D-braneworld cosmology

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We discuss D-braneworld cosmology; that is, the brane is described by the Born-Infeld action. Compared with the usual Randall-Sundrum braneworld cosmology where the brane action is the Nambu-Goto one, we can see some drastic changes in the very early universe: (i) the universe may experience a rapidly accelerating phase; (ii) the closed universe may avoid the initial singularity. We also briefly address the dynamics of the cosmology in the open string metric, which might be more favored than the induced metric from the viewpoint of the D-brane.

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I. INTRODUCTION

One of the motivations for the braneworld model originated through the genius of a D-brane; that is, open strings describing the standard model particles stick to the brane. So if one is serious about this, we must employ the effective action for the D-brane, the Born-Infeld action, not the Nambu-Goto membrane action [1]. In the presence of matter on the brane, a difference between the above two actions will appear.

The simplest braneworld model was proposed by Randall and Sundrum [2,3] and subsequently extended to the cosmology by many peoples [4–6]. However, the action for the brane is often assumed to be the Nambu-Goto action and the action for the matter on the brane is simply added by direct sum. In this paper we explore the cosmology on the D-brane (related to the tachyon condensation; the tachyon matter on the brane has also been considered in the braneworld [7]). As we shall see soon, our starting point is the five-dimensional Einstein-Hilbert action plus the Born-Infeld action (we call this the D-braneworld). Then we consider the radiation dominated universe on the D-brane. In this situation the brane action is described by the Born-Infeld one where the matter term is automatically included.

Here, we recall that there is a nontrivial aspect of the interpretation of the D-brane. According to Seiberg and Witten [8], the metric for the gauge field on the brane is given by $\overset{s}{g}_{\mu\nu}=g_{\mu\nu}-(2\pi\alpha')^2(F^2)_{\mu\nu}$, not just the induced metric $g_{\mu\nu}$. F represents the field strength of the gauge fields on the brane. For this we will discuss the cosmology in both metrics. See Ref. [9] for the causality issue.

Another motivation to think of the D-braneworld is the magnetogenesis in the very early universe (see Ref. [10] for a comprehensive review and references). The coherent magnetic field over the horizon scale currently has the limit $B<10^{-9}$ G from the cosmic microwave background anisotropy [11]. If such a magnetic field with a long coherence length actually exists, we must seek for its primordial origin, especially in the inflating stage of the universe. It is well known that the magnetic field cannot be generated due to the conformal invariance of the Maxwell theory in four dimensions [12]. But the conformal invariance is broken due to the nonlinearity in the Born-Infeld theory. Thus we might be able to have a magnetogenesis scenario during inflation on the D-braneworld.

The rest of this paper is organized as follows. In Sec. II, we describe the setup for the D-braneworld. We also comment on holographic aspects in the D-braneworld. In Sec. III, we focus on the radiation dominated cosmology in the induced metric. First, we average the energy-momentum tensor over the volume to obtain the equation of state, and then we see that the universe may be accelerating at the very early stage and a closed universe could avoid the initial singularities. In Sec. IV, we briefly reconsider the cosmology in the open string metric. In Sec. V, we give a summary and discussion. See Refs. [13–15] for other issues in the Born-Infeld cosmology/black holes and Ref. [16] for the D-brane effect in a different context.

II. SETUP

The total action is composed of the bulk and the D-brane action:

$$S=S_{\rm bulk}+S_{\rm BI}, \tag{1}$$

where $S_{\rm bulk}$ is the five-dimensional Einstein-Hilbert action with a negative cosmological constant and $S_{\rm BI}$ is the Born-Infeld action for a D-brane:

$$S_{\rm BI}=-\sigma\int d^4x\sqrt{-\det[g_{\mu\nu}+(2\pi\alpha')F_{\mu\nu}]}, \tag{2}$$

where $F_{\mu\nu}$ is the field strength of the Maxwell theory on the brane. So $F_{\mu\nu}$ will correspond to the cosmic microwave background radiation if one thinks of a homogeneous and isotropic universe as discussed later. In the Born-Infeld action the matter part is automatically included. In the usual braneworld, on the other hand, we assume that the action on the brane is given by the Nambu-Goto action $S_{\rm NG}\sim-\sigma\int d^4\sqrt{-g}$ plus the matter action $S_{\rm matter}$, in particular, $S_{\rm matter}\propto\int d^4x\sqrt{-g}F^2$ for the Maxwell field on the brane.

In four dimensions $S_{\rm BI}$ becomes

$$\begin{aligned}S_{\rm BI}&=-\sigma\int d^4x\sqrt{-g}\Bigg[1-\frac{1}{2}(2\pi\alpha')^2{\rm Tr}(F^2)\\&\quad+\frac{1}{8}(2\pi\alpha')^4[{\rm Tr}(F^2)]^2-\frac{1}{4}(2\pi\alpha')^4{\rm Tr}(F^4)\Bigg]^{1/2}\\&=-\sigma\int d^4x\sqrt{-g}\Bigg[1-\frac{1}{4}(2\pi\alpha')^2{\rm Tr}(F^2)\\&\quad+\frac{1}{32}(2\pi\alpha')^4[{\rm Tr}(F^2)]^2-\frac{1}{8}(2\pi\alpha')^4{\rm Tr}(F^4)\\&\quad+O(\alpha'^6)\Bigg].\end{aligned}\tag{3}$$

From the first to the second line we expanded the square root and wrote down the expression up to the order of $O(\alpha'^4)$. For practicality and simplicity, hereafter, we employ this approximate action to discuss the *whole* history of cosmology. Since the action for a self-gravitating D-brane is still not known, the present treatment is conservative. The energy-momentum tensor on the brane is given by

$$\begin{aligned}\overset{(\rm BI)}{T}{}_{\mu\nu}&=-\sigma g_{\mu\nu}+4\pi\sigma(2\pi\alpha')^2\overset{(\rm em)}{T}{}_{\mu\nu}\\&\quad+\frac{1}{4}\sigma(2\pi\alpha')^4{\rm Tr}(F^2)\left[(F^2)_{\mu\nu}-\frac{1}{8}g_{\mu\nu}{\rm Tr}(F^2)\right]\\&\quad-\sigma(2\pi\alpha')^4\left[(F^4)_{\mu\nu}-\frac{1}{8}g_{\mu\nu}{\rm Tr}(F^4)\right]\\&=:-\sigma g_{\mu\nu}+T_{\mu\nu},\end{aligned}\tag{4}$$

where

$$\overset{(\rm em)}{T}{}_{\mu\nu}=\frac{1}{4\pi}\left(F^\alpha_\mu F_{\nu\alpha}-\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right).\tag{5}$$

Hereafter we call $T_{\mu\nu}$ the energy-momentum tensor of the Born-Infeld matter. To regard the above $\overset{(\rm em)}{T}{}_{\mu\nu}$ as the energy-momentum tensor of the usual Maxwell field on the brane, we set

$$\sigma(2\pi\alpha')^2=1.\tag{6}$$

At the low energy limit $T_{\mu\nu}$ becomes $T_{\mu\nu}\simeq-\sigma g_{\mu\nu}+\overset{(\rm em)}{T}{}_{\mu\nu}$, which is often used in the usual braneworld.

Since we are interested in what happens on the brane, it is useful to consult the gravitational equation on the brane [4] (see also [5]):

$${}^{(4)}G_{\mu\nu}=8\pi GT_{\mu\nu}+\kappa^4\pi_{\mu\nu}-E_{\mu\nu},\tag{7}$$

where

$$8\pi G=\frac{\kappa^2}{\ell},\tag{8}$$

$$\pi_{\mu\nu}=-\frac{1}{4}T_{\mu\alpha}T^\alpha_\nu+\frac{1}{12}TT_{\mu\nu}+\frac{1}{8}g_{\mu\nu}T^\alpha_\beta T^\beta_\alpha-\frac{1}{24}g_{\mu\nu}T^2,\tag{9}$$

and

$$E_{\mu\nu}=C_{\mu\alpha\nu\beta}n^\alpha n^\beta.\tag{10}$$

In the above we supposed the Randall-Sundrum fine-tuning, that is,

$$\frac{1}{\ell}=\frac{\kappa^2}{6}\sigma,\tag{11}$$

where ℓ is the curvature length of the five dimensional anti-de Sitter spacetime. Under this tuning, the net cosmological constant on the brane vanishes. We stress, however, that the tuning is not necessary for the discussion in this paper.

In general the above system is not closed on the brane except for the homogeneous isotropic universe due to the presence of a part of the five dimensional Weyl tensor $E_{\mu\nu}$. In the weak field limit, we can check that the four dimensional Einstein gravity can be recovered [17].

Finally, it is worth noting that the trace part of $\pi_{\mu\nu}$ is related to the trace part of $T_{\mu\nu}$ as

$$-\frac{\kappa^4}{3}\pi^\mu_\mu=\frac{\kappa^2}{\ell}T^\mu_\mu=-\frac{\kappa^4}{12}\left[{\rm Tr}(F^4)-\frac{1}{4}[{\rm Tr}(F^2)]^2\right].\tag{12}$$

This is a realization of holography in the braneworld, that is, π^μ_μ represents a part of the quantum correction to the electromagnetic field theory. This is because the Born-Infeld theory is a sort of phenomenological theory for quantum electrodynamics [18]. In some cases we can show that π^μ_μ is identical to the trace anomaly of the quantum field theory on the brane [19,20].

III. COSMOLOGICAL MODELS

Let us focus on a homogeneous and isotropic universe. For simplicity, we consider the single brane model. Then the metric on the brane is

$$ds^2=-dt^2+a^2(t)\gamma_{ij}dx^idx^j,\tag{13}$$

where γ_{ij} is the metric of a three dimensional unit sphere or unit hyperboloid or flat space. In this case we know $E^\mu_\nu=a^{-4}{\rm diag}(3\mu,-\mu,-\mu,-\mu)$ and μ is proportional to the mass of the five dimensional Schwarzschild–anti-de Sitter

spacetime, which is the bulk geometry. Moreover, the gravitational equation is closed on the brane, that is, it is completely written in terms of the four dimensional quantities on the brane. From now on we set $\mu=0$ which means that the bulk geometry is exactly the anti–de Sitter spacetime.[1] Thus, the modified Friedmann equation becomes

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{\kappa^2}{3\ell}\rho_{\rm BI}+\frac{\kappa^4}{36}\rho_{\rm BI}^2-\frac{K}{a^2} \tag{14}$$

and

$$\frac{\ddot{a}}{a}=-\frac{\kappa^2}{6\ell}(\rho_{\rm BI}+3P_{\rm BI})-\frac{\kappa^4}{36}\rho_{\rm BI}(2\rho_{\rm BI}+3P_{\rm BI}). \tag{15}$$

By defining the electric and magnetic fields by

$$E^i=F^i_0 \quad \text{and} \quad B^i=\frac{1}{2}\epsilon^{ijk}F_{jk}, \tag{16}$$

$T_{\mu\nu}$ can be rewritten as

$$T_{00}=\frac{1}{2}(E^2+B^2)+\frac{\kappa^2\ell}{12}(E_iB^i)^2 + \frac{\kappa^2\ell}{12}(E^2-B^2)\left[E^2-\frac{1}{4}(E^2-B^2)\right], \tag{17}$$

$$T_{0i}=\epsilon_{ijk}E^jB^k\left[1+\frac{\kappa^2\ell}{12}(E^2-B^2)\right], \tag{18}$$

and

$$T_{ij}=-\left[E_iE_j+B_iB_j-\frac{1}{2}g_{ij}(E^2+B^2)\right]-\frac{\kappa^2\ell}{12}(E^2-B^2) \times\left[E_iE_j+B_iB_j-B^2g_{ij}-\frac{1}{4}g_{ij}(E^2-B^2)\right] -\frac{\kappa^2\ell}{12}g_{ij}(E_kB^k)^2. \tag{19}$$

Since we identify the Maxwell field as the background radiation, the energy density and the pressure of the Born-Infeld matter should be evaluated by averaging over volume as[2]

$$\rho_{\rm BI}:=\langle T_{00}\rangle =\frac{1}{2}(E^2+B^2)+\frac{\kappa^2\ell}{36}B^2E^2 + \frac{\kappa^2\ell}{12}(E^2-B^2)\left[E^2-\frac{1}{4}(E^2-B^2)\right] \tag{20}$$

and

$$P_{\rm BI}:=\frac{1}{3}\langle T^i_i\rangle =\frac{1}{6}(E^2+B^2)-\frac{\kappa^2\ell}{36}B^2E^2 - \frac{\kappa^2\ell}{144}(E^2-B^2)(E^2-5B^2). \tag{21}$$

We assumed that E_i and B_i are random fields and the coherence lengths are much shorter than the cosmological horizon scales. In the above we assumed $\langle E_iE_j\rangle=(1/3)g_{ij}E^2$, $\langle B_iB_j\rangle=(1/3)g_{ij}B^2$, $\langle E_i\rangle=\langle B_i\rangle=0$, and $\langle E_iB_j\rangle=0$, which are natural in a homogeneous and isotropic universe. In addition, it is natural to assume "equipartition"[3]

$$E^2(t)=B^2(t)=:\epsilon. \tag{22}$$

Thus the energy density and the pressure are simply given by

$$\rho_{\rm BI}=\epsilon+\frac{\kappa^2\ell}{36}\epsilon^2 \tag{23}$$

and

$$P_{\rm BI}=\frac{1}{3}\epsilon-\frac{\kappa^2\ell}{36}\epsilon^2. \tag{24}$$

By combining Eqs. (23) and (24), we obtain the equation of state

$$P_{\rm BI}=(\gamma_{BI}-1)\rho_{BI}, \tag{25}$$

with an effective adiabatic index

$$\gamma_{BI}=\frac{4}{3}\frac{1}{1+(\kappa^2\ell/36)\epsilon}. \tag{26}$$

At a low energy scale such $\epsilon\ll 36/\kappa^2\ell$, the Born-Infeld matter behaves just as a radiation fluid with $\gamma_{\rm BI}\sim 4/3$ and $\rho_{\rm BI}$

[1] If a deviation from the anti–de Sitter or Schwarzschild–anti–de Sitter spacetime occurs, the gravitational equation is not closed on the brane [4].

[2] If we consider full order terms of α', the averaged energy-momentum tensor is written by an infinite number of terms and cannot be represented by elementary functions. This is because averaging is not commutative with the expansion in the operation. As stressed before, we will not consider all the terms because we do not know the action for the self-gravitating D-brane. The action that we know is only for probe one.

[3] Properly speaking, we must confirm this following the process to the equilibrium state based on the Boltzmann-like equation. If $F_{\mu\nu}$ corresponds to the primordial magnetic field, it is natural to assume $E^2=0$ and $B^2=2\epsilon\neq 0$. In this situation, the energy density and pressure are given by $\rho_{\rm BI}=\epsilon-(\kappa^2\ell/12)\epsilon^2$ and $P_{\rm BI}=1/3\epsilon-(5\kappa^2\ell/36)\epsilon^2$. In Ref. [13] a similar Born-Infeld fluid was considered, but the authors did not considered the D-braneworld, just cosmology with a nonlinear Maxwell field.

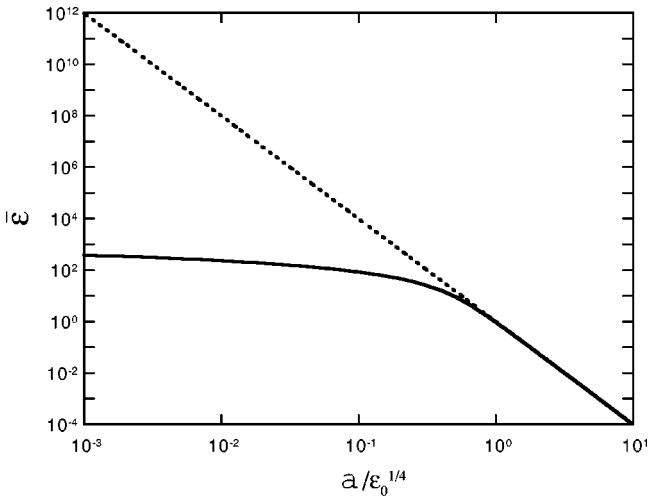

FIG. 1. The log-plot of ϵ. The horizontal axis is the scale factor. The dotted line is the case of the ordinary radiation fluid. The solid line is the case of the Born-Infeld matter.

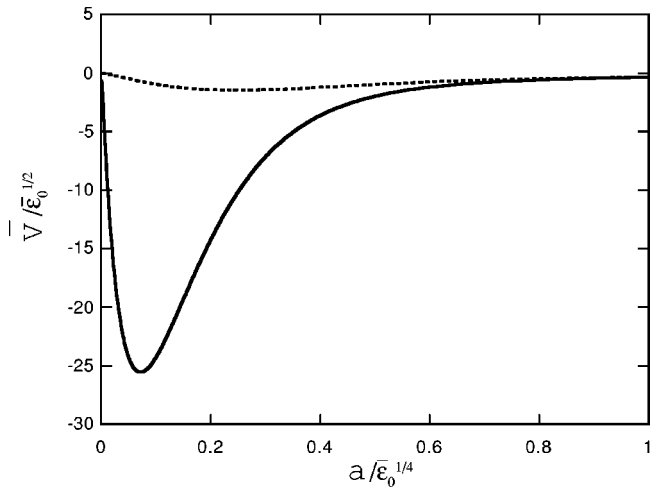

FIG. 2. The potential profile of $\bar{V}(a) \equiv V\ell^2$. The potential has a minimum and vanishes at $a=0$. We also plot the four-dimensional case with the BI matter (dashed line).

$\sim(1/3)P_{\rm BI}$. Note that the Born-Infeld matter looks like a time-dependent cosmological constant if the second terms are dominant, i.e., $\gamma_{\rm BI}\sim 0$.

Now we have the equations of motion (14) and (15) and the equation of state (25) for the Born-Infeld matter. It should be noted that these equations can be scaled by defining new variables $\bar{t} \equiv t/\ell$, $\bar{K} \equiv K\ell^2$, $\bar{\epsilon} \equiv \kappa^2\ell\epsilon$, $\bar{\rho}_{\rm BI} \equiv \kappa^2\ell\rho_{\rm BI}$, and $\bar{P}_{\rm BI} \equiv \kappa^2\ell P_{\rm BI}$. We can see the scale factor dependence of the energy density using the energy-conservation law $\dot{\rho}_{\rm BI}+3H(\rho_{\rm BI}+P_{\rm BI})=0$ on the brane:

$$\left(1+\frac{\kappa^2\ell}{18}\epsilon\right)\dot{\epsilon}=-4H\epsilon. \tag{27}$$

It is easy to integrate the above equation and then

$$\epsilon e^{(\kappa^2\ell/18)\epsilon}=\frac{\epsilon_0}{a^4}. \tag{28}$$

See Fig. 1 for ϵ. Therein we also draw the ordinary radiation case of $\epsilon\propto a^{-4}$. In the early universe, the Born-Infeld matter is significantly suppressed compared to the ordinary radiation fluid. We would stress that the drastic changes from the ordinary braneworld come from these features of the equation of state and the scale factor dependence of the energy density ϵ. For example, if one considers the radiation dominated cosmology in the ordinary braneworld, the correction terms to the Friedmann equation will be given by $\pi^{\mu}_{\nu}=(\rho^2/12)(-1,1,1,1)$ and it will not play a role as a vacuum energy.

Here note that the matter does not always satisfy the "strong energy condition" $\rho+3P\geqslant 0$. For the D-brane matter,

$$\rho_{\rm BI}+3P_{\rm BI}=2\epsilon-\frac{\kappa^2\ell}{18}\epsilon^2 \tag{29}$$

and

$$2\rho_{\rm BI}+3P_{\rm BI}=3\epsilon-\frac{\kappa^2\ell}{36}\epsilon^2. \tag{30}$$

When $\gamma_{BI}<2/3$, i.e., $\epsilon>36/\kappa^2\ell\sim(10^3\ {\rm GeV})^4(M_5/10^8\ {\rm GeV})^6$, the "strong energy condition" is presumably broken.[4] Thus the universe is accelerating during this period, $\ddot{a}/a>0$. Furthermore, we have the opportunity that the initial singularity can be avoided.

To see the quantitative features of the dynamics of the cosmology it is useful to write down the generalized Friedmann equation as usual:

$$\dot{a}^2+V(a)=-K, \tag{31}$$

where

$$V(a) \equiv -\frac{\kappa^2}{3\ell}a^2\rho_{\rm BI}\left(1+\frac{\kappa^2\ell}{12}\rho_{\rm BI}\right). \tag{32}$$

See Fig. 2 for the potential profile. Surprisingly, there is a minimum. In addition, as shown analytically, the potential is zero at $a=0$. Let us look around $a=0$. ϵ can be approximately solved as

$$\frac{\kappa^2\ell}{18}\epsilon\sim -4\log a. \tag{33}$$

We can see that the potential, indeed, is zero at $a=0$ as seen in Fig. 2. In the current approximation, we see that

[4] M_5 is the fundamental scale of five dimensional gravity and thus $M_5\sim\kappa^{-1/3}\sim(M_{\rm pl}^2\ell^{-1})^{1/3}\sim 10^8(1\ {\rm mm}/\ell)^{1/3}\ {\rm GeV}$ in the single brane models. The string length becomes $\ell_s=\sqrt{\alpha'}\sim\sigma^{-1/4}\sim(\ell_{\rm pl}\ell)^{1/2}\sim 10^{-17}\times(\ell/1\ {\rm mm})^{1/2}$ cm.

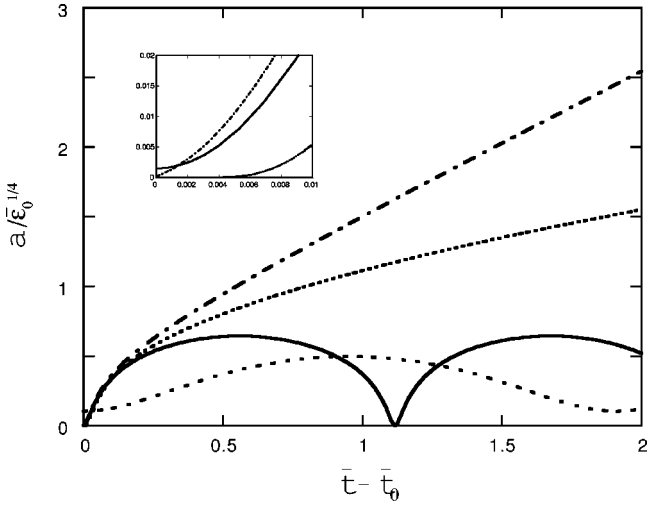

FIG. 3. The behavior of the scale factors of the closed (solid line), flat (dashed line), and open (dot-dashed line) cases in the D-braneworld. We also plot the four-dimensional case with the BI matter for the closed universe (dotted line).

$$V(a)\sim -\frac{576}{\ell^2}a^2(\log a)^4\to 0 \quad (a\to 0). \tag{34}$$

As a result the closed universe is bouncing around $a=0$. The flat or open universe has an initial singularity. Near $a=0$, the behavior of the scale factor in the flat universe becomes

$$a(t)\sim e^{-\ell/24t}, \tag{35}$$

and then $\ddot{a}/a\sim(\ell^2/576)(1/t^2)>0$, that is, the universe is accelerating. The bouncing behavior of the closed universe is owing to the existence of the BI field, as we can observe a similar form of the potential without the quadratic terms π^μ_ν in Fig. 2. As we will see soon, however, the quantative behavior of the scale factor is quite different especially around the bounce, where the energy density becomes large.

For $\epsilon\ll 36/\kappa^2\ell$, as it should be, the universe is described by the ordinary radiation dominated model. See Fig. 3 for the behavior of the scale factor.

IV. COSMOLOGY IN STRINGY VIEW

So far we investigated the D-braneworld in terms of the induced metric $g_{\mu\nu}$. For a gauge field on the brane like photons, however, the propagation of the field is described by the metric

$$\overset{s}{g}_{\mu\nu}=g_{\mu\nu}-(2\pi\alpha')^2(F^2)_{\mu\nu}. \tag{36}$$

Hence it is fair to consider the gravitational equation and cosmology in terms of $\overset{s}{g}_{\mu\nu}$. In the case of the radiation dominated universe, the corresponding metric is given by

$$\begin{aligned} d\tilde{s}^2&=\overset{s}{g}_{\mu\nu}dx^\mu dx^\nu \\ &=-d\tilde{t}^2+\tilde{a}(\tilde{t})^2\gamma_{ij}dx^i dx^j, \\ &=-[1+(2\pi\alpha')^2\langle(F^2)_{00}\rangle]dt^2 \\ &\quad+[g_{ij}-(2\pi\alpha')^2\langle(F^2)_{ij}\rangle]dx^i dx^j, \\ &=-\left(1-\frac{\kappa^2\ell}{6}\epsilon\right)dt^2+a^2\left(1+\frac{\kappa^2\ell}{18}\epsilon\right)\gamma_{ij}dx^i dx^j, \end{aligned} \tag{37}$$

where $\tilde{t}$ and $\tilde{a}(\tilde{t})$ are the cosmic time and the scale factor of the open string metric, respectively. As discussed in Ref. [9], the light cone with respect to the open string metric is smaller than that in the induced metric. Let n^μ to be null vector for the induced metric. For concreteness, $n=\partial_t+(1/a)\partial_x$. Then

$$\begin{aligned} \overset{s}{g}_{\mu\nu}n^\mu n^\nu&=-(2\pi\alpha')^2(F^2)_{\mu\nu}n^\mu n^\nu \\ &=\frac{2\kappa^2\ell}{9}\epsilon>0, \end{aligned} \tag{38}$$

and this means that n is spacelike in the open string metric.

We point out that the singularity appears at a *finite* value of $\epsilon=\epsilon_c:=12/\kappa^2\ell$. The physical energy measured also diverges because it is proportional to $1/\sqrt{1-\kappa^2\ell\epsilon/12}$. Anyway the universe evolves, keeping $\epsilon<\epsilon_c$. This means that there are no drastic changes in the open string metric, but slight modifications from the ordinary radiation dominated universe.

At first glance, we cannot examine the interesting region where the feature of the Born-Infeld action becomes essential. However, it might be better to say that this is because of the limitation of the stringy metric.

V. SUMMARY AND DISCUSSION

In this paper we have considered the Randall-Sundrum D-braneworld cosmology. Therein the matter on the brane is described by the Born-Infeld action. As a first step, we considered only the $U(1)$ gauge field and treated it as a sort of radiation fluid. Then we examined the radiation dominated universe on the D-brane. We found that the strong energy condition is broken in the very early stage and the universe is more accelerated than in ordinary inflation. Furthermore, the initial singularity is avoided in a closed universes. Thus we can conclude that we have a different history for the early stage from the ordinary braneworld scenario if we are living on the D-brane, not on the Nambu-Goto membrane. In the acceleration phase, the ratio of the two scale factors at different times is given by $a(t_f)/a(t_i)\sim e^{(\kappa^2\ell/72)(\epsilon_i-\epsilon_f)}$. For the horizon problem $a(t_f)/a(t_i)>10^{13}\times(100\ \mathrm{km/s/Mpc}/H_0)\times(T/10^3\ \mathrm{GeV})$ is required. Then if $(\kappa^2\ell/72)\Delta\epsilon>30$ the horizon problem is solved. However, the origin of the density fluctuation is not provided in this scenario.

We should remark that we employed an approximate action in the second line of Eq. (3). We discussed the high

energy regime where the expansion is broken and the approximate action may not be appropriate. Without such approximation, however, we must treat an infinite series expansion due to the volume averaging. Although our treatment contains this kind of problem, we could obtain the important trends. Future improvement for the treatment of the higher derivative terms is desirable.

We also addressed the D-braneworld in the open string metric, that is, the *stringy view*. As a result, the singularity appears just at the time when the nontrivial effects of the D-brane are dominant. In this sense we might not be able to expect a significant contribution from the D-braneworld specialties in the stringy view.

Since we obtained the result that the Born-Infeld matter contains a vacuum/dark energy part, the current accelerating universe can be explained in the D-braneworld without introducing additional exotic fields like quintessence.

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