Dimensional reduction of a Lorentz- and *CPT*-violating Maxwell-Chern-Simons model

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(Received 27 January 2003; published 18 June 2003)

Taking as a starting point a Lorentz and *CPT* noninvariant Chern-Simons-like model defined in $1+3$ dimensions, we proceed to realize its dimensional reduction to $D=1+2$. One then obtains a new planar model, composed by the Maxwell-Chern-Simons (MCS) sector, a Klein-Gordon massless scalar field, and a coupling term that mixes the gauge field to the external vector v^{μ}. In spite of breaking Lorentz invariance in the particle frame, this model may preserve *CPT* symmetry for a single particular choice of v^{μ}. Analyzing the dispersion relations, one verifies that the reduced model exhibits stability, but the causality can be jeopardized by some modes. The unitarity of the gauge sector is assured without any restriction, while the scalar sector is unitary only in the spacelike case.

DOI: 10.1103/PhysRevD.67.125011 PACS number(s): 11.10.Kk, 11.30.Cp, 11.30.Er, 12.60.−i

I. INTRODUCTION

It is usually assumed that a consistent quantum field theory (QFT) must respect at least two symmetries: Lorentz covariance and *CPT* invariance. The traditional framework of a local QFT, from which we derive the standard model that sets the physics inherent to the fundamental particles, satisfies both these symmetries. In the beginning of the 1990s, a new work [1] proposing a correction term to conventional Maxwell electrodynamics that preserves gauge invariance, despite breaking Lorentz, *CPT*, and parity symmetries, was first analyzed. The correction term, composed of the gauge potential A_{μ} and an external background 4-vector v_{μ}, has a Chern-Simons-like structure, $\epsilon^{\mu\nu\kappa\lambda}v_{\mu}A_{\nu}F_{\kappa\lambda}$, and is responsible for inducing an optical activity of the vacuum—or birefringence—among other effects. In this same work, however, it was shown that astrophysical data do not support the birefringence and impose stringent limits on the value of the constant vector, v_{μ}, reducing it to a negligible correction term. Similar conclusions, also based on astrophysical observations, were also confirmed by Goldhaber and Timble [2]. Some time later, Colladay and Kostelecky [3] adopted a quantum field-theoretical framework to address the issue of *CPT* and Lorentz breakdown as a spontaneous violation. In this sense, they constructed an extension of the minimal standard model, which maintains unaffected the $SU(3)\times SU(2)\times U(1)$ gauge structure of the usual theory, and incorporates *CPT* violation as an active feature of the effective low-energy broken action. They started from a usual *CPT* and Lorentz-invariant action as defining the properties of an eventual underlying theory at the Planck scale [4]. It then undergoes a spontaneous breaking of both symmetries. In the broken phase, there arises an effective action, with *CPT* and Lorentz symmetries broken, but covariance preserved for the inertial observer frame. The Lorentz invariance is spoiled at the level of the particle system, which can be viewed in terms of the noninvariance of the fields under boosts and Lorentz rotations (relative inertial observer frames). This covariance breakdown is also manifest whenever analyzing the dispersion relations, extracted from the propagators.

Investigations concerning the unitarity, causality, and consistency of a QFT with violation of Lorentz and *CPT* symmetries (induced by a Chern-Simons term) were carried out by Adam and Klinkhamer [5]. As a result, it was verified that the causality and unitarity of the model may be preserved whenever the fixed (background) 4-vector is spacelike, and spoiled whenever it is timelike or null. An analysis of the consistency put forward in the presence of spontaneous symmetry breaking (SSB) [6] has confirmed the results obtained in Ref. [5]: the spacelike case is free from unitarity illnesses; on the other hand, they are present in the timelike and lightlike cases.

The active development of Lorentz- and *CPT*-violating theories in $D=1+3$ has led to the inquiry regarding the structure of a similar model in $1+2$ dimensions and its possible implications. To study a planar theory endowed with Lorentz- and *CPT* violation, we have decided to adopt a dimensional reduction procedure: we start from the original Chern-Simons-like term, $\epsilon^{\mu\nu\kappa\lambda}v_{\mu}A_{\nu}F_{\kappa\lambda}$, and perform its reduction to $D=1+2$, which yields a pure Chern-Simons term along with a breaking mixed term. Our purpose, therefore, is to end up with a planar model, whose structure is derived from a known counterpart defined in $1+3$ dimensions. Next, we investigate some of its features, such as propagators, dispersion relations, causality, stability, and unitarity.

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The motivation to study the planar descent of the Carroll-Field-Jackiw model [1] is based on two main aspects: (i) the theoretical relevance of investigating a new planar system and comparing its features with the ones of its four-dimensional counterpart; (ii) the establishment of a new theoretical framework with perspectives for applications on low-dimension systems. In connection with the latter, we point out that the Lorentz breaking, explicitly realized by the external vector v^{μ}, may account for at least two usual facts for condensed matter systems: the nonrelativistic regime (inherent to these systems) and the presence of an anisotropy in the wave functions of some condensed matter systems. A spacelike background may effectively induce a spatial anisotropy which appears clearly in the solutions of the potentials discussed in the work of Ref. [7]. Such anisotropy would play a relevant role if it could be identified with the ones present in several condensed matter systems, such as the one in an $e^{-}e^{-}$ pair condensate of a class of high-T_c superconductors. Therefore, the study of the planar version of the Lorentz- and *CPT*-breaking model may be adopted not only to shed light on the four-dimensional parent model, but also to offer a possibility to fit phenomenological aspects of planar condensed matter systems. To pursue our investigation, we perform the dimensional reduction to $1+2$ dimensions of an Abelian gauge model with nonconservation of the Lorentz and *CPT* symmetries [1,5] induced by the term $\epsilon^{\mu\nu\kappa\lambda}v_{\mu}A_{\nu}F_{\kappa\lambda}$. The resulting planar quantum electrodynamics (QED_3) is composed of a Maxwell-Chern-Simons gauge field (A_{μ}), a scalar (φ), a scalar parameter (s) without dynamics (the Chern-Simons mass), and a fixed 3-vector (v^{μ}). Besides the MCS sector, this Lagrangian has a massless scalar sector, represented by the field φ, which also works out as the coupling constant in the Chern-Simons-like structure that mixes the gauge field to the 3-vector v^{μ} (where one gauge field is replaced by v^{μ}). This latter is responsible for the Lorentz noninvariance. Therefore, the reduced Lagrangian is endowed with three coupled sectors: a MCS sector, a massless Klein-Gordon sector, and a mixing Lorentz-violating one. As is well known, the MCS sector breaks both parity and time-reversal symmetries, but it preserves the Lorentz and *CPT* ones. The scalar sector preserves all discrete symmetries and Lorentz covariance, whereas the mixing sector, as it will be seen, breaks Lorentz invariance (in relation to the particle frame), keeps conserved parity and charge-conjugation symmetries conserved, but may break (or preserve) time-reversal symmetry. This implies that both conservation (for a purely spacelike v^{μ}) and violation (for v^{μ} timelike and lightlike) of *CPT* invariance may take place.

This paper is outlined as follows. In Sec. II, we perform the dimensional reduction, leading to the reduced model. With the new planar Lagrangian, we devote some effort to derive the propagators of the gauge and scalar fields; this requires a closed algebra composed by 11 spin operators, as displayed in Table I. In Sec. III, we investigate the stability and the causal structure of the theory. We discuss the causality by looking at the dispersion relations extracted from the poles of the propagators, which reveal the existence of both causal and noncausal modes. All the modes, nevertheless, present positive definite energy (positivity) relative to any Lorentz frame, which implies an overall stability. In Sec. IV, we carry out the unitarity analysis, based on the residue matrix evaluated at the poles of the propagators. Unitarity is ensured only in the case of a purely spacelike background vector v^{μ}. In Sec. V, we present our concluding comments.

II. THE DIMENSIONALLY REDUCED MODEL

We start from the Maxwell Lagrangian[1] in $1+3$ dimensions supplemented by a term that couples the dual electromagnetic tensor to a fixed 4-vector v^{μ} as it appears in Refs. [1], [2], [5]:

$$\mathcal{L}_{1+3}=\left\{-\frac{1}{4}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}}+\frac{1}{4}\epsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}}v_{\hat{\mu}}A_{\hat{\nu}}F_{\hat{\kappa}\hat{\lambda}}+A_{\hat{\nu}}J^{\hat{\nu}}\right\}, \quad (1)$$

with the additional presence of the coupling between the gauge field and the external current, $A_{\hat{\nu}}J^{\hat{\nu}}$.

This model (in its free version) is gauge invariant but does not preserve Lorentz and *CPT* symmetries relative to the particle frame. For the observer system, the Chern-Simons-like term transforms covariantly, once the background also is changed under an observer boost: $v^{\hat{\mu}}\rightarrow v^{\hat{\mu}'}=\Lambda^{\mu}_{\alpha}v^{\alpha}$. In connection with the particle system, however, when one applies a boost on the particle, the background 4-vector is supposed to remain unaffected, behaving like a set of four independent numbers, which configures the breaking of the covariance. This term also breaks the parity symmetry, but keeps the invariance under charge conjugation and time reversal.

To study this model in $1+2$ dimensions, we perform its dimensional reduction, which is based on the following ansatz over any 4-vector: (i) one keeps its 0-, 1-, and 2-components; (ii) one identifies its third component with a scalar in $(1+2)$ and makes the assumption that there is no dependence on the third spatial dimension: $\partial_3(\text{anything})\rightarrow 0$. Applying this prescription to the gauge 4-vector $A^{\hat{\mu}}$ and to the fixed external 4-vector $v^{\hat{\mu}}$, and to the 4-current $J^{\hat{\mu}}$, one has

$$A^{\hat{\mu}}\rightarrow(A^{\mu};\varphi), \quad (2)$$

$$v^{\hat{\mu}}\rightarrow(v^{\mu};s), \quad (3)$$

$$J^{\hat{\mu}}\rightarrow(J^{\mu};J), \quad (4)$$

where $A^{(3)}=\varphi$, $v^{(3)}=s$, $J^{(3)}=J$, and $\mu=0, 1, 2$. According to this process, there appear two scalars: the scalar field φ that exhibits dynamics, and s, a constant scalar (without dynamics). Carrying out this prescription for Eq. (1), one then obtains

[1]Here one has adopted the following metric conventions: $g_{\mu\nu}=(+,-,-,-)$ in $D=1+3$ and $g_{\mu\nu}=(+,-,-)$ in $D=1+2$. The greek letters (with hat) $\hat{\mu}$ run from 0 to 3, while the pure greek letters μ run from 0 to 2.

$$\mathcal{L}_{1+2}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi+\frac{s}{2}\epsilon_{\mu\nu k}A^\mu\partial^\nu A^k$$
$$-\varphi\epsilon_{\mu\nu k}v^\mu\partial^\nu A^k-\frac{1}{2\alpha}(\partial_\mu A^\mu)^2+A_\mu J^\mu+\varphi J, \quad (5)$$

where the last free term represents the gauge-fixing term, added up after the dimensional reduction. The scalar field φ exhibits a typical Klein-Gordon massless dynamics and it also appears as the coupling constant that links the fixed v^μ to the gauge sector of the model, by means of the new term: $\varphi\epsilon_{\mu\nu k}v^\mu\partial^\nu A^k$. In spite of being covariant in form, this kind of term breaks the Lorentz symmetry in the particle frame, since the 3-vector v^μ is not sensitive to particle Lorentz boost, behaving like a set of three scalars.

The Lagrangian (1), originally proposed by Carroll, Field, and Jackiw [1], has the property of breaking parity symmetry, even though it conserves time-reversal and charge-conjugation symmetries (for a pure timelike v^μ), resulting in nonconservation of the *CPT* symmetry. Simultaneously, the Lorentz invariance is spoiled, since the fixed 4-vector v^μ breaks the rotational and boost invariances. On the other hand, the reduced model, given by Eq. (5), does not necessarily jeopardize the *CPT* conservation, which depends truly on the character of the fixed vector v^μ. As it is known, the parity transformation ($\mathcal{P}$) in $1+2$ dimensions is characterized by the inversion of only one of the spatial axes: $x^\mu \overset{\mathcal{P}}{\rightarrow} x'^\mu=(x_o,-x,y)$, the same being valid for the 3-potential: $A^\mu \overset{\mathcal{P}}{\rightarrow} A'^\mu=(A_0,-A^{(1)},A^{(2)})$.

The time-reversal transformation ($\mathcal{T}$) must keep unchanged the dynamics of the system, so that we must have $x^\mu \overset{\mathcal{T}}{\rightarrow} x'^\mu=(-x_o,x,y)$, $\quad A^\mu \overset{\mathcal{T}}{\rightarrow} A'^\mu=(A_0,-A^{(1)},-A^{(2)})$, while the charge conjugation determines $x^\mu \overset{\mathcal{C}}{\rightarrow} x'^\mu=x^\mu$, $A^\mu \overset{\mathcal{C}}{\rightarrow} A'^\mu=-A^\mu$. We know that the Chern-Simons term breaks both parity and time-reversal symmetries and keeps conserved the charge conjugation, which assures the global *CPT* invariance. The new term, $\varphi\epsilon_{\mu\nu k}v^\mu\partial^\nu A^k$, however, will manifest a nonsymmetric behavior before $\mathcal{T}$ transformation: there will occur conservation if one works with a purely spacelike external vector [$v^\mu=(0,\vec{v})$], or breakdown if v^μ is purely timelike. Under parity and charge-conjugation transformations, in turn, this term will evidence noninvariance for any adopted v^μ, thereby one can state that *CPT* conservation will occur when v^μ is purely spacelike, and *CPT* violation otherwise. Here, the field φ was considered as having a scalar character under the parity transformation. Yet if this field behaves like a pseudoscalar,[2] the *CPT* conservation will be assured for a purely timelike v^μ. For a lightlike v^μ, there will always occur time-reversal noninvariance, and consequently *CPT* violation.

[2]The adoption of a pseudoscalar field can be justified by looking at the vector character of the potential ($\vec{A}\overset{\mathcal{P}}{\rightarrow}-\vec{A}$) before the dimensional reduction. If one assumes that the field φ maintains the same behavior of its ancestral (A_3), one has a pseudoscalar.

Neglecting divergence terms, we can write the linearized free action in an explicitly quadratic form, namely

$$\Sigma_{1+2}=\int d^3x\,\frac{1}{2}\{A^\mu[M_{\mu\nu}]A^\nu-\varphi\Box\varphi$$
$$-\varphi[\epsilon_{\mu\alpha\nu}v^\mu\partial^\alpha]A^\nu+A^\mu[\epsilon_{\nu\alpha\mu}v^\nu\partial^\alpha]\varphi\}, \quad (6)$$

which can also appear in the matrix form:

$$\Sigma_{1+2}=\int d^3x\,\frac{1}{2}(A^\mu\varphi)\begin{bmatrix} M_{\mu\nu} & T_\mu \\ -T_\nu & -\Box \end{bmatrix}\begin{pmatrix} A^\nu \\ \varphi \end{pmatrix}. \quad (7)$$

The action (7) has as its nucleus a square matrix, P, composed of the quadratic operators of the initial action. The mass dimensions of the physical parameters and tensors are $[A^\mu]=[\varphi]=1/2$, $[v^\mu]=[s]=1$, $[T_\mu]=[M_{\mu\nu}]=2$. Here, some definitions are necessary:

$$M_{\mu\nu}=\Box\theta_{\mu\nu}+sS_{\mu\nu}+\frac{\Box}{\alpha}\omega_{\mu\nu}, \quad T_\nu=S_{\mu\nu}v^\mu, \quad (8)$$

$$S_{\mu\nu}=\varepsilon_{\mu\kappa\nu}\partial^\kappa, \quad \theta_{\mu\nu}=\eta_{\mu\nu}-\omega_{\mu\nu}, \quad \omega_{\mu\nu}=\frac{\partial_\mu\partial_\nu}{\Box}, \quad (9)$$

where $\theta_{\mu\nu},\omega_{\mu\nu},S_{\mu\nu}$ stand, respectively, for the transverse, longitudinal, and Chern-Simons dimensionless projectors, while $M_{\mu\nu}$ is the quadratic operator associated with the MCS sector. The inverse of the square matrix, P, given at the action (7) yields the propagators of the gauge and the scalar fields, which are also written in a matrix form, the propagator matrix (Δ):

$$\Delta=P^{-1}=\frac{-1}{(\Box M_{\mu\nu}-T_\mu T_\nu)}\begin{bmatrix} -\Box & T_\nu \\ -T_\mu & M_{\mu\nu} \end{bmatrix}. \quad (10)$$

The propagators of the gauge field, Δ_{11}, and scalar field, Δ_{22}, are written as

$$(\Delta_{11})^{\mu\nu}=\left[\Box\theta_{\mu\nu}+sS_{\mu\nu}+\frac{\Box}{\alpha}\omega_{\mu\nu}-\frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \quad (11)$$

$$(\Delta_{22})=-\frac{M_{\mu\nu}}{\Box}\left[\Box\theta_{\mu\nu}+sS_{\mu\nu}+\frac{\Box}{\alpha}\omega_{\mu\nu}-\frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \quad (12)$$

$$(\Delta_{12})^{\mu}=-\frac{T_\nu}{\Box}\left[\Box\theta_{\mu\nu}+sS_{\mu\nu}+\frac{\Box}{\alpha}\omega_{\mu\nu}-\frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \quad (13)$$

$$(\Delta_{21})^{\nu}=\frac{T_\mu}{\Box}\left[\Box\theta_{\mu\nu}+sS_{\mu\nu}+\frac{\Box}{\alpha}\omega_{\mu\nu}-\frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \quad (14)$$

while the terms Δ_{12}, Δ_{21} are related to the mixed propagators $\langle A_\mu\varphi\rangle$, $\langle\varphi A_\mu\rangle$ that indicate a scalar mediator turning into a gauge mediator and vice versa. Here, for future purposes, it is useful to present the inverse of the tensor $M_{\mu\nu}$, that is, the propagator of the pure MCS Lagrangian:

TABLE I. Multiplicative operator algebra fulfilled by θ, ω, S, Λ, TT, Q, Σ, and Φ. The products are supposed to be in the ordering "row times column."

	$\theta_{\mu\nu}$	$\omega_{\mu\nu}$	$S_{\mu\nu}$	$\Lambda_{\mu\nu}$	$T_\mu T_\nu$	$Q_{\mu\nu}$	$Q_{\nu\mu}$	$\Sigma_{\mu\nu}$	$\Sigma_{\nu\mu}$	$\Phi_{\mu\nu}$	$\Phi_{\nu\mu}$
$\theta^{\nu\alpha}$	${\theta_\mu}^\alpha$	0	${S_\mu}^\alpha$	${\Lambda_\mu}^\alpha + -\frac{\lambda}{\Box}{\Sigma_\mu}^\alpha$	$T_\mu T^\alpha$	${Q_\mu}^\alpha$	${Q^\alpha}_\mu + -\frac{\lambda}{\Box}{\Phi_\mu}^\alpha$	0	${\Sigma^\alpha}_\mu + -\lambda\Box{\omega^\alpha}_\mu$	0	${\Phi^\alpha}_\mu$
$\omega^{\nu\alpha}$	0	${\omega_\mu}^\alpha$	0	$\frac{\lambda}{\Box}{\Sigma_\mu}^\alpha$	0	0	$\frac{\lambda}{\Box}{\Phi_\mu}^\alpha$	${\Sigma_\mu}^\alpha$	$\lambda{\omega^\alpha}_\mu$	${\Phi^\alpha}_\mu$	0
$S^{\nu\alpha}$	${S_\mu}^\alpha$	0	$-\Box{\theta_\mu}^\alpha$	${Q_\mu}^\alpha$	$\lambda{\Phi_\mu}^\alpha + -\Box{Q^\alpha}_\mu$	$\lambda{\Sigma_\mu}^\alpha + -{\Lambda_\mu}^\alpha\Box$	$-T_\mu T^\alpha$	0	$\partial_\mu T^\alpha$	0	$\Box({\omega_\mu}^\alpha + -{\Sigma^\alpha}_\mu)$
$\Lambda^{\nu\alpha}$	${\Lambda_\mu}^\alpha + -\frac{\lambda}{\Box}{\Sigma^\alpha}_\mu$	$\frac{\lambda}{\Box}{\Sigma^\alpha}_\mu$	$-{Q^\alpha}_\mu$	$v^2{\Lambda_\mu}^\alpha$	0	0	$v^2{Q^\alpha}_\mu$	$\lambda{\Lambda_\mu}^\alpha$	$v^2{\Sigma^\alpha}_\mu$	$\lambda{Q^\alpha}_\mu$	0
$T^\nu T^\alpha$	$T_\mu T^\alpha$	0	$\Box{Q_\mu}^\alpha + -\lambda{\Phi^\alpha}_\mu$	0	$T^2 T_\mu T^\alpha$	$T^2 Q_\mu^\alpha$	0	0	0	0	$T^2{Q_\mu}^\alpha$
$Q^{\nu\alpha}$	${Q_\mu}^\alpha + -\frac{\lambda}{\Box}{\Phi^\alpha}_\mu$	$\frac{\lambda}{\Box}{\Phi^\alpha}_\mu$	$-T_\mu T^\alpha$	$v^2{Q_\mu}^\alpha$	0	0	$v^2 T_\mu T^\alpha$	$\lambda{Q_\mu}^\alpha$	$v^2\partial_\mu T^\alpha$	$\lambda T_\mu T^\alpha$	0
$Q^{\alpha\nu}$	${Q^\alpha}_\mu$	0	$\Box{\Lambda_\mu}^\alpha + -\lambda{\Sigma^\alpha}_\mu$	0	$T^2{Q^\alpha}_\mu$	$T^2{\Lambda^\alpha}_\mu$	0	0	0	0	$T^2{\Sigma^\alpha}_\mu$
$\Sigma^{\nu\alpha}$	${\Sigma_\mu}^\alpha + -\lambda{\omega_\mu}^\alpha$	$\lambda{\omega_\mu}^\alpha$	$-{\Phi_\mu}^\alpha$	$v^2{\Sigma_\mu}^\alpha$	0	0	$v^2{\Phi_\mu}^\alpha$	$\lambda{\Sigma_\mu}^\alpha$	$v^2{\Lambda_\mu}^\alpha$	$\lambda{\Phi_\mu}^\alpha$	0
$\Sigma^{\alpha\nu}$	0	${\Sigma^\alpha}_\mu$	0	$\lambda{\Lambda_\mu}^\alpha$	0	0	$\lambda{Q^\alpha}_\mu$	$\Box{\Lambda_\mu}^\alpha$	$v^2{\Lambda_\mu}^\alpha$	$\Box{Q_\mu}^\alpha$	0
$\Phi^{\nu\alpha}$	${\Phi_\mu}^\alpha$	0	$\Box({\Sigma_\mu}^\alpha + -\lambda{\omega_\mu}^\alpha)$	0	$T^2{\Phi_\mu}^\alpha$	$T^2{\Sigma_\mu}^\alpha$	0	0	0	0	$\Box T^2\omega_\mu^\alpha$
$\Phi^{\alpha\nu}$	0	${\Phi^\alpha}_\mu$	0	$\lambda{\Phi^\alpha}_\mu$	0	0	$\lambda T_\mu T^\alpha$	$\Box{\Phi^\alpha}_\mu$	$\lambda{\Phi^\alpha}_\mu$	$\Box T_\mu T^\alpha$	0

$$(M_{\mu\nu})^{-1} = \frac{1}{\Box + s^2}\,\theta^{\mu\nu} - \frac{s}{\Box(\Box + s^2)}S^{\mu\nu} + \frac{\alpha}{\Box}\,\omega^{\nu\mu}. \tag{15}$$

To perform the inversion of the above operator, we need to define some new operators, since the ones known so far do not form a closed algebra, as is shown below:

$$S_{\mu\nu}T^\nu T^\alpha = \Box v_\mu T^\alpha - \lambda T^\alpha \partial_\mu = \Box {Q_\mu}^\alpha - \lambda {\Phi^\alpha}_\mu, \tag{16}$$

$$Q_{\mu\nu}Q^{\alpha\nu} = T^2 v^\alpha v_\mu = T^2 {\Lambda^\alpha}_\mu, \tag{17}$$

$$Q_{\mu\nu}\Phi^{\nu\alpha} = T^2 v_\mu \partial^\alpha = T^2 {\Sigma_\mu}^\alpha, \tag{18}$$

where the new operators are

$$Q_{\mu\nu} = v_\mu T_\nu, \quad \Lambda_{\mu\nu} = v_\mu v_\nu, \quad \Sigma_{\mu\nu} = v_\mu \partial_\nu, \quad \Phi_{\mu\nu} = T_\mu \partial_\nu, \tag{19}$$

and

$$\lambda \equiv \Sigma_\mu^\mu = v_\mu \partial^\mu, \quad T^2 = T_\alpha T^\alpha = (v^2\Box - \lambda^2). \tag{20}$$

Their mass dimensions are $[\Lambda_{\mu\nu}] = 2$, $[Q_{\mu\nu}] = 3$, $[\Sigma_{\mu\nu}] = 2$, $[\Phi_{\mu\nu}] = 3$.

Three of these new terms exhibit a nonsymmetric structure, which suggests they should be taken in pairs, namely $Q_{\mu\nu}$, $Q_{\nu\mu}$; $\Sigma_{\mu\nu}$, $\Sigma_{\nu\mu}$; $\Phi_{\mu\nu}$, $\Phi_{\nu\mu}$. The inversion of the operator Δ_{11} is done according to the expression $(\Delta_{11}^{-1})_{\mu\nu}(\Delta_{11})^{\nu\alpha} = \delta_\mu^\alpha$, where the operator $(\Delta_{11})^{\nu\alpha}$ is composed of all the possible tensor combinations (of rank 2) involving $T_\mu, v_\mu, \partial_\alpha$. In so doing, the proposed propagator will consist, at first glance, of 11 terms:

$$(\Delta_{11})^{\nu\alpha} = a_1\theta^{\nu\alpha} + a_2\omega^{\nu\alpha} + a_3 S^{\nu\alpha} + a_4\Lambda^{\nu\alpha} + a_5 T^\nu T^\alpha + a_6 Q^{\nu\alpha} + a_7 Q^{\alpha\nu} + a_8\Sigma^{\nu\alpha} + a_9\Sigma^{\nu\alpha} + a_{10}\Phi^{\nu\alpha} + a_{11}\Phi^{\alpha\nu}, \tag{21}$$

which are displayed in Table I, where we observe explicitly the closure of the operator algebra.

Using the data contained in Table I, we find out that the gauge-field propagator assumes the form

$$(\Delta_{11})^{\mu\nu}=\frac{1}{\Box+s^2}\,\theta^{\mu\nu}+\frac{\alpha(\Box+s^2)\boxtimes-\lambda^2s^2}{\Box(\Box+s^2)\boxtimes}\,\omega^{\mu\nu}-\frac{s}{\Box(\Box+s^2)}\,S^{\mu\nu}-\frac{s^2}{(\Box+s^2)\boxtimes}\,\Lambda^{\mu\nu}+\frac{1}{(\Box+s^2)\boxtimes}\,T^\mu T^\nu-\frac{s}{(\Box+s^2)\boxtimes}\,Q^{\mu\nu}$$
$$+\frac{s}{(\Box+s^2)\boxtimes}\,Q^{\nu\mu}+\frac{\lambda s^2}{\Box(\Box+s^2)\boxtimes}\,\Sigma^{\mu\nu}+\frac{\lambda s^2}{\Box(\Box+s^2)\boxtimes}\,\Sigma^{\nu\mu}-\frac{s\lambda}{\Box(\Box+s^2)\boxtimes}\,\Phi^{\mu\nu}+\frac{s\lambda}{\Box(\Box+s^2)\boxtimes}\,\Phi^{\nu\mu},$$

where $\boxtimes=(\Box^2+s^2\Box-T^2)$.

By the same procedure, we evaluate the mixed propagator, $(\Delta_{12})^{\alpha}=-T_\nu/\Box(\Delta_{11})^{\nu\alpha}$, which can be written in the following form:

$$(\Delta_{12})^{\nu}=-\frac{1}{\boxtimes}\left[T^\nu+sv^\nu-\frac{s\lambda}{\Box}\,\partial^\nu\right], \tag{22}$$

whereas the propagator $(\Delta_{21})^{\nu}$, in turn, becomes

$$(\Delta_{21})^{\nu}=-\frac{1}{\boxtimes}\left[-T^\nu+sv^\nu-\frac{s\lambda}{\Box}\,\partial^\nu\right].$$

In order to compute the propagator of the scalar field,

$$(\Delta_{22})=-\frac{1}{\Box}\left[1-\frac{1}{\Box}\,T_\mu(M_{\mu\nu})^{-1}T_\nu\right]^{-1}, \tag{23}$$

we make use of the inverse of the tensor $M_{\mu\nu}$, given by Eq. (15), so that $T_\mu(M^{-1})^{\mu\nu}T_\nu=(\Box+s^2)^{-1}T^2$. In such a way, a compact scalar propagator arises:

$$(\Delta_{22})=-\frac{\Box+s^2}{\boxtimes}. \tag{24}$$

In momentum space, the photon propagator takes the final expression

$$\langle A^\mu(k)A^\nu(k)\rangle=i\Bigg\{-\frac{1}{k^2-s^2}\,\theta^{\mu\nu}-\frac{\alpha(k^2-s^2)\boxtimes(k)+s^2(v\cdot k)^2}{k^2(k^2-s^2)\boxtimes(k)}\,\omega^{\mu\nu}-\frac{s}{k^2(k^2-s^2)}\,S^{\mu\nu}+\frac{s^2}{(k^2-s^2)\boxtimes(k)}\,\Lambda^{\mu\nu}$$
$$-\frac{1}{(k^2-s^2)\boxtimes(k)}\,T^\mu T^\nu+\frac{s}{(k^2-s^2)\boxtimes(k)}\,Q^{\mu\nu}-\frac{s}{(k^2-s^2)\boxtimes(k)}\,Q^{\nu\mu}+\frac{is^2(v\cdot k)}{k^2(k^2-s^2)\boxtimes(k)}\,\Sigma^{\mu\nu}$$
$$+\frac{is^2(v\cdot k)}{k^2(k^2-s^2)\boxtimes(k)}\,\Sigma^{\nu\mu}-\frac{is(v\cdot k)}{k^2(k^2-s^2)\boxtimes(k)}\,\Phi^{\mu\nu}+\frac{is(v\cdot k)}{k^2(k^2-s^2)\boxtimes(k)}\,\Phi^{\nu\mu}\Bigg\}, \tag{25}$$

while the scalar and the mixed propagators read as

$$\langle\varphi\varphi\rangle=\frac{i}{\boxtimes(k)}[k^2-s^2], \tag{26}$$

$$\langle A^\alpha(k)\varphi\rangle=-\frac{i}{\boxtimes(k)}\left[T^\alpha+sv^\alpha-\frac{s(v\cdot k)}{k^2}\,k^\alpha\right], \tag{27}$$

$$\langle\varphi A^\alpha(k)\rangle=-\frac{i}{\boxtimes(k)}\left[-T^\alpha+sv^\alpha-\frac{s(v\cdot k)}{k^2}\,k^\alpha\right], \tag{28}$$

where $\boxtimes(k)=[k^4-(s^2-v\cdot v)k^2-(v\cdot k)^2]$. These results show that the factor $\boxtimes$ is present in the denominator of all propagators, so that the scalar and the gauge field will share the pole structure, and consequently, the physical excitations associated to the poles of $\boxtimes(k)$. This common dependence on $1/\boxtimes$ also amounts to similarities on the causal structure of the scalar and gauge sectors of this model, as will be discussed in Sec. III.

III. DISPERSION RELATIONS, STABILITY, AND CAUSALITY ANALYSIS

Some references in the literature [5,6,8,9] have dealt with the issues of stability, causality, and unitarity concerning Lorentz- and *CPT*-violating theories. Causality is usually addressed as a quantum feature that requires commutation between observables separated by a spacelike interval, which one calls microcausality in field theory [10]. In this section, however, we analyze causality at the level of the classical theory. The starting point for all investigation is the propagator, whose poles are associated to dispersion relations (DR) that provide information about the stability and causality of the model. The causality analysis is then related to the sign of the propagator poles, given in terms of k^2, so that one must have $k^2 \geqslant 0$ in order to preserve it (circumventing the existence of tachyons). In the second quantization framework, stability is related to the positivity of the energy of the Fock states for any momentum. Here, stability is directly associated with the positivity of the energy of each mode arising from the DR.

The field propagators, given by Eqs. (25)–(27), present three families of poles at k^2:

$$k^2=0,\ \ k^2-s^2=0,\ \ k^4-(s^2-v\cdot v)k^2-(v\cdot k)^2=0, \tag{29}$$

from which we straightforwardly infer the DR derived from the Lagrangian (5), namely

$$k_{0(1)}^2=\vec{k}^2,\ \ k_{0(2)}^2=\vec{k}^2+s^2,$$

$$k_{0(3)}^2=\vec{k}^2+\frac{1}{2}[(s^2-v\cdot v)\pm\sqrt{(s^2-v\cdot v)^2+4(v\cdot k)^2}]. \tag{30}$$

The first dispersion relation, $k_0=\pm|\vec{k}|$, stands for a massless photon mode, which carries no degree of freedom, since the Lagrangian (5) involves a massive photon. The second DR represents the Chern-Simons massive mode, $k_0=\pm\sqrt{s^2+|\vec{k}|^2}$, which propagates only one degree of freedom (in the Maxwell-Chern-Simons electrodynamics, the scalar magnetic field encloses all information of the electromagnetic field, which justifies the existence of a single degree of freedom). These first two poles apparently respect the causality condition, since $k^2 \geqslant 0$ for them. Once the causality is set up, the stability comes up as a direct consequence.

Concerning the third DR, corresponding to the roots of $\boxtimes(k)$, it may provide both massless and massive modes for some specific $\vec{k}$ values, but in general, the mode is massive. By remembering that $\vec{k}$ is the transfer momentum, whose values are generally integrated from zero to infinity, we conclude it does not make much sense to fix any value for $\vec{k}$ in order to obtain a particular dispersion relation. Remarking that the term $\boxtimes(k)$ is ubiquitous in the denominator of all propagators, as is explicit in Eqs. (25)–(27), we conclude the causal structure entailed in the poles of $1/\boxtimes$ will be common to these three propagators. Specifically, for a purely spacelike 3-vector, $v^\mu=(0,\vec{v})$, this DR is written as

$$k_{0\pm}^2=\vec{k}^2+\frac{1}{2}[(s^2+\vec{v}^2)\pm\sqrt{(s^2+\vec{v}^2)^2+4(\vec{v}\cdot\vec{k})^2}]. \tag{31}$$

A simple analysis of this expression indicates that both k_{0+}^2 and k_{0-}^2 are positive-energy modes for any $\vec{k}$ value (and for any Lorentz observer), which ensures the stability of these modes. This fact may suggest that the causal structure of the spacelike sector of this model remains preserved, as was observed by Adam and Klinkhamer [5] in the context of the four-dimensional version of this theory, which is endowed with a dispersion relation very similar to Eq. (31) (this conclusion was also supported by the attainment of a group velocity, associated with this mode, smaller than 1). Concerning the pole analysis, however, we have $k_+^2>0$ for arbitrary $\vec{k}$ and $k_-^2<0$ (unless $\vec{k}\perp\vec{v}$ or $\vec{k}=0$, which implies $k_-^2=0$). So, while the mode k_+^2 preserves the causality and stability, the mode k_-^2, in spite of assuring stability, will be in general noncausal, preserving causality only for $\vec{k}\perp\vec{v}$ or $\vec{k}=0$.

In the case of a purely timelike 3-vector, $v^\mu=(v_0,\vec{0})$, the DR assumes the form

$$k_{0\pm}^2=\frac{1}{2}[(s^2+2\vec{k}^2)\pm\sqrt{s^4+4v_0^2\vec{k}^2}], \tag{32}$$

where one observes a similar behavior: the mode k_{0+}^2 will exhibit stability and causality, while the mode k_{0-}^2 will present energy positivity (for an arbitrary $\vec{k}$ value) whenever the condition $s^2-v_0^2>0$ is fulfilled. From now on, we must assume the validity of this condition, so that the mode k_{0-}^2 can be taken stable. This latter mode is noncausal for any $\vec{k}\neq 0$. Assuming the coefficients for Lorentz violation are small near the Chern-Simons mass ($s^2 \gg v_0^2,|\vec{v}|^2$), we obtain an entirely causal theory (at least at zero order in v^2/s^2). This is consistent with some results [9] concerning some quantum theories containing Lorentz-violating terms, which evidence the preservation of causality when the breaking factors are small.

Hence, the modes $k_{0\pm}^2$ exhibit positive energy both in spacelike and timelike cases, which also implies these two modes can be written as an expansion in terms of positive and negative frequency terms. This separation allows the definition of particle and antiparticle states, a necessary condition for the quantization of this theory. Nevertheless, the existence of noncausal modes, both in the timelike and spacelike case, may be seen already at the classical level, as a prediction of the impossibility of realizing a consistent quantization of this model, an issue that will be properly addressed when one analyzes the unitarity at these noncausal poles. Therefore, the existence of quantization illness will be solved by investigating the unitarity of the model, a matter that will be discussed in the next section.

The similarity underlying the dispersion relations of our reduced model and its four-dimensional counterpart entails some common properties. The stability, for instance, is ensured in the same way. As for the causality issue, we report here on the existence of noncausal modes (k_-^2) for both timelike and spacelike backgrounds. On the other hand, following the approach of Adam and Klinkhamer [5], it is possible to argue that these specific noncausal modes do not harm the causal structure of the overall reduced model, once the group velocity associated with all modes (for both cases) is always less than 1 ($u_g<1$). Hence, in our planar model, the causal structure is preserved for both the timelike and spacelike cases. Instead, in the four-dimensional parent model [5], this occurs only for the spacelike background. The explanation for this peculiarity is the same one used to justify the property of unitarity for the gauge sector of the reduced model in both timelike and spacelike cases.

In a Lorentz covariant framework, k^2 is a Lorentz scalar, which assures a unique value for all Lorentz frames. In such a case, if k^2 represents a causal mode for one observer, the same will be true for all observers. The fact that k^2 does not have a positive definite value in an arbitrary Lorentz frame is unequivocally indicative of the Lorentz covariance breakdown.

IV. UNITARITY ANALYSIS

In order to analyze the unitarity of the model at tree level, we have adopted the method which consists in saturating the propagators with external currents. The fact that our model possesses two sectors (scalar and gauge) implies that we must saturate the scalar propagator and the gauge propagator individually. In such a way, we write the two saturated propagators, namely

$$SP_{\langle A_\mu A_\nu\rangle}=J^{*\mu}\langle A_\mu(k)A_\nu(k)\rangle J^\nu,$$

$$SP_{\langle\varphi\varphi\rangle}=J^*\langle\varphi\varphi\rangle J,$$

where the gauge current J^μ must obey the conservation law valid for the gauge sector of the system,[3] whereas the scalar current J is not subject to any constraint. The unitarity analysis is based on the residues of *SP*, namely the unitarity is ensured whenever the imaginary part of the residues of *SP* at the poles of each propagator is positive. It is easy to notice that the saturated propagator in the momentum space is the current-current transition amplitude.

A. Scalar sector

We can initiate our analysis by the scalar sector, whose saturated amplitude is given by $SP_{\langle\varphi\varphi\rangle}=J^*\langle\varphi\varphi\rangle J$, or more explicitly,

$$SP_{\langle\varphi\varphi\rangle}=J^*\frac{i(k^2-s^2)}{\boxtimes(k)}J.$$

This expression presents two poles, k_+^2, k_-^2, the roots of $\boxtimes(k)=0$. In the purely timelike case, $v^\mu=(v_0,\vec{0})$, these poles are exactly the ones given by Eq. (32): $k_\pm^2=1/2[s^2\pm\sqrt{s^4+4v_0^2\vec{k}^2}]$. Evaluating the residues of $SP_{\langle\varphi\varphi\rangle}$ at the pole k_+^2 we achieve a positive imaginary result, while at the pole k_-^2 a positive result appears only under the condition $\vec{k}^2<(v_0^2+s^2)$. In such a way, we conclude that the unitarity of the scalar sector, in the timelike case, is not assured.

Considering now the purely spacelike case $v^\mu=(0,\vec{v})$, the poles of $SP_{\langle\varphi\varphi\rangle}$ are given by Eq. (31): $k_\pm^2=1/2[(s^2+\vec{v}^2)\pm\sqrt{(s^2+\vec{v}^2)^2+4(\vec{v}\cdot\vec{k})^2}]$. The residues associated with these two poles exhibit a positive-definite imaginary part, so we can state that the unitarity of the scalar sector, at the spacelike case, is generically preserved.

B. Gauge-field sector

The continuity equation $\partial_\mu J^\mu=0$ in the k space is read as $k_\mu J^\mu=0$; it allows us to write the current in the form $J^\mu=(j^0,0,(k_0/k_2)j^{(0)})$. The conservation constraint $j^{(2)}=(k_0/k_2)j^{(0)}$ appears whenever one adopts $k^\mu=(k_0,0,k_2)$ as the momentum. The current-conservation law also reduces to six the number of terms of the photon propagator that contributes to the evaluation of the saturated propagator:

$$SP_{\langle A_\mu A_\nu\rangle}=J_\mu^*(k)\left\{\frac{i}{D}(\Box\boxtimes g^{\mu\nu}-s\boxtimes S^{\mu\nu}-s^2\Box\Lambda^{\mu\nu}+\Box T^\mu T^\nu-s\Box Q^{\mu\nu}+s\Box Q^{\mu\nu})\right\}J_\nu(k), \tag{33}$$

where $D=\Box(\Box+s^2)\boxtimes$. Writing this expression in the momentum space, one obtains

$$SP_{\langle A_\mu A_\nu\rangle}=J^{*\mu}(k)\{iB_{\mu\nu}\}J^\nu(k), \tag{34}$$

where $D=k^2(k^2-s^2)\boxtimes$, with $\boxtimes(k)=k^4-(s^2-v\cdot v)k^2-(v\cdot k)^2$.

1. Timelike case

We start by analyzing the unitarity in the case corresponding to a timelike background vector: $v^\mu=(v_0,\vec{0})$. In this situation, the 2-rank tensor $B_{\mu\nu}$ can be put in the form

$$B_{\mu\nu}(k)=\frac{1}{D(k)}\begin{bmatrix} k^2(s^2v_0^2-\boxtimes) & ik^{(2)}(s\boxtimes-v_0^2s^2k^2) & ik^{(1)}(-s\boxtimes+v_0^2s^2k^2)\\ ik^{(2)}(-s\boxtimes+v_0^2s^2k^2) & k^2(\boxtimes+v_0^2k_2^2) & is\boxtimes k^{(0)}-v_0^2k^2k^{(1)}k^{(2)}\\ ik^{(1)}(s\boxtimes-v_0^2s^2k^2) & -is\boxtimes k^{(0)}-v_0^2k^2k^{(1)}k^{(2)} & k^2(\boxtimes+v_0^2k_1^2)\end{bmatrix}, \tag{35}$$

[3] By applying the differential operator ∂_μ on the equation of motion derived from Lagrangian (5), there results the following equation (see Ref. [7]) for the gauge current: $\partial_\mu J^\mu=-\varepsilon^{\mu\nu\rho}\partial_\mu v_\nu\partial_\rho\varphi$, which reduces to the conventional current-conservation law, $\partial_\mu J^\mu=0$, whenever v^μ is constant or has a null rotational ($\varepsilon^{\mu\nu\rho}\partial_\mu v_\nu=0$).

where $\boxtimes = k^4-(s^2-v_0^2)k^2-k_0^2$.

For the pole $k^2=0$, with $k^\mu=(k_0,0,k_0)$, we have the following residue matrix:

$$B_{\mu\nu}|_{(k^2=0)}=\frac{1}{s^2}\begin{bmatrix} 0 & -isk_0 & 0 \\ isk_0 & 0 & -isk_0 \\ 0 & isk_0 & 0 \end{bmatrix}, \tag{36}$$

which is reduced to a null matrix when saturated with the conserved current, $J^\mu=(j^0,0,(k_0/k_2)j^{(0)})$, implying also a null saturation $(SP=0)$. This fact indicates that the mode associated with the pole $k^2=0$ carries no physical degree of freedom, and further, it does not jeopardize the unitarity.

For the pole $k^2=s^2$, with $k^\mu=(k_0,0,k_2)$, the matrix takes the form

$$B_{\mu\nu}|_{(k^2=s^2)}=-\frac{1}{s^2k_2^2}\begin{bmatrix} s^2k_0^2 & -isk^{(2)}k_0^2 & 0 \\ isk^{(2)}k_0^2 & 0 & -isk_0k_2^2 \\ 0 & isk_0k_2^2 & -s^2k_2^2 \end{bmatrix}. \tag{37}$$

This matrix, whenever saturated with the external current $J^\mu=(j^0,0,(k_0/k_2)j^{(0)})$, leads to a trivial saturation $(SP=0)$, which is compatible with unitarity requirements. The vanishing of the current-current amplitude at this pole indicates that the massive excitation $k^2=s^2$ is not dynamical for the timelike background.

At the pole $k_+^2=1/2[s^2+\sqrt{s^4+4v_0^2\vec{k}^2}]$, the residue matrix reads

$$B_{\mu\nu}|_{(k^2=k_+^2)}=\frac{v_0^2}{(k_+^2-s^2)(k_+^2-k_-^2)}\begin{bmatrix} s^2 & -is^2k^{(2)} & 0 \\ is^2k^{(2)} & k_2^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{38}$$

which has as eigenvalues $\lambda_1=0$, $\lambda_2=0$, $\lambda_3=k_2^2+s^2$. Consequently, one has $SP>0$ (unitarity preservation).

At the pole k_-^2, a similar behavior occurs: we obtain a residue matrix similar to the one given above. The difference rests only on the coefficient appearing in front of the matrix, in this case $1/D(k_-)=v_0^2[(k_-^2-s^2)(k_-^2-k_+^2)]^{-1}>0$. The fact that this last coefficient turns out to be positive indicates that the unitarity is also preserved at the pole $k^2=k_-^2$, once one has the same eigenvalues.

Here, we observe that, for a timelike background vector, our planar system presents no ghost states that spoil unitarity. This is a sensitive difference with respect to its $(1+3)$-dimensional parent model, described in the works of Refs. [5], [6]. For the latter, the gauge sector is always plagued by ghost states which cannot be removed by any gauge choice, actually spoiling the unitarity. We justify this peculiarity for the scalar and gauge-field sectors, in the timelike case, by noticing that the scalar φ identified with $A^{(3)}$ becomes (upon dimensional reduction) a gauge-invariant field in $(1+2)$ dimensions. The gauge transformation it is acted upon in $(1+3)$D is no longer present in the planar case, by virtue of our dimensional reduction ansatz. This is why our reduced model inherits a ghostlike excitation in the scalar sector; the gauge-field ghost of $(1+3)$D migrates to the planar space-time through the scalar field. Next, for the spacelike case, no ghosts are present in the $(1+3)$D case; naturally, in $(1+2)$ dimensions, φ cannot inherit any ghost state.

2. Spacelike case

In this case, taking $v^\mu=(0,0,v)$, the tensor $B_{\mu\nu}$ is given as follows:

$$B_{\mu\nu}(k)=\frac{1}{D(k)}\begin{bmatrix} -k^2(\boxtimes-v^2k_1^2) & is\boxtimes k^{(2)}-k^2v^2k_0k^{(1)} & ik^{(1)}(-s\boxtimes-sk^2v^2) \\ -is\boxtimes k^{(2)}-k^2v^2k_0k^{(1)} & k^2(\boxtimes+v^2k_0^2) & isk_0(\boxtimes+v^2k^2) \\ ik^{(1)}(s\boxtimes+sk^2v^2) & -is\boxtimes k_0+iskv^2k_0 & k^2(\boxtimes+v^2s^2) \end{bmatrix}, \tag{39}$$

where $\boxtimes=k^4-(s^2-v^2)k^2-v^2k_2^2$.

For the pole $k^2=0$, with $k^\mu=(k_0,0,k_0)$, we obtain the same matrix attained in the timelike case, given by Eq. (36). Exactly for the same reasons presented in the preceding section, we can assert that the unitarity is preserved at this pole.

For the pole $k^2=s^2$, with $k^\mu=(k_0,0,k_2)$, the resulting matrix is identical to one given by Eq. (37), so that the conclusions established in the timelike case are also valid here. The vanishing of the saturated propagator at the pole $k^2=s^2$, in both cases, indicates that the massive excitation $k^2=s^2$ is not dynamical in our model.

For the pole $k_+^2=1/2[(s^2+\vec{v}^2)\pm\sqrt{(s^2+\vec{v}^2)^2+4(\vec{v}\cdot\vec{k})^2}]$, with $k^\mu=(k_0,0,k_2)$, the residue matrix is reduced to

$$B_{\mu\nu}|_{(k^2=k_+^2)}=\frac{v^2}{(k_+^2-s^2)(k_+^2-k_-^2)}\begin{bmatrix} 0 & 0 & 0 \\ 0 & k_0^2 & isk_0 \\ 0 & -isk_0 & s^2 \end{bmatrix}, \tag{40}$$

where $(k_+^2-k_-^2)=\sqrt{(s^2+v^2)^2+4v^2k_2^2}$. The eigenvalues of this matrix are $\lambda_1=0$, $\lambda_2=0$, $\lambda_3=k_0^2+s^2$, which leads to a positive saturation ($SP>0$), and then unitarity is guaranteed at this pole. For the pole k_-^2, unitarity is also ensured; this may be seen in a similar way to the one performed in the timelike case.

Taking into account all results concerning the gauge sector of this model, we conclude that the unitarity is preserved in both timelike and spacelike cases (at all the poles of the gauge propagator) without any restriction. Considering the restriction on the unitarity of the scalar sector at the timelike case, we can state that our entire model preserves unitarity only in the spacelike case. It is also interesting to note that the unitarity of the gauge sector is guaranteed even at the noncausal poles k_-^2, which confirms the consistency of our model.

V. CONCLUDING COMMENTS

We have accomplished the dimensional reduction to $1+2$ dimensions of a gauge-invariant, Lorentz- and *CPT*-violating model, defined by the Carroll-Field-Jackiw term, $\varepsilon^{\mu\nu\kappa\lambda}v_\mu A_\nu F_{\kappa\lambda}$. We then obtain a Maxwell-Chern-Simons planar Lagrangian in the presence of a Lorentz-breaking term and a massless scalar field. Concerning this reduced model, the *CPT* symmetry is conserved for a purely spacelike v^μ, and spoiled otherwise. The propagators of this model are evaluated and exhibit a common pole structure (bound to the dependence on $1/\boxtimes$), which is used as starting point for the analysis of causality, stability, and unitarity. Concerning the dispersion relations, we verify that the modes have positive-definite energy, which ensures stability. The causality is assured for all modes of the theory, except for k_-^2 (both in spacelike and timelike case), which does not pose a physical problem, once these modes are stable and the causal structure of the global model remains preserved. In connection with the unitarity of this model, we have analyzed the scalar and the gauge sectors separately, by means of the saturation of the residue matrix. The gauge sector has been shown to be unitary for timelike and spacelike background vectors, whereas the scalar sector has been shown to preserve unitarity only in the spacelike case. We should now pay attention to a special property of 3-space-time dimensions, namely the absence of ghosts in the gauge-field spectrum for a timelike, v^μ. Unitarity is a relevant matter and an essential condition for a consistent quantization of any theory. Once the unitarity is ensured here, this model may become a useful and interesting tool to analyze planar systems (including condensed-matter ones) with anisotropic properties.

The fact that the pole $k^2=s^2$ is not associated with a physical degree of freedom reflects an interesting new feature of this planar system. In a usual MCS model, this pole stands for a massive degree of freedom leading to a considerable screening of the corresponding potentials and correlation functions derived from the MCS Lagrangian. In spite of the presence of the MCS terms in our case, the nondynamical character of the pole $k^2=s^2$ reveals that this sector decouples from the physical spectrum. Indeed, a good perception of this fact is provided by evaluating the potentials and correlation functions [7]. The attainment of results typical of a massless theory confirms the absence of physical content concerning the MCS sector.

A new version of this work [11] may address the dimensional reduction of a gauge-Higgs model [6] in the presence of the Carroll-Field-Jackiw term. In this case, the reduced model will be composed of two scalar fields (one stemming from the dimensional reduction, the other being the Higgs scalar), by a Maxwell-Chern-Simons-Proca gauge field, and by the Lorentz-violating mixing term. The introduction of the Higgs sector may shed light on new interesting issues concerning planar systems, such as the investigation of vortexlike configurations in the framework of a Lorentz-breaking model.

Another interesting question concerns the stability of three-dimensional QED (QED_3) with the scalar φ against quantum corrections if fermions are coupled to both the gauge field (minimal coupling) and φ (Yukawa coupling). Should Lorentz and *CPT* symmetries be broken by a fermionic term of the form $\bar{\Psi}\gamma^\mu\Psi b_\mu$ [12,13], one-loop fermionic corrections would induce the terms $s\varepsilon_{\mu\nu k}A^\mu F^{\nu k}$ and $\varphi\varepsilon_{\mu\nu k}v^\mu F^{\nu k}$ as the planar counterpart of the similar mechanism in 4D. So, a Lorentz breaking in the fermionic sector radiactively induces Lorentz breaking in terms of the gauge and scalar fields. As for this issue, see also Andrianov *et al.* [5].

Another natural investigation consists also in studying the solutions to the classical equations of motion (the extended Maxwell equations) and wave equations (for the potential A^μ) corresponding to the reduced Lagrangian. It is possible that such equations reveal a similar structure (but more complex) to the MCS conventional electrodynamics, since the reduced Lagrangian indeed contains the MCS sector. The solution to these equations may unveil some interesting aspects, such as the property of anisotropy (induced by a spacelike background $\vec{v}$) and the role of the CS term on the interaction potential. This issue is actually being investigated and we shall report on it in a forthcoming paper [7]. The theoretical framework developed here may also be useful to address the issue of electron-electron pairing and the symmetry of the order parameter representing the correlated electron pairs, which exhibit a *d*-wave pattern. This can be carried out by studying the Möller scattering in the nonrelativistic limit.

ACKNOWLEDGMENTS

M.M.F. is grateful to the Centro Brasileiro de Pesquisas Físicas (CBPF) for the kind hospitality. J.A.H.-N. acknowledges the CNPq for financial support. The authors are grateful to Álvaro L. M. de Almeida Nogueira, Marcelo A. N. Botta Cantcheff, and Gustavo D. Barbosa for relevant discussions.

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