

Supergravity pp -wave solutions with 28 and 24 supercharges

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We conduct an exhaustive search for solutions of type IIA and IIB supergravity with augmented supersymmetry. We find a two-parameter family of type IIB solutions preserving 28 supercharges, as well as several other type IIA and IIB families of solutions with 24 supercharges. Given the simplicity of the pp -wave solution, the algorithm described here represents a systematic way of classifying all such solutions with augmented supersymmetry. By T dualizing some of these solutions we obtain exact non- pp -wave supergravity solutions (with 8 or 16 supercharges), which can be interpreted as perturbations of the AdS-CFT correspondence with irrelevant operators.

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I. INTRODUCTION

Plane waves are among the simplest solutions of the supergravity equations of motion. Because of the existence of a null Killing field, they are also solutions of string theory to all orders in the sigma model perturbation theory [1,2].

In addition to the three well-known supergravity solutions with 32 supercharges ($\text{AdS}_{4,5,7} \times S^{7,5,4}$), it is possible to construct two more [3–5]. Even though originally these solutions were constructed by solving the equations of motion, it later turned out that they can be obtained as Penrose-Güven limits [6] of the former. One of these solutions [3,4] is a pp wave in 11 dimensional supergravity, has a nonzero four-form field strength F_4 , and is the Penrose-Güven limit of both $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$. The other solution [5] is a pp wave in 10 dimensional type IIB supergravity, has a nonzero self-dual five-form field strength F_5 , and is the Penrose-Güven limit of $\text{AdS}_5 \times S^5$. These important observations provided the link between plane wave solutions of supergravity equations of motion and the AdS conformal field theory (CFT) correspondence. Thus, string theory in the plane wave geometry is dual to a sector with a large R charge on the gauge theory side [7].

The ensuing burst of interest in plane wave geometries prompted the construction of solutions [8–11] generalizing the original ones and preserving more supersymmetries than the standard 16 of any plane wave. The plane wave geometry seems simple enough to attempt a classification of these augmented supersymmetry solutions. In this paper we perform this analysis for type IIB and IIA supergravity with the surprising result that in the type IIB theory there exist solutions preserving 28 supercharges. These solutions turn out to be related to Penrose limits of coset spaces [12]. Our method is powerful enough to allow the classification of all solutions with 24 supercharges as well. We construct a fairly large number of them, both in type IIA and type IIB supergravity. Even though we do not prove here that our analysis exhausts all these solutions, we believe it is quite likely that it does. In the type IIA theory we also find solutions preserving

(p, q), $p \neq q$ supercharges. Four dimensional solutions with this property were also constructed in Ref. [13].

In the presence of general form fields, the dilatino variation is proportional to the contraction of these forms with the Dirac Γ matrices acting on the supersymmetry parameter ϵ . Therefore, in order to obtain more preserved supersymmetries, one needs the Γ matrices to combine into commuting projectors. In order for this to happen one needs to turn on appropriate forms with appropriate coefficients.

If the dilatino variation takes the form

$$\delta\lambda = M(\Gamma_0\Gamma_-)(1+M_1)(1+M_2)\epsilon, \quad (1.1)$$

where M is a matrix, M_1 and M_2 are independent, commuting and unipotent ($M_i^2 = 1$) combinations of Γ matrices, each of the three projectors will annihilate half of the spinors it acts upon. Since we assumed them to be independent and commuting they will annihilate *different* sets of spinors and thus the right hand side of Eq. (1.1) will vanish for $16+8+4=28$ spinors. If instead of three projectors we only have two, then only $16+8=24$ spinors give a zero dilatino variation.

Once we have these candidates for Killing spinors, the next step is to test whether the gravitino supersymmetry variation vanishes. For plane wave would-be solutions this completely fixes the metric, as well as the dependence of the spinors on the coordinates. In some cases all the 24 or 28 spinors give a zero gravitino variation, so they are Killing spinors. In other cases, the number of Killing spinors is smaller.

In the next section of this paper we describe the pp -wave geometry and the form of the dilatino and gravitino supersymmetry variations. We then explore the types and combinations of form fields that can be turned on in order for projectors to appear in these equations. Then, we use the dilatino variation to make a number of educated guesses for solutions with enhanced supersymmetry, both in type IIA and in type IIB supergravity. In Sec. IV we test these *Ansätze* against the gravitino variation, and find the full solutions.

We first describe two families of type IIB backgrounds with 28 Killing spinors. These backgrounds have nonvanishing self-dual F_5 flux, as well as nonzero Ramond-Ramond (RR) or Neveu-Schwarz-Neveu-Schwarz (NSNS) three

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forms in particular combinations. The relative strength of the five form and RR or NSNS three form is a free parameter, so each of the two solutions is in fact a one parameter family.¹

We then list the other type IIA and type IIB backgrounds with more than 16 Killing spinors which we obtain by this procedure. We list solutions with 24 supercharges involving F_3+F_5 , H_3+F_5 , $F_3+H_3+F_5$, F_4+H_3 , F_4+F_2 , H_3+F_2 , as well as solutions preserving chiral supersymmetry.

Some of the solutions we analyze have some Killing spinors independent of the coordinate along the direction of propagation of the wave. Thus, it is possible to T dualize along this direction and still have a solution preserving some supersymmetry. We find that the dual geometries can be interpreted as arising from smeared strings or D-branes deformed with transverse fluxes, and explain them in light of the AdS-CFT correspondence. In the process we construct exact nonsingular flows from brane near-horizon geometries in the IR to certain nontrivial geometries in the UV. The results are described in Sec. V.

It is also interesting to ask what is the highest number of supersymmetries than can be preserved by a pp -wave background in type II theories. To obtain 32 supercharges one needs the dilatino variation to vanish, in order to impose no constraints on the supersymmetry parameters. Thus the only form field we can have is the type IIB self-dual five-form. The maximally supersymmetric pp -wave background obtained in Ref. [5] is the only such solution.

A solution preserving 30 supercharges would have a dilatino variation containing a product of four independent projectors. As we will show in Sec. VI, it is not possible to combine the fields of type IIA and IIB supergravity to form so many projectors. Thus, in addition to the maximally supersymmetric solution of type IIB supergravity, the solutions with 28 supercharges described here have the largest possible amount of supersymmetry one can obtain in a pp -wave background in 10 dimensions.²

II. SUPERSYMMETRIES AND PROJECTORS

In this section we will describe in detail a general way of constructing wave solutions of the supergravity equations of motion with enhanced supersymmetry. As it is known, the metric and forms of a pp wave are quite simple, yet nontrivial. We choose a metric of the form

$$ds^2 = -2dx^+ dx^- - A_{ab}(x^+) z^a z^b (dx^+)^2 + (dz^a)^2, \quad (2.1)$$

and the only nonzero component of the field strengths of the RR and NSNS fields is $F_{+i_1 \dots i_p}(x^+)$. Because they only

¹Of course these two solutions are related by S duality, and are just the end points of an entire family of solutions generated by rotating F_3 and H_3 into each other via S duality. This gives in the end a 2-parameter family of 28 supercharge solutions.

²It would be interesting to see if the methods we use for finding pp -wave solutions with augmented supersymmetry (combining the forms to form projectors) can be used to find M-theory or lower dimensional supergravity solutions with 28 supersymmetries.

depend on x^+ , the forms satisfy the equations of motion and Bianchi identities by construction.

Choosing ($\eta_{+-} = -1$), the vielbeine are

$$e^+ = dx^+, \quad e^a = (d\phi, dx^i, dy^i) \equiv dz^a, \\ e^- = dx^- + \frac{1}{2} A_{ab}(x^+) z^a z^b dx^+, \quad (2.2)$$

and the spin connection (defined by $de^A + \omega^A_B \wedge e^B = 0$) is

$$\omega^{-c} = A_{cb}(x^+) z^b dx^+. \quad (2.3)$$

The supercovariant derivative is therefore given by

$$\nabla_i = \partial_i, \quad \nabla_- = \partial_-, \quad \nabla_+ = \partial_+ + \frac{1}{2} A_{ab}(x^+) z^b \Gamma_- \Gamma_a, \quad (2.4)$$

and the Ricci tensor is just

$$R_{++} = A^a_a(x^+). \quad (2.5)$$

Thus, the only equation of motion our backgrounds have to satisfy is

$$R_{++} = A^a_a(x^+) = \frac{1}{2} \sum_p \frac{1}{p!} F_{+i_1 \dots i_p} F^{+i_1 \dots i_p}, \quad (2.6)$$

where $F_{+i_1 \dots i_p}$ are the field strengths of the various RR and NSNS p forms present and self-dual fields enter only once.

We will use the conventions of Ref. [14] for the type II supersymmetry transformation rules. In these conventions we will work with two Dirac spinors (thus, all Dirac matrices will be 32-dimensional) obeying appropriate chirality conditions and forming a 2-dimensional representation of an auxiliary $SL(2, \mathbb{R})$. Defining $\mathbf{F}_{(n)} = (1/n!) \Gamma^{N_1 \dots N_n} F_{N_1 \dots N_n}$, the supersymmetry transformations are for type IIA,

$$\delta\lambda = \frac{1}{2} \Gamma^M \partial_M \phi \epsilon - \frac{1}{4} \not{H} \sigma^3 \epsilon + \frac{1}{2} e^\phi [5F_{(0)} \sigma^1 + 3\mathbf{F}_{(2)}(i\sigma^2) \\ + \mathbf{F}'_{(4)} \sigma^1] \epsilon, \quad (2.7)$$

$$\delta\Psi_M = \nabla_M \epsilon - \frac{1}{8} \Gamma^{NP} H_{MNP} \sigma^3 \epsilon + \frac{1}{8} e^\phi [F_{(0)} \Gamma_M \sigma^1 \\ + \mathbf{F}_{(2)} \Gamma_M(i\sigma^2) + \mathbf{F}'_{(4)} \Gamma_M \sigma^1] \epsilon,$$

and for type IIB,

$$\delta\lambda = \frac{1}{2} \Gamma^M \partial_M \phi \epsilon - \frac{1}{4} \not{H} \sigma^3 \epsilon - \frac{1}{2} e^\phi \left[\mathbf{F}_{(1)}(i\sigma^2) \epsilon + \frac{1}{2} \mathbf{F}'_{(3)} \sigma^1 \epsilon \right], \quad (2.8)$$

$$\delta\Psi_M = \nabla_M \epsilon - \frac{1}{8} \Gamma^{NP} H_{MNP} \sigma^3 \epsilon + \frac{e^\phi}{8} \left[\mathbb{F}_{(1)} \Gamma_M (i\sigma^2) + \mathbb{F}'_{(3)} \Gamma_M \sigma^1 + \frac{1}{2} \mathbb{F}'_{(5)} \Gamma_M (i\sigma^2) \right] \epsilon$$

with the modified field strengths F' given by

$$\begin{aligned} F'_{(3)} &= F_{(3)} - CH_{(3)}, & F'_{(5)} &= F_{(5)} - H_{(3)} \wedge C_{(2)}, \\ F'_{(4)} &= F_{(4)} - H_{(3)} \wedge C_{(1)}. \end{aligned} \quad (2.9)$$

Preserved supersymmetry appears in the form of spinors that are annihilated by a set of projectors when the above transformations are evaluated on solutions to the equations of motion. Thus, a classification of all possible solutions preserving some supersymmetry becomes a three-step process. The first step requires a classification of projectors that can be built out of supergravity fields in the dilatino transformation rule. The next step requires checking whether these field configurations are compatible with the gravitino supersymmetry transformation (Killing spinor equation) and the third step involves checking whether the equations of motion are satisfied.

The first step in the procedure outlined above can be performed in quite some generality. In the notation we are using here a generic projector looks similar to

$$P = \frac{1}{2} (1 + \Gamma \otimes \sigma), \quad (\Gamma \otimes \sigma)^2 = 1, \quad (2.10)$$

where Γ is some combination of Dirac matrices and σ is one of the $\mathfrak{gl}(2, \mathbb{R})$ generators. We will loosely refer to the σ dependence of various terms as their $\mathfrak{gl}(2)$ structure. Half of the eigenvalues of such a projector vanish. Thus, one such projector will preserve one half of the supersymmetries. The only way to find more preserved supersymmetries is to have the dilatino variation be proportional to a product of commuting projectors. This observation allows us to find the maximum number of supersymmetries that can be preserved by a solution of the equations of motion which has a non-trivial supersymmetry transformation of the dilatino.³

Due to the fact that we are considering wave solutions, each term in the dilatino variation is proportional to the Dirac matrix pointing along the (null) direction of propagation of the wave (this direction will be denoted by x^+). This matrix is proportional to a projector (2.10) in which $\sigma = 1$. It is easy to see that this projector commutes with any other projector that can be constructed from the remaining Dirac matrices appearing in the supersymmetry transformation rules. Thus a wave solution will always preserve sixteen supercharges.

In the next two sections we will follow the steps outlined above. We will begin by describing several field configurations that factorize the dilatino variation into projectors.

³For the maximally supersymmetric wave each term in the dilatino variation vanishes separately.

These field configurations have the potential of producing wave solutions preserving 28 supercharges. We will then proceed in Sec. IV to analyze the Killing spinor equation and the equations of motion.

III. POTENTIAL SOLUTIONS

Given the simplicity of the wave metric and the fact that all field strengths carry one null index, it is easy to find field configurations such that the dilatino transformation is proportional to a product of commuting projectors. We will begin with the type IIB supergravity. It will be argued in Sec. VI that the dilaton and the axion cannot have nontrivial values if more than 16 supercharges are to be preserved. Thus, we will look for field configurations involving only the 3-form field strengths.

We will first discuss potential solutions with either one of $H_{(3)}$ or $F'_{(3)}$ nonvanishing. It is very easy to see that, after factorizing the Dirac matrix pointing along the direction of propagation of the wave, both $H_{(3)}$ and $F'_{(3)}$ will contribute two Dirac matrices that must be further combined in projectors. Since for the time being we are considering only one type of field, the $\mathfrak{gl}(2)$ component of the supersymmetry transformation rule will factorize. The only possibility is then to find projectors constructed out of four Dirac matrices. It turns out to be possible to have

$$\delta\lambda \sim \Gamma_{-} (1 - \beta\Gamma_{1234}) (1 - \gamma\Gamma_{1256}) \epsilon, \quad \beta^2 = \gamma^2 = 1 \quad (3.1)$$

which vanishes for 28 different spinors. The field configuration realizing this setup is the following:

$$\begin{aligned} H_{+12} &= \beta H_{+34} = \gamma H_{+56} = \alpha\beta\gamma H_{+78} = f(x^+), \\ \Gamma_{-1} \epsilon &= \alpha \epsilon, \quad \alpha^2 = 1. \end{aligned} \quad (3.2)$$

Here $f(x^+)$ is for the time being an arbitrary function of x^+ while Γ_{-1} is the 10-dimensional chirality operator. As stated in the beginning, we are free to replace H with F' . This function will be fixed in the next section using the gravitino variation as well as the equations of motion.

To show that this field configuration indeed reproduces Eq. (3.1) we need to make use of the fact that both supersymmetry parameters have the same chirality. After pulling out $\Gamma_{-}\Gamma_{12}$ as common factor the dilatino variation becomes

$$\begin{aligned} \delta\lambda &= \frac{f(x^+)}{4} \Gamma_{-}\Gamma_{12} \otimes \sigma^l [1 - \beta\Gamma_{1234} - \gamma\Gamma_{1256} - \alpha\beta\gamma\Gamma_{1278}] \epsilon \\ &= \frac{f(x^+)}{4} \Gamma_{-}\Gamma_{12} \otimes \sigma^l [1 - \beta\Gamma_{1234} - \gamma\Gamma_{1256} \\ &\quad - \alpha\beta\gamma\Gamma_{3456}\Gamma_{-1}] \epsilon \\ &= \frac{f(x^+)}{4} \Gamma_{-}\Gamma_{12} \otimes \sigma^l (1 - \beta\Gamma_{1234}) (1 - \gamma\Gamma_{1256}) \epsilon, \\ & \quad l = 1, 3. \end{aligned} \quad (3.3)$$

Here we used the definition of the chirality operator⁴ $\Gamma_{-1} = -\frac{1}{2}[\Gamma_+, \Gamma_-]\Gamma_{12345678}$ and the definition of α in Eq. (3.2). The choices $l=1$ and $l=3$ correspond to having a nontrivial RR 3-form and NSNS 3-form, respectively. S duality continuously interpolates between these two solutions.

Next we discuss a possible solution of type IIB supergravity which preserves 28 supercharges, contains both $F_{(3)}$ and $H_{(3)}$ and is not S dual to the solutions considered above (neither $F_{(3)}$ nor $H_{(3)}$ can be S -dualized away). It is clear that, after pulling out a common factor, some terms will be left with the identity operator as their $gl(2)$ component while others will have $(i\sigma^2)$. Since $(i\sigma^2)$ can appear in a projector only tensored with two or three Dirac matrices, it is easy to see that a possible combination of projection operators is

$$\delta\lambda \sim \Gamma_- [1 - \beta\Gamma_{14}(i\sigma^2)][1 - \gamma\Gamma_{23}(i\sigma^2)]\epsilon, \quad \beta^2 = \gamma^2 = 1. \quad (3.4)$$

The field configuration producing this dilatino variation is

$$\gamma H_{+13} = -\beta H_{+24} = F'_{+12} = \beta\gamma F'_{+34} = f(x^+). \quad (3.5)$$

As before, $f(x^+)$ is for the time being arbitrary and will be determined by the gravitino variation and equations of motion. One can in principle construct these field strengths from several different potentials. However, we choose the gauge in which the potentials do not carry the $+$ index. The reason for this gauge choice is to make sure that the modified 5-form field strength remains trivial. As promised, the dilatino variation is

$$\begin{aligned} \delta\lambda &= -\frac{f(x^+)}{4}\Gamma_- [(\gamma\Gamma_{13} - \beta\Gamma_{24})\sigma^3 + (\Gamma_{12} + \beta\gamma\Gamma_{34})\sigma^1]\epsilon \\ &= -\frac{f(x^+)}{4}\Gamma_- \Gamma_{12}\sigma^1 [1 - \beta\Gamma_{14}(i\sigma^2)][1 - \gamma\Gamma_{23}(i\sigma^2)]\epsilon. \end{aligned} \quad (3.6)$$

We will show in Sec. VI that, up to a relabeling of coordinates, the field configurations described above are the only ones that lead to a product of three projectors (two if one ignores Γ_-) in the dilatino supersymmetry transformation rule.

We now turn to possible solutions of type IIA supergravity. As in type IIB supergravity, any wave solution preserving more than 16 supercharges has a trivial dilaton. Even with this simplification, the situation is substantially more complicated than in type IIB theory since there are three different types of fields contributing to the dilatino transformation rule. Deferring the detailed analysis to Sec. VI, we present here several examples.

The only possible (up to relabeling and reshuffling of terms) projector that could preserve 28 supercharges is

$$\delta\lambda \sim \Gamma_- [1 + \beta\Gamma_{148}(i\sigma^2)][1 + \gamma\Gamma_{245}(i\sigma^2)] \quad (3.7)$$

and the field configuration generating it is

$$\frac{1}{2}H_{+12} = \frac{1}{2}\beta\gamma H_{+58} = -\beta F_{+248} = \gamma F_{+145} = f(x^+). \quad (3.8)$$

As before $f(x^+)$ is an arbitrary function to be determined by the Killing spinor equation and equations of motion. The dilatino variation generated by this field configuration is indeed proportional to Eq. (3.7)

$$\delta\lambda = \Gamma_- \Gamma_{12} \otimes \sigma^3 [1 + \beta\Gamma_{148} \otimes (i\sigma^2)][1 + \gamma\Gamma_{245} \otimes (i\sigma^2)]. \quad (3.9)$$

It is possible to add a further projector to the product above. However, this requires use of Γ_{-1} and thus it enhances supersymmetry only in the right-handed sector while breaking it in the left-handed sector.

Finding solutions preserving 24 supercharges is also easy in this approach. The projector

$$\Gamma_- (1 + \beta\Gamma_1 \otimes \sigma^1) \quad (3.10)$$

can appear in a solution with nonzero $H_{(3)}$ and $F_{(2)}$

$$3F_{+1} = -\frac{1}{2}H_{+12} = f(x^+). \quad (3.11)$$

This gives the dilatino variation

$$\begin{aligned} \delta\lambda &= \frac{f(x^+)}{2}\Gamma_- [\Gamma_{12}\sigma^3 + \Gamma_1(i\sigma^2)] \\ &= -\frac{f(x^+)}{2}\Gamma_1(i\sigma^2)\Gamma_- (1 - \Gamma_2\sigma^1), \end{aligned} \quad (3.12)$$

which contains the projector promised above.

Another example of potential solutions preserving 24 supercharges is built on the projector

$$\Gamma_- (1 + \Gamma_{1234}\sigma^3). \quad (3.13)$$

The field configuration that can generate this projector contains $F_{(4)}$ and $F_{(2)}$:

$$3F_{+1} = F_{+234} = f(x^+). \quad (3.14)$$

This leads to the dilatino variation

$$\begin{aligned} \delta\lambda &= \frac{f(x^+)}{2}\Gamma_- [\Gamma_1(i\sigma^2) + \Gamma_{234}\sigma^1] \\ &= -\frac{f(x^+)}{2}\Gamma_1(i\sigma^2)\Gamma_- (1 + \Gamma_{1234}\sigma^3). \end{aligned} \quad (3.15)$$

To summarize, we have described how the study of the dilatino variation can yield field configurations that have the potential of preserving large amounts of supersymmetry. The final word in this matter belongs however to the Killing spinor equation and the supergravity equation of motion. We proceed with their analysis, thus completing the second and third steps of the program outlined in Sec. II.

⁴ $\Gamma_{\pm} = (1/\sqrt{2})(\Gamma_0 \pm \Gamma_9)$; $\{\Gamma_+, \Gamma_-\} = -2$.

IV. GRAVITINO VARIATION AND EQUATIONS OF MOTION

A. Generalities

The strategy for solving the Killing spinor equations in plane wave backgrounds was discussed in some detail in Refs. [4,8]. Here we will go beyond their analysis and cast these equations in a form suitable for the setup discussed in the previous sections.

The generic structure of the gravitino transformation is

$$\delta\Psi_M = \nabla_M \epsilon + \Omega_M(x^+) \epsilon, \quad (4.1)$$

where $\Omega_M(x^+)$ is the torsion part of the spin connection and represents the contribution of the various form fields. If all RR fields vanish then Ω_M is just the standard torsion induced by the NSNS 3-form field strength. It is not hard to see from the gravitino variations (2.7) and (2.8) that $\Omega_i(x^+)$ is proportional to Γ_- . Therefore

$$\Gamma_- \Omega_i(x^+) = \Omega_i(x^+) \Gamma_- = \Omega_i(x^+) \Omega_j(x^+) = 0 \quad (4.2)$$

because Γ_- is nilpotent. On the other hand, Ω_+ does not satisfy these relations because it contains the combination $\Gamma_- \Gamma_+$ which is not nilpotent.

Since the spin connection vanishes along the transverse directions, it is trivial to solve the corresponding equations

$$\partial_i \epsilon + \Omega_i(x^+) \epsilon = 0 \rightarrow \epsilon = [1 - x^i \Omega_i(x^+)] \chi, \quad (4.3)$$

where χ is an unconstrained spinor depending only on x^+ . The remaining nontrivial equation corresponds to the $+$ direction. In the following we will suppress the dependence on x^+ , with the understanding that both Ω and A are x^+ dependent:

$$\partial_+ [(1 - x^i \Omega_i) \chi] + \frac{1}{2} A_{ij} x^j \Gamma_- \Gamma_i \chi + \Omega_+ (1 - x^i \Omega_i) \chi = 0. \quad (4.4)$$

It is clear that the terms with different x^i dependence should cancel separately. Thus, the equation above splits in two parts, one of which can be used to remove from the other one terms with derivatives acting on the spinor. The final result is

$$0 = \partial_+ \chi + \Omega_+ \chi, \quad (4.5)$$

$$0 = -(\partial_+ \Omega_i) \chi + [\Omega_i, \Omega_+] \chi + \frac{1}{2} A_{ij} \Gamma_- \Gamma_j \chi.$$

Being a first order differential equation, the first equation always has the solution

$$\chi = e^{-\int dx^+ \Omega_+} \rho, \quad (4.6)$$

where ρ is an unconstrained, constant spinor.

The second equation is more restrictive. Consider a wave solution supported by both NSNS flux $H_{(3)}$ as well as RR fluxes which we will generically denote as $F \equiv \sum_p F'_{(p+1)}$. Then, Ω_M is given by

$$\Omega_M = -\frac{1}{8} \Gamma^{NP} H_{MNP} \otimes \sigma^3 + \frac{e^\phi}{8} \mathbb{F} \Gamma_M, \quad (4.7)$$

where $\mathbb{F} \equiv \sum_p F'_{(p+1)} \otimes \sigma^{l(p)}$ and $l(p)$ is determined from the supersymmetry transformation rules (2.7),(2.8).

Defining $\mathbb{h}_{(2)}$ and $\mathbf{f}_{(p)}$ as

$$\mathbb{h}_{(3)} \otimes \sigma^3 \equiv \Gamma_- \mathbb{h}_{(2)}, \quad \mathbb{F}_{(p+1)} \otimes \sigma^{l(p)} \equiv \Gamma_- \mathbf{f}_{(p)}, \quad (4.8)$$

and $\mathbf{f} = \sum_p \mathbf{f}_{(p)}$, the torsion Ω_M decomposes into transverse and lightlike components as

$$\Omega_i = \frac{1}{8} \Gamma_- [\mathbb{h}_{(2)}, \Gamma_i] - \frac{1}{8} \Gamma_- \mathbf{f} \Gamma_i,$$

$$\Omega_+ = -\frac{1}{4} \mathbb{h}_{(2)} - \frac{(-)^p}{8} \Gamma_- \Gamma_+ \mathbf{f}, \quad (4.9)$$

while $\Omega_- = 0$. We also lowered the upper $+$ index on the Dirac matrices and this leads to the various sign differences between Eqs. (4.9) and (4.7). Then, the commutator appearing in Eq. (4.5) becomes

$$[\Omega_i, \Omega_+] = \frac{1}{32} \Gamma_- [(\mathbb{h}_{(2)}^2 + \mathbf{f}^2) \Gamma_i - \{f, \mathbb{h}_{(2)}\} \Gamma_i + 2\mathbf{f} \Gamma_i \mathbb{h}_{(2)} - 2\mathbb{h}_{(2)} \Gamma_i \mathbb{h}_{(2)} + \Gamma_i \mathbb{h}_{(2)}^2]. \quad (4.10)$$

Consider now the case when the NSNS field $H_{(3)}$ and only one of the RR fields, $F_{(p+1)}$ are turned on, and both have exactly one nonvanishing, constant component. Then, the first two terms above represent the right-hand side of the equation of motion, while the last two terms give a traceless contribution to A_{ij} . Therefore, the remaining two terms must give a traceless (or vanishing) contribution to A_{ij} if the equation of motion is to be satisfied.

Since $F_{(p+1)}$ and $H_{(3)}$ combine to form a projector in the dilatino variation, it is not hard to see that \mathbf{f} and $\mathbb{h}_{(2)}$ commute, which implies that the two terms we are interested in can be written as

$$2\mathbf{f} [\Gamma_i, \mathbb{h}_{(2)}]. \quad (4.11)$$

Moreover, the vanishing dilatino variation implies that $\mathbf{f}_{(p)} \chi$ and $\mathbb{h}_{(2)} \chi$ are proportional. Therefore, the object above can always be written as $C_{ij} \Gamma_- \Gamma_j$, where C_{ij} is a constant matrix. Its trace is the obstruction to constructing a solution of the field equation with 24 supercharges and the NSNS and RR fluxes described above, and it vanishes.

An important question is whether any wave solution preserving more than 16 supercharges can have x^+ -dependent form fields. If such a field existed, it would follow that $\partial_+ \Omega_i$ in Eq. (4.5) is nonvanishing. Its Dirac matrix structure allows a contribution of $F_{(2)}$ be canceled by introducing off-diagonal entries of the coefficient matrix A_{ij} . However, the differences between the $\mathfrak{gl}(2)$ structures of the two terms prevents this cancellation. Thus, we conclude that all form fields must be constant.

B. Solutions with 28 supercharges

We now analyze the field configurations put forward in Sec. III. Of the potential solutions with 28 supercharges, some do not solve the Killing spinor equations. Those which solve it exist in type IIB and can be extended to include the 5-form field strength as well.

Let us begin with the type IIB theory and discuss the fields in Eq. (3.2) and its S -dual version. These fields do not satisfy the assumptions introduced at the end of the previous subsection, so we must start with Eq. (4.10). Consider first the field configuration in Eq. (3.2). Since \mathbf{f} vanishes, the second and third terms in Eq. (4.10) are absent. Furthermore, from the previous section we know that the dilatino variation is proportional to $\mathbf{h}_{(2)}$ which is

$$\mathbf{h}_{(2)} = f(x^+) (\Gamma_{12} + \beta \Gamma_{34} + \gamma \Gamma_{56} + \alpha \beta \gamma \Gamma_{78}) \otimes \sigma^3. \quad (4.12)$$

Thus, taking χ to be the spinors that annihilate the dilatino variation, the only terms that survive in the second term of Eq. (4.5) are

$$0 = \frac{1}{32} \Gamma_- \mathbf{h}_{(2)}^2 \Gamma_i \chi + \frac{1}{2} A_{ij} \Gamma_- \Gamma_j \chi. \quad (4.13)$$

To find A_{ij} it is helpful to notice that, for any choice of the index i in Eq. (4.13), passing Γ_i through $\mathbf{h}_{(2)}$ changes the sign of exactly one of the four terms in $\mathbf{h}_{(2)}$. Then, the fact that $\mathbf{h}_{(2)}$ annihilates χ implies that the three terms with the sign unchanged can be replaced by the fourth one, whose square is proportional to the identity matrix. For example, for $i=1,2$ we have

$$\begin{aligned} (\mathbf{h}_{(2)})^2 \Gamma_i \chi &= f^2 \Gamma_i (-\Gamma_{12} + \beta \Gamma_{34} + \gamma \Gamma_{56} + \alpha \beta \gamma \Gamma_{78})^2 \chi \\ &= f^2 \Gamma_i (-2\Gamma_{12})^2 \chi = -4f^2 \Gamma_i. \end{aligned} \quad (4.14)$$

Thus, the Eq. (4.13) implies that

$$A_{ij} = \frac{1}{4} \delta_{ij} f^2. \quad (4.15)$$

It is trivial to check that the equation of motion (2.6) is satisfied.

The same analysis applies with only cosmetic changes to any of the S duals of Eq. (3.2). Since the Dirac matrix structure of \mathbf{f} and $\mathbf{h}_{(2)}$ is identical, and $\mathbf{h}_{(2)} \chi = 0$, then only the first two terms in Eq. (4.10) survive; for both of them the discussion above Eq. (4.14) applies without change.

This family of S -dual solutions can be further extended to a two 2-parameter one by including the 5-form field strength. This is possible because the 5-form field strength does not appear in the dilatino supersymmetry transformation rule. Consider the following addition to Eq. (3.2):

$$\begin{aligned} F_{(5)} &= g dx^+ \wedge (dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 \\ &\quad + \alpha dx^5 \wedge dx^6 \wedge dx^7 \wedge dx^8), \quad \Gamma_- \chi = \alpha \chi. \end{aligned} \quad (4.16)$$

Under these circumstances, $\mathbf{f}_{(p)}$ in Eq. (4.10) must be replaced with $\mathbf{f}_{(2)} + \frac{1}{2} \mathbf{f}_{(4)}$ and this leads to

$$0 = \frac{1}{32} \Gamma_- \left(\mathbf{f}_{(2)} + \frac{1}{2} \mathbf{f}_{(4)} \right)^2 \Gamma_i \chi + \frac{1}{2} A_{ij} \Gamma_- \Gamma_j \chi, \quad (4.17)$$

where $\mathbf{f}_{(4)}$ is given by

$$\mathbf{f}_{(4)} = g (\Gamma_{1234} + \alpha \Gamma_{5678}). \quad (4.18)$$

Since $\mathbf{f}_{(2)}$ and $\mathbf{f}_{(4)}$ anticommute, Eq. (4.17) becomes

$$0 = \frac{1}{32} \Gamma_- \left(\mathbf{f}_{(2)}^2 + \frac{1}{4} \mathbf{f}_{(4)}^2 \right) \Gamma_i \chi + \frac{1}{2} A_{ij} \Gamma_- \Gamma_j \chi. \quad (4.19)$$

Thus, each of the two RR field strengths gives an independent contribution to the coefficients A_{ij} . This shows that under certain circumstances plane wave solutions can be superposed without breaking supersymmetry.

The $\mathbf{f}_{(2)}$ dependence is treated as above while the $\mathbf{f}_{(4)}$ is analyzed as in the case of the maximally supersymmetric plane wave solution, which we now repeat for the reader's convenience. The important observation is that for each choice of the index i , pushing Γ_i past $\mathbf{f}_{(4)}$ changes the relative sign between the two terms in $\mathbf{f}_{(4)}$. Then, using the chirality operator, the term with changed sign can be mapped into the one that did not. Since each of the two terms square to $-g^2$ we find

$$\mathbf{f}_{(4)} = \mathbf{f}_{(4)}^1 + \mathbf{f}_{(4)}^2, \quad (\mathbf{f}_{(4)}^I)^2 = -g^2,$$

$$\begin{aligned} \Gamma_- (\mathbf{f}_{(4)}^1 + \mathbf{f}_{(4)}^2)^2 \Gamma_i \chi &= \Gamma_- \Gamma_i (\mathbf{f}_{(4)}^1 - \mathbf{f}_{(4)}^2)^2 \chi = 4 \Gamma_- \Gamma_i \mathbf{f}_{(4)}^1{}^2 \chi \\ &= -4g^2 \Gamma_- \Gamma_i \chi. \end{aligned} \quad (4.20)$$

Thus, the coefficients A_{ij} now become the sum of the F_3 and F_5 contribution, and the solution is

$$\begin{aligned} F_{(3)} &= f dx^+ \wedge (dx^1 \wedge dx^2 + \beta dx^3 \wedge dx^4 \\ &\quad + \gamma dx^5 \wedge dx^6 + \alpha \beta \gamma dx^7 \wedge dx^8), \end{aligned}$$

$$\begin{aligned} F_{(5)} &= g dx^+ \wedge (dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 \\ &\quad + \alpha dx^5 \wedge dx^6 \wedge dx^7 \wedge dx^8), \end{aligned}$$

$$A_{ij} = \frac{1}{4} \left(f^2 + \frac{1}{4} g^2 \right) \delta_{ij}. \quad (4.21)$$

Even though f and g appear in the metric only in the combination $(f^2 + \frac{1}{4} g^2)$, the field strengths retain information on f and g separately. Using S duality one can reconstruct the full 2-parameter family of solutions (by rotating F_3 into H_3 by any angle). The maximal rotation corresponds to solutions with only H_3 and F_5 .

To conclude this discussion, we formulate a superposition rule for wave solutions: *Adding any two plane wave solutions with RR fields $F_{(p+1)}$ and $F_{(q+1)}$ leads to a new solu-*

tion. If the corresponding $\mathbf{f}_{(p)}$ and $\mathbf{f}_{(q)}$ anticommute, the common supernumerary Killing spinors are inherited by the resulting solution.

This statement allows one to immediately decide whether the direct sum of two wave solutions remains a solution by just looking at the directions covered by the various excited field strengths. The final amount of supersymmetry is given by the number of Killing spinors common to both solutions, which can be found from the dilatino variation only.

We now turn to the other candidate solution preserving 28 supercharges (3.5). The building blocks of Eq. (4.10) are in this case

$$\mathbf{h}_{(2)} = f(\gamma\Gamma_{13} - \beta\Gamma_{24}) \otimes \sigma^3, \quad \mathbf{f}_{(2)} = f(\Gamma_{12} + \beta\gamma\Gamma_{34}) \otimes \sigma^1. \quad (4.22)$$

Unfortunately, Eq. (3.5) cannot source a solution that preserves more than 20 supercharges. Indeed, \mathbf{f}^2 contains a term of the form $\beta\gamma\Gamma_{1234} \otimes 1$, which cannot be canceled either by a choice of A_{ij} or by introducing other fields. This further restricts the extra Killing spinors to be eigenvectors of Γ_{1234} with the same eigenvalue, and thus reduces them to 4. A 5-form field strength can also be added to this configuration without further reducing its supersymmetry. Solutions containing $F_{(5)}$ and the fields in Eq. (3.5) were explored in Ref. [15] and obtained as Penrose limits of the Pilch-Warner flow [16].

We can also analyze the possible type IIA solution preserving 28 supercharges (3.8). The discussion is similar to the one above; unfortunately, these solutions do not preserve more than 20 supercharges.

The solution sourced by NSNS fluxes found in type IIB supergravity is a solution of the type IIA theory as well. However, in the latter case it preserves only (14,8) supercharges, because the two fermions have opposite chiralities.

To summarize the results thus far, we have constructed in type IIB supergravity a two-parameter family of wave solutions preserving 28 supercharges. They are S dual to each other and are constructed by adding 5-form flux to the field configurations suggested by the projector analysis of Sec. III.

C. Solutions with 24 supercharges

Deformations of solutions with 28 supercharges. Any of the solutions discussed in the previous section and preserving 28 supercharges can be deformed to solutions preserving only 24. Indeed, if one modifies one of the two commuting projectors in the dilatino equation $(1+M)(1+N)$, the other one is still a projector, and still annihilates half the spinors it acts upon. Thus, all the families of solutions of the form (4.21) with $\beta^2=1, \gamma^2 \neq 1$ or vice-versa preserve 24 supercharges. Adding one structure of five-form field strength can be done without paying any cost. Thus we find a two 2-parameter family of pp -wave solutions with 24 supercharges

$$F_{+12} = F_{+56} = f, \quad F_{+34} = \alpha F_{+78} = \beta f,$$

$$F_{+1234} = \alpha F_{+5678} = g,$$

$$A_{11} = A_{22} = A_{55} = A_{66} = \frac{g^2 + 4f^2}{16},$$

$$A_{33} = A_{44} = A_{77} = A_{88} = \frac{g^2 + 4f^2\beta^2}{16}, \quad (4.23)$$

together with its S -dual cousins.

The ratio f/g , β and the S -duality parameter are unconstrained. In the case $\beta=0$ we recover the Penrose-Güven limit of $\text{AdS}_3 \times \text{S}^3 \times \text{T}^4$ [7], which is also the T dual of the solutions with 24 supercharges obtained in type IIA by Ref. [9]. If the five-form field is vanishing and $\beta=0$, then the “most distant” S -dual cousin of Eq. (4.23) (involving only $H_{(3)}$) is a 24 supercharge solution of type IIA supergravity.

In Sec. III we also discussed some projectors that preserve 24 supercharges and cannot be extended to preserve 28. We now construct the supergravity solutions which realize them.

Type IIB supergravity. As we saw in the previous section, the field configuration (3.5) cannot be completed to a full supergravity solution with 28 supercharges. Nevertheless, it is possible to use it for constructing solutions with 24 supercharges by truncating it to

$$\alpha H_{+13} = F'_{+12} = f(x^+), \quad \alpha^2 = 1. \quad (4.24)$$

The dilatino variation is

$$\begin{aligned} \delta\lambda &= -\frac{f(x^+)}{4} \Gamma_- [\alpha\Gamma_{13}\sigma^3 + \Gamma_{12}\sigma^1] \epsilon \\ &= -\frac{f(x^+)}{4} \Gamma_- \Gamma_{12}\sigma^1 [1 + \alpha\Gamma_{23}(i\sigma^2)] \epsilon \end{aligned} \quad (4.25)$$

and the gravitino variation fixes the metric coefficients to

$$A_{ij} = \frac{f^2}{16} \text{diag}(9, 1, 1, 1, 1, 1, 1, 1). \quad (4.26)$$

As expected, the equation of motion

$$R_{++} = \text{Tr} A = f^2 = \frac{1}{2} (H_{+13} H_{+13} + F_{+12} F_{+12}) \quad (4.27)$$

is also satisfied.

Type IIA supergravity. A similar solution to the one obtained above involves $F_{(4)}$ and $H_{(3)}$:

$$\frac{1}{2} H_{+12} = \alpha F_{+145} = f(x^+), \quad \alpha^2 = 1, \quad (4.28)$$

and thus

$$\begin{aligned} \delta\lambda &= \frac{f(x^+)}{2} \Gamma_- (\Gamma_{12}\sigma^3 + \alpha\Gamma_{145}\sigma^1) \\ &= \frac{f(x^+)}{2} \Gamma_- \Gamma_{12}\sigma^3 [1 - \alpha\Gamma_{245}(i\sigma^2)]; \end{aligned} \quad (4.29)$$

the gravitino variation fixes the metric coefficients to

$$A_{ij} = \frac{f^2}{16} \text{diag}(9, 25, 1, 1, 1, 1, 1, 1) \quad (4.30)$$

and the equation of motion

$$R_{++} = \text{Tr}A = 5f^2/2 = \frac{1}{2}(H_{+12}H_+{}^{12} + F_{+145}F_+{}^{145}) \quad (4.31)$$

is also satisfied.

Another type IIA solution preserving 24 supercharges can be obtained by combining $H_{(3)}$ and $F_{(2)}$:

$$3F_{+1} = -\frac{\alpha}{2}H_{+12} = f(x^+), \quad \alpha^2 = 1. \quad (4.32)$$

The dilatino variation is given by Eq. (3.12), and the metric is given by the gravitino variation to be

$$A_{ij} = -\frac{2}{9} \text{diag}(121, 169, 1, 1, 1, 1, 1, 1). \quad (4.33)$$

Despite these rather bizarre numbers, the equation of motion is also satisfied:

$$R_{++} = \text{Tr}A = \frac{37}{18} = \frac{1}{2} \left(\frac{1}{3^2} + 2^2 \right) = \frac{1}{2}(F_{+1}F_+{}^1 + H_{+12}H_+{}^{12}). \quad (4.34)$$

The last solution discussed in Sec. III [Eq. (3.14)]:

$$3F_{+1} = F_{+234} = f(x^+) \quad (4.35)$$

also preserves 24 supercharges. The dilatino variation is Eq. (3.15), and the metric determined by the gravitino variation

$$A_{ij} = \frac{f^2}{9 \times 4} \begin{pmatrix} 4\mathbb{1}_4 & 0 \\ 0 & \mathbb{1}_4 \end{pmatrix} \quad (4.36)$$

satisfies the equation of motion

$$R_{++} = \frac{5}{9}f^2 = \frac{1}{2}(F_{+1}^2 + F_{+234}^2). \quad (4.37)$$

Upon lifting this solution to M-theory one obtains the maximally supersymmetric solution found in Ref. [5]. We can also use the superposition principle formulated in the previous section to add to this solution the identical solution with fields along different directions. As we explained, the gravitino variation equation is satisfied if the fields anticommute, and the dilatino variation becomes the sum of two projectors. Thus the superposition solution with

$$3F_{+1} = F_{+234} = f, \quad 3F_{+2} = F_{+156} = g, \quad (4.38)$$

and the corresponding A_{ij} preserves 4 supernumerary Killing spinors and thus has 20 supercharges.

V. T DUALITY

It is interesting to explore the metrics one obtains by T dualizing some of the solutions with augmented supersymmetry found in the previous sections. The Killing spinors that survive the T -duality transformation are those which commute with the Killing vector defining the duality direction. Equation (4.3) implies that all spinors depend on the transverse coordinates, therefore these directions cannot be used for our purpose. We thus explore duality transformations along x^+ , which is the most interesting of the remaining directions.

However, as one can see from the solutions described in the previous chapter, all A_{ij} giving augmented supersymmetry are positive, and therefore Eq. (2.1) implies that x^+ is timelike. Unfortunately, timelike T duality is hard to interpret physically since it yields RR-field kinetic terms with the wrong sign [8,17]. Thus, it can only be used as a solution-generating technique, and only for spacetimes with NSNS fields.

There are two ways we can circumvent this problem. The first one is to T dualize only the solutions with NSNS flux. We have one such solution with 28 supercharges, as well as 2 families of solutions with 24. The second [8] is to perform a coordinate transformation $x^- \rightarrow x^- - (c/2)x^+$, where c is a positive constant. The metric (2.1) becomes

$$ds^2 = -2dx^+dx^- + c(dx^+)^2 - A_{ab}(x^+)z^a z^b(dx^+)^2 + (dz^a)^2, \quad (5.1)$$

and thus for any c there exists a region of space where x^+ is spacelike and T duality can be performed. The same shift can be performed for the spacetimes containing only NSNS fields. It is rather straightforward to take any of the solutions we have and T dualize it using the rules in Ref. [18].

As explained in the beginning of this section, not all of the original supersymmetries survive the T -duality procedure. Only those Killing spinors which are independent of x^+ remain Killing spinors of the new geometries. From Eq. (4.5) we can see that these spinors satisfy $\Omega_+\chi = 0$.

Unfortunately, for the solutions with 28 supercharges, Ω_+ is not proportional to any projector from the dilatino variation. This is because f and h are no longer multiplied from the left by Γ_- (as in the dilatino variation), and therefore the chirality of the spinors cannot be used to combine the Γ matrices into products of projectors. It is, however, not hard to see that, when T dualizing the solutions with 28 supercharges, all the supernumerary Killing spinors disappear, and only 6 of the 16 annihilated by Γ_- remain.

pp -wave solutions with only two nonzero structures of F_3 or H_3 (preserving 24 supercharges) have more x^+ independent Killing spinors. Indeed, in both cases Ω_+ contains one projector, and thus all the 8 supernumerary Killing spinors and 8 of the 16 regular ones survive the T duality. The result of the duality transformation along x^+ is a non- pp -wave solution of type IIA and 11 dimensional (11D) supergravity with 16 supercharges.

Let us first consider a solution containing only NSNS fluxes [similar to the S dual of Eq. (4.23)]:

$$\begin{aligned}
H_{+12}=H_{+34}=h, \quad \rightarrow B_{+1}=hx^2, \quad B_{+3}=hx^4, \\
H=c-\frac{h}{4}[(x^1)^2+(x^2)^2+(x^3)^2+(x^4)^2].
\end{aligned}
\tag{5.2}$$

The T dual of this geometry is

$$\begin{aligned}
ds^2 &= \frac{1}{H}[(dx^+ + hx^2 dx^1 + hx^4 dx^3)^2 - (dx^-)^2] + (dx^i)^2, \\
e^{2\Phi} &= \frac{1}{|H|}, \quad B = \frac{1}{H}(dx^+ + hx^2 dx^1 + hx^4 dx^3) \wedge dx^-,
\end{aligned}
\tag{5.3}$$

which is exactly the metric of smeared F strings perturbed with transverse fluxes. The solution diverges at finite distance from the origin.

Since Eq. (5.3) only contains NSNS fields, it makes sense as a solution when H is negative. The only difference is that x^- becomes spacelike, x^+ becomes timelike, and the B field switches sign. Since $|H|$ can be chosen to be nowhere vanishing (by choosing $c < 0$), this solution is regular everywhere.

It is quite surprising that these metrics preserves 16 supercharges, and it is even more surprising that such metrics are T dual to that of a pp wave. Very similar solutions can be obtained by T dualizing the solution with H_3 and 28 supercharges. In that case only 6 of the original 28 supercharges survive T duality; however, it is possible that the resulting solution preserves a larger amount of supersymmetry, of which only 6 supercharges commute with T duality. We did not investigate this possibility.

For positive H we can also T dualize the solution with nontrivial F_3 :

$$\begin{aligned}
F_{+12}=F_{+34}=f, \quad \rightarrow C_{+1}=fx^2, \quad C_{+3}=fx^4, \\
H=c-\frac{f}{4}[(x^1)^2+(x^2)^2+(x^3)^2+(x^4)^2],
\end{aligned}
\tag{5.4}$$

and obtain a solution corresponding to smeared F strings perturbed with transverse RR 2-form

$$\begin{aligned}
ds^2 &= \frac{1}{H}[(dx^+)^2 - (dx^-)^2] + (dx^i)^2, \\
e^{2\Phi} &= \frac{1}{|H|}, \quad B_{+-} = \frac{1}{H}, \quad C_1 = fx^2, \quad C_3 = fx^4.
\end{aligned}
\tag{5.5}$$

Upon lifting this solution to M theory we can obtain the supergravity solution of smeared M2 branes (with the harmonic function H), perturbed with off-diagonal components of the metric. Similar to the previous solutions obtained by spacelike T duality, these solutions become divergent at finite radius.

One can T dualize the other solutions we found and obtain geometries corresponding to F1 strings and M2 branes de-

formed with transverse forms. It is also possible to add to H the regular harmonic function N/r^6 , in which case the supernumerary Killing spinors disappear, but a fraction of the regular ones survives the T duality. One can thus obtain more realistic perturbed M2 brane solutions.

A. The AdS-CFT interpretation of the divergences

All the solutions we found by spacelike T duality, as well as the solutions found in Ref. [8] have the generic property that the curvature diverges at a finite radius. Since all these solutions correspond to smeared F1 strings or M2 branes perturbed with transverse fluxes, it is possible to give them a very interesting interpretation from the point of view of the AdS-CFT correspondence.

To do this, we first add the usual harmonic function N/r^6 to H . The metrics obtained above are still solutions, but they only have 8 supercharges. Nevertheless, it now becomes possible to interpret them as near-horizon geometries of F1 strings or M2 branes perturbed with constant transverse F_2 , off-diagonal metric components, or transverse F_4 . It is quite straightforward⁵ to see that these perturbations correspond to turning on an irrelevant operator in the boundary theory. In the case of the M2 branes, the transverse perturbation with constant F_4 corresponds to a boundary operator of dimension 5, of the form $F^2\Psi\Psi$.⁶

Since the operator is irrelevant, if one turns on a finite perturbation in the UV, it flows to zero in the IR. Conversely, if one turns on a finite perturbation in the IR, it diverges in the UV. Thus, the only solutions which are regular at infinity are those with $f=0$, which is exactly what the solutions (5.5) and the ones discussed in Sec. VII of Ref. [8] imply.

This singularity can also be seen as coming from ‘‘negative mass’’ smeared M2 branes effectively created by the combination of the transverse 4-form (or F_2 and F_6 in the F1 string case) via the Chern-Simons term of the 11D supergravity Lagrangian. When one puts enough real M2 branes in the geometry (by adding to the harmonic function a constant or N/r^6), the supergravity is regular up to the radius where H become zero, which is the radius where the ‘‘negative mass’’ M2 branes overtake the real ones. Such setups are very reminiscent of the ones where an enhancement mechanism is responsible for the removal of singularities [19], and it would be interesting to explore if this is also the fate of the singularities present here.

In addition to these divergent solutions we can also obtain metrics which are everywhere regular by adding to H the function $-N/r^6$ and performing timelike T duality. Of course, the wrong sign of $-N/r^6$ is unphysical in the original pp -wave metric, but since we are only using timelike T duality for solution generating we do not worry about this. We obtain the metric (5.3) with $-H=|H|=N/r^6$

⁵See Ref. [20] for the AdS-CFT analysis of the perturbation of F1 strings with transverse F_2 and Ref. [21] for the AdS-CFT analysis of the perturbation of M2 branes with transverse F_4 .

⁶This can be seen from Eqs. (16) and (17) in Ref. [21], and is similar to Eq. (50) in Ref. [22].

$+(f/4)\Sigma_i(x^i)^2$, which can again be interpreted as the near-horizon of F1 strings or M2 branes perturbed with off-diagonal metric components and $B_{(2)}$.

Unlike its cousin obtained by spacelike T duality, this solution does not diverge at finite radius. The two solutions correspond to turning on *different* perturbations in the IR (in one case the $B_{(2)}$ perturbation contains a timelike direction and in the other it does not). Therefore it is not surprising that these perturbations give rise to different UV physics.

In the regular case, the metric in the UV becomes (in the string frame)

$$ds^2 = \frac{f}{2u} [-(dt + fx^2 dx^1 + fx^4 dx^3)^2 + (dx^-)^2 + du^2 + 4u^2 d\Omega_3^2] + \sum_{i=5}^8 (dx^i)^2, \quad (5.6)$$

where $u = (f/2)\Sigma_{i=1}^4(x^i)^2$. The nontrivial part of this metric is conformal to a fibration of a \mathbb{Z}_2 orbifold of a 4-plane, and does not appear to be singular. If only one structure of $H_{(3)}$ is turned on, the metric resembles that of a wave. This flow can easily be lifted to M theory, or dualized to other flows. Thus we obtained a nonsingular supergravity flow, starting from $\text{AdS}_4 \times S^7$ (or the near horizon F1 string metric) in the IR and ending with the geometry (5.6) in the UV. It would be very interesting to find if this geometry has a field theory dual, and learn more about these irrelevant perturbations. Moreover, by using T and S duality it is possible to construct similar nonsingular flows from $\text{AdS}_5 \times S^5$ in the IR to a metric similar to Eq. (5.6) in the UV.

These types of flows are reminiscent of the one obtained by turning on the dimension 6 operator B_{12} in the $\text{AdS}_5 \times S^5$ dual of the $\mathcal{N}=4$ Yang-Mills theory. In that case one also flows to a nontrivial UV geometry, which is dual to a noncommutative field theory.

B. pp waves as solution factories

As we have seen in the beginning of this section, by T -dualizing pp -wave solutions with fluxes one can obtain metrics corresponding to branes and strings perturbed with constant fluxes. It is quite trivial to further use T duality and S duality on these solutions to generate the solutions corresponding to other branes perturbed with transverse fluxes.

However, since we consider wave backgrounds in which the form field strengths only depend on x^+ (otherwise constructing solutions of the equation of motion and Bianchi identities becomes more challenging), the resulting fluxes will not depend on the transverse directions, and will generically correspond to turning on irrelevant operators in the boundary theory. Thus, most of these solutions have either singularities at a finite distance from the origin, or very nontrivial UV completions.

Since the fluxes do not depend on transverse directions, it does not appear possible to obtain from the simple pp -wave ansatz the full solutions corresponding to perturbations of the AdS-CFT correspondence with *relevant* operators (in fact, it seems quite remarkable that pp -wave backgrounds are T dual

to a perturbation of the AdS-CFT duality in the first place). A possible direction toward obtaining these full solutions would be to go backward along the chain of dualities, to first obtain a more generic wave and then use its simple features to try to find the full solution.

The metrics obtained by T duality can be easily made time dependent. As we explained in the previous chapter, adding x^+ dependence to the forms and the metric removes the supernumerary Killing spinors. Nevertheless, a certain fraction of the 16 spinors annihilated by Γ_- (1/2 or 3/4, depending on the fluxes) survive the T duality. Thus, we obtain time-dependent metrics with nontrivial fluxes, and some supersymmetry (8 or 12 supercharges).

VI. A GENERAL ANALYSIS

In this section we will prove that a wave solution with nontrivial dilatino variation cannot preserve more than 28 supercharges and that the field configurations analyzed in Secs. III and IV are the only ones with this property. As discussed in the beginning of this paper, a systematic way of constructing all solutions with more than 16 supercharges is to start from the dilatino variation and ask for field configurations that organize it as a product of commuting projectors. Thus

$$\delta\lambda = \Gamma_- M \left(\prod_{i=1}^n P_i \right) \epsilon, \quad P_i = \frac{1}{2}(1 + A_i), \quad (6.1)$$

where M is some combination of Dirac matrices and P_i are a set of commuting projectors.

An upper bound on the number of preserved supercharges translates into an upper bound on the number of projectors that can be generated in the dilatino variation by the fields present in the theory.

The basic observation that will help us reach our goal is that *any* two terms in the dilatino variation must form a projector, up to a common factor. It is easy to see that this is the case by expanding the brackets in Eq. (6.1). Furthermore, any of these terms has to be generated by one of the form fields appearing in Eqs. (2.7), (2.8). This implies that, for the cases we are interested in, the dilaton cannot contribute to the dilatino variation. Indeed, after factoring out the Dirac matrix Γ_- which is common to all fields, the contribution of any component of any form field squares to -1 while the dilaton contribution squares to one. Similar arguments lead to the conclusion that the axion cannot contribute either. Since the 0-form field strength cannot contribute to a wave solution⁷ we are left to consider $H_{(3)}$, $F_{(2)}$, and $F_{(4)}$ in the type IIA theory and the two three-forms of the type IIB theory. We will now argue that it is not possible for the dilatino variation to contain more than two projectors in addition to Γ_- .

⁷The equations of motion are not satisfied in the presence of a cosmological constant unless form fields are allowed to have non-null nonvanishing components.

A. The type IIB theory

We begin by discussing the type IIB theory. Because all spinors appearing in this theory have the same chirality, we can use the chirality operator Γ_{-1} to rewrite a product of m Dirac matrices as a product of $(8-m)$.

A simple inspection of the available form fields reveals that in type IIB the prefactor M in Eq. (6.1) must be a product of two Dirac matrices tensored with either σ^1 or σ^3 . Then, the $\mathfrak{gl}(2)$ component of any of the projectors in Eq. (6.1) is either the identity matrix or $i\sigma^2$ depending, respectively, on whether only one or both types of fields are excited. Therefore, the Dirac matrix part of all A_i 's in Eq. (6.1) must commute. Furthermore, they can be either products of two or four Dirac matrices. These observations set an upper bound on the number n of projectors. In particular, there are at most three *independent* commuting products of two Dirac matrices⁸ and only two *independent* commuting products of four Dirac matrices. We will now discuss separately the possible constructions of projectors.

(1) The easiest to analyze is the case in which all A_i 's are built out of products of four Dirac matrices. Since there are only two such independent combinations, it follows that $n \leq 2$ which in turn implies that there are at most 28 preserved supercharges. This product of projectors, which can be generated using either one of the two 3-form field strengths present in the theory, was analyzed in Secs. III and IV.

(2) Consider next the situation when all projectors are constructed out of products of two Dirac matrices. The product of projectors can be expanded as

$$M \sum_{k=0}^n \sum_{\sigma_k \in C_n^k} \prod_{j \in \sigma_k} A_j, \quad (6.2)$$

where C_n^k is the collection of sets σ_k of k elements picked out of n . Since all A_i 's are different, they will commute with each other if and only if no two have common Dirac matrices. The only way for this to come from a sum of bilinears of Dirac matrices of the type appearing in the dilatino transformation rule, is that exactly one of the matrices building any A_i appears in M . Indeed, if this were not the case, the product between M and the corresponding A_i would contain four Dirac matrices and this cannot be generated by one of the available fields.⁹ Furthermore, such a term cannot be canceled using Γ_{-1} since all terms in the sum above are proportional to M and the use of Γ_{-1} would produce terms without this property. Since, as argued above M must be a bilinear in Dirac matrices, we can have at most two projectors of this type in Eq. (6.1) and therefore there are at most 28 super-

charges. Such products of projectors can be generated using combinations of the two 3-forms and were analyzed in Secs. III and IV.

(3) The last possibility is to have some projectors constructed out of products of four Dirac matrices while the others of products of two. The requirement that they commute implies that there must be an even number of common Dirac matrices between any two A_i and A_j . If one product of two Dirac matrices, call it B_2 , is not contained in one product of four of them, call it B_4 , then, expanding the brackets in Eq. (6.1) implies that we need a form field to supply a term of the type MB_2B_4 . But such a field does not appear in the dilatino transformation rule unless one of the Dirac matrices appearing in M also appears either in B_2 , in B_4 or in both. Indeed, if this were the case, then MB_2B_4 will become a product of six Γ matrices and using Γ_{-1} can be rewritten as a product of two of them. Furthermore, similar arguments applied on the terms MB_4 implies that M and B_4 cannot have a common Dirac matrix. We are therefore left with the following possibility:

$$\Gamma_{ab}(1+\Gamma_{bc})(1+\Gamma_{defg}) \quad (6.3)$$

This combination has the potential of preserving 28 supercharges and was analyzed in Secs. III and IV. Since there does not seem to be any obstruction to adding more projectors we will attempt to do so. It is easy to see that a projector constructed out of four Dirac matrices that satisfies both (1) and (3) will anticommute with Γ_{defg} and thus is not allowed. We are thus left with the possibility of adding a projector constructed out of two Dirac matrices. This will have to comply with both the restrictions of point (2) as well as with those of point (3). Thus, it seems possible to insert

$$1+\Gamma_{ah}, \quad (6.4)$$

where the index h represents the matrix which does not already appear in Eq. (6.3). Nevertheless, the three projectors are not independent because $\Gamma_{-1}\Gamma_{ah}\Gamma_{-1}=\Gamma_{-1}\Gamma_{bc}\Gamma_{defg}$ and thus the third projector does not lead to more preserved supersymmetry.

This concludes the analysis of the type IIB theory with the result that any solution of the equations of motion which leads to a nontrivial dilatino variation will preserve at most 28 supercharges. Because the 5-form field strength does not appear in the dilatino variation, it can be used to enlarge the set of fields producing the projectors discussed above. This possibility was discussed in detail in Sec. IV.

B. The type IIA theory

We now turn to the analysis of the type IIA theory. The discussion is complicated by having fields contributing different numbers of Dirac matrices but is also simplified by the fact that we are no longer allowed to use the chirality operator Γ_{-1} to map products of Γ matrices into each other. Indeed, any projector constructed by using Γ_{-1} would lead to enhanced supersymmetry in one sector while breaking it in the other sector. Furthermore, in the type IIA theory all fields

⁸Products of two Dirac matrices generate $SO(8)$ whose rank is four. Due to the fact that the chirality of both spinors is the same and that the dilatino variation is proportional to Γ_{-1} , it follows that in the dilatino variation in the type IIB theory one of the four Cartan generators of $SO(8)$ can be expressed in terms of the other three and the chirality operator Γ_{-1} .

⁹The 5-form field strength does not appear in the dilatino variation.

appear in the dilatino variation. Thus, the set of fields leading to projectors in this variation cannot be enlarged.

As discussed before, the dilaton cannot be excited in a wave solution with augmented supersymmetry. As a first step in answering the question of how many independent commuting projectors can appear in the dilatino variation, we first study if it is possible to have two projectors. Thus,

$$\delta\lambda \sim M(1+A_1)(1+A_2)\epsilon = [M + M(A_1+A_2) + MA_1A_2]\epsilon. \quad (6.5)$$

As noticed before, each term above must be produced by one of the fields present in the background. We have at our disposal products of one, two and three Dirac matrices.

Let us now discuss case by case the possible matrices M and for each of them the allowed projectors A .

(1) M is generated by the 2-form field strength, i.e., $M = \Gamma_a \otimes (i\sigma^2)$. The fact that MA_i must be generated by one of the fields implies that the Dirac matrix component of A_i is constructed out of one, two, or three¹⁰ matrices and the requirement of $(1+A)$ being a projector fixes the $\mathfrak{gl}(2)$ component. Combining everything we are left with the following possibilities: $\Gamma_b \otimes g$ with $g=1, \sigma^1, \sigma^3$, $\Gamma_{bc} \otimes (i\sigma^2)$ and $\Gamma_{abc} \otimes (i\sigma^2)$. It is easy to see that some of these possibilities cannot be generated by the available fields. Indeed, the products $\Gamma_a \otimes (i\sigma^2)\Gamma_{bc} \otimes (i\sigma^2)$ and $\Gamma_a \otimes (i\sigma^2)\Gamma_{abc} \otimes (i\sigma^2)$ have the identity matrix as $\mathfrak{gl}(2)$ component and there is no field with this property except for the dilaton which does not contribute any Γ matrix. The remaining possibility is $A = \Gamma_b \otimes g$ with $g=1, \sigma^1, \sigma^3$. Inserting these remaining combinations in Eq. (6.5) we find that the $\mathfrak{gl}(2)$ component is fixed by the terms MA to be $g = \sigma^1$. This last possibility is nevertheless eliminated by considering MA_1A_2 . Thus, we conclude that if M is generated by the 2-form field strength, the dilatino variation contains at most one projector besides Γ_- , and thus no more than 24 supercharges can exist.

(2) The next possibility is for M to be generated by the NSNS 3-form field strength, i.e., $M = \Gamma_{ab} \otimes \sigma^3$. Then, requiring that $1+A$ is a projector, we have the following possibilities.¹¹

$A = \Gamma_c \otimes \{1, \sigma^{1,3}\}$. The case with 1 and σ^3 cannot be generated due to the $\mathfrak{gl}(2)$ component while the other one can be generated using the 2-form field strength if $c=a$ or $c=b$.

$A = \Gamma_{ac} \otimes (i\sigma^2)$. One of the Dirac matrices that appear in A must also appear in M since otherwise there would be four Γ matrices in MA . It then follows that this projector cannot be generated due to the $\mathfrak{gl}(2)$ component of the product MA .

$A = \Gamma_{acd} \otimes (i\sigma^2)$. One of the Dirac matrices that appear in A must also appear in M since otherwise there would be five Γ matrices in MA . Then the $\mathfrak{gl}(2)$ component of the product MA requires that $c \neq b$ and $d \neq b$ for this projector to be generated.

¹⁰This last possibility occurs when A and M have one common Dirac matrix.

¹¹We will put from the outset some common Dirac matrices between A and M . This is due to the fact that the product MA must have at most three Dirac matrices for such a term to be generated.

$A = \Gamma_{abcd} \otimes \{1, \sigma^{1,3}\}$. Two of the Dirac matrices that appear in A must also appear in M since otherwise there would be four Γ matrices in MA . This in turn fixes the $\mathfrak{gl}(2)$ component of A to be the identity matrix. Such a term can be generated by the NSNS 3-form field strength.

$A = \Gamma_{abcde} \otimes \{1, \sigma^{1,3}\}$. Two of the Dirac matrices that appear in A must also appear in M since otherwise there would be more than three Γ matrices in MA . Then, the $\mathfrak{gl}(2)$ component prevents this term from being generated.

Thus, A_1 and A_2 must be of the type $\Gamma_{acd} \otimes (i\sigma^2)$ with $c \neq b$ and $d \neq b$ (i.e., they must have one Dirac matrix common with M) or of the type $\Gamma_{abef} \otimes 1$. The $\mathfrak{gl}(2)$ component of MA_1A_2 forbids both A 's be of the second type. Thus, we have two choices.

If both are of the first type then, due to the $\mathfrak{gl}(2)$ component of MA_1A_2 being σ^3 , this term must be generated by the NSNS 3-form and thus must contain exactly two Dirac matrices. The only possibility for this to happen is if the Dirac matrices that are common between A_i and M are different and there is one more Dirac matrix common between the two A_i 's. Thus, the only solution is

$$\Gamma_{ab} \otimes \sigma^3 [1 + \Gamma_{acd} \otimes (i\sigma^2)] [1 + \Gamma_{bce} \otimes (i\sigma^2)]. \quad (6.6)$$

This projector is generated by the following choice of fields:

$$\frac{1}{2}H_{+ab} = -\frac{1}{2}H_{+de} = F_{+bcd} = F_{+ace} \quad (6.7)$$

which is none other than the field configuration discussed in Eq. (3.8).

If the two A_i 's are of different type, then the $\mathfrak{gl}(2)$ component of MA_1A_2 implies that this term is generated by the 4-form field strength. Thus, there must be another Dirac matrix common between A_1 and A_2 which uniquely identifies the projectors as

$$\Gamma_{ab} \otimes \sigma^3 [1 + \Gamma_{acd} \otimes (i\sigma^2)] (1 + \Gamma_{abce} \otimes 1), \quad (6.8)$$

which is just a rewriting of Eq. (6.6).

It is now easy to analyze the problem of adding more projectors to either one of Eqs. (6.6) or (6.8). Let us discuss Eq. (6.6). If A_3 is of the same type as A_1 and A_2 , then the Dirac matrix that is common between A_3 and M must be different from the ones common between A_1 and M and A_2 and M . However, M is constructed out of only two Γ matrices. Therefore, a third projector of the first type is forbidden. If A_3 is of the second type, it follows from Eq. (6.8) that it must have one common Dirac matrix with A_1 and A_2 for $MA_1A_2A_3$ to be generated. Furthermore, this matrix cannot be common between A_1 and A_2 because otherwise $MA_1A_2A_3$, which is proportional to σ^3 , could not be generated. Thus, we are left with

$$A_3 = \Gamma_{abde} = A_1A_2. \quad (6.9)$$

But this does not lead to an independent projector

$$\begin{aligned}
& (1+A_1)(1+A_2)(1+A_1A_2) \\
& = (1+A_1+A_2+A_1A_2)(1+A_1A_2) \\
& = 2(1+A_1)(1+A_2) \tag{6.10}
\end{aligned}$$

since $A_i^2=1$. Thus, if M is generated by the NSNS 3-form field strength, the dilatino variation contains at most two projectors in addition to Γ_- and there are at most 28 preserved supercharges.

(3) The third and last possibility is for M to be generated by the 4-form field strength, i.e., $M=\Gamma_{abc}\otimes\sigma^1$. As in the previous case, there are several possibilities for A_i .¹²

$A=\Gamma_d\otimes\{1, \sigma^{1,3}\}$. There is no field that can give this contribution. Indeed, MA is a product of either four or two Dirac matrices tensored with $1, \sigma^1$ or $(i\sigma^2)$. None of these terms can be generated by the available fields.

$A=\Gamma_{ad}\otimes(i\sigma^2)$. This leads to an MA with σ^3 as the $\mathfrak{gl}(2)$ component, which requires two Dirac matrices. There is no choice of d that can do this, and thus such an A is not allowed.

$A=\Gamma_{abd}\otimes(i\sigma^2)$. This leads again to σ^3 being the $\mathfrak{gl}(2)$ component of MA . If $d\neq c$ it also contains a product of two Dirac matrices. This can be generated using the NSNS 3-form field strength.

$A=\Gamma_{abde}\otimes\{1, \sigma^{1,3}\}$. The choices 1 and σ^1 as $\mathfrak{gl}(2)$ components cannot be generated while the choice σ^3 can be generated using $F_{(2)}$ if $d=c$.

$A=\Gamma_{abcdef}\otimes\{1, \sigma^{1,3}\}$. The first possibility is viable if $d=c$, since it can be generated by the NSNS 3-form. The other two cases cannot be generated due to a mismatch between the $\mathfrak{gl}(2)$ component and the Dirac matrix component of MA .

$A=\Gamma_{abcdefg}\otimes(i\sigma^2)$. The Dirac matrix component requires $d=c$, leading to a product of three matrices which does not match with the $\mathfrak{gl}(2)$ component which is σ^3 . Therefore, this combination is not allowed.

Thus, if $M=\Gamma_{abc}\otimes\sigma^1$ both A_1 and A_2 can be either of the type $\Gamma_{abd}\otimes(i\sigma^2)$ with $d\neq c$, $\Gamma_{abce}\otimes\sigma^3$ or $\Gamma_{abcf}\otimes 1$. By analyzing the six inequivalent combinations it follows that A_1 and A_2 cannot be of different types because of a mismatch between the $\mathfrak{gl}(2)$ component and the number of Dirac matrices that can be generated in MA_1A_2 . Therefore, they must be of the same type, which requires that MA_1A_2 be built out of three Dirac matrices since its $\mathfrak{gl}(2)$ component is σ^1 . Since $(\Gamma_{abc})(\Gamma_{abce})(\Gamma_{abcf})$ contains five Dirac matrices, while $(\Gamma_{abc})(\Gamma_{abceg})(\Gamma_{abcfh})$ contains either five or seven Dirac matrices, it follows that the only possibility is

$$\Gamma_{abc}\otimes\sigma^1[1+\Gamma_{abd}\otimes(i\sigma^2)][1+\Gamma_{bce}\otimes(i\sigma^2)], \quad c\neq d\neq e, \tag{6.11}$$

¹²As in previous discussions, we will put from the outset some common Dirac matrices between A and M . This is due to the fact that the product MA must have at most three Dirac matrices for such a term to be generated.

i.e., A_1 and A_2 each have two common Γ matrices with M and one common between themselves. This projector is just the rewriting of Eq. (6.6).

In this case there exists $A_3=\Gamma_{acf}$ which has the same properties as A_1 and A_2 . However, adding it to Eq. (6.11) does not lead to enhanced supersymmetry. Indeed, the term $MA_1A_2A_3$ will contain six Dirac matrices. Such a term can be generated only using Γ_- to map it to a product of two Γ matrices, but this operation enhances supersymmetry in the left-handed sector while breaking it in the right-handed sector. The required field configuration has the potential of preserving (15,8) supercharges.

This concludes the analysis of the dilatino variation in type IIA theory with the result that there exist field configurations leading to variations proportional to Eqs. (6.6) and (6.11) which potentially preserve 28 supercharges. Unlike the case of the type IIB theory where the 5-form does not appear in the dilatino variation, in the type IIA theory we cannot enlarge the set of fields that lead to enhanced supersymmetry.

To summarize, we have shown that a wave solution of the supergravity equations of motion with a nontrivial dilatino variation preserves at most 28 supercharges. The candidates are given by the projectors analyzed in Secs. III and IV.

VII. SUMMARY AND CONCLUSIONS

Using the relative simplicity of pp -wave geometries we have explored wavelike solutions of type IIA and type IIB supergravity with augmented amounts of supersymmetry. Making use of the chirality of the fermions of type IIB supergravity, and expressing the dilatino variation as a product of commuting projectors, we found one two-parameter family of type IIB solutions with 28 supercharges, as well as one type IIB three-parameter family of solutions with 24 supercharges. We also found several individual solutions of type IIA supergravity preserving 24 supercharges, as well as solutions preserving (14,8) supersymmetry.

In the process of doing this, we formulated a superposition rule for wave solutions, giving an easy way of testing when a direct sum of wave solutions still possesses enhanced supersymmetry. We also conducted a rigorous exploration of the possibility of constructing solutions with augmented supersymmetry, and concluded that 28 supercharges is the most one can find when the dilatino variation is nontrivial.

By T dualizing some of our pp -wave solutions we obtained solutions with 16 supercharges similar to smeared F1 strings perturbed with transverse fluxes. After adding an extra term to the harmonic function we interpreted these solutions as perturbations of the AdS-CFT duality with irrelevant operators. This allowed us to give a field theoretical interpretation to the singularities some of these solutions generically have at finite distance from the origin.

We also obtained an exact nonsingular IR \rightarrow UV flow from AdS₄ \times S⁷ to an intriguing UV geometry. Similar procedures can be used to construct flows from AdS₅ \times S⁵ and other near-horizon geometries.

Since our solutions are exact, the careful investigation of these geometries can yield further insights into the role of

these irrelevant operators. The similarity of these flows to the flows to geometries dual to noncommutative field theories deserves further investigation, and could yield interesting physics.

The work presented here can be extended in several different directions. One would be to try to realize these pp -wave solutions as Penrose-Güven limits of other supergravity backgrounds. This does not seem straightforward, especially for the solutions with 28 supercharges; their multifarious mix of fields makes them hard to obtain as such limits. Nevertheless, some of their cousin solutions (like the one with 24 supercharges and only two nonzero structures of F_3) can easily be obtained as limits of the $\text{AdS}_3 \times \text{S}^3$ geometry [7,23–27], so it is not implausible that a careful analysis could in the end find a “mother background.” If this background were found and it had a dual field theory, our backgrounds would be dual to limits of the field theory with 28 supercharges, which can yield interesting insights into that theory.

Another possibility would be to look for waves whose duals correspond to *relevant* perturbations of the AdS-CFT correspondence. As explained in Sec. V, such waves would have r dependent fields, and they would not be as simple as the ones discussed here. However, it is conceivable that the equations of motions would take a more transparent form, which could be more amenable to finding exact solutions.

The duality between pp waves and irrelevant perturbations of the AdS-CFT correspondence could be further refined. As seen in Ref. [7], string theory on a pp -wave background is dual to a large R -charge sector of a field theory. After T dualization, the resulting background can only be interpreted as a perturbed near-horizon geometry (which is

dual to another field theory) only if one adds by hand a term of the form Q/r^6 in the harmonic function. However, this spoils the original duality with the large angular momentum sector of the first field theory. It thus appears that when Q goes to zero, the dual field theory changes drastically, although this is not such a drastic change from the point of view of the supergravity. This phenomenon deserved further study, and might even be a link toward establishing a more direct relation between the field theories at the ends of the “broken” duality chain.

Yet another direction involves investigating and maybe expanding the exact nonsingular IR to UV flows we constructed, finding possible field theory duals of the UV geometry, and understanding their similarity to the flows to noncommutative theories.

Last but not least, arguments similar to those leading to the equivalence of the $\mathcal{N}=3$ and $\mathcal{N}=4$ vector multiplets in 4 dimensions imply the equivalence of $\mathcal{N}=7$ (28 supercharges) and $\mathcal{N}=8$ gravity multiplets. It would be interesting to see if this structure is preserved by interactions in the background of the waves with 28 supercharges.

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