2.5 times from their expected values. Due to this, we suggested in Refs. [4,6,9] that the early prediction of the NRQM [14] $|\mu(p)/\mu(n)|=3/2$ is coincidental. Indeed [Eq. (10)] it is

$$|\mu(p)/\mu(n)|=-(3/2)[\tilde{g}_1/(\tilde{g}_1-\tilde{g}_3)], \tag{17}$$

so that $|\mu(p)/\mu(n)|$ depends critically on $\tilde{g}_3$, the coefficient of the second (nonadditive) term in Eq. (16).

Recently Leinweber *et al.* [7] reached the same conclusion that the almost perfect 3/2 prediction is coincidental. In Ref. [7] $|\mu(p)/\mu(n)|$ is calculated in a chiral QCD perturbation theory, dynamically broken by pions; the above ratio varies from 1.37 to 1.55 as the pion mass varies from 0 to $\simeq$280 MeV (corresponding to "a variation of current quark mass from 0 to just 20 MeV"). *Again, as with the work of Durand et al., the chiral conclusion agrees with that from the GP*. We try, however, to clarify some statements in Ref. [7] and in Cloet *et al.* [8].

(1) The assertion in Ref. [7] that "within the constituent quark model the ratio $|\mu(p)/\mu(n)|$ would remain constant at 3/2, independent of the change of the quark mass" is correct only in an additive model, such as the original NRQM [14]. The GP expression (16) for the p,n magnetic moments in a constituent model obtained from QCD shows that this is not additive (for the importance of nonadditivity see also Ref. [19]).

(2) Cloet *et al.* [8], referring to the GP as "something a little more sophisticated than the simplest constituent quark model," add a statement on "the need to incorporate meson cloud effects into conventional constituent quark models." There is no doubt that constituent quarks must be dressed, but this was already there in the old additive NRQM [14]. The GP incorporates all $q\bar{q}$ and gluon effects [3]; in particular, the magnetic moments of the **8** baryons in the GP are those of *any possible* constituent quark model compatible with QCD, endowed with the correct "dressing" of the (constituent) quarks.

To conclude, the chiral Lagrangians of Durand *et al.* and of Leinweber *et al.* produce results in agreement with the GP. As to the question, what about the pion field that appears in the chiral Lagrangians of Refs. [1] and [7], but not in the QCD Lagrangian (where the pion is not an independent field), this has been answered in Sec. IV for the magnetic moments and a similar argument should be true for the masses (as shown by the results of Ref. [1]).

VI. PARAMETRIZING THE MAGNETIC MOMENTS OF THE DECUPLET: p,n,Δ

We apply now the GP to the magnetic moments of the Δ's in addition to p,n. This clarifies further the mechanism producing accidentally a small value of $\tilde{g}_3$ (noted in the past section) and thus the coincidental nature of $|\mu(p)/\mu(n)|=3/2$. Also we obtain some results on the Δ's. The general QCD spin-flavor structure of the magnetic moments of p,n,Δ's is [3,9]

$$\mu(B)=\sum_{perm}[\alpha Q_1+\delta(Q_2+Q_3)]\boldsymbol{\sigma}_{1z}+[\beta Q_1+\gamma(Q_2+Q_3)]\boldsymbol{\sigma}_{1z}(\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3). \tag{18}$$

Equation (18) is the same as Eq. (62) of Ref. [3]; $\alpha,\delta,\beta,\gamma$ are four real parameters. The sum over perm(utations) in Eq. (18) means that to term (123) one adds (321) and (231).[4]

We adopt the "standard" hierarchy for the parameters $\alpha,\delta,\beta,\gamma$ with the reduction factor 0.3 for one more gluon exchange; but because this factor for the magnetic moments is between 0.2 and 0.3, see the remarks in Sec. IV after Eq. (12), we widen the error in δ, the largest parameter after the dominant one, α:

$$|\delta/\alpha|=0.2\leftrightarrow0.3,\quad |\beta/\delta|\approx0.3,\quad |\gamma/\delta|\approx0.3. \tag{19}$$

From Eqs. (18), (16) we obtain for $\tilde{g}_1$ and $\tilde{g}_3$,

$$\tilde{g}_1=\alpha-3\beta-2\gamma,\quad \tilde{g}_3=\delta-\beta-4\gamma. \tag{20}$$

[4]In Ref. [3] correct the following misprints: In Eq. (63) insert (-2γ) in the second square brackets; in Eq. (66) write $F=\delta-\beta-4\gamma$; in Eq. (64) (Q term) replace -2γ with $+4\gamma$.

From Eq. (16) one has for the magnetic moments (p,n), now indicated by the particle symbols, expressed in proton magnetons:

$$p=(\alpha-3\beta-2\gamma),\quad n=-(2/3)(\alpha-\delta-2\beta+2\gamma). \tag{21}$$

Hence

$$(n/p)=-2/3[1+(-\delta+\beta+4\gamma)/p]. \tag{22}$$

Thus the deviation of $|n/p|$ from (2/3) is determined by the term in square brackets in Eq. (22). If it were not for the *second-order terms* $(\beta+4\gamma)$ with β and γ of order $(0.3)^2$ (three indices), the dominant deviation would be of order $|\delta/p|=0.25\pm0.05$; that is, 20–30 % of the "perfect" value 2/3. To summarize, the mechanism giving to $|(p/n)|$ a value so near to (3/2) is this: In Eq. (22) $(\beta+4\gamma)$ almost cancels $(-\delta)$ (which is >0), producing, accidentally, $(n/p)=-(2/3)$ to a few percent. One can also show that β and γ must have opposite signs and $\gamma<0$. This is derived by using the $(\Delta\to p\gamma)_0$ matrix element extrapolated to vanishing transferred photon momentum $(k=0)$ that we know experimentally. We do not enter on this here (compare [9] where, however, some data must be changed). Here we just give the approximate values of $\alpha,\delta,\beta,\gamma$. It is $\alpha\simeq3$ and the values of δ,β,γ [affected by the errors stated in Eq. (19)] are $\delta=-0.75$, $\beta=0.25$, $\gamma=-0.25$.

One more point: From Eq. (18) one can also express in terms of $\alpha,\delta,\beta,\gamma$ the magnetic moments $\mu(\Delta)$ of the Δ's. It is

$$\mu(\Delta)=(\alpha+2\delta+\beta+2\gamma)Q_\Delta=[\mu(p)+2\delta+4\beta+4\gamma]Q_\Delta. \tag{23}$$

In view of the above, the magnetic moment of the singly charged Δ^+, $\mu(\Delta^+)$ (the coefficient of J_z), is expected to be appreciably smaller than $\mu(p)$, but the error on its value is large. Of course, it is $\mu(\Delta^{++})\simeq2\mu(\Delta^+)$. We stress that, in general, it is $\mu(\Delta^Q)=\mathrm{k}Q+\xi$, but ξ is negligible [17] because it is a Trace term strongly depressed by the exchange of several gluons needed by the Furry theorem.

VII. SOME REMARKS ON A SUM RULE FOR THE BARYON OCTET MAGNETIC MOMENTS

Long ago Franklin [12] suggested a sum rule for the baryon octet magnetic moments that is often called the Coleman-Glashow rule, since it has the same form as the Coleman-Glashow rule for the electromagnetic mass differences. Franklin's rule is

$$\Sigma_\mu=0, \tag{24}$$

where it is

$$\Sigma_\mu\equiv\mu(p)-\mu(n)+\mu(\Sigma^-)-\mu(\Sigma^+)+\mu(\Xi^0)-\mu(\Xi^-). \tag{25}$$

Because in chiral models of baryons of the Manohar-Georgi type (χQM) the rule of Eq. (24) should be satisfied [10], but in reality it is violated:

$$\Sigma_\mu=0.49\pm0.03, \tag{26}$$

some work has been done [10,11] to understand the reason for this fact. After showing that in a χQM the rule (24) is satisfied, Linde *et al.* consider several extensions of the χQM (several phenomenological models) that break the rule. We refer to Refs. [10,11] for many references and a description of the models.

Here we will only show that it follows from the GP that the rule is necessarily broken by two specific first-order flavor breaking terms, $\tilde{g}_5$ and $\tilde{g}_6$. Indeed, using Eqs. (10), (12), it is

$$\Sigma_\mu\equiv2(\tilde{g}_5-\tilde{g}_6). \tag{27}$$

With the $\tilde{g}_\nu$'s given in Eq. (12) the rhs of Eq. (27) is in fact (0.49 ± 0.03).

The interest of the above deduction stays in its conclusion:

Any Lagrangian or phenomenological model (chiral or nonchiral) designed to reproduce the result of the exact QCD Lagrangian violates the Franklin (Coleman-Glashow) sum rule for the octet baryon magnetic moments *if and only if* the coefficients of the flavor breaking terms $\mathbf{G}_5$ and $\mathbf{G}_6$ do not vanish and their difference does not vanish. In such case the model must be built in such a way that $2(g_5-g_6)$ is equal to the experimental value 0.49 ± 0.03. All other parameters, multiplying $\mathbf{G}_\nu$ with $\nu\neq5,6$, even the flavor breaking ones with $\nu=2,4,7$, do not produce violations of the rule.

VIII. CONCLUSION

In Ref. [6] we stated that the general QCD parametrization (GP) explains why a large variety of different theories and models, *including relativistic chiral theories*, may work successfully. Now, thanks especially to the treatment by Durand *et al.* [1] of the baryon masses and magnetic moments, we have an explicit detailed confirmation that the GP covers (see also Ref. [7]) the case of the relativistic chiral field theories, provided that such theories (or models) are compatible with the general properties of QCD.

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