Thermal self-energies using light-front quantization

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A recent paper by Alves, Das, and Perez contains a calculation of the one-loop self-energy in ϕ^3 field theory at $T\neq 0$ using light-front quantization and concludes that the self-energy is different than the conventional answer and is not rotationally invariant. The changes of variable displayed below show that despite the complicated appearance of the thermal self-energy in light-front variables, it is exactly the same as the conventional result.

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In Ref. [1], Alves, Das, and Perez introduce the technique of light-front quantization into thermal field theory using a heat bath at rest. As shown in [2] the appropriate quantization evolves the system in the $x^+=(x^0+x^3)/\sqrt{2}$ coordinate while keeping constant x^1,x^2, and x^3. [Normal light-front quantization keeps x^1,x^2, and $x^-=(x^0-x^3)/\sqrt{2}$ constant.] The momenta conjugate to x^+,x^1,x^2,x^3 are k^0,k^1,k^2,k^+ as can be seen from the identity

$$k^0x^0-k^3x^3-\mathbf{k}_\perp\cdot\mathbf{x}_\perp=\sqrt{2}k^0x^+-\sqrt{2}k^+x^3-\mathbf{k}_\perp\cdot\mathbf{x}_\perp .$$

In the imaginary time propagator, x^+ is made negative imaginary: $-i\beta\leqslant\sqrt{2}x^+\leqslant 0$. In the Fourier transform of the propagator $k^0=i2\pi nT$ whereas k^+ and $\mathbf{k}_\perp$ are real. The relation

$$(k^0)^2-(k^3)^2-k_\perp^2=2\sqrt{2}k^0k^+-2(k^+)^2-k_\perp^2$$

immediately leads to the propagator

$$G(k^+,k_\perp,n)=\frac{1}{i4\sqrt{2}\pi nk^+-2(k^+)^2-\omega_k^2},$$

where $\omega_k^2=k_\perp^2+m^2$ is the transverse energy.

One of the interesting calculations performed by Alves, Das, and Perez [1] using this propagator is the one-loop self-energy for a scalar field theory with self-interaction $g\phi^3/3!$. The result of performing the summation over the loop integer n is given in Eq. (40) of Ref. [1]:

$$\Pi(p)=\frac{g^2}{8}\int\frac{dk^+d^2k_\perp}{(2\pi)^3}\frac{\coth(X_1/2T)-\coth(X_2/2T)}{Y}. \tag{1}$$

During the summation the external variable p^0 is an integer multiple of $2\pi iT$, but after the summation p^0 is continued to real values. The quantities X_1,X_2, and Y are complicated functions of the integration variables $k^+,\mathbf{k}_\perp$ and of the external variables $p^0,p^+,\mathbf{p}_\perp$:

$$X_1=\frac{\omega_k^2+2(k^+)^2}{2\sqrt{2}k^+}$$

$$X_2=\frac{\omega_{k+p}^2+2(k^++p^+)^2}{2\sqrt{2}(k^++p^+)}$$

$$Y=2\sqrt{2}k^+(k^++p^+)[-X_1+X_2-p^0].$$

The self-energy (1) looks quite different from the usual result and is not manifestly invariant under $O(3)$ rotations of the external momentum $\mathbf{p}$.

The following will describe a change of integration variable from k^+ to a new variable k^3 that is chosen to make the self-energy a function of the two variables p^0 and $\mathbf{p}^2=p_\perp^2+(\sqrt{2}p^+-p^0)^2$. The final answer is the sum of Eqs. (2), (3), (4), and (5).

(1a) For the term $\cosh(X_1/2T)/Y$ in Eq. (1), when k^+ is positive change to a new integration variable k^3 defined by

$$k^+=\frac{1}{\sqrt{2}}[k^3+\sqrt{m^2+k_\perp^2+(k^3)^2}].$$

The range of k^3 is $-\infty\leqslant k^3\leqslant\infty$. The Jacobian of the transformation is $dk^+/dk^3=k^+/E_k$, where $E_k=\sqrt{m^2+k^2}$ is the square root displayed above. Under this change,

$$X_1=E_k,\quad Y=k^+[(E_{k+p})^2-(p^0+E_k)^2],$$

where $E_{k+p}=\sqrt{m^2+(\mathbf{k}+\mathbf{p})^2}$. The factor k^+ in the Jacobian cancels a similar factor in Y and yields a contribution to the self-energy;

$$\Pi_{1a}=\frac{g^2}{8}\int\frac{d^2k_\perp}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dk^3}{E_k}\frac{\coth(E_k/2T)}{(E_{k+p})^2-(p^0+E_k)^2}. \tag{2}$$

This integrand is invariant invariant under simultaneous rotations of the vectors $\mathbf{k}$ and $\mathbf{p}$. Thus Π_{1a} depends only on $|\mathbf{p}|$ and p^0.

(1b) For the term $\cosh(X_1/2T)/Y$ in Eq. (1), when k^+ is negative make the change of variable

$$k^+=\frac{1}{\sqrt{2}}[k^3-\sqrt{m^2+k_\perp^2+(k^3)^2}],$$

where $-\infty\leqslant k^3\leqslant\infty$. The Jacobian of the transformation is $dk^+/dk^3=-k^+/E_k$ and

$$X_1=-E_k,\quad Y=k^+[(E_{k+p})^2-(p^0-E_k)^2].$$

The corresponding self-energy contribution is

$$\Pi_{1b}=\frac{g^2}{8}\int\frac{d^2k_\perp}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dk^3}{E_k}\frac{\coth(E_k/2T)}{(E_{k+p})^2-(p^0-E_k)^2}. \qquad (3)$$

The sum of Eqs. (2) and (3) is an even function of p^0.

(2a) In the second term in Eq. (1), $\cosh(X_2/2T)/Y$, when $k^++p^+>0$ then change to k^3 given by

$$k^+=\frac{1}{\sqrt{2}}[k^3-p^0+\sqrt{m^2+(\mathbf{k}_\perp+\mathbf{p}_\perp)^2+(k^3+p^3)^2}],$$

where $-\infty\leqslant k^3\leqslant\infty$. Using $dk^+/dk^3=(k^++p^+)/E_{k+p}$, and

$$X_2=E_{k+p},\quad Y=(k^++p^+)[(p^0-E_{k+p})^2-E_k^2],$$

this contribution is

$$\Pi_{2a}=\frac{g^2}{8}\int\frac{d^2k_\perp}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dk^3}{E_{k+p}}\frac{\coth(E_{k+p}/2T)}{E_k^2-(p^0-E_{k+p})^2}. \qquad (4)$$

(2b) In the second term in Eq. (1), $\cosh(X_2/2T)/Y$, if $k^++p^+<0$ make the change of variable

$$k^+=\frac{1}{\sqrt{2}}[k^3-p^0-\sqrt{m^2+(\mathbf{k}_\perp+\mathbf{p}_\perp)^2+(k^3+p^3)^2}],$$

where $-\infty\leqslant k^3\leqslant\infty$. Since $dk^+/dk^3=-(k^++p^+)/E_{k+p}$, and

$$X_2=-E_{k+p},\quad Y=(k^++p^+)[(p^0+E_{k+p})^2-E_k^2],$$

the contribution to the self-energy is

$$\Pi_{2b}=\frac{g^2}{8}\int\frac{d^2k_\perp}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dk^3}{E_{k+p}}\frac{\coth(E_{k+p}/2T)}{E_k^2-(p^0+E_{k+p})^2}. \qquad (5)$$

The sum of Eqs. (4) and (5) is an even function of p^0.

The sum of these four contributions, Eqs. (2), (3), (4), and (5), is the standard answer for the self-energy [3]. Therefore the light-front formulation is a different, and in some cases a more efficient [2], way of organizing the calculation, but the results are the same.

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