Thermal one- and two-graviton Green's functions in the temporal gauge

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The thermal one- and two-graviton Green's functions are computed using a temporal gauge. In order to handle the extra poles which are present in the propagator, we employ an ambiguity-free technique in the imaginary-time formalism. For temperatures T high compared with the external momentum, we obtain the leading T^4 as well as the subleading T^2 and $\log(T)$ contributions to the graviton self-energy. The gauge fixing independence of the leading T^4 terms as well as the Ward identity relating the self-energy with the one-point function are explicitly verified. We also verify the 't Hooft identities for the subleading T^2 terms and show that the logarithmic part has the same structure as the residue of the ultraviolet pole of the zero temperature graviton self-energy. We explicitly compute the extra terms generated by the *prescription poles* and verify that they do not change the behavior of the leading and sub-leading contributions from the *hard thermal loop* region. We discuss the modification of the solutions of the dispersion relations in the graviton plasma induced by the subleading T^2 contributions.

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I. INTRODUCTION

One of the main motivations for the first attempts to compute the self-energy at finite temperature was the study of dispersion relations of a graviton plasma and the related interesting phenomena of antidamping and wave propagation [1,2]. For temperatures T high compared with the external momentum, but well below the Planck scale, the complete tensor structure of the leading one-loop contributions, proportional to T^4, was calculated for the first time in Ref. [2]. Later some subleading contributions of order T^2 were computed, including the contributions of thermal scalar matter and radiation [3], and subsequently all terms proportional to T^4, T^2 and $\log(T)$ were computed taking into account thermal loops of gravitons [4].

When the internal graviton lines are included, the gauge dependence which arises from the choice of gauge fixing in the gravitational action becomes an issue. In Ref. [4] the graviton self-energy was computed employing the Feynman–de Donder gauge with an arbitrary gauge fixing parameter. While the subleading contributions are gauge dependent, the leading T^4 contributions to the self-energy as well as to the one-point function are gauge fixing independent and satisfy the Ward identity. This last property is also true for the contributions from matter and radiation, being consistent with a gauge invariant effective action for hard thermal loops interacting with gravity.

One can go further into the question of gauge dependence by considering a class of non-covariant gauges of the kind that has been employed in gravity at zero temperature [5–10]. At finite temperature non-covariant temporal gauges would be even more appropriate, since Lorentz covariance is already broken by the heat bath but the rotational invariance is preserved. Despite the other well known advantages of the temporal gauge, finite temperature calculations have been performed only in Yang-Mills theories both in imaginary and in the real time formalisms [11–17]. This can be partially understood in view of the complexity of the gravitational interaction and so explicit calculations in non-covariant gauges have been restricted to the zero temperature case. Another reason for the lack of popularity of the temporal gauge in gravity is that, in contrast with the situation in Yang-Mills theory, the zero temperature graviton self-energy is not transverse [7,8]. However, this should not be a very important concern in the finite temperature case where the transversality property is expected to be violated in general. A more important difficulty in the temporal gauge is the problem of spurious singularities arising from the $n=0$ terms in the Matsubara sums [18], which is even more severe in the case of gravity in view of the higher powers of n in the denominator of the temporal gauge graviton propagator. This situation was improved after the development of an ambiguity-free technique to perform perturbative calculations at finite temperature in the temporal gauge [15,16]. Originally this technique was tested using zeta functions to compute the Matsubara sums and later it was applied to the calculation of the gluon self-energy using the standard method of introducing thermal distributions by replacing the Matsubara frequency sum with a contour integral in the complex plane of the zero component of the internal momentum [19].

The purpose of the present work is to apply the Leibbrandt's prescription to the calculation of the thermal one- and two-graviton Green's functions in a class of temporal gauges. We will show explicitly how this approach leads to a well defined result which can be expressed in terms of forward scattering amplitudes of thermal gravitons [20] plus contributions from prescription poles. We will also show how the ghost interactions effectively decouple leaving only thermal gravitons in the forward scattering amplitudes. We provide the explicit results for the leading and sub-leading hard thermal loop contributions and show that the prescription poles do not change the hard thermal loop behavior.

In Sec. II we will present the Lagrangian for the graviton field and the corresponding Feynman rules in a class of temporal gauges. We will also illustrate the basic approach with the simplest one-loop calculation, namely the one-graviton function (tadpole). In Sec. III we describe how the thermal graviton self-energy can be split in two parts. The first part arises from the on-shell poles of thermal graviton, and is expressed in terms of forward scattering amplitudes, while the second part is generated by poles in the complex energy plane which are characteristic of the temporal gauge prescription. We obtain from the forward scattering amplitudes the leading T^4 and the subleading T^2 and $\log T$ contributions. In Sec. IV we explicitly calculate the contributions from the prescription poles and compare the results with the high temperature limit of the forward scattering expression. In Sec. V we employ the hard thermal loop results, up to the subleading T^2 contributions, to investigate the modification of the solutions of the dispersion relations in a gravitational plasma. We will discuss our results in Sec. VI.

II. LAGRANGIAN, FEYNMAN RULES AND BASIC DEFINITIONS

The graviton field $\phi_{\mu\nu}$ can be defined as a small perturbation around the flat space-time metric, $\eta_{\mu\nu}$, as follows:

$$g_{\mu\nu}(x)=\eta_{\mu\nu}+\kappa\phi_{\mu\nu}(x), \quad \kappa^2=32\pi G. \tag{2.1}$$

Here G is Newton's constant and $g_{\mu\nu}$ is the metric tensor. The Einstein Lagrangian density is given by

$$\mathcal{L}=\frac{2}{\kappa^2}\sqrt{-g}g^{\mu\nu}R_{\mu\nu} \tag{2.2}$$

where $R_{\mu\nu}$ is the Ricci tensor given by

$$R_{\mu\nu}=\partial_\nu\Gamma^\alpha_{\mu\alpha}-\partial_\alpha\Gamma^\alpha_{\mu\nu}-\Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\alpha\beta}+\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha}$$

$$\Gamma^\alpha_{\mu\nu}=\frac{1}{2}g^{\alpha\beta}(\partial_\mu g_{\beta\nu}+\partial_\nu g_{\beta\mu}-\partial_\beta g_{\mu\nu}). \tag{2.3}$$

It is clear from the previous expressions that the Einstein Lagrangian is an infinity series in powers of κ (an infinity number of terms arises both from the inverse metric $g^{\mu\nu}$ and from the determinant g). Each power κ^n will come out multiplied by a combination of tensor scalar products of n tensor fields ϕ and two derivatives $\partial\phi$. Performing a systematic expansion in powers of the coupling constant κ, it is straightforward to obtain the tree-level Feynman rules corresponding to the terms which are quadratic, cubic, etc. [30]. Before we show the explicit form of these vertices, let us recall that the invariance of the Einstein action under general coordinate transformations (gauge transformations) imply the existence of Ward identities relating all the vertices down to the quadratic term (see Appendix A). The identity given by Eq. (A10) shows explicitly the usual problem of inverting the free quadratic part of a gauge invariant Lagrangian. Following the standard procedure of introducing a gauge fixing condition and ghost fields, we add the following two terms to the Einstein Lagrangian [21]

$$\mathcal{L}_{fix}=-\frac{1}{2\alpha}\eta^{\mu\nu}(n^\rho\phi_{\rho\mu})(n^\sigma\phi_{\sigma\nu}) \tag{2.4}$$

and

TABLE I. The 14 independent tensors built from $\eta_{\mu\nu}$, k_μ and $u_\mu\equiv n_\mu/n_0$ and satisfying the symmetry conditions $\mathcal{T}^i_{\mu\nu,\rho\sigma}=\mathcal{T}^i_{\nu\mu,\rho\sigma}=\mathcal{T}^i_{\mu\nu,\sigma\rho}=\mathcal{T}^i_{\rho\sigma,\mu\nu}$.

$\mathcal{T}^1_{\mu\nu,\rho\sigma}=\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}$
$\mathcal{T}^2_{\mu\nu,\rho\sigma}=\eta_{\mu\rho}u_\nu u_\sigma+\eta_{\mu\sigma}u_\nu u_\rho+\eta_{\nu\rho}u_\mu u_\sigma+\eta_{\nu\sigma}u_\mu u_\rho$
$\mathcal{T}^3_{\mu\nu,\rho\sigma}=u_\mu u_\nu u_\rho u_\sigma$
$\mathcal{T}^4_{\mu\nu,\rho\sigma}=\eta_{\mu\nu}\eta_{\rho\sigma}$
$\mathcal{T}^5_{\mu\nu,\rho\sigma}=\eta_{\mu\nu}u_\rho u_\sigma+\eta_{\rho\sigma}u_\mu u_\nu$
$\mathcal{T}^6_{\mu\nu,\rho\sigma}=\frac{1}{k\cdot u}[(\eta_{\mu\rho}k_\nu+\eta_{\nu\rho}k_\mu)u_\sigma+(\eta_{\mu\sigma}k_\nu+\eta_{\nu\sigma}k_\mu)u_\rho+(\eta_{\mu\rho}u_\nu+\eta_{\nu\rho}u_\mu)k_\sigma+(\eta_{\mu\sigma}u_\nu+\eta_{\nu\sigma}u_\mu)k_\rho]$
$\mathcal{T}^7_{\mu\nu,\rho\sigma}=\frac{1}{k\cdot u}(k_\mu u_\nu u_\rho u_\sigma+k_\nu u_\mu u_\rho u_\sigma+k_\rho u_\mu u_\nu u_\sigma+k_\sigma u_\mu u_\nu u_\rho)$
$\mathcal{T}^8_{\mu\nu,\rho\sigma}=\frac{1}{k^2}(\eta_{\mu\rho}k_\nu k_\sigma+\eta_{\mu\sigma}k_\nu k_\rho+\eta_{\nu\rho}k_\mu k_\sigma+\eta_{\nu\sigma}k_\mu k_\rho)$
$\mathcal{T}^9_{\mu\nu,\rho\sigma}=\frac{1}{k^2}(k_\mu k_\nu u_\rho u_\sigma+k_\rho k_\sigma u_\mu u_\nu)$
$\mathcal{T}^{10}_{\mu\nu,\rho\sigma}=\frac{1}{(k\cdot u)^2}[(k_\mu u_\nu+k_\nu u_\mu)(k_\rho u_\sigma+k_\sigma u_\rho)]$
$\mathcal{T}^{11}_{\mu\nu,\rho\sigma}=\frac{1}{k^2 k\cdot u}(u_\mu k_\nu k_\rho k_\sigma+u_\nu k_\mu k_\rho k_\sigma+u_\rho k_\mu k_\nu k_\sigma+u_\sigma k_\mu k_\nu k_\rho)$
$\mathcal{T}^{12}_{\mu\nu,\rho\sigma}=\frac{1}{k^4}k_\mu k_\nu k_\rho k_\sigma$
$\mathcal{T}^{13}_{\mu\nu,\rho\sigma}=\frac{1}{k^2}(\eta_{\mu\nu}k_\rho k_\sigma+\eta_{\rho\sigma}k_\mu k_\nu)$
$\mathcal{T}^{14}_{\mu\nu,\rho\sigma}=\frac{1}{k\cdot u}[\eta_{\mu\nu}(k_\rho u_\sigma+u_\rho k_\sigma)+\eta_{\rho\sigma}(k_\mu u_\nu+u_\mu k_\nu)]$

$$\mathcal{L}_{ghost}=-n^\mu\chi^\nu\frac{\delta\phi_{\mu\nu}}{\delta\varepsilon^\lambda}\eta^\lambda, \tag{2.5}$$

where χ^μ and η^μ are the gravitational Faddeev-Popov ghost vector fields, n^μ is the axial vector and α is a constant gauge parameter. Using Eq. (A3), we obtain the following explicitly form for the ghost Lagrangian:

$$\mathcal{L}_{ghost}=\chi^\nu\{n_\lambda\partial_\nu+\eta_{\lambda\nu}n\cdot\partial+\kappa[n^\mu\phi_{\mu\lambda}\partial_\nu+\phi_{\nu\lambda}n\cdot\partial+n^\mu(\partial_\lambda\phi_{\mu\nu})]\}\eta^\lambda. \tag{2.6}$$

Notice that, unlike Yang-Mills theory, ghosts remain coupled to the gravitons even for the choice $\alpha=0$. However, our explicit calculation will show that the decoupling occurs when the loop integrations are performed.

We have now all the basic ingredients to perform perturbative calculations in thermal gravity. The graviton propagator can now be obtained inverting the quadratic term of $\mathcal{L}+\mathcal{L}_{fix}$. Our choice of gauge fixing is such that even the bare graviton propagator is already dependent on 14 independent tensors as shown in Table I (we will employ this same basis in order to obtain the tensor structure of the thermal self-energy) [31]. Using this tensor basis it is possible to obtain the following compact form for the graviton propagator

$$\mathcal{D}_{\lambda\beta,\rho\sigma}(k)=\frac{1}{(k^2+i\epsilon)}\left\{I^1_{\lambda\beta,\rho\sigma}-\frac{1}{D-2}I^2_{\lambda\beta,\rho\sigma}+\alpha\frac{k^4}{n_0^2(k\cdot u)^2}\left[\mathcal{T}^8_{\lambda\beta,\rho\sigma}+\frac{k^2}{(k\cdot u)^2}\mathcal{T}^{12}_{\lambda\beta,\rho\sigma}-\mathcal{T}^{11}_{\lambda\beta,\rho\sigma}\right]\right\}, \tag{2.7}$$

where

$$I^1_{\mu\nu,\rho\sigma}=\frac{1}{4}(d_{\mu\kappa}d_{\nu\lambda}+d_{\mu\lambda}d_{\nu\kappa})(d_{\rho\kappa}d_{\sigma\lambda}+d_{\rho\lambda}d_{\sigma\kappa}),$$

$$I^2_{\mu\nu,\rho\sigma}=d_{\mu\kappa}d_{\nu\kappa}d_{\rho\lambda}d_{\sigma\lambda}, \quad d_{\mu\nu}=\eta_{\mu\nu}-\frac{k_\mu u_\nu}{k\cdot u},$$

are convenient linear combinations of the tensors in Table I. As we can see the graviton propagator has the usual poles at $k^2=0$ as well as the poles at $k\cdot u=k_0=0$. The first and second order terms in κ yield the following three and four graviton vertices, respectively:

$$\begin{aligned}V^3_{\alpha\beta,\rho\lambda,\delta\gamma}(k_1,k_2,k_3)=&\frac{\kappa}{4}(\{k_2\cdot k_3[\eta_{\alpha\beta}(\eta_{\rho\lambda}\eta_{\delta\gamma}-\eta_{\rho\delta}\eta_{\lambda\gamma})+4\eta_{\alpha\delta}(\eta_{\beta\rho}\eta_{\gamma\lambda}-\eta_{\rho\lambda}\eta_{\beta\gamma})]\\&+2k_{2\alpha}[k_{3\beta}(\eta_{\lambda\gamma}\eta_{\rho\delta}-\eta_{\rho\lambda}\eta_{\delta\gamma})+2k_{3\rho}(\eta_{\beta\lambda}\eta_{\delta\gamma}-2\eta_{\beta\delta}\eta_{\lambda\gamma})]\\&+2k_{2\rho}[2k_{3\alpha}\eta_{\beta\lambda}\eta_{\delta\gamma}+k_{3\lambda}(2\eta_{\beta\gamma}\eta_{\alpha\delta}-\eta_{\alpha\beta}\eta_{\delta\gamma})]+2k_{2\delta}k_{3\rho}(\eta_{\alpha\beta}\eta_{\lambda\gamma}-2\eta_{\beta\lambda}\eta_{\alpha\gamma})\}\\&+\text{symmet. on}(\alpha\leftrightarrow\beta),(\rho\leftrightarrow\lambda),(\delta\leftrightarrow\gamma))+\text{permut. of}(k_1,\alpha,\beta),(k_2,\rho,\lambda),(k_3,\delta,\gamma),\end{aligned} \tag{2.8}$$

$$\begin{aligned}V^4_{\alpha\beta,\rho\lambda,\delta\gamma,\tau\sigma}(k_1,k_2,k_3,k_4)=&\frac{\kappa^2}{16}(\{k_3\cdot k_4[(\eta_{\alpha\beta}\eta_{\rho\lambda}-2\eta_{\alpha\rho}\eta_{\beta\lambda})(\eta_{\delta\gamma}\eta_{\tau\sigma}-\eta_{\delta\tau}\eta_{\gamma\sigma})+8(\eta_{\alpha\delta}\eta_{\beta\rho}+\eta_{\alpha\rho}\eta_{\beta\delta}-\eta_{\alpha\beta}\eta_{\delta\rho})\\&\times(\eta_{\lambda\gamma}\eta_{\tau\sigma}-\eta_{\sigma\gamma}\eta_{\tau\lambda})+8\eta_{\rho\tau}\eta_{\alpha\delta}(\eta_{\beta\gamma}\eta_{\sigma\lambda}-\eta_{\beta\sigma}\eta_{\gamma\lambda})]+4k_{3\alpha}(2k_{4\rho}\eta_{\beta\lambda}-k_{4\beta}\eta_{\rho\lambda})\\&\times(\eta_{\delta\gamma}\eta_{\tau\sigma}-\eta_{\delta\tau}\eta_{\gamma\sigma})+16(k_{3\rho}k_{4\alpha}\eta_{\beta\delta}-k_{3\alpha}k_{4\beta}\eta_{\delta\rho})(\eta_{\gamma\sigma}\eta_{\lambda\tau}-\eta_{\gamma\lambda}\eta_{\tau\sigma})+8(k_{3\alpha}k_{4\delta}+k_{3\delta}k_{4\alpha})\\&\times(\eta_{\rho\lambda}\eta_{\beta\gamma}-2\eta_{\gamma\lambda}\eta_{\beta\rho})\eta_{\tau\sigma}+16k_{3\alpha}k_{4\delta}[\eta_{\rho\tau}(2\eta_{\beta\sigma}\eta_{\gamma\lambda}-\eta_{\beta\gamma}\eta_{\sigma\lambda})+\eta_{\gamma\sigma}(2\eta_{\rho\tau}\eta_{\beta\lambda}-\eta_{\rho\lambda}\eta_{\beta\tau})]\\&-16k_{3\delta}k_{4\alpha}\eta_{\rho\tau}\eta_{\beta\gamma}\eta_{\sigma\lambda}+2(k_{3\tau}k_{4\delta}\eta_{\gamma\sigma}-k_{3\delta}k_{4\gamma}\eta_{\tau\sigma})(\eta_{\alpha\beta}\eta_{\rho\lambda}-2\eta_{\alpha\rho}\eta_{\beta\lambda})\\&+8(k_{3\tau}k_{4\delta}\eta_{\gamma\lambda}-k_{3\delta}k_{4\gamma}\eta_{\lambda\tau})(2\eta_{\beta\sigma}\eta_{\alpha\rho}-\eta_{\rho\sigma}\eta_{\alpha\beta})\}\\&+\text{symmet. on}(\alpha\leftrightarrow\beta),(\rho\leftrightarrow\lambda),(\delta\leftrightarrow\gamma),(\tau\leftrightarrow\sigma))\\&+\text{permut. of}(k_1,\alpha,\beta),(k_2,\rho,\lambda),(k_3,\delta,\gamma),(k_4,\tau\sigma).\end{aligned} \tag{2.9}$$

We have verified that these vertices are in agreement with the Ward identities described in Appendix A.

Finally, the quadratic and the interacting term in the ghost Lagrangian (2.6) yields the ghost propagator

$$\mathcal{D}^{\text{ghost}}_{\lambda\mu}(k)=i\left[\frac{1}{2(n.k)^2}k_\lambda n_\mu-\frac{1}{n.k}\eta_{\lambda\mu}\right] \tag{2.10}$$

and the graviton-ghost-ghost vertex

$$\begin{aligned}V^{Ggg}_{\mu\kappa,\rho\nu}(k_1,k_2,k_3)=i\kappa(&\eta_{\rho\mu}\eta_{\nu\kappa}n.k_2+\eta_{\rho\kappa}n_\mu k_{2\nu}\\&+\eta_{\nu\kappa}n_\mu k_{1\rho})+\mu\leftrightarrow\kappa,\end{aligned} \tag{2.11}$$

respectively.

A. The one-point function

In order to introduce our notation and the basic method of calculation we will rederive here the result for the thermal one-point function. The one-point function is interesting by

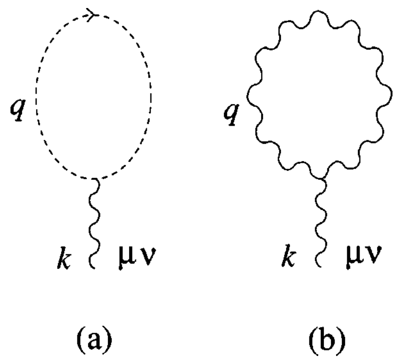


FIG. 1. Diagrams contributing to the one-graviton function in the one-loop approximation. The curly lines represent gravitons and the dashed lines represent ghosts.

itself, since it is directly related to the energy momentum tensor derived from the effective action [2]. It also provides the simplest non-trivial example of a one-loop calculation in gravity. Indeed, in contrast with the zero temperature case, the finite temperature one-point function is non-zero, being exactly proportional to T^4. For that reason it will play an important rôle in the Ward identities obeyed by the hard thermal loop Green's functions.

The relevant diagrams are shown in Fig. 1. Using the imaginary time formalism [18], Eqs. (2.10) and (2.11) give the following contribution for the ghost loop diagram shown in Fig. 1(a):

$$\Gamma^{\text{ghost}}_{\mu\nu}=\kappa T\sum_{q_0}\int\frac{d^{D-1}\vec{q}}{(2\pi)^{D-1}}\eta_{\mu\nu},\quad q_0=2\pi i n T,\quad n=0,\pm 1,\pm 2,\ldots . \tag{2.12}$$

Throughout this work the Matsubara sums like that one in Eq. (2.12) will be computed using the standard and elegant relation [18]

$$\begin{aligned}T\sum_{q_0}f(q_0)&=\int_{-i\infty+\delta}^{i\infty+\delta}\frac{\mathrm{d}q_0}{2\pi i}[f(q_0)+f(-q_0)]\frac{1}{2}\coth\left(\frac{q_0}{2T}\right)\\&=\int_{-i\infty+\delta}^{i\infty+\delta}\frac{\mathrm{d}q_0}{2\pi i}[f(q_0)+f(-q_0)]\\&\quad\times\left(\frac{1}{2}+\frac{1}{\mathrm{e}^{q_0/T}-1}\right).\end{aligned}\tag{2.13}$$

In general, the vacuum part of the amplitudes [terms which arise from the factor 1/2 inside the parentheses of Eq. (2.13)] may be divergent in the limit $D\to 4$ and so the arbitrary dimension D provides a regulator for the vacuum piece of the thermal Green's functions as usual [22].

The tadpole diagram provides the simplest example of an effective decoupling of the ghost graviton interaction in the temporal gauge (this is not a trivial property at non-zero temperature). Indeed, substituting Eq. (2.13) into Eq. (2.12) we can see that the vacuum piece vanishes as a consequence of the identity

$$\int d^{D-1}\vec{q}\,|\vec{q}|^r=0. \tag{2.14}$$

The thermal piece also vanishes since we can close the contour in the right-hand side of the q_0 plane without enfolding any poles.

The contribution from the graviton loop in Fig. 1(b) is a little bit more involved. After some straightforward tensor algebra we obtain from Eqs. (2.7) and (2.8) the following result:

$$\Gamma_{\mu\nu}=\kappa T\sum_{q_0}\int\frac{d^{D-1}\vec{q}}{(2\pi)^{D-1}}\frac{D}{8}\times\left[2(D-3)\frac{q_\mu q_\nu}{q^2}-(D-5)\eta_{\mu\nu}\right]. \tag{2.15}$$

It is interesting to notice that the gauge parameter α from the graviton propagator has already canceled out at the integrand level.

Let us now compute Eq. (2.15) with the help of formula (2.13). As in the case of the ghost loop diagram, the contribution proportional to $\eta_{\mu\nu}$ vanishes. The dimensional regularized vacuum piece will also vanish and we are left with only the following expression:

$$\Gamma_{\mu\nu}=2\kappa\int_{-i\infty+\delta}^{i\infty+\delta}\frac{\mathrm{d}q_0}{2\pi i}\frac{1}{\mathrm{e}^{q_0/T}-1}\int\frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\frac{q_\mu q_\nu}{q^2}. \tag{2.16}$$

Closing the contour in the right hand side plane the pole at $q_0=|\vec{q}|$ gives the following contribution

$$\Gamma_{\mu\nu}=-\kappa\int_0^\infty\frac{\mathrm{d}|\vec{q}|}{(2\pi)^3}\frac{|\vec{q}|^3}{\mathrm{e}^{|\vec{q}|/T}-1}\int\mathrm{d}\Omega\,\hat{q}_\mu\hat{q}_\nu\bigg|_{q_0=|\vec{q}|}, \tag{2.17}$$

where we have introduced $\hat{q}_\mu=q_\mu/|\vec{q}|$ and $\int\mathrm{d}\Omega$ is the integration over all directions of $\vec{q}$. Finally, using the formula [23]

$$\int_0^\infty\frac{x^{\nu-1}}{\mathrm{e}^{x/T}-1}\mathrm{d}x=\Gamma(\nu)\zeta(\nu)T^\nu \tag{2.18}$$

we obtain

$$\begin{aligned}\Gamma_{\mu\nu}&=-\kappa\frac{\pi^2T^4}{30}\int\frac{\mathrm{d}\Omega}{4\pi}\hat{q}_\mu\hat{q}_\nu\bigg|_{q_0=|\vec{q}|}\\&=-\kappa\frac{\pi^2T^4}{90}(4u_\mu u_\nu-\eta_{\mu\nu}),\end{aligned}\tag{2.19}$$

where we have employed the quantity $u\equiv(1,0,0,0)$, which coincides with the vector representing the local rest frame of the plasma and was introduced in Table I.

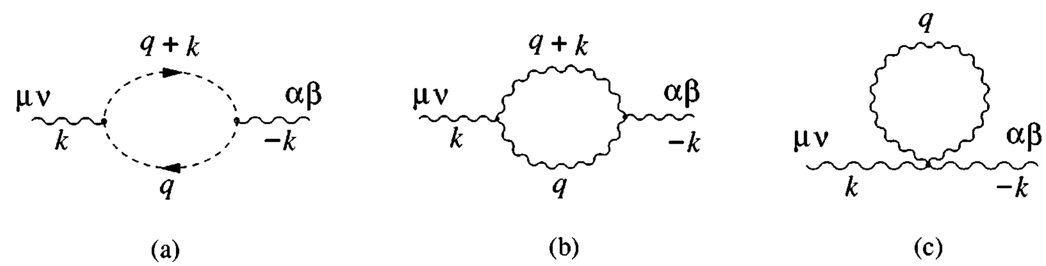


FIG. 2. Diagrams contributing to the graviton self-energy. The curly lines represent gravitons and the dashed lines represent ghosts. The external momentum k is inward.

III. THERMAL FORWARD SCATTERING CONTRIBUTIONS TO THE GRAVITON SELF-ENERGY

The diagrams which contribute to the graviton self-energy are shown in Fig. 2. The relevant Feynman rules for the propagators and vertices are all given in the previous section. Let us first consider the ghost loop diagram shown in Fig. 2(a). As we can see from the structure of the ghost propagator in Eq. (2.10) the integrand will involve a combination of fractions of the following type:

$$\frac{1}{(q\cdot u)^m[(k+q)\cdot u]^n}, \quad m,n=0,1,2. \tag{3.1}$$

Before trying to perform the loop momentum integrations explicitly it is convenient to simplify the integrand using well known algebraic identities and change of variables which may reduce the number of terms considerably. Indeed, we have found that using a partial fraction decomposition of the quantities shown in Eq. (3.1) and a shift $q\rightarrow q-k$ in the resulting partial fractions containing powers of $[(q+k)\cdot u]^{-1}$, leads to the simplest possible result given by

$$\begin{aligned}\Pi^{gh}_{\mu\nu,\alpha\beta}=T\sum_{q_0}\int\frac{dq^{D-1}}{(2\pi)^{D-1}}\Bigg[&\frac{k^2k\cdot q u_\mu u_\nu u_\alpha u_\beta}{k\cdot u^3 q\cdot u}-\frac{1}{2}\frac{k^2u_\mu u_\nu q_\alpha u_\beta}{k\cdot u^2q\cdot u}+\frac{1}{2}\frac{u_\mu k_\nu u_\alpha q_\beta}{k\cdot uq\cdot u}-\frac{1}{2}\frac{k\cdot qu_\mu k_\nu u_\alpha u_\beta}{k\cdot u^2q\cdot u}-\frac{1}{2}\frac{u_\mu q_\nu u_\alpha q_\beta}{q\cdot u^2}\\
&+\frac{1}{2}\frac{u_\mu q_\nu k_\alpha u_\beta}{k\cdot uq\cdot u}+\frac{1}{2}\eta_{\alpha\mu}\eta_{\beta\nu}+\frac{1}{2}\frac{q_\mu u_\nu k_\alpha u_\beta}{k\cdot uq\cdot u}-\frac{1}{2}\frac{k\cdot qu_\mu u_\nu k_\alpha u_\beta}{k\cdot u^2q\cdot u}+\frac{1}{2}\frac{k\cdot qq_\mu u_\nu u_\alpha u_\beta}{k\cdot uq\cdot u^2}-\frac{1}{2}\frac{k^2u_\mu q_\nu u_\alpha u_\beta}{k\cdot u^2q\cdot u}\\
&+\frac{1}{2}\frac{k\cdot qu_\mu q_\nu u_\alpha u_\beta}{k\cdot uq\cdot u^2}-\frac{1}{2}\frac{q_\mu u_\nu q_\alpha u_\beta}{q\cdot u^2}-\frac{1}{2}\frac{q_\mu u_\nu u_\alpha q_\beta}{q\cdot u^2}-\frac{1}{2}\frac{u_\mu q_\nu q_\alpha u_\beta}{q\cdot u^2}+\frac{1}{2}\eta_{\alpha\nu}\eta_{\beta\mu}-\frac{1}{2}\frac{k^2q_\mu u_\nu u_\alpha u_\beta}{k\cdot u^2q\cdot u}+\frac{1}{2}\frac{u_\mu k_\nu q_\alpha u_\beta}{k\cdot uq\cdot u}\\
&+\frac{1}{2}\frac{k_\mu u_\nu u_\alpha q_\beta}{k\cdot uq\cdot u}+\frac{1}{2}\frac{k_\mu u_\nu q_\alpha u_\beta}{k\cdot uq\cdot u}-\frac{1}{2}\frac{k\cdot qk_\mu u_\nu u_\alpha u_\beta}{k\cdot u^2q\cdot u}+\frac{1}{2}\frac{u_\mu q_\nu u_\alpha k_\beta}{k\cdot uq\cdot u}-\frac{1}{2}\frac{k\cdot qu_\mu u_\nu u_\alpha k_\beta}{k\cdot u^2q\cdot u}+\frac{1}{2}\frac{q_\mu u_\nu u_\alpha k_\beta}{k\cdot uq\cdot u}\\
&-\frac{1}{2}\frac{k^2u_\mu u_\nu u_\alpha q_\beta}{k\cdot u^2q\cdot u}+\frac{1}{2}\frac{k\cdot qu_\mu u_\nu u_\alpha q_\beta}{k\cdot uq\cdot u^2}-\frac{1}{2}\frac{k\cdot q^2u_\mu u_\nu u_\alpha u_\beta}{k\cdot u^2q\cdot u^2}+\frac{1}{2}\frac{k\cdot qu_\mu u_\nu q_\alpha u_\beta}{k\cdot uq\cdot u^2}\Bigg].\end{aligned} \tag{3.2}$$

This procedure (partial fractions and then shifts) has been employed previously in the case of the Yang-Mills theory [17]. In contrast with the present thermal gravity result given by Eq. (3.2), the axial gauge Yang-Mills ghost loop vanishes at the integrand level. Notice that the partial fraction decomposition is justified since the integrands are regularized accordingly.

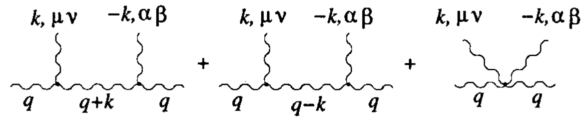
Let us now consider the diagrams shown in Figs. 2(b) and 2(c). An important difference between these diagrams and the ghost loop is that while the ghost loop contains *only* the poles at $q_0=0$, the structure of the graviton propagator in Eq. (2.7) is such that there are *also* the usual simple poles in the right hand side plane located at $q_0=|\vec{q}|$ and $q_0=|\vec{q}+\vec{k}|-k_0$ for the diagram in Fig. 2(b) and at $q_0=|\vec{q}|$ for the diagram in Fig. 2(c) (notice that k_0 is an imaginary quantity at this stage of the calculation). In order to use the contour method of integration described in Sec. II A, we will employ the following prescription for the poles at $q_0=0$ [15]:

$$\frac{1}{q_0^r}\rightarrow\lim_{\mu\rightarrow 0}\frac{q_0^r}{(q_0^2-\mu^2)^r}. \tag{3.3}$$

With this prescription the temporal gauge poles are moved away from the imaginary axis and we are allowed to employ the formula (2.13). The q_0 integral can then be performed closing the contour of integration in the right hand side of the q_0 plane, as we did in the previous section in the case of the one point function. The contributions from the *prescription poles* located at $q_0=\mu$ will be analyzed in the next section.

We now follow the steps explained in Appendix A of Ref. [17]. Basically this consists of the use of Eq. (2.13) taking into account only the contributions from the poles located on the right hand side plane at $q_0=|\vec{q}|$ and $q_0=|\vec{q}+\vec{k}|-k_0$. Then, in the residues from the poles at $|\vec{q}+\vec{k}|-k_0$ we perform the shift $\vec{q}\to\vec{q}-\vec{k}$ and use the property $\coth(x+k_0)=\coth(x)$. This yields the following expression in terms of thermal *forward scattering amplitudes*:

$$\Pi_{\mu\nu,\alpha\beta}|_{FS} = -\frac{1}{(2\pi)^3}\int \frac{d^3q}{2|\vec{q}|}\frac{1}{e^{\frac{|\vec{q}|}{T}}-1}\frac{1}{2}\left\{ \ldots + q\leftrightarrow -q \right\}_{q_0=|\vec{q}|}, \tag{3.4}$$



where the factor 1/2 in front of the curly brackets takes into account the symmetry of the graphs in Figs. 2(b) and 2(c). It is understood that the external graviton lines with momentum q are contracted with the tensor given by the curly bracket of Eq. (2.7).

We remark that the gauge parameter dependence of Eq. (3.4) involves only linear terms in α. This can be understood since the quadratic powers of α which could in principle arise from the propagator in Eq. (2.7) do not have the on-shell poles. Another interesting property of Eq. (3.4) is that it does not involve *thermal ghosts*.

The forward scattering expression in Eq. (3.4) is very convenient when considering the *hard thermal loop* contributions which arise from the region where the internal momentum q is of the order of the temperature T, which is large compared to the external momentum k. In this regime we can expand the denominators in Eq. (3.4) as follows:

$$\frac{1}{k^2\pm 2k\cdot q}=\pm\frac{1}{2k\cdot q}-\frac{k^2}{(2k\cdot q)^2}+\cdots. \tag{3.5}$$

The leading hard thermal loop contribution is obtained by considering all the integrands which are of degree two in the internal momenta q. After some straightforward but very tedious algebra we were able to express the leading contribution in the following rather compact form:

$$\Pi^{\rm lead}_{\mu\nu,\alpha\beta}|_{\rm FS}=-\kappa^2\frac{\pi^2T^4}{30}\int\frac{d\Omega}{4\pi}\frac{1}{2}\left[\left(k\cdot\frac{\partial}{\partial\hat{q}}\right)\frac{\hat{q}_\mu\hat{q}_\nu\hat{q}_\alpha\hat{q}_\beta}{\hat{q}\cdot k} - \eta_{\mu\alpha}\hat{q}_\nu\hat{q}_\beta-\eta_{\nu\alpha}\hat{q}_\mu\hat{q}_\beta-\eta_{\mu\beta}\hat{q}_\nu\hat{q}_\alpha - \eta_{\nu\beta}\hat{q}_\mu\hat{q}_\alpha\right]_{q_0=|\vec{q}|}, \tag{3.6}$$

where we have employed the formula (2.18) and $\hat{q}$ have the same meaning as in Eq. (2.19).

One can easily verify that this leading T^4 contribution is related to the one-graviton function in Eq. (2.19) by the Ward identity in Eq. (A5) (this result is also in agreement with the calculations performed in the Feynman–de Donder gauge [2,4]). Since we expect that the leading T^4 contributions are generated by a gauge independent effective action, the contributions from the prescription poles in Eq. (3.3) should not modify the leading T^4 behavior. This will be confirmed by our explicit calculation in the next section.

Let us now consider the subleading contributions which are generated when we expand the integrand of Eq. (3.4) up to terms of degree zero in q. By power counting these will be of order T^2. In order to obtain the full tensor structure generated by the expression (3.4) it is convenient to use the following tensor decomposition:

$$\Pi^{\mu\nu\alpha\beta}=\sum_{l=1}^{14} C_l \mathcal{T}_l^{\mu\nu,\alpha\beta}, \tag{3.7}$$

where the tensors $\mathcal{T}_l^{\mu\nu,\alpha\beta}$ are given in Table I. The coefficients C_l are obtained solving the system of 14 equations

$$\sum_{l=1}^{14}(\mathcal{T}_i^{\mu\nu,\alpha\beta}\mathcal{T}_{l\mu\nu,\alpha\beta})C_l=\Pi_i, \quad i=1,2,\ldots,14, \tag{3.8}$$

where the quantities Π_i are the following projections of the graviton self-energy:

$$\Pi_i=\Pi_{\mu\nu,\alpha\beta}\mathcal{T}_i^{\mu\nu,\alpha\beta}, \quad i=1,\ldots,14. \tag{3.9}$$

Each one of these projections can be expanded using Eq. (3.5). The integrals over the modulus of $\vec{q}$ can be easily performed using Eq. (2.18) (they yield the T^2 factor) and the angular integrals are all straightforward. Inserting the results for Π_i into Eq. (3.8) and solving for C_l, we obtain

$$C_1^{T^2}=\left[\frac{k^4L(k)}{\vec{k}^2}\left(\frac{1}{12}+\frac{5}{192}\frac{k^2}{\vec{k}^2}\right)+\frac{1}{36}\vec{k}^2-\frac{5}{576}\frac{k^4}{\vec{k}^2}-\frac{1}{144}k^2-\frac{1}{30}\frac{\alpha}{n_0^2}k^4\right]\kappa^2T^2$$

$$C_2^{T^2}=\left[\frac{k^6L(k)}{\vec{k}^4}\left(\frac{7}{32}+\frac{25}{192}\frac{k^2}{\vec{k}^2}\right)-\frac{25}{576}\frac{k^6}{\vec{k}^4}-\frac{1}{18}\frac{k^4}{\vec{k}^2}-\frac{1}{36}k^2+\frac{4}{45}\frac{\alpha}{n_0^2}k^4\right]\kappa^2T^2$$

$$C_3^{T^2}=\left[\frac{k^8L(k)}{\vec{k}^6}\left(\frac{15}{16}+\frac{175}{192}\frac{k^2}{\vec{k}^2}\right)-\frac{175}{576}\frac{k^8}{\vec{k}^6}-\frac{55}{288}\frac{k^6}{\vec{k}^4}+\frac{1}{18}\frac{k^4}{\vec{k}^2}+\frac{1}{9}k^2-\frac{4}{15}\frac{\alpha}{n_0^2}k^4\right]\kappa^2T^2$$

$$C_4^{T^2}=\left[\left(-\frac{1}{16}+\frac{5}{192}\frac{k^2}{\vec{k}^2}\right)\frac{k^4L(k)}{\vec{k}^2}-\frac{4}{15}\frac{\alpha}{n_0^2}\left(\vec{k}^4+k^2\vec{k}^2-\frac{1}{12}k^4\right)-\frac{5}{576}\frac{k^4}{\vec{k}^2}+\frac{5}{288}k^2\right]\kappa^2T^2$$

$$C_5^{T^2}=\left[\frac{25}{192}\frac{k^8L(k)}{\vec{k}^6}+\frac{1}{18}\vec{k}^2-\frac{25}{576}\frac{k^6}{\vec{k}^4}+\frac{5}{288}\frac{k^4}{\vec{k}^2}+\frac{1}{9}k^2-\frac{1}{15}\frac{\alpha}{n_0^2}\left(4k^2\vec{k}^2+\frac{14}{3}k^4\right)\right]\kappa^2T^2$$

$$C_6^{T^2}=k_0^2\left[-\left(\frac{7}{32}+\frac{25}{192}\frac{k^2}{\vec{k}^2}\right)\frac{k^4L(k)}{\vec{k}^4}+\frac{25}{576}\frac{k^4}{\vec{k}^4}+\frac{1}{18}\frac{k^2}{\vec{k}^2}-\frac{4}{45}\frac{\alpha}{n_0^2}k^2\right]\kappa^2T^2$$

$$C_7^{T^2}=k_0^2\left[-\left(\frac{15}{16}+\frac{175}{192}\frac{k^2}{\vec{k}^2}\right)\frac{k^6L(k)}{\vec{k}^6}+\frac{175}{576}\frac{k^6}{\vec{k}^6}+\frac{55}{288}\frac{k^4}{\vec{k}^4}-\frac{1}{18}\frac{k^2}{\vec{k}^2}-\frac{1}{18}+\frac{4}{15}\frac{\alpha}{n_0^2}k^2\right]\kappa^2T^2$$

$$C_8^{T^2}=k^2\left[\left(\frac{13}{96}+\frac{31}{96}\frac{k^2}{\vec{k}^2}+\frac{25}{192}\frac{k^4}{\vec{k}^4}\right)\frac{k^2L(k)}{\vec{k}^2}-\frac{25}{576}\frac{k^4}{\vec{k}^4}-\frac{13}{144}\frac{k^2}{\vec{k}^2}-\frac{1}{48}+\left(\frac{4}{45}\vec{k}^2+\frac{11}{90}k^2\right)\frac{\alpha}{n_0^2}\right]\kappa^2T^2$$

$$C_9^{T^2}=k^2\left[\left(\frac{15}{16}+\frac{55}{32}\frac{k^2}{\vec{k}^2}+\frac{175}{192}\frac{k^4}{\vec{k}^4}\right)\frac{k^4L(k)}{\vec{k}^4}-\frac{175}{576}\frac{k^6}{\vec{k}^6}-\frac{65}{144}\frac{k^4}{\vec{k}^4}-\frac{11}{72}\frac{k^2}{\vec{k}^2}+\frac{2}{45}\frac{\alpha}{n_0^2}k^2\right]\kappa^2T^2$$

$$C_{10}^{T^2}=k_0^2\left[\left(\frac{23}{32}+\frac{55}{32}\frac{k^2}{\vec{k}^2}+\frac{175}{192}\frac{k^4}{\vec{k}^4}\right)\frac{k^4L(k)}{\vec{k}^4}-\frac{175}{576}\frac{k^6}{\vec{k}^6}-\frac{65}{144}\frac{k^4}{\vec{k}^4}-\frac{23}{288}\frac{k^2}{\vec{k}^2}+\frac{1}{12}-\frac{4}{15}\frac{\alpha}{n_0^2}\left(\vec{k}^2+\frac{4}{3}k^2\right)\right]\kappa^2T^2$$

$$C_{11}^{T^2}=\frac{k^2k_0^2}{\vec{k}^2}\left[-\left(\frac{1}{2}+\frac{35}{24}\frac{k^2}{\vec{k}^2}+\frac{175}{192}\frac{k^4}{\vec{k}^4}\right)\frac{k^2L(k)}{\vec{k}^2}+\frac{175}{576}\frac{k^4}{\vec{k}^4}+\frac{35}{96}\frac{k^2}{\vec{k}^2}+\frac{1}{24}+\frac{2}{15}\frac{\alpha}{n_0^2}\vec{k}^2\right]\kappa^2T^2$$

$$C_{12}^{T^2}=\frac{k^4}{\vec{k}^2}\left[\left(\frac{1}{6}+\frac{29}{24}\frac{k^2}{\vec{k}^2}+\frac{95}{48}\frac{k^4}{\vec{k}^4}+\frac{175}{192}\frac{k^6}{\vec{k}^6}\right)L(k)-\frac{175}{576}\frac{k^4}{\vec{k}^4}-\frac{155}{288}\frac{k^2}{\vec{k}^2}-\frac{5}{24}-\frac{2}{45}\frac{\alpha}{n_0^2}\vec{k}^2\right]\kappa^2T^2$$

$$C_{13}^{T^2}=k^2\left[\left(\frac{1}{16}+\frac{5}{48}\frac{k^2}{\vec{k}^2}+\frac{25}{192}\frac{k^4}{\vec{k}^4}\right)\frac{k^2L(k)}{\vec{k}^2}-\frac{25}{576}\frac{k^4}{\vec{k}^4}-\frac{5}{288}\frac{k^2}{\vec{k}^2}-\left(\frac{14}{45}\vec{k}^2+\frac{1}{3}k^2\right)\frac{\alpha}{n_0^2}\right]\kappa^2T^2$$

$$C_{14}^{T^2}=k_0^2\left[-\frac{25}{192}\frac{k^6L(k)}{\vec{k}^6}+\frac{25}{576}\frac{k^4}{\vec{k}^4}-\frac{5}{288}\frac{k^2}{\vec{k}^2}-\frac{1}{18}+\frac{1}{15}\frac{\alpha}{n_0^2}\left(4\vec{k}^2+\frac{14}{3}k^2\right)\right]\kappa^2T^2 \qquad (3.10)$$

where

$$L(k)=\frac{k_0}{2|\vec{k}|}\log\frac{k_0+|\vec{k}|}{k_0-|\vec{k}|}-1. \qquad (3.11)$$

There are some properties of the T^2 contributions which are worth stressing. Firstly, the T^2 contributions show their gauge dependence explicitly through the gauge parameter α. Each of these gauge parameter dependent terms have two powers of momentum relative to the corresponding α-independent ones (the correct mass dimension is provided by n_0^2 in the denominators). Secondly, the simple Ward iden-

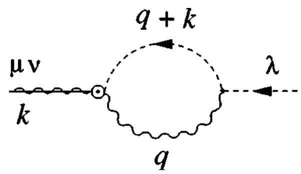


FIG. 3. The source-ghost diagram. The solid and wavy line on the left represents the external source.

tity satisfied by the leading T^4 contributions is no longer true for the sub-leading contributions. In Appendix B we derive the more general 't Hooft identities and we verify that the following identity is satisfied:

$$\chi^{(0)}_{\mu\nu\lambda}\Pi^{\mu\nu\alpha\beta}_{T^2} = -\chi^{(1)T^2}_{\mu\nu\lambda}V^{2\mu\nu\alpha\beta} \tag{3.12}$$

where $\chi^{(1)}_{\mu\nu\lambda}$ is represented by the diagram shown in Fig. 3. The T^2 contribution to $\chi^{(1)}_{\mu\nu\lambda}$ is computed in detail in Appendix B.

We have proceeded even further with the hard thermal loop expansion of Eq. (3.4) and computed the contributions from the integrands of degree minus 2 in q. After integration these yield the $\log T$ terms. We have verified that the $\log T$ contributions of all the projections Π_i [see Eq. (3.9)] are simply related to the corresponding projections of the ultraviolet divergent part of zero temperature graviton self-energy. The zero temperature results were computed using the gauge choice $\alpha=0$ in [8]. Setting $\alpha=0$ in our general result we have verified that

$$\Pi^{\log}_{\mu\nu,\alpha\beta}\big|_{\rm FS} = \log(T)\Pi^{\epsilon}_{\mu\nu,\alpha\beta}, \tag{3.13}$$

where $\Pi^{\epsilon}_{\mu\nu,\alpha\beta}$ is the residue of the ultraviolet divergent zero temperature contribution computed in $D=4-2\epsilon$ dimensions. The verification of this property in the case of gravity formulated in the temporal gauge complements similar results obtained in the Feynman–de Donder gauge [4] as well as in the case of the Yang-Mills theory [17,24]. Since our calculation has been performed for arbitrary values of α, we present complete results in Appendix C.

IV. THE CONTRIBUTIONS FROM PRESCRIPTION POLES

Let us now consider the terms that arise from the poles located at $q_0=\mu$, where μ is the quantity introduced in Eq. (3.3). It is convenient to express these contributions directly in terms of the projections defined by Eq. (3.9). Each one of the 14 projections can be expressed as follows:

$$\Pi_i^{\rm presc} = \lim_{\mu\to 0}\sum_{r=1}^{4} T\sum_{q_0}\int d^{D-1}\vec{q}\left[\frac{f_i^r(k_0,\vec{k}\cdot\vec{q},\vec{q}^2,\vec{k}^2)}{q^2(q+k)^2} + g_i^r(k_0,\vec{k}\cdot\vec{q},\vec{q}^2,\vec{k}^2)\right]\frac{(q_0)^r}{(q_0^2-\mu^2)^r},\quad i=1,2,\ldots,14, \tag{4.1}$$

where f_i^r and g_i^r are polynomials in their arguments, and the denominators q_0^r have been replaced according to the prescription (3.3). This is the most general form of the integrands from the diagrams with gluon or ghost loops. Notice that, in particular, the ghost loop expression given by Eq. (3.2) yields, after projection, contributions of the kind given by the g_i^r terms above, which contains no on-shell poles.

The parameter μ regulates the originally ill defined sums and also makes possible the use of the formula (2.13), since now the integrand is regular along the imaginary q_0 axis. Hence Eq. (4.1) can be rewritten as

$$\Pi_i^{\rm presc} = \lim_{\mu\to 0}\sum_{r=1}^{4}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}\frac{dq_0}{2\pi i}\int d^{D-1}\vec{q}\,\frac{1}{2}\coth\left(\frac{q_0}{2T}\right)\times\left\{\frac{(q_0)^r}{(q_0^2-\mu^2)^r}\left[\frac{f_i^r(k_0,\vec{k}\cdot\vec{q},\vec{q}^2,\vec{k}^2)}{q^2(q+k)^2}\right]+q\leftrightarrow -q\right\}. \tag{4.2}$$

We have employed Eq. (2.14) and so the dimensionally regularized integration of the g_i^r terms has vanished. This important property shows how the ghosts are effectively decoupled at finite temperature.

Performing the q_0 integration by closing the integration contour at right-hand side plane, we obtain, from Eq. (4.2),

$$\Pi_i^{\rm presc} = \lim_{\mu\to 0}\sum_{r=1}^{4}\frac{d^{r-1}}{d\mu^{r-1}}\coth\left(\frac{\mu}{2T}\right)\times\int d^{D-1}\vec{q}\,G^r(\mu,k_0,\vec{k}\cdot\vec{q},\vec{q}^2,\vec{k}^2). \tag{4.3}$$

In order to obtain the limit $\mu\to 0$, we need the following expansions of the coth and its derivatives:

$$\coth\left(\frac{\mu}{2T}\right) = \frac{2T}{\mu} + \mathcal{O}(\mu),$$

$$\frac{d}{d\mu}\coth\left(\frac{\mu}{2T}\right) = -\frac{2T}{\mu^2} + \frac{1}{6}\frac{1}{T} + \mathcal{O}(\mu^2),$$

$$\frac{d^2}{d\mu^2}\coth\left(\frac{\mu}{2T}\right) = \frac{4T}{\mu^3} + \mathcal{O}(\mu),$$

$$\frac{d^3}{d\mu^3}\coth\left(\frac{\mu}{2T}\right) = -\frac{12T}{\mu^4} - \frac{1}{60}\frac{1}{T^3} + \mathcal{O}(\mu^2). \tag{4.4}$$

An important property of the contribution of the prescription poles is that all the temperature dependence arises only from the expansions of the hyperbolic cotangent shown above as *odd powers* of T. Therefore, the results obtained in the previous section for the leading T^4 and sub-leading T^2 and $\log(T)$ are not modified by the temporal gauge prescription.

It remains to be verified that the limit $\mu\to 0$ is well defined. Our explicit calculations show that the results for all projections do not involve inverse powers of μ. Using the symmetry of the angular integrals all the inverse powers of μ cancel and we obtain finite results given by

$$\Pi_i^{\rm presc}=T\int d^{D-1}\vec{q}\frac{F_i^1(k_0,\vec{k}\cdot\vec{q},\vec{q}^2,\vec{k}^2)}{\vec{q}^6[-k_0^2+(\vec{q}+\vec{k})^2]^5[-k_0^2+(\vec{q}-\vec{k})^2]^5}+\frac{1}{T}\int d^{D-1}\vec{q}\frac{F_i^2(k_0,\vec{k}\cdot\vec{q},\vec{q}^2,\vec{k}^2)}{\vec{q}^4[-k_0^2+(\vec{q}+\vec{k})^2]^3[-k_0^2+(\vec{q}-\vec{k})^2]^3}+\frac{1}{T^3}\int d^{D-1}\vec{q}\frac{F_i^3(k_0,\vec{k}\cdot\vec{q},\vec{q}^2,\vec{k}^2)}{\vec{q}^2[-k_0^2+(\vec{q}+\vec{k})^2][-k_0^2+(\vec{q}-\vec{k})^2]}. \tag{4.5}$$

All these integrals are regular and can be done. In Appendix D we show in an explicit example a closed form result. For $i=8,10,11,12$ we obtain $\Pi_i^{\rm presc}=0$, which is in agreement with the 't Hooft identity given by Eq. (B5). We also show that $\Pi_i^{\rm presc}=0$ for $i=7,9,13,14$. Though the non-vanishing integrals ($i=1,2,3,4,5,6$) introduce an extra temperature dependence, it is clear that they do not change the behavior of the hard thermal loop expressions obtained in the previous section.

We remark that these non-vanishing integrals include both the thermal and the zero temperature contributions (notice that the integrands contains a coth instead of the purely thermal part involving the Bose-Einstein distribution). Had we computed the thermal part separately we would be left with contributions which are divergent when $\mu\to 0$ as well as the inverse powers of T. Since the dimensional regularization is employed only for the space part of the integrals, the vacuum part also contains inverse powers of μ (only the fully dimensionally regularized zero temperature calculation is well defined in the limit $\mu\to 0$ [15]). It is remarkable that the inverse powers of μ in the thermal part are exactly canceled by the corresponding ones in the vacuum part and we are left only with the inverse powers of the temperature. This property indicates that some of the ill-defined inverse powers of μ have been replaced by a thermal regulated expression. In order to understand why these prescription dependent parts are not well defined when $T\to 0$, one should notice that μ is a dimensionful parameter which was made "small" in the sense that $\mu\ll T$. Therefore, the prescription-dependent results cannot be extended to the region $T\to 0$.

V. DISPERSION RELATIONS IN A GRAVITON PLASMA

The sub-leading hard thermal loop contributions proportional to T^2 will produce modifications in the solution of the dispersion relations describing the wave propagation in a graviton plasma. The dispersion relations were carefully investigated in the case of the leading T^4 contributions [2]. The inclusion of sub-leading contributions has been considered in the Feynman–de Donder gauge [4]. Although the sub-leading modification of the solutions of the dispersion relations are suppressed by a factor $GT^2\ll 1$, in relation to the order one part arising from the T^4 contributions, one may be interested to know how the gauge dependence of the graviton self-energy will affect these solutions (the one-graviton function, which also contributes to the dispersion relations, has no sub-leading gauge dependent contributions at the one-loop order considered here). In Yang-Mills theories, the problem of gauge-(in) dependence is well understood since a theorem was proved by Kobes, Kunstatter and Rebhan (KKR) [25]. In a one-loop calculation, the gauge dependences of the location of the poles of the gluon propagator are explained in terms of the KKR identities. A well known example of this problem is the gauge dependence of the plasmon damping constant (see [26] for a recent review) and its solution by the Braaten and Pisarski resummation scheme [27]. As far as we know, a complete analysis of this problem, in the case of gravity, is still missing. Therefore, we believe that it is important to investigate how gauge dependent the graviton propagator is and whether it is possible to extract gauge independent information. In this regard, it is remarkable that the one-loop calculations of the QCD damping constant, in the axial gauge, though incomplete, satisfy some of the necessary conditions required by any physical quantity, being both gauge independent and positive [12].

With this motivation, let us apply our axial gauge results in the dispersion relations associated with the *transverse traceless components* of the *Jacobi equation* for small disturbances in the graviton plasma [2]. Proceeding as in Ref. [4], the results given in Eq. (3.10) (as well as the corresponding leading T^4 contributions) yields the following dispersion relations for the three transverse traceless modes:

$$k^2\left[1+\frac{16\pi GT^2k^2}{15}\frac{\alpha}{n_0^2}\right]=16\pi G\rho\left[\left(\frac{5}{9}+\frac{1}{2}r^4L-\frac{1}{6}r^2\right)+\frac{5k^2}{\pi^2T^2}\left(r^2L+\frac{5}{16}r^4L-\frac{5}{48}r^2-\frac{1}{12}+\frac{1}{3}\frac{1}{r^2}\right)\right],$$

$$k^2\left[1+\frac{16\pi GT^2k^2}{15}\frac{\alpha}{n_0^2}\left(1+\frac{8}{3}\frac{1}{r^2}\right)\right]=16\pi G\rho\left[\left(\frac{2}{9}-2r^4L+\frac{2}{3}r^2+\frac{10}{9}\frac{1}{r^2}\right)+\frac{5k^2}{\pi^2T^2}\left(-\frac{13}{8}r^2L-\frac{5}{4}r^4L+\frac{5}{12}r^2+\frac{7}{12}+\frac{2}{3}\frac{1}{r^2}\right)\right],\quad \rho=\frac{\pi^2}{30}T^4,$$

$$k^2\left[1+\frac{16\pi GT^2k^2}{15}\frac{\alpha}{n_0^2}\left(1+\frac{8}{3}\frac{1}{r^4}+\frac{32}{9}\frac{1}{r^2}\right)\right]=16\pi G\rho\left[\left(\frac{8}{9}+3r^4L-r^2+\frac{28}{27}\frac{1}{r^2}\right)+\frac{5k^2}{\pi^2T^2}\left(\frac{5}{4}r^2L+\frac{15}{8}r^4L-\frac{5}{8}r^2+\frac{1}{24}+\frac{1}{r^2}+\frac{4}{9}\frac{1}{r^4}\right)\right],\quad r^2\equiv\frac{k^2}{\vec{k}^2},\tag{5.1}$$

where $L(k)$ is given by Eq. (3.11).

Let us now solve these relations in the region of real values of k_0 and $\vec{k}$, which is relevant for the propagation of waves, and then compare with the corresponding solutions previously obtained in the Feynman–de Donder gauge. It is convenient to introduce the dimensionless quantities $\overline{k^2}\equiv|\vec{k}|^2/(16\pi G\rho)$, $\overline{\omega^2}\equiv\omega^2/(16\pi G\rho)$ and $\overline{n_0^2}\equiv n_0^2/(16\pi G\rho)$. We will also choose $n_0^2=\omega^2$ so that the scale of the gauge fixing is compatible with the momentum scale. For intermediate values of $\overline{k}$ and $\overline{\omega}$, the dispersion relations have to be solved numerically and the results are qualitatively similar to the ones shown in Fig. 3 of Ref. [4]. In order to discuss the specific issue of gauge dependence in terms of well defined analytic expressions, we will consider the asymptotic regions of very small and very large values of $\overline{k},\overline{\omega}$. In the limit $\overline{k}\rightarrow 0$ the solution of the dispersion relations (5.1) gives the following result for the *plasma frequency* (this is the minimum frequency above which propagating waves are supported by the plasma)

$$(\overline{\omega}_{\rm pl}^{axial})^2=\frac{22}{45}\left[1+\left(\frac{25}{6}-8\alpha\right)\frac{2\pi GT^2}{15}\right],\tag{5.2}$$

where we have neglected higher powers of GT^2. From Eqs. (4.15) of Ref. [4] the same limit $\overline{k}\rightarrow 0$ yields

$$(\overline{\omega}_{\rm pl}^{cov.})^2=\frac{22}{45}\left[1-\left(\frac{14}{5}+(1-\xi)\right)\frac{32\pi GT^2}{15}\right],\tag{5.3}$$

were ξ is the gauge parameter in the class of covariant gauges and $\xi=1$ defines the Feynman–de Donder gauge employed in Ref. [4]. An important property of these results is that, in both classes of gauges, there is the same strong dependence on the gauge parameter. In order to understand this behavior, let us reintroduce the dimensionful parameter $16\pi G\rho=(8/15)\pi^3GT^4$. Then, in both classes of gauges, one can see that the gauge dependent subleading correction is of order $(GT^4)(GT^2)$ which is of the same order as the *two-loop corrections*, not included in this calculation. Therefore, the subleading contributions to the plasma frequency constitute only a partial result at the one-loop order.

In the limit of high frequencies, $\overline{\omega^2}\sim\overline{k^2}\gg 1$ the asymptotic behavior of the solutions is described by the *thermal masses* $\overline{m_{\rm I}^2}\equiv\overline{\omega_{\rm I}^2}-\overline{k_{\rm I}^2}$ $(I=A,B,C)$. Using Eqs. (5.1) it is straightforward to obtain the following results:

$$(\overline{m}_{\rm A}^{axial})^2=\frac{5}{9}\left[1+\frac{8}{5}\overline{k^2}\pi GT^2\right],\tag{5.4}$$

$$(\overline{m}_{\rm B}^{axial})^2=\frac{\sqrt{10}}{3}\left[1+\sqrt{10}\overline{k}\frac{4\pi GT^2}{15}\right]\overline{k}\tag{5.5}$$

and

$$(\overline{m}_{\rm C}^{axial})^2=\frac{2\sqrt{21}}{9}\left[1+\frac{4}{7}\overline{k^2}\pi GT^2\right]\overline{k}.\tag{5.6}$$

When the same derivation is performed using Eqs. (4.15) of Ref. [4], for arbitrary values of the gauge parameter ξ, one obtains the following results:

$$(\overline{m}_{\rm A}^{cov.})^2=\frac{5}{9}\left[1-(9-\xi)\frac{32\pi GT^2}{15}\right],\tag{5.7}$$

$$(\overline{m}_{\rm B}^{cov.})^2=\frac{\sqrt{10}}{3}\left[1-(1-\xi)\frac{16\pi GT^2}{15}\right]\overline{k}\tag{5.8}$$

and

$$(\overline{m}_{\rm C}^{cov.})^2=\frac{2\sqrt{21}}{9}\left[1+\xi\frac{16\pi GT^2}{15}\right]\overline{k}.\tag{5.9}$$

All these explicit examples clearly show the main differences between these two distinct classes of gauges. It is remarkable that the axial gauge subleading contributions contain extra powers of $\overline{k}$ which makes them larger than the corresponding corrections in the covariant gauges, of order $(GT^4)(GT^2)$. Notice, however, that the hard thermal loop condition, $k^2\ll T^2$, implies that $\overline{k^2}GT^2\ll 1$ so that the subleading contributions will not exceed the leading ones. As far as the gauge dependences are concerned, we remark that there are no gauge parameter dependences in the axial gauge results (for the choice $n_0^2=\omega^2\simeq|\vec{k}|^2$). While this property is consistent with the necessary requirement that any physical quantity should satisfy, the same is not true when the masses $\overline{m}_I$ $(I=A,B,C)$ are computed in the covariant gauges.

VI. DISCUSSION

In this paper we have explicitly computed the thermal one- and two-graviton functions in the temporal gauge. We have applied Leibbrandt's [15] prescription to deal with the temporal gauge poles at finite temperature. This calculation provides a rather non-trivial explicit verification of the gauge invariance properties of the hard thermal loop contributions. Indeed, the leading T^4 behavior is in agreement with previous calculations in covariant gauges. The subleading contributions of order T^2 have a gauge dependence in agreement with the 't Hooft identities. We have also compared our $\log T$ contributions with the residue of the ultraviolet pole of the dimensionally regularized zero temperature graviton self-energy, given in Ref. [8], and found that they are the same (this property has also been verified in the Feynman–de Donder gauge [4]). Our results also include the full gauge parameter dependence, as shown in Appendix C.

Our explicit calculation indicates that the temporal gauge may be consistently employed even in the highly non-trivial case of thermal gravity. The form of the prescription poles in Eq. (3.3) do not change the hard thermal loop behavior of our main result given by Eq. (3.4). An important property of this forward scattering amplitude is that, as opposite to the covariant gauges, it does not involve *thermal ghosts* and the gauge parameter dependence is linear in α.

In the analysis of the dispersion relations we have included the hard thermal loop subleading contributions proportional to T^2 and compared the structure of the gauge dependence with similar calculations which were performed earlier in the Feynman gauge. As expected from general formal arguments there are gauge dependent contributions which arises from the subleading T^2 terms in the graviton self-energy. By power counting, some of the gauge dependent terms are of the same order as the two-loop contributions. However, the subleading terms, when computed in the axial gauge, are such that their contributions to the asymptotic masses are enhanced by extra powers of $\overline{k}$ and have a weaker gauge dependence. This behavior is analogous to what happens in QCD, where the plasmon damping constant (which also is subleading in the temperature) has a weaker gauge dependence when computed in non-covariant gauges.

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APPENDIX A: WARD IDENTITIES

In this appendix we derive the identities which must be satisfied by the vertex functions generated from an action which is invariant under coordinate transformations. These identities provide an important consistency check of the gravitational Feynman rules as well as the leading high temperature thermal Green's functions.

The invariance of an action S can be expressed as follows:

$$\delta S=\int \mathrm{d}^4x\frac{\delta\mathcal{L}(x)}{\delta\phi_{\mu\nu}(x)}\,\delta\phi_{\mu\nu}(x)=0. \tag{A1}$$

Let us choose the following coordinate transformation with an infinitesimal parameter $\delta\varepsilon_\mu(x)$:

$$x'^{\mu}=x^{\mu}+\delta\varepsilon^{\mu}(x).$$

Performing the transformation in the metric

$$\begin{aligned}g'_{\mu\nu}(x')&=\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x^{\beta}}{\partial x'^{\nu}}g_{\alpha\beta}\\&=g_{\mu\nu}(x)-g_{\alpha\nu}(x)\partial_\mu\delta\varepsilon^{\alpha}-g_{\alpha\mu}(x)\partial_\nu\delta\varepsilon^{\alpha}\\&=g'_{\mu\nu}(x)+\delta\varepsilon^{\lambda}\partial_\lambda g'_{\mu\nu}(x),\end{aligned} \tag{A2}$$

we obtain

$$\begin{aligned}g'_{\mu\nu}(x)-g_{\mu\nu}(x)&\equiv\delta g_{\mu\nu}\\&=\kappa\,\delta\phi_{\mu\nu}\\&=-g_{\mu\sigma}\partial_\nu\delta\varepsilon^{\sigma}-g_{\nu\sigma}\partial_\mu\delta\varepsilon^{\sigma}-\delta\varepsilon_\lambda(\partial^\lambda g_{\mu\nu})\\&=-\partial_\nu\delta\varepsilon_\mu-\partial_\mu\delta\varepsilon_\nu-\kappa[\phi_{\mu\sigma}(\partial_\nu\delta\varepsilon^{\sigma})\\&\quad+\phi_{\nu\sigma}(\partial_\mu\delta\varepsilon^{\sigma})+\delta\varepsilon^{\lambda}(\partial_\lambda\phi_{\mu\nu})],\end{aligned} \tag{A3}$$

where we have used Eq. (2.1). Inserting Eq. (A3) into Eq. (A1) and using integration by parts, we obtain

$$\begin{aligned}&\int \mathrm{d}^4x\,\delta\varepsilon^{\lambda}(\eta_{\nu\lambda}\partial_\mu+\eta_{\mu\lambda}\partial_\nu)\frac{\delta\mathcal{L}}{\delta\phi_{\mu\nu}}\\&\quad=-\kappa\int \mathrm{d}^4x\,\delta\varepsilon^{\lambda}[\partial_\nu\phi_{\mu\lambda}+\partial_\mu\phi_{\nu\lambda}+(\partial_\lambda\phi_{\mu\nu})]\frac{\delta\mathcal{L}}{\delta\phi_{\mu\nu}}.\end{aligned} \tag{A4}$$

Taking functional derivatives of Eq. (A4) one obtains the following Ward identities in momentum space:

$$\begin{aligned}&\frac{1}{\kappa}\chi^{0}_{\mu\nu\lambda}(k_1)V^{2\mu\nu}_{\alpha\beta}(k_1,k_2)\\&\quad=-\chi^{1}_{\mu\nu\alpha\beta\lambda}(k_1,k_2)V^{1\mu\nu}(k_1+k_2=0)\end{aligned} \tag{A5}$$

$$\begin{aligned}&\frac{1}{\kappa}\chi^{0}_{\mu\nu\lambda}(k_1)V^{3\mu\nu}{}_{\alpha\beta\delta\gamma}(k_1,k_2,k_3)\\&\quad=-\chi^{1}_{\mu\nu\alpha\beta\lambda}(k_1,k_2)V^{2\mu\nu}{}_{\delta\gamma}(k_1+k_2,k_3)\\&\qquad-\chi^{1}_{\mu\nu\delta\gamma\lambda}(k_1,k_3)V^{2\mu\nu}{}_{\alpha\beta}(k_1+k_3,k_2)\end{aligned} \tag{A6}$$

$$\begin{aligned}&\frac{1}{\kappa}\chi^{0}_{\mu\nu\lambda}(k_1)V^{4\mu\nu}{}_{\alpha\beta\delta\gamma\tau\sigma}(k_1,k_2,k_3,k_4)\\&\quad=-\chi^{1}_{\mu\nu\alpha\beta\lambda}(k_1,k_2)V^{3\mu\nu}{}_{\delta\gamma\tau\sigma}(k_1+k_2,k_3,k_4)\\&\qquad-\chi^{1}_{\mu\nu\delta\gamma\lambda}(k_1,k_3)V^{3\mu\nu}{}_{\alpha\beta\tau\sigma}(k_1+k_3,k_2,k_4)\\&\qquad-\chi^{1}_{\mu\nu\tau\sigma\lambda}(k_1,k_4)V^{3\mu\nu}{}_{\alpha\beta\delta\gamma}(k_1+k_4,k_2,k_3),\end{aligned} \tag{A7}$$

where

$$\chi^{0}_{\mu\nu\lambda}(k_1)=k_{1\mu}\eta_{\nu\lambda}+k_{1\nu}\eta_{\mu\lambda} \tag{A8}$$

$$\begin{aligned}\chi^{1}_{\mu\nu\alpha\beta\lambda}(k_1,k_2)&=k_{1\nu}\eta_{\alpha\mu}\eta_{\lambda\beta}+k_{1\mu}\eta_{\alpha\lambda}\eta_{\nu\beta}\\&\quad+k_{2\lambda}\eta_{\alpha\mu}\eta_{\beta\nu},\end{aligned} \tag{A9}$$

and the vertices V^n are the momentum space expressions for the nth functional derivatives computed at $\phi_{\mu\nu}=0$ (momentum conservation is understood in all identities).

In the case of a tree-level action there is no one-point "vertex" $V^{1\mu\nu}$ and so the quadratic term satisfies the transversality condition

$$\chi^{0}_{\mu\nu\lambda}(k_1)V^{2\mu\nu}_{\alpha\beta}(k_1,k_2)=0. \tag{A10}$$

This may not be the case for an effective gauge invariant Lagrangian. Indeed, it is well known that the one-graviton function is non-zero at finite temperature.

APPENDIX B: GRAVITATIONAL 'T HOOFT IDENTITIES

The imaginary time formalism at finite temperature follows closely the corresponding formalism at $T=0$. Consequently, the 't Hooft identities at finite T would be identical to the ones at $T=0$, if there were no 1-particle tadpole contributions (such terms vanish at $T=0$ in the dimensional regularization scheme). However, since the tadpole is exactly proportional to T^4, it will not affect the identities involving the sub-leading contributions. To derive these, we start from the action

$$I=\int d^4x d^4y\, \phi_{\mu\nu}(x) S^{\mu\nu\alpha\beta}_{\rm sub}(x-y)\,\phi_{\alpha\beta}(y) + \int d^4x d^4y\, J^{\mu\nu}(x) X_{\mu\nu\lambda}(x-y)\,\eta^{\lambda}(y)+\cdots. \tag{B1}$$

Here $S^{\mu\nu\alpha\beta}_{\rm sub}$ denotes the tree order quadratic term plus the sub-leading contributions to the graviton 2-point function and $X_{\mu\nu\lambda}$ represents the tensor generated by a gauge transformation of the graviton field which is given to lowest order, in the momentum space, by Eq. (A8). $J^{\mu\nu}$ is an external source, η^{λ} represents the ghost field and the $\cdots$ stand for terms which are not relevant for our purpose. The 't Hooft identity involving the graviton self-energy function is a consequence of the Becchi-Rouet-Stora-Tyutin (BRST) invariance of the action I:

$$\int d^4x \frac{\delta I}{\delta J^{\mu\nu}(x)}\frac{\delta I}{\delta \phi_{\mu\nu}(x)}=0. \tag{B2}$$

In general, Eq. (B2) implies the 't Hooft identity

$$X_{\mu\nu\lambda}S^{\mu\nu\alpha\beta}_{\rm sub}=0, \tag{B3}$$

which can be written to second order as

$$X^{(0)}_{\mu\nu\lambda}\Pi^{\mu\nu\alpha\beta}_{\rm sub}=-X^{(1)}_{\mu\nu\lambda}V^{2\mu\nu\alpha\beta}, \tag{B4}$$

where $V^{2\mu\nu\alpha\beta}$ satisfies the identity (A10). Using Eq. (A10) we see that Eq. (B4) leads immediately to the 't Hooft identity

$$\chi^{(0)}_{\mu\nu\lambda}(k)\Pi^{\mu\nu\alpha\beta}_{\rm sub}(k,u)\chi^{(0)}_{\alpha\beta\delta}(k)=0. \tag{B5}$$

TABLE II. Ten independent tensors base.

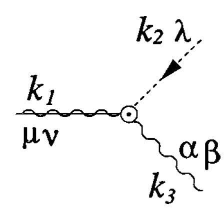
$k_\mu k_\nu k_\lambda$
$u_\mu u_\nu u_\lambda$
$k_\lambda u_\mu u_\nu$
$u_\lambda k_\mu k_\nu$
$k_\mu u_\nu u_\lambda + k_\nu u_\mu u_\lambda$
$u_\mu k_\nu k_\lambda + u_\nu k_\mu k_\lambda$
$\eta_{\mu\nu}k_\lambda$
$\eta_{\mu\nu}u_\lambda$
$k_\mu \eta_{\nu\lambda} + k_\nu \eta_{\mu\lambda}$
$u_\mu \eta_{\nu\lambda} + u_\nu \eta_{\mu\lambda}$

It is straightforward to show that the identity (B5) implies that $\Pi_{\rm sub8}=\Pi_{\rm sub10}=\Pi_{\rm sub11}=\Pi_{\rm sub12}=0$ and so these projections have a temperature behavior which is at most proportional to T^4.

In order to verify (3.12) we need to calculate the tensor $\chi^{(1)}_{\mu\nu\lambda}$ which appears in (B4). In this way we need the source-graviton-ghost vertex which can be obtained from the Lagrangian [28]

$$\mathcal{L}_S=\kappa J^{\mu\nu}D_{\mu\nu\lambda}\epsilon^{\lambda}. \tag{B6}$$

Using the transformation $\delta g_{\mu\nu}=\kappa D_{\mu\nu\lambda}\epsilon^{\lambda}$ we obtain



$$=-i\frac{\kappa}{2}[\eta_{\alpha\lambda}\eta_{\beta\nu}k_{2\mu}+\eta_{\alpha\mu}\eta_{\beta\lambda}k_{2\nu}+\eta_{\alpha\mu}\eta_{\beta\nu}k_{3\lambda}]+\alpha\leftrightarrow\beta. \tag{B7}$$

The diagram in Fig. 3 can now be calculated using vertex (B7) and Feynman rules (2.7), (2.10) and (2.11). Expanding $\chi^{(1)}_{\mu\nu\lambda}$ in the base shown in Table II and using the forward scattering method as we did for the one- and two-graviton functions, we obtain the following *leading* T^2 contribution for $\chi^{(1)}_{\mu\nu\lambda}$:

$$\chi^{(1)}_{\mu\nu\lambda}=\frac{T^2}{18}k_0[\eta_{\lambda\nu}u_\mu+\eta_{\lambda\mu}u_\nu-2u_\mu u_\nu u_\lambda]. \tag{B8}$$

Contracting (B8)) with $V^{2\mu\nu\alpha\beta}$ yields

$$\chi^{(1)}_{\mu\nu\lambda}V^{2\mu\nu\alpha\beta}=\frac{T^2}{18}k_0\{2k\cdot u(k_\alpha u_\beta+k_\beta u_\alpha)+k\cdot u \times(2\eta_{\alpha\beta}k_\lambda-\eta_{\lambda\beta}k_\alpha-\eta_{\lambda\alpha}k_\beta)+k^2(\eta_{\alpha\lambda}u_\beta + \eta_{\beta\lambda}u_\alpha-2u_\alpha u_\beta u_\lambda)-k_\lambda(k_\alpha u_\beta+k_\beta u_\alpha) -2k\cdot u^2\eta_{\alpha\beta}u_\lambda\}. \tag{B9}$$

Using the result for $\Pi_{T^2}^{\mu\nu\alpha\beta}$, which can be obtained inserting Eq. (3.10) into Eq. (3.7), we have verified that the contraction with $\chi^{(0)}_{\mu\nu\lambda}$ yields Eq. (B9) with opposite sign in complete agreement with Eq. (3.12).

APPENDIX C: log T CONTRIBUTIONS FOR ARBITRARY VALUES OF α

In this appendix we complement the result presented in Eq. (3.13) and include the contributions proportional to the gauge parameter α. We have obtained the following results for the 14 projections [see Eq. (3.9)]:

$$\Pi_1^{\log}=\frac{\kappa^2k^4}{15\pi^2}\log(T)\left[\frac{1}{32}(1312y^2-304y-363)-\frac{1}{28}\frac{\alpha}{n_0^2}k^2(4336y^3-1948y^2-1272y+39)\right]$$

$$\Pi_2^{\log}=-\frac{\kappa^2k^4}{15\pi^2}(y-1)\log(T)\left[\frac{1}{16}(8y-113)-\frac{1}{14}\frac{\alpha}{n_0^2}k^2(1332y^2-928y-19)\right]$$

$$\Pi_3^{\log}=\frac{\kappa^2k^4}{15\pi^2}(y-1)^2\log(T)\left[\frac{17}{16}-\frac{1}{28}\frac{\alpha}{n_0^2}k^2(76y+1)\right]$$

$$\Pi_4^{\log}=\frac{\kappa^2k^4}{15\pi^2}\log(T)\left[\frac{1}{32}(352y^2-464y-23)-\frac{1}{7}\frac{\alpha}{n_0^2}k^2(y-1)(1084y^2-243y-1)\right]$$

$$\Pi_5^{\log}=-\frac{\kappa^2k^4}{15\pi^2}(y-1)\log(T)\left[\frac{1}{16}(56y+59)+\frac{1}{7}\frac{\alpha}{n_0^2}k^2(y-1)(174y+1)\right]$$

$$\Pi_6^{\log}=\frac{4\kappa^2k^4}{3\pi^2}\log(T)y(y-1)$$

$$\Pi_7^{\log}=\Pi_8^{\log}=\Pi_9^{\log}=\Pi_{10}^{\log}=\Pi_{11}^{\log}=\Pi_{12}^{\log}=0$$

$$\Pi_{13}^{\log}=\frac{2\kappa^2k^4}{3\pi^2}\log(T)y^2(y-1)$$

$$\Pi_{14}^{\log}=0, \tag{C1}$$

where we are using the quantity $y\equiv k_0^2/k^2$ in order to compare with the zero temperature results of Ref. [8].

In terms of these projections, the coefficients of the transverse traceless components of the graviton self-energy, as defined, for instance, in Ref. [2], can be written as

$$c_{\rm A}=\frac{\kappa^2k^4}{15\pi^2}\log(T)\left[\frac{5}{2}y^2+2y+\frac{19}{64}-\alpha\frac{k_0^2}{n_0^2}\left(\frac{24}{7}+\frac{39}{56}\frac{1}{y}\right)\right]$$

$$c_{\rm B}=\frac{\kappa^2k^4}{15\pi^2}\log(T)\left[5y^2+\frac{1}{8}y-\frac{181}{64}+\alpha\frac{k_0^2}{n_0^2}\left(\frac{135}{7}+\frac{3}{8}\frac{1}{y}-\frac{333y}{14}\right)\right]$$

$$c_{\rm C}=\frac{\kappa^2k^4}{15\pi^2}\log(T)\left[\frac{11}{6}y^2-\frac{25}{6}y-\frac{71}{192}+\alpha\frac{k_0^2}{n_0^2}\left(\frac{53}{21}-\frac{168}{56}\frac{1}{y}+\frac{115}{6}y-\frac{542}{21}y^2\right)\right]. \tag{C2}$$

These expressions show that the dispersion relations associated with the transverse traceless modes will, in general, be gauge dependent at this order of perturbation theory.

APPENDIX D

As an example of the calculation shown in Sec. IV B we will calculate a contribution of the prescription poles for the projection $\Pi_6^{\rm presc}$. This contribution can be written as

$$\Pi_6^{\rm presc}=\frac{\kappa^2}{k_0}T\sum_{q_0}\int\frac{d^{D-1}\vec{q}}{(2\pi)^{D-1}}\frac{1}{2}\frac{(D-3)D}{D-2}\left[\frac{\vec{q}^4+(5-2D)k_0^4+(2D-6)k_0^2(\vec{k}^2+\vec{q}^2)+\vec{k}^4-2\vec{q}^2\vec{k}^2}{q_0(q+k)^2}\right]. \tag{D1}$$

Using the prescription (3.3) and Eq. (2.13) we obtain

$$\Pi_6^{\rm presc}=\frac{\kappa^2}{k_0}\lim_{\mu\to 0}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}\frac{dq_0}{4\pi i}\int\frac{d^{D-1}\vec{q}}{(2\pi i)^{D-1}}\coth\left(\frac{q_0}{2T}\right)\frac{D}{2}\frac{(D-3)}{D-2}\left\{\frac{q_0[\vec{q}^4+(5-2D)k_0^4+(2D-6)k_0^2(\vec{k}^2+\vec{q}^2)+\vec{k}^4-2\vec{q}^2\vec{k}^2]}{(q_0^2-\mu^2)(q+k)^2}\right.$$
$$\left.+q\leftrightarrow -q\right\}. \tag{D2}$$

Closing the integration contour at right-hand side plane and expanding in terms of a power series in μ we obtain

$$\Pi_6^{\rm presc}=\kappa^2T\int\frac{d^{D-1}\vec{q}}{(2\pi i)^{D-1}}\frac{D}{2}\frac{(D-3)}{D-2}\left\{\frac{(2D-6)k_0^2(\vec{q}^2+\vec{k}^2)+(5-2D)k_0^4+(\vec{k}^2-\vec{q}^2)^2}{[k_0^2-(\vec{q}+\vec{k})^2]^2}+\vec{k}\leftrightarrow-\vec{k}\right\}$$
$$=\kappa^2T\int\frac{d^{D-1}\vec{q}}{(2\pi i)^{D-1}}\frac{D(D-3)}{D-2}\left[\frac{-(2D-6)k_4^2(\vec{q}^2+2\vec{k}^2)+(5-2D)k_4^4+\vec{q}^4+4(\vec{k}\cdot\vec{q})^2}{(k_4^2+\vec{q}^2)^2}\right] \tag{D3}$$

where $k_4=ik_0$ and we have performed a shift $\vec{q}\to\vec{q}-\vec{k}$. In the limit $D\to 4$ we obtain

$$\Pi_6^{\rm presc}=\frac{2\kappa^2T}{\pi}|k_4|(|k_4|^2-\vec{k}^2). \tag{D4}$$

The contributions to the projections $\Pi_7^{\rm presc}$, $\Pi_9^{\rm presc}$, $\Pi_{13}^{\rm presc}$ and $\Pi_{14}^{\rm presc}$ are obtained in a similar way and we find that they vanish. However, for the projections $\Pi_1^{\rm presc}$, $\Pi_2^{\rm presc}$, $\Pi_3^{\rm presc}$, $\Pi_4^{\rm presc}$ and $\Pi_5^{\rm presc}$ we have more involved expressions containing inverse powers of T as in Eq. (4.5).

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