ρ polarization and model independent extraction of $|V_{ub}|/|V_{cd}|$ from $D\rightarrow\rho\ell\nu$ and $B\rightarrow\rho\ell\nu$

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We briefly discuss the predictions of the heavy quark effective theory for the semileptonic decays of a heavy pseudoscalar to a light one, or to a light vector meson. We point out that measurement of combinations of differential helicity decay rates at CLEO-c and the B factories can provide a model independent means of extracting the ratio $|V_{ub}|/|V_{cd}|$. We briefly discuss the corrections to this prediction.

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I. INTRODUCTION AND MOTIVATION

Extraction of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub} is one of the crucial ingredients needed to determine the source of *CP* violation. A number of methods have been suggested for extraction of this matrix element, from both inclusive and exclusive decays. In this paper, we point out that, modulo $1/m_c$, $1/m_b$ and isospin corrections, the ratio $|V_{ub}|/|V_{cd}|$ can be determined using the predictions of the heavy quark effective theory (HQET), and the measurement of some polarization observables in the decays $D\rightarrow\rho\ell\nu$ and $B\rightarrow\rho\ell\nu$, in a model independent way.

Among the many measurements that may be done at the proposed CLEO-c are precision studies of the semileptonic decays $D\rightarrow\pi\ell\nu$ and $D\rightarrow\rho\ell\nu$: it is expected that the ratio $|V_{cd}|/|V_{cs}|$ can be measured to 1.3%. In particular, there should be ample statistics to study the polarization of the vector mesons produced in the latter reaction. If this is the case, and if similar measurements can be done at the B factories, we propose measurements that can provide the advertised ratio of CKM matrix elements with very small theoretical errors.

In the next section we briefly discuss the predictions of HQET for these heavy $\rightarrow$ light transitions, at leading order. Section III discusses a number of experimental observables from these processes, and their dependence on the form factors describing these semileptonic decays. In that section, we also present the observables that allow extraction of the advertised ratio. In Sec. IV we briefly discuss the $1/m_Q$ corrections, and in Sec. V we present our conclusions.

II. HQET AND FORM FACTORS

The hadronic matrix elements for the decays $D\rightarrow\pi\ell\nu$ and $D\rightarrow\rho\ell\nu$ are

$$\langle\pi(p)|\overline{q}\gamma_\mu c|D(P)\rangle = f_+^{D\pi}(P+p)_\mu + f_-^{D\pi}(P-p)_\mu,$$

$$\langle\pi(p)|\overline{q}\gamma_\mu\gamma_5 c|D(P)\rangle = 0,$$

$$\langle\rho(p,\epsilon)|\overline{q}\gamma_\mu c|D(P)\rangle = ig^{D\rho}\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}(P+p)^\alpha(P-p)^\beta,$$

$$\langle\rho(p,\epsilon)|\overline{q}\gamma_\mu\gamma_5 c|D(P)\rangle = f^{D\rho}\epsilon^*_\mu + a_+^{D\rho}\epsilon^*\cdot P(P+p)_\mu + a_-^{D\rho}\epsilon^*\cdot P(P-p)_\mu, \quad (1)$$

where the light quark q can be either u or d. These decays are thus described in terms of six independent, *a priori* unknown form factors. The terms in f_- and a_- are unimportant when the lepton mass is ignored, since

$$(P-p)_\mu\overline{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell = (k_\nu+k_\ell)_\mu\overline{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell = m_\ell\overline{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell. \quad (2)$$

For the transitions from B mesons, an analogous set of matrix elements and form factors are necessary.

Using the Dirac matrix representation of heavy mesons, we may treat heavy-to-light transitions using the same trace formalism that has been applied to heavy-to-heavy transitions [1,2]. In the effective theory, a heavy pseudoscalar meson (D or B), denoted P, traveling with velocity v is represented as [1,2]

$$|\mathcal{P}(v)\rangle = -\frac{1}{\sqrt{2}}\frac{1+\not{v}}{2}\gamma_5 \equiv \mathcal{M}_P(v). \quad (3)$$

These states are normalized so that

$$\langle\mathcal{P}(v')|\mathcal{P}(v)\rangle = 2v_0\delta^3(\mathbf{p}-\mathbf{p}'). \quad (4)$$

The states of QCD and HQET are therefore related by

$$|P(v)\rangle = \sqrt{m_P}|\mathcal{P}(v)\rangle, \quad (5)$$

where m_P is the mass of the pseudoscalar meson.

For the semileptonic transitions between such a heavy meson (P meson) and a light pseudoscalar (π), the matrix element of interest is [3]

$$\langle\pi(p)|\overline{q}\Gamma h_v^{(Q)}|\mathcal{P}(v)\rangle = \mathrm{Tr}[\gamma_5(\xi_1+\not{p}\xi_2)\gamma_\mu\mathcal{M}_P(v)], \quad (6)$$

where $h_v^{(Q)}$ is the heavy quark in the effective theory, and q denotes a light quark (u or d). For the transition to a light vector (ρ), the matrix elements are written

$$\langle\rho(p,\varepsilon)|\bar{q}\Gamma h_v^{(Q)}|\mathcal{P}(v)\rangle=\mathrm{Tr}[\{(\xi_3+\not p\xi_4)\varepsilon^*\cdot v+\not\varepsilon^*(\xi_5+\not p\xi_6)\}\Gamma\mathcal{M}_P(v)], \quad (7)$$

where Γ denotes either γ_μ or $\gamma_\mu\gamma_5$.

The form factors ξ_i are kinematic functions of $v\cdot p$. They are independent of the mass of the heavy quark, and are therefore universal functions of the kinematic variable $v\cdot p$. Thus, they are valid for $D\to\pi(\rho)$ decays, as well as for $B\to\pi(\rho)$ decays. This independence of the quark mass allows us to deduce, in a relatively straightforward manner, the scaling behavior of the usual form factors that describe these transitions [4]. More precisely, they allow us to relate the form factors for D transitions to those for B transitions. The relationships are

$$f_+^B(v\cdot p)=\frac{1}{2}\left(\frac{m_B}{m_D}\right)^{1/2}\left[f_+^D(v\cdot p)\left(1+\frac{m_D}{m_B}\right)+f_-^D(v\cdot p)\left(\frac{m_D}{m_B}-1\right)\right],$$

$$f_-^B(v\cdot p)=\frac{1}{2}\left(\frac{m_B}{m_D}\right)^{1/2}\left[f_-^D(v\cdot p)\left(1+\frac{m_D}{m_B}\right)+f_+^D(v\cdot p)\left(\frac{m_D}{m_B}-1\right)\right],$$

$$f^B(v\cdot p)=\left(\frac{m_B}{m_D}\right)^{1/2}f^D(v\cdot p),$$

$$g^B(v\cdot p)=\left(\frac{m_D}{m_B}\right)^{1/2}g^D(v\cdot p),$$

$$a_+^B(v\cdot p)=\frac{1}{2}\left(\frac{m_D}{m_B}\right)^{1/2}\left[a_+^D(v\cdot p)\left(1+\frac{m_D}{m_B}\right)+a_-^D(v\cdot p)\left(\frac{m_D}{m_B}-1\right)\right],$$

$$a_-^B(v\cdot p)=\frac{1}{2}\left(\frac{m_D}{m_B}\right)^{1/2}\left[a_-^D(v\cdot p)\left(1+\frac{m_D}{m_B}\right)+a_+^D(v\cdot p)\left(\frac{m_D}{m_B}-1\right)\right], \quad (8)$$

where f_+^D is the form factor appropriate to the $D\to\pi$ transition, while f_+^B is the form factor appropriate to the $B\to\pi$ transition, and quantities on the left-hand sides of Eqs. (8) are evaluated at the same values of $v\cdot p$ as those on the right-hand sides. Omitted from each of Eqs. (8) is a QCD scaling factor.

We point out here that Kramer *et al.* [5] have looked at these heavy $\to$ light transitions, but the relationships that they obtain among the form factors are not the same as those discussed elsewhere in the literature [6]. In particular, their a_+^B does not involve an admixture of a_+^D and a_-^D, but is related to a_+^D only. In addition, it must be noted that the usual limiting forms of the matrix element for heavy to heavy transitions are not recoverable from the expressions that they write down, while this limiting form is recoverable from the expressions in Eq. (8).

In the limit of a heavy b quark, the full current of QCD is replaced by [8]

$$\bar{q}\Gamma b\to\bar{q}\Gamma h_v^{(b)}\left[\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right]^{-6/25}. \quad (9)$$

This arises from integrating out the b quark, and matching the resulting effective theory onto full QCD at the scale m_b, at one loop level. At the scale m_c, we must also integrate out the c quark, but there is also the effect due to running between m_b and m_c. The net effect of this is that the form factors ξ_i appropriate to the $b\to u$ transitions are related to those for the $c\to d$ transitions by

$$\xi_i^{b\to u}=\xi_i^{c\to d}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}. \quad (10)$$

III. OBSERVABLES

In the differential decay rate of $D(B)\to\pi\ell\nu$, terms that depend on the form factor f_- are proportional to the mass of the lepton, and are thus very difficult to extract from experiment. Detecting the polarization of the charged lepton offers the only possibility of extracting this form factor, but this is apparently a very remote prospect with muons or electrons. Because of the predicted mixing of f_+ and f_- in HQET, it becomes very difficult to say anything about $B\to\pi$ observables based on information extracted from the corresponding $D\to\pi$ observables. Assumptions about the form of f_- can be and have been made, but this introduces some model dependence into any information extracted.

For the decay to the ρ, helicity amplitudes $H_\pm$ and H_0 can be defined as

$$H_\pm(q^2)=f(q^2)\pm m_P k_\rho g(q^2),$$

$$H_0(q^2)=\frac{1}{2m_\rho\sqrt{q^2}}[(m_P^2-m_\rho^2-q^2)f(q^2)+4m_P^2k_\rho^2a_+(q^2)], \quad (11)$$

where k_ρ is the momentum of the daughter ρ in the rest frame of the parent pseudoscalar. In terms of these, the differential decay rate is written [7]

$$\frac{d\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_\rho d\chi} = \frac{3G_F^2|V_{qQ}|^2 k_\rho q^2}{8(4\pi)^4 m_P^2}\{[(1+\eta\cos\theta_\ell)^2|H_+(q^2)|^2+(1-\eta\cos\theta_\ell)^2|H_-(q^2)|^2]\sin^2\theta_\rho + 4\sin^2\theta_\ell\cos^2\theta_\rho|H_0(q^2)|^2 - 2\sin^2\theta_\ell\sin^2\theta_\rho\cos 2\chi H_+(q^2)H_-(q^2) + 4\eta\sin\theta_\ell\sin\theta_\rho\cos\theta_\rho\cos\chi H_0(q^2)[(1-\eta\cos\theta_\ell)H_-(q^2)-(1+\eta\cos\theta_\ell)H_+(q^2)]\}, \tag{12}$$

where $\eta=+1$ for B decays, and -1 for D decays. The angles θ_ℓ, θ_ρ and χ are explained in Fig. 1.

With sufficient statistics in both B and D decays, this differential decay rate can provide means of extracting the ratio $|V_{ub}|/|V_{cd}|$. The differential decay rates into specific helicity states of the ρ can be written as

$$\frac{d\Gamma_i}{dq^2} = \frac{G_F^2|V_q Q|^2}{96\pi^3} k_\rho \frac{q^2}{m_P^2}|H_i(q^2)|^2. \tag{13}$$

Much of the difficulty of saying anything about the total rate in $B\to\rho\ell\nu$, based on measurements of $D\to\rho\ell\nu$, remains because of the mixing of a_+ and a_-. As with the decays to pions, a_- is, for the most part, only accessible through measurement of the polarization of the charged lepton. However, decays to transversely polarized ρ mesons are independent of this form factor, depending only on f and g.

If $d\Gamma^D_\pm/dq^2$ can be measured or extracted at CLEO-c, and $d\Gamma^B_\pm/dq^2$ at B factories, the ratio of these two differential decay rates, at the same kinematic point $v\cdot p$ ($=E_\rho$ in the rest frame of the parent hadron), depends only on known or measurable kinematic quantities, as the form factor dependence drops out at leading order in HQET. Actually, this is not quite true. This ratio will depend on the form factor ratio r defined as

$$r(v\cdot p) = \frac{g^D(v\cdot p)}{f^D(v\cdot p)}, \tag{14}$$

but this ratio should be extractable from other observables in the $D\to\rho\ell\nu$ decay. More explicitly,

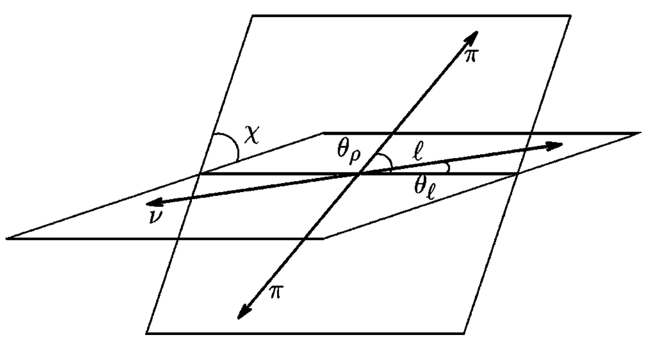

FIG. 1. The angles of Eq. (12). θ_ℓ and θ_ρ are defined in the rest frames of the lepton pair and the ρ, respectively. χ is the angle between the lepton plane and the hadron plane.

$$\frac{d\Gamma^B_\pm/dq^2}{d\Gamma^D_\pm/dq^2} = \frac{|V_{ub}|^2}{|V_{cd}|^2}\frac{q_B^2 m_D^2}{q_D^2 m_B^2} \times \frac{m_B/m_D + m_B m_D (k_\rho^B)^2 r^2(v\cdot p) \pm 2 m_B k_\rho^B r(v\cdot p)}{1+m_D^2(k_\rho^D)^2 r^2(v\cdot p) \pm 2 m_D k_\rho^D r(v\cdot p)}. \tag{15}$$

Since the numerator and denominator of this ratio are evaluated at the same value of E_ρ, $k_\rho^D = k_\rho^B$. In the expression above,

$$q_B^2 = m_B^2 + m_\rho^2 - 2m_B v\cdot p, \quad q_D^2 = m_D^2 + m_\rho^2 - 2 m_D v\cdot p. \tag{16}$$

The combination $d\Gamma_+/dq^2 + d\Gamma_-/dq^2$ is more easily accessible in these experiments [as the term proportional to $(1+\cos^2\theta_\ell)\sin^2\theta_\rho$ in the expression for the decay rate]. The ratio of this quantity for B decays to that for D decays is

$$\frac{d\Gamma^B_+/dq^2 + d\Gamma^B_-/dq^2}{d\Gamma^D_+/dq^2 + d\Gamma^D_-/dq^2} = \frac{|V_{ub}|^2}{|V_{cd}|^2}\frac{q_B^2 m_D^2}{q_D^2 m_B^2}\frac{m_B/m_D + m_B m_D (k_\rho^B)^2 r^2(v\cdot p)}{1+m_D^2(k_\rho^D)^2 r^2(v\cdot p)}. \tag{17}$$

In addition, the differential rate $d\Gamma_{+-}/dq^2$, proportional to $H_+(q^2)H_-(q^2)$, may also be accessible (as the term proportional to $\cos 2\chi$ in the expression for the decay rate). If this is measured in B and D decays, the ratio is

$$\frac{d\Gamma^B_{+-}/dq^2}{d\Gamma^D_{+-}/dq^2} = \frac{|V_{ub}|^2}{|V_{cd}|^2}\frac{q_B^2 m_D^2}{q_D^2 m_B^2}\frac{m_B/m_D - m_B m_D (k_\rho^B)^2 r^2(v\cdot p)}{1-m_D^2(k_\rho^D)^2 r^2(v\cdot p)}. \tag{18}$$

This ratio again depends only on known or measurable kinematic quantities and the form factor ratio $r(v\cdot p)$.

Far more intriguing is the ratio of the difference of these differential helicity decay rates, related to the differential lepton forward-backward asymmetry, and proportional to $\cos\theta_\ell\sin^2\theta_\rho$. This ratio takes the form

$$\frac{d\Gamma_+^B/dq^2-d\Gamma_-^B/dq^2}{d\Gamma_+^D/dq^2-d\Gamma_-^D/dq^2}=-\frac{|V_{ub}|^2}{|V_{cd}|^2}\frac{q_B^2 m_D}{q_D^2 m_B}, \tag{19}$$

where a factor of k_ρ^B/k_ρ^D has been set to unity, as explained above. Inclusion of the lowest order radiative corrections means that the ratios above must be multiplied by the factor $[\alpha_s(m_b)/\alpha_s(m_c)]^{-12/25}$.

At leading order, the last ratio of observables is completely independent of any form factor, and so should provide a very good means of extracting the ratio $|V_{ub}|/|V_{cd}|$. It is to be emphasized that implied in Eq. (19) is the fact that the measurement of the ratio need only be made at a single value of $v\cdot p$.

It must be emphasized what "completely independent of any form factor" means for the model dependence in the extraction of the ratio $|V_{ub}|/|V_{cd}|$. Since model dependence in the extraction of this ratio arises from the model dependence that is present in the "parametrization" of the form factors of unknown form and unknown normalization, use of Eq. (19) yields a truly model independent means of extracting the ratio $|V_{ub}|/|V_{cd}|$. This means that, if there were no $1/m_Q$ corrections to Eq. (19), the model dependence in the extraction of $|V_{ub}|/|V_{cd}|$ would be reduced to zero.

IV. $1/m_Q$ CORRECTIONS AND MODEL DEPENDENCE

In order for these predictions to be of any use, we must know how robust they are in the face of $1/m_Q$ corrections. More importantly, it is crucial to know how model dependent the $1/m_Q$ corrections to Eq. (19) are, for instance. In this section, we briefly examine these corrections for Eq. (19), as this prediction is model independent at leading order in HQET.

This equation is understood as having a factor of $f^{B\to\rho}g^{B\to\rho}/f^{D\to\rho}g^{D\to\rho}$ multiplying it, and this factor is unity at leading order in HQET. At order $1/m_Q$, Huang *et al.* [9] state that a total of 22 new universal functions are needed to describe the semileptonic decays of a heavy pseudoscalar meson to a light vector. However, for the purposes of this analysis, we can do something much simpler. Whatever the form of the $1/m_Q$ corrections, the form factors f and g can be written as

$$f=\sqrt{m_P}\left(\tilde f_0+\frac{1}{m_Q}\tilde f_1\right),$$

$$g=\frac{1}{\sqrt{m_P}}\left(\tilde g_0+\frac{1}{m_Q}\tilde g_1\right), \tag{20}$$

where m_P is the mass of the heavy pseudoscalar, $\tilde f_0$ and $\tilde g_0$ are the leading order universal functions, and $\tilde f_1$ and $\tilde g_1$ are themselves universal functions, and are the appropriate linear combinations of the 22 universal functions discussed by Huang *et al.* [9]. Using this, we can then expand the form factor ratio multiplying Eq. (19) as

$$\frac{f^{B\to\rho}g^{B\to\rho}}{f^{D\to\rho}g^{D\to\rho}}=\frac{[\tilde f_0+(1/m_b)\tilde f_1][\tilde g_0+(1/m_b)\tilde g_1]}{[\tilde f_0+(1/m_c)\tilde f_1][\tilde g_0+(1/m_c)\tilde g_1]}=1+\left(\frac{1}{m_b}-\frac{1}{m_c}\right)\left(\frac{\tilde f_1}{\tilde f_0}+\frac{\tilde g_1}{\tilde g_0}\right)+\mathcal{O}\left(\frac{1}{m_c^2}\right). \tag{21}$$

The prediction of Eq. (19) would then become

$$\frac{d\Gamma_+^B/dq^2-d\Gamma_-^B/dq^2}{d\Gamma_+^D/dq^2-d\Gamma_-^D/dq^2}=-\frac{|V_{ub}|^2}{|V_{cd}|^2}\frac{q_B^2 m_D}{q_D^2 m_B}\left[1+\left(\frac{1}{m_b}-\frac{1}{m_c}\right)\times\left(\frac{\tilde f_1}{\tilde f_0}+\frac{\tilde g_1}{\tilde g_0}\right)+\mathcal{O}\left(\frac{1}{m_c^2}\right)\right]. \tag{22}$$

Using Eqs. (62)–(66) of Isgur, Scora, Grinstein, and Wise (ISGWII) [10], our crude and conservative estimate is that the correction to the leading order prediction is of the order of 25%, at the non-recoil point ($v\cdot p=m_\rho$). Note, however, that this is *not* the number that constrains the model dependent uncertainty. What is more interesting is the variation in this number due to model dependence, and this is determined by the model dependence in $\tilde f_1/\tilde f_0+\tilde g_1/\tilde g_0$. If, in some other model, this number is 50% larger than the ISGWII number, this would represent about a 10% change in the value of the ratio $f^{B\to\rho}g^{B\to\rho}/f^{D\to\rho}g^{D\to\rho}$, and therefore about a 5% uncertainty in the ratio $|V_{ub}|/|V_{cd}|$. This is much smaller than the current model dependence in V_{ub}, which is of the order of 20%. Furthermore, our perhaps naive expectations are that the model dependence in $\tilde f_1/\tilde f_0+\tilde g_1/\tilde g_0$ should be much smaller than the 50% guessed above, meaning that the model dependence in $|V_{ub}|/|V_{cd}|$ should be of the order of a few percent (e.g., a 10% uncertainty in $\tilde f_1/\tilde f_0+\tilde g_1/\tilde g_0$ leads to about 1% uncertainty in $|V_{ub}|/|V_{cd}|$). This discussion does not take into account the uncertainties that arise in the values chosen for the quark masses, for instance. However, the important point here is that the model dependence now occurs not in the leading prediction, but in a potentially small correction.

Similar analyses can be performed for the quantities in Eqs. (14)–(18). Model dependence in these might be expected to be significantly larger than discussed above, mainly because these quantities depend on the form factor ratio r, at leading order.

V. DISCUSSION AND CONCLUSION

We have suggested three measurements that can allow the extraction of $|V_{ub}|/|V_{cd}|$ in an absolutely model independent manner. However, our results are subject to a number of corrections, which we discuss briefly below.

One of the corrections that must be taken into account is isospin breaking. For the D mesons, the possible decay modes are $D^0 \to \rho^- \ell^+ \nu_\ell$ and $D^+ \to \rho^0 \ell^+ \nu_\ell$, while the corresponding B decay modes are $\overline{B}^0 \to \rho^+ \ell \overline{\nu_\ell}$ and $B^- \to \rho^0 \ell \overline{\nu}_\ell$. The predictions given in the previous section implicitly assume that the light component of the heavy meson is the same for both the D and B decays. This means that comparisons can be made between the decays of the D^0 and those of the B^-, or between the decays of the D^+ and those of the $\overline{B}^0$. In each of these comparisons, the daughter ρ is different, so some assumption of isospin invariance among the form factors has to be made. Departures from this invariance can be expected to be small. There is also an implicit assumption of isospin invariance in the discussion of the radiative factor of $[\alpha_s(m_b)/\alpha_s(m_c)]^{-12/25}$.

By far the larger corrections are expected to come from the $1/m_c$ and $1/m_b$ contributions to the matrix elements of interest. We have briefly discussed these in the previous section, and have suggested that, for the ratio $d\Gamma_+^B/dq^2 - d\Gamma_-^B/dq^2/d\Gamma_+^D/dq^2 - d\Gamma_-^D/dq^2$, model dependence in the universal functions that arise at order $1/m_Q$ should lead to uncertainty in the extraction of V_{ub} of no more than a few percent.

We conclude with a comment comparing the method proposed for extracting V_{ub} with other methods, such as the ratios of differential decay rates to unpolarized ρ mesons. Any such ratio will depend, in general, on the form factors f, g and a_+. Since neither these form factors nor their normalizations are known, models of some sort must be used in the extraction of V_{ub} from such measurements. This introduces large model dependences, even at leading order in the heavy quark expansion. As pointed out at the end of the last section, this is true of the quantities discussed in Eqs. (15), (17) and (18), for instance.

The quantity proposed in Eq. (19) is different in that there is no form factor dependence in the leading order prediction. This means that our lack of knowledge of the form factors at leading order does not affect the extraction of V_{ub} from that expression. Of course, this prediction is subject to $1/m_c$ and $1/m_b$ corrections, which now introduce model dependence into the extraction [see Eq. (22)]. However, the model dependence now appears only in a correction to the leading order prediction, not in the leading order prediction itself. If the correction to the leading order prediction is relatively small, the model dependence in the overall prediction should also be small. Our estimates are that the model dependence in the extraction of V_{ub} should be reduced to a few percent. It is to be emphasized that this is the estimate of the model uncertainty in the extraction of V_{ub}, not an estimate of the size of the corrections to the leading order prediction.

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