Angular conditions, relations between the Breit and light-front frames, and subleading power corrections

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We analyze the current matrix elements in the general collinear (Breit) frames and find the relation between the ordinary (or canonical) helicity amplitudes and the light-front helicity amplitudes. Using the conservation of angular momentum, we derive a general angular condition which should be satisfied by the light-front helicity amplitudes for any spin system. In addition, we obtain the light-front parity and time-reversal relations for the light-front helicity amplitudes. Applying these relations to the spin-1 form factor analysis, we note that the general angular condition relating the five helicity amplitudes is reduced to the usual angular condition relating the four helicity amplitudes due to the light-front time-reversal condition. We make some comments on the consequences of the angular condition for the analysis of the high-Q^2 deuteron electromagnetic form factors, and we further apply the general angular condition to the electromagnetic transition between spin-1/2 and spin-3/2 systems and find a relation useful for the analysis of the N-Δ transition form factors. We also discuss the scaling law and the subleading power corrections in the Breit and light-front frames.

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I. INTRODUCTION

A relativistic treatment is one of the essential ingredients that should be incorporated in describing hadronic systems. The hadrons have an intrinsically relativistic nature since the quantum chromodynamics (QCD) governing the quarks and gluons inside the hadrons has *a priori* a strong interaction coupling and the characteristic momenta of quarks and gluons are of the same order, or even very much larger, than the masses of the particles involved. It has also been realized that a parametrization of nuclear reactions in terms of nonrelativistic wave functions must fail. In principle, a manifestly covariant framework such as the Bethe-Salpeter approach and its covariant equivalents can be taken for the description of hadrons. However, in practice, such tools are intractable because of the relative time dependence and the difficulty of systematically including higher order kernels. A different and more intuitive framework is the relativistic Hamiltonian approach. With the recent advances in the Hamiltonian renormalization program, a promising technique to impose the relativistic treatment of hadrons appears to be light-front dynamics (LFD), in which a Fock-space expansion of bound states is made at equal light-front time $\tau = t + z/c$. The reasons that make LFD so attractive to solve bound-state problems in field theory make it also useful for a relativistic description of nuclear systems.

Light-front quantization [1,2] has already been applied successfully in the context of current algebra [3] and the parton model [4] in the past. For the analysis of exclusive processes involving hadrons, the framework of light-front (LF) quantization [5] is also one of the most popular formulations. In particular, the light-front or Drell-Yan-West ($q^+ = q^0 + q^3 = 0$) frame has been extensively used in the calculation of various electroweak form factors and decay processes [6–8]. In this frame [9], one can derive a first-principle formulation for the exclusive amplitudes by choosing judiciously the component of the light-front current. As an example, only the parton-number-conserving (valence) Fock state contribution is needed in the $q^+ = 0$ frame when a ''good'' component of the current, J^+ or $\mathbf{J}_\perp = (J_x, J_y)$, is used for the spacelike electromagnetic form factor calculation of pseudoscalar mesons. One does not need to suffer from complicated vacuum fluctuations in the equal τ formulation due to the rational dispersion relation. The zero-mode contribution may also be avoided in the Drell-Yan-West (DYW) frame by using the plus component of current [10]. The perturbative QCD (PQCD) factorization theorem for the exclusive amplitudes at asymptotically large momentum transfer can also be proved in LFD formulated in the DYW frame.

However, caution is needed in applying the established Drell-Yan-West formalism to other frames because the current components do mix under the transformation of the reference frame [11]. Especially for the spin systems, the light-front helicity states are in general different from the ordinary (or canonical) helicity states which may be more appropriate degrees of freedom to discuss the angular momentum conservation. As the spin of the system becomes larger, the number of current matrix elements gets larger than the number of physical form factors and the conditions that the current matrix elements must satisfy are essential to test the underlying theoretical model for the hadrons. Thus, it is crucial to find the relations between the ordinary helicity amplitudes and

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the light-front helicity amplitudes in the frame that they are computed.

In this work we use the general collinear frames which cover both Breit and target-rest frames to find the relations between the ordinary helicity amplitudes and the light-front helicity amplitudes. Using the conservation of angular momentum, we derive a general angular condition which can be applied for any spin system. The relations among the light-front helicity amplitudes are further constrained by the light-front parity and time-reversal consideration. For example, the spin-1 form factor analysis requires in general nine light-front helicity amplitudes although there are only three physical form factors. Thus, there must be six conditions for the helicity amplitudes. Using the light-front parity relation, one can reduce the number of helicity amplitudes down to five. The general angular condition gives one relation among the five light-front helicity amplitudes, leaving four of them independent. One more relation comes by applying the light-front time-reversal relation, also having the effect that the general angular condition can be reduced to the usual angular condition relating only four helicity amplitudes. Consequently, only three helicity amplitudes are independent of each other, as it should be because there are only three physical form factors in spin-1 systems. We also apply the general angular condition to the electromagnetic transition between spin-1/2 and spin-3/2 systems and find the relation among the helicity amplitudes that can be used in the analysis of the N-Δ transition. In particular, the angular condition provides a strong constraint to the N-Δ transition, indicating that the suppression of the helicity flip amplitude with respect to the helicity non-flip amplitude for the momentum transfer Q in PQCD is in an order of m/Q or M/Q rather than Λ_{QCD}/Q, where both nucleon mass m and delta mass M are much larger than the QCD scale Λ_{QCD}. Thus, one may expect that the applicability of leading PQCD could be postponed to a larger Q^2 region than one may naively anticipate from leading PQCD. The same consideration can apply for the deuteron form factor analysis from the spin-1 angular condition. This work presents further discussions on the scaling law and the subleading power corrections in the Breit and light-front frames.

The paper is organized as follows. In the next section (Sec. II), we present the derivation of transformation laws between the ordinary helicity amplitudes and the light-front helicity amplitudes and obtain a general angular condition on the current matrix elements using the rotational covariance of the current operator. Since we start from the definition of states in a general collinear frame, our derivation may be more physically transparent than any other formal derivation. In Sec. III we present the light-front discrete symmetries and derive the parity and time-reversal rules for the helicity amplitudes. In Sec. IV we discuss the consequences from these findings of the general angular condition and the light-front discrete symmetry relations. The reduction of the number of independent helicity amplitudes is shown for a few example spin systems. The current matrix elements of the spin-1 system and the spin-1/2 to spin-3/2 transition are shown as explicit examples. The subleading power corrections are obtained from the general angular condition and the scaling laws are derived for the ordinary Breit frame helicity amplitudes and the light-front helicity amplitudes. A summary and conclusion follow in Sec. V.

II. FRAME RELATIONS AND GENERAL ANGULAR CONDITION

Our subject is relations among matrix elements or helicity amplitudes for the process $\gamma^*(q)+h(p)\to h'(p')$, where γ^* is an off-shell photon of momentum q, and h and h' are hadrons with momenta p and p', respectively. (Results will be easily extendable for other incoming vector bosons.)

Calculations may be done in the light-front frame, which is characterized by having $q^+\equiv q^0+q^3=0$, and may be done in the Breit frame, which is characterized by having the photon and hadron 3-momenta along a single line. Each frame has its advantages. In the light front frame with $q^+=0$, and for matrix elements of the current component J^+, the photon only couples to forward moving constituents (quarks) of the hadrons and never produces a quark-antiquark pair. Thus one only needs wave functions for hadrons turning into constituents going forward in (light-front) time, and can develop a simple parton picture of the interaction. On the other hand, the Breit frame, being a collinear frame, makes it easy to add up the helicities of the incoming and outgoing particles and to count the number of independent non-zero amplitudes.

By transforming efficiently back and forth one can realize the advantages of both frames. Hence our first goal in this section will be to find the relations between the light-front and Breit frame helicity amplitudes, and then to use those relations to derive in a transparent way the general relation among the light-front amplitudes that is usually referred to as the ''angular condition.''

A. Relations among helicity amplitudes

Connecting light-front and Breit helicity amplitudes is facilitated by finding frames that are both simultaneously realized. One excellent and easy example is the particular light-front frame where the target is at rest. This is also a Breit frame, since with the target 3-momentum zero, the remaining momenta must lie along the same line. We are perhaps extending the idea of a Breit frame, but are doing so in a way that leaves invariant the Breit frame helicity amplitude. That is, one normally thinks of a Breit frame as one where the incoming and outgoing hadron have oppositely directed momenta, along the same line. Sometimes one specifies that the line is the z axis. However, since helicities are unaffected by rotations [12], one can choose any line at all. Further, helicities are unaffected by collinear boosts [12] that do not change the particle's momentum direction. One can also boost along the direction of motion until one of the hadrons is at rest, provided one defines the positive helicity direction for the particle at rest to be parallel to the momentum the particle would have in a conventional Breit frame. With this natural helicity direction choice, the Breit frame helicity amplitude in a target rest frame is (if we use relativistic normalization conventions, as we shall always do) precisely the

same as the Breit frame amplitude in a conventional Breit frame with the same helicity labels.

Thus, the light-front frame with the target at rest is both a Breit frame and a light-front frame. There is a continuum of such frames. Another useful example is a Breit frame with the incoming and outgoing hadrons moving in the negative and positive x directions, adjusted to have equal incoming and outgoing energies. In this case, q^0 and q^3 are individually zero, so that $q^+=0$ and we have also a light-front frame.

Even in a frame that is simultaneously light-front and Breit, the connection between the two types of helicity amplitudes can be a bit involved. This is because the definitions of the light-front and ordinary helicity states are not the same and the general conversion between them for a moving state involves a rotation by an angle that is not trivial to determine.

Our plan will be to use the rest of this section to define our notation, state the main result for the light-front to Breit and vice-versa helicity amplitude conversion formulas, and show how one obtains the general angular condition from this result. Then in the next section we will give the details of the derivation.

For light-front amplitudes one uses light-front helicity states, which for a momentum p are defined by taking a state at rest with the spin projection along the z direction equal to the desired helicity, then boosting in the z direction to get the desired p^+, and then doing a light-front transverse boost to get the desired transverse momentum $p_\perp$. We call this state

$$|p,\lambda\rangle_L, \tag{2.1}$$

and it is defined by formula in the next section. The spin of the particle, j, is understood but not usually written, λ is the light-front helicity of the particle, and the normalization is

$$\begin{aligned}&{}_L\langle p_2,\lambda_2|p_1,\lambda_1\rangle_L\\&\quad=(2\pi)^3 2p_1^+\delta(p_1^+-p_2^+)\delta^2(p_{2\perp}-p_{1\perp})\delta_{\lambda_1\lambda_2}.\end{aligned} \tag{2.2}$$

The light-front helicity amplitude G_L is a matrix element of the electromagnetic current J^ν given by

$$G^\nu_{L\lambda'\lambda}={}_L\langle p',\lambda'|J^\nu|p,\lambda\rangle_L. \tag{2.3}$$

In the Breit frame, we use ordinary helicity states, which are defined by starting with a state at rest having a spin projection along the z direction equal to the desired helicity, then boosting in the z direction to get the desired $|\vec{p}|$, and then rotating to get the momentum and spin projection in the desired direction. (We shall generally keep our momenta in the x-z plane, so we do not need to worry about the distinction between, for example, the Jacob-Wick [12] helicity states and the somewhat later Wick states [13].) The state will be denoted

$$|p,\mu\rangle_B, \tag{2.4}$$

where μ is the helicity or spin projection in the direction of motion, and the subscript "B" reminds us which frame we use these states in. Except for momenta directly along the z axis, the light-front helicity and regular helicity states are not the same, but if the 4-momenta are the same they can be related by a rotation. The Breit frame helicity amplitude G_B is

$$G^\nu_{B\mu'\mu}={}_B\langle p',\mu'|J^\nu|p,\mu\rangle_B, \tag{2.5}$$

where J^ν is the same electromagnetic current.

A main result is the relation between G_L and G_B, which is

$$G^\nu_{B\mu'\mu}=d^{j'}_{\lambda'\mu'}(-\theta')G^\nu_{L\lambda'\lambda}d^j_{\mu\lambda}(\theta). \tag{2.6}$$

A sum on repeated helicity indices is implied. The d functions are the usual representations of rotations about the y axis for particles whose spins are given by the superscript. The angles are given by

$$\tan\frac{\theta}{2}=\frac{Q_+Q_--Q^2-M^2+m^2}{2mQ} \tag{2.7}$$

and

$$\tan\frac{\theta'}{2}=-\frac{Q_+Q_--Q^2+M^2-m^2}{2MQ}, \tag{2.8}$$

where m is the mass of the incoming hadron, M is the mass of the outgoing hadron, $Q=\sqrt{Q^2}$,

$$Q^2=-q^2=-(p-p')^2, \tag{2.9}$$

and

$$Q_\pm=[Q^2+(M\pm m)^2]^{1/2}, \tag{2.10}$$

and we assume that q^2 is spacelike (negative, in our metric). For the elastic case, $M=m$, the angles θ and $-\theta'$ are the same. [It may seem peculiar to have a minus sign inserted twice, as it appears in Eqs. (2.6) and (2.8), but it will seem more sensible when one sees how the angles arise, in the next section.]

In the Breit frame, since it is collinear, the sum of spin projections along the direction of motion must be conserved, so that if λ_γ is the helicity of the photon,

$$\lambda_\gamma=\mu+\mu'. \tag{2.11}$$

Even if the photon is off-shell, it cannot have more than one unit of helicity in magnitude. Hence there is a constraint on the Breit frame helicity amplitude,

$$G^\nu_{B\mu'\mu}=0 \quad \text{if} \quad |\mu'+\mu|\geqslant 2. \tag{2.12}$$

This induces a constraint on the light-front amplitudes, and this constraint is what is called the angular condition, given generally by [14]

$$d^{j'}_{\lambda'\mu'}(-\theta')G^\nu_{L\lambda'\lambda}d^j_{\mu\lambda}(\theta)=0 \quad \text{if} \quad |\mu'+\mu|\geqslant 2, \tag{2.13}$$

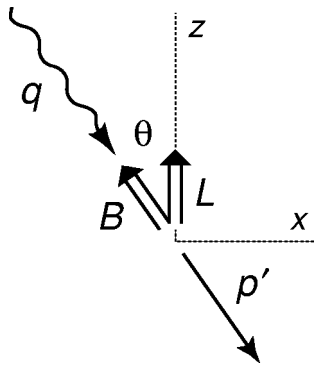

FIG. 1. Photon with $q^+=0$ absorbed on particle at rest. Two choices for the target spin axis are indicated. In the Breit frame (B), the helicity state positive direction is opposite to the direction of the entering photon. The light-front state (L), in this case, is identical to the rest state quantized along the positive z axis. The angle θ between the photon 3-momentum direction and the (negative) z axis is also the angle between the two choices of spin quantization axis.

and most often applied when the Lorentz index ν is $+$. One sees that the result follows from angular momentum conservation and the limited helicity of the photon. We will check in Sec. IV that upon expressing the d functions in terms of Q^2 and mass, one obtains the known angular condition for electron-deuteron elastic scattering, and we will also obtain in terms of masses and Q^2 the angular condition for the N-$\Delta(1232)$ electromagnetic transition.

B. Deriving the light-front to Breit relation

We gave two examples of frames that were simultaneously Breit and light-front frames. It turns out that half the work we need to do is very easy in one of these frames, and that visualizing one ensuing equality is quite easy in the other.

Clearly, one can write

$$G^{\nu}_{B\mu'\mu} = {}_B\langle p',\mu'|p',\lambda'\rangle_L G^{\nu}_{L\lambda'\lambda\,L}\langle p,\lambda|p,\mu\rangle_B, \tag{2.14}$$

so that the problem reduces to finding the overlaps of the light-front helicity and ordinary helicity amplitudes.

We shall start using the light-front frame with the target at rest. For the initial state, being at rest, the light-front helicity state is identical to the state with spin quantized in the positive z direction,

$$|p,\lambda\rangle_L = |\text{rest},\lambda\rangle_z. \tag{2.15}$$

The helicity state, however, should be quantized along a direction antiparallel to the momentum of the entering photon; see Fig. 1. The photon four-momentum is

$$q=(q^+,q^-,q_\perp)=\left(0,\frac{Q^2+M^2-m^2}{m},Q\right), \tag{2.16}$$

and it makes an angle θ with the z axis, where

$$\tan\theta = \frac{2mQ}{Q^2+M^2-m^2}, \tag{2.17}$$

equivalent to the half-angle version given earlier, Eq. (2.7). With θ taken positive, it is also the rotation angle from the Breit frame helicity state to the light-front state,

$$|p,\mu\rangle_L = R_y(\theta)|p,\mu\rangle_B, \tag{2.18}$$

which leads to

$${}_L\langle p,\lambda|p,\mu\rangle_B = d^j_{\lambda\mu}(-\theta) = d^j_{\mu\lambda}(\theta). \tag{2.19}$$

For the outgoing hadron, the helicity state is (see Fig. 1 to get the angle),

$$|p',\mu'\rangle_B = R_y(\pi-\theta)e^{-iK_3\xi}|\text{rest},\mu'\rangle_z, \tag{2.20}$$

where K_3 is the boost operator for the z direction and ξ is a rapidity given in terms of the energy E' of the outgoing hadron,

$$\xi = \operatorname{arccosh}\frac{E'}{M} = \operatorname{arccosh}\frac{Q^2+M^2+m^2}{2mM}. \tag{2.21}$$

The light-front state is given by first boosting to the correct $(p')^+=m$ for outgoing hadron (a boost in the negative z direction, if $Q\neq 0$), followed by a boost to get the correct transverse momentum, which is the same as for the photon. One has

$$|p',\lambda'\rangle_L = e^{-iQE_1/m}e^{-iK_3\xi'}|\text{rest},\lambda'\rangle_z, \tag{2.22}$$

where

$$\xi' = -\operatorname{arccosh}\frac{M^2+m^2}{2mM} \tag{2.23}$$

and E_1 is the light-front transverse boost

$$E_1 = K_1 + J_2. \tag{2.24}$$

Thus the overlap is

$$\begin{aligned} &{}_B\langle p',\mu'|p',\lambda'\rangle_L \\ &\quad = {}_z\langle \text{rest},\mu'|e^{iK_3\xi}e^{iJ_2(\pi-\theta)}e^{-iQE_1/m}e^{-iK_3\xi'}|\text{rest},\lambda'\rangle_z \\ &\quad \equiv {}_z\langle \text{rest},\mu'|R_y(-\theta')|\text{rest},\lambda'\rangle_z = d^{j'}_{\mu'\lambda'}(-\theta'), \end{aligned} \tag{2.25}$$

where we know the product of the four operators can only be a rotation because the rest four-momentum is undisturbed. Consistent with our previous choice, we define θ' as the angle rotating from the rest state connected to the Breit frame helicity state to the corresponding state connected to the light-front state. Our method for finding θ' is to choose a representation for the operators, namely,

$$J_2=\frac{1}{2}\sigma_2, \quad K_3=\frac{i}{2}\sigma_3, \quad \text{and} \quad E_1=\frac{1}{2}(i\sigma_1+\sigma_2), \tag{2.26}$$

where the σ_i are the usual 2×2 Pauli matrices, and then to multiply the operators out explicitly. The result is

$$\tan\theta' = -\frac{2MQ}{Q^2-M^2+m^2}, \tag{2.27}$$

equivalent to the useful half-angle version given earlier, Eq. (2.8).

Putting the pieces together gives the light-front to Breit frame helicity amplitude conversion formula, quoted in Eq. (2.6). The inverse of this relation follows using

$$d^j_{\mu\lambda_1}(\theta)d^j_{\mu\lambda}(\theta) = \delta_{\lambda_1\lambda}, \tag{2.28}$$

and is

$$G^\nu_{L\lambda'\lambda} = d^{j'}_{\lambda'\mu'}(-\theta')G^\nu_{B\mu'\mu}d^j_{\mu\lambda}(\theta). \tag{2.29}$$

III. LIGHT-FRONT DISCRETE SYMMETRY

The discrete symmetries of parity inversion and time reversal are not compatible with the light-front requirement that $q^+=0$. However, putting all momenta in the x-z plane, we can compound the usual parity and time reversal operators with 180° rotations about the y axis to produce useful and applicable light-front parity and time reversal operators [15].

A. Light-front parity

Let $\mathbb{P}$ be the ordinary unitary parity operator that takes $\vec{x}\to-\vec{x}$ and $t\to t$. Define the light-front parity operator by [12,15]

$$Y_P = R_y(\pi)\mathbb{P}. \tag{3.1}$$

Since Y_P commutes with operators E_1 and K_3, one has that Y_P acting on a light-front state gives

$$\begin{aligned} Y_P|p,\lambda\rangle_L &= Y_P e^{-iE_1p_\perp/p^+}e^{-iK_3\xi}|\text{rest},\lambda\rangle_z \\ &= e^{-iE_1p_\perp/p^+}e^{-iK_3\xi}Y_P|\text{rest},\lambda\rangle_z. \end{aligned} \tag{3.2}$$

Further,

$$Y_P|\text{rest},\lambda\rangle_z = \eta_P R_y(\pi)|\text{rest},\lambda\rangle_z = \eta_P|\text{rest},\lambda'\rangle_z d^j_{\lambda'\lambda}(\pi), \tag{3.3}$$

where η_P is the intrinsic parity of the state. Then using $d^j_{\lambda'\lambda}(\pi)=(-1)^{j+\lambda}\delta_{\lambda',-\lambda}$, one gets for the states

$$Y_P|p,\lambda\rangle_L = \eta_P(-1)^{j+\lambda}|p,-\lambda\rangle_L. \tag{3.4}$$

For current component J^+, since Y_P is unitary,

$$\begin{aligned} &{}_L\langle p',\lambda'|J^+|p,\lambda\rangle_L \\ &\quad = {}_L\langle Y_P(p',\lambda')|Y_PJ^+Y_P^\dagger Y_P|p,\lambda\rangle_L \\ &\quad = \eta'_P\eta_P(-1)^{j'-j+\lambda'-\lambda}{}_L\langle p',-\lambda'|J^+|p,-\lambda\rangle_L. \end{aligned} \tag{3.5}$$

Hence, the parity relation for light-front helicity amplitudes is

$$G^+_{L,-\lambda',-\lambda} = \eta'_P\eta_P(-1)^{j'-j+\lambda'-\lambda}G^+_{L\lambda'\lambda}. \tag{3.6}$$

The parity relation for the usual (Breit frame) helicity amplitudes is known [12], and is usually given in terms of amplitudes with definite photon helicity, which we define in Sec. IV E. We shall only note that we can derive the relation from the light-front result just above, and quote for completeness,

$$G^{-\lambda_\gamma}_{B,-\mu',-\mu} = \eta'_P\eta_P(-1)^{j'+j}G^{\lambda_\gamma}_{B\mu'\mu}. \tag{3.7}$$

B. Light-front time reversal

Let $\mathbb{T}$ be the ordinary time reversal operator which takes $t\to-t$ and $\vec{x}\to\vec{x}$ and which is antiunitary. By known arguments, time reversal acting on a state at rest reverses the spin projection, and one has

$$\mathbb{T}|\text{rest},\lambda\rangle_z = (-1)^{j-\lambda}|\text{rest},-\lambda\rangle_z. \tag{3.8}$$

[By way of review, one starts with $\mathbb{T}|\text{rest},\lambda\rangle_z=\eta_T(\lambda)|\text{rest},-\lambda\rangle_z$, and recalls that the states with different λ are related by the angular momentum raising and lowering operators $J_\pm$. One shows that $\eta_T(\lambda)$ changes sign as the spin projection changes by one unit by considering how $\mathbb{T}$ commutes with the raising and lowering operators. That only leaves $\eta_T(j)$ to be fixed. Since $\mathbb{T}$ is antiunitary, one can change $\eta_T(j)$ by changing the phase of the state, and one chooses the phase of the state so that $\eta_T(j)$ is one.]

Define a light-front time reversal operator by

$$Y_T = R_y(\pi)\mathbb{T}, \tag{3.9}$$

giving

$$Y_T|\text{rest},\lambda\rangle_z = |\text{rest},\lambda\rangle_z. \tag{3.10}$$

This also works for moving light-front states. Since Y_T is antiunitary,

$$Y_TiK_3Y_T^{-1} = iK_3 \quad\text{and}\quad Y_TiE_1Y_T^{-1} = iE_1, \tag{3.11}$$

from which we see

$$Y_T|p,\lambda\rangle_L = Y_Te^{-iE_1p_\perp/p^+}e^{-iK_3\xi}|\text{rest},\lambda\rangle_z = |p,\lambda\rangle_L. \tag{3.12}$$

We use time reversal first to show that the light-front amplitudes are real, for current component J^+, still remembering that Y_T is antiunitary,

$$\begin{aligned} {}_L\langle p',\lambda'|J^+|p,\lambda\rangle_L &= {}_L\langle Y_T(p',\lambda')|Y_TJ^+Y_T^{-1}Y_T|p,\lambda\rangle^*_L \\ &= {}_L\langle p',\lambda'|J^+|p,\lambda\rangle^*_L, \end{aligned} \tag{3.13}$$

or

$$G^+_{L\lambda'\lambda} = (G^+_{L\lambda'\lambda})^* \tag{3.14}$$

(for momenta in the x-z plane).

In general, it is not useful to reverse the initial and final states because the particles are different. But for the elastic case we can use further time reversal to relate amplitudes with interchanged helicity. First note that for the light-front frame with the target at rest, the initial and final particles have the same p^+, so to get a state with the final momentum requires just the transverse boost,

$$|p',\lambda\rangle_L = e^{-iQE_1/p^+}|p,\lambda\rangle_L . \tag{3.15}$$

Beginning by applying the previous time reversal result to the elastic case, and recalling that E_1 commutes with "+" components of four-vectors, leads to

$$\begin{aligned} G^+_{L\lambda'\lambda} &= {}_L\langle p',\lambda'|J^+|p,\lambda\rangle_L = {}_L\langle p,\lambda|J^+|p',\lambda'\rangle_L \\ &= {}_L\langle p,\lambda|J^+ e^{-iQE_1/p^+}|p,\lambda'\rangle_L \\ &= {}_L\langle p,\lambda|e^{-iQE_1/p^+}J^+|p,\lambda'\rangle_L \\ &= {}_L\langle p,\lambda|e^{-iJ_3\pi}e^{+iQE_1/p^+}e^{iJ_3\pi}J^+|p,\lambda'\rangle_L \\ &= (-1)^{\lambda'-\lambda}{}_L\langle p',\lambda|J^+|p,\lambda'\rangle_L . \end{aligned} \tag{3.16}$$

Thus when the incoming and outgoing particles have the same identity, time reversal gives

$$G^+_{L\lambda'\lambda} = (-1)^{\lambda'-\lambda} G^+_{L\lambda\lambda'} . \tag{3.17}$$

Similarly to the close of the last section, we record the time reversal result for the helicity amplitudes in the Breit frame, for identical incoming and outgoing particles,

$$G^{\lambda_\gamma}_{B\mu'\mu} = (-1)^{\mu'-\mu} G^{\lambda_\gamma}_{B\mu\mu'} . \tag{3.18}$$

C. The x-Breit frame

Note that $\theta = -\theta'$ for the equal mass case. While some of the transformations are easy in the target rest frame, where we calculated, visualizing this result is not. For this purpose, the x-Breit frame, where the incoming and outgoing particles are both along the x direction, works well.

The momenta are, in (p^0,p^1,p^2,p^3) notation,

$$\begin{aligned} p &= (\sqrt{m^2+Q^2/4}, -Q/2, 0, 0), \\ p' &= (\sqrt{m^2+Q^2/4}, Q/2, 0, 0), \\ q &= (0, Q, 0, 0), \end{aligned} \tag{3.19}$$

and the incoming states are defined by

$$\begin{aligned} |p,\lambda\rangle_L &= e^{iE_1 Q/2p^+} e^{-iK_3\xi_1}|\text{rest},\lambda\rangle_z , \\ |p,\lambda\rangle_B &= R_y(-\pi/2) e^{-iK_3\xi}|\text{rest},\lambda\rangle_z . \end{aligned} \tag{3.20}$$

The outgoing states have the same longitudinal boosts, but have opposite transformations for getting the transverse momentum. (The boost parameters are not the same. The transformation with ξ gives a momentum along the z direction with the final energy; the transformation with ξ_1 gives a momentum along the z direction with the final p^+, but with energy and p^z different from the final ones. From the kinematics given by Eq. (3.19), we find $\xi_1 = \text{arccosh}[(m^2 + Q^2/8)/m\sqrt{m^2+Q^2/4}]$ and $\xi = \text{arccosh}(\sqrt{m^2+Q^2/4}/m)$.)

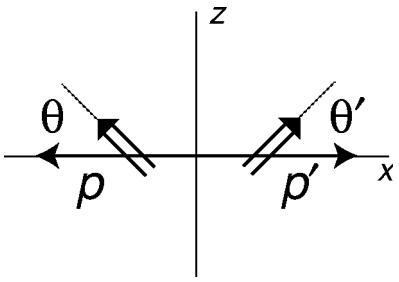

FIG. 2. Momenta and spin directions for light-front helicity states in the x-Breit frame. The momenta are in the $\pm x$ direction and the spin directions for the light-front states are indicated by the doubled lines.

Formally, one defines angle θ from

$${}_L\langle p,\lambda|p,\mu\rangle_B = {}_z\langle \text{rest},\lambda|e^{-iJ_2\theta}|\text{rest},\mu\rangle_z , \tag{3.21}$$

with a corresponding equation involving the final states and angle θ'. Using the representation given earlier in Eq. (2.26),

$$e^{-i\sigma_2\theta/2} = e^{-\sigma_3\xi_1/2} e^{(\sigma_1 - i\sigma_2)Q/4p^+} e^{i\sigma_2\pi/4} e^{\sigma_3\xi/2} . \tag{3.22}$$

Conjugating the above equation with σ_3 (i.e., taking $\sigma_3 \ldots \sigma_3$) gives

$$e^{+i\sigma_2\theta/2} = e^{-\sigma_3\xi_1/2} e^{-(\sigma_1 - i\sigma_2)Q/4p^+} e^{-i\sigma_2\pi/4} e^{\sigma_3\xi/2} = e^{-i\sigma_2\theta'/2}, \tag{3.23}$$

and $\theta = -\theta'$.

Pictorially, we draw the momenta in Fig. 2, and for the helicity states the particle spins point along the direction of the momenta. The incoming and outgoing light front states both start with a boost in the z direction, and then receive symmetrically opposite transverse boosts which rotate the spin vectors in opposite directions by the same amount. The angles θ and θ' are indicated in the figure. One can see both that the size of the angles should be the same and that the senses should be opposite.

IV. CONSEQUENCES

A. Light-front parity and the angular condition

The general angular condition for current component J^+ reads

$$d^{j'}_{\lambda'\mu'}(-\theta') G^+_{L\lambda'\lambda} d^j_{\mu\lambda}(\theta) = 0 \quad \text{for} \quad |\mu'+\mu| \geqslant 2. \tag{4.1}$$

Say that $\mu + \mu' \geqslant 2$. By changing the sign of both μ and μ' it looks like we could get another angular condition,

$$d^{j'}_{\lambda',-\mu'}(-\theta')G^{+}_{L\lambda'\lambda}d^{j}_{-\mu,\lambda}(\theta)=0. \tag{4.2}$$

However, using the first of the identities

$$d^{j}_{m'm}(\theta)=(-1)^{m-m'}d^{j}_{-m',-m}(\theta)=d^{j}_{-m,-m'}(\theta)$$
$$=(-1)^{m-m'}d^{j}_{mm'}(\theta) \tag{4.3}$$

and the light-front parity relation, Eq. (3.6), one can show by a series of reversible steps that each angular condition with $\mu+\mu'\leqslant -2$ is equivalent to one with $\mu+\mu'\geqslant 2$. Hence, we only need to consider cases where $\mu+\mu'\geqslant 2$.

B. The angular condition for deuterons

We shall implement the general angular condition in a couple of special cases, rewriting the angular dependence in terms of Q^2 and masses. For the deuteron, the angular condition comes only from $\mu=\mu'=1$ and we have

$$d^{1}_{\lambda'1}(-\theta')G^{+}_{L\lambda'\lambda}d^{1}_{1\lambda}(\theta)=0. \tag{4.4}$$

For the equal mass case, the arguments of the d functions are the same,

$$\tan\theta=-\tan\theta'=\frac{2m_d}{Q} \tag{4.5}$$

(m_d is the deuteron mass). Using light-front parity, Eq. (3.6), and the d-function identities, Eq. (4.3), one gets

$$G^{+}_{L++}[(d^{1}_{11})^2+(d^{1}_{1,-1})^2]-(G^{+}_{L0+}-G^{+}_{L+0})d^{1}_{10}(d^{1}_{11}-d^{1}_{1,-1})$$
$$+G^{+}_{L+-}2d^{1}_{11}d^{1}_{1,-1}-G^{+}_{L00}(d^{1}_{10})^2=0. \tag{4.6}$$

Substituting for the d^1's and $\tan\theta$, and using the light-front time reversal result $G^{+}_{L+0}=-G^{+}_{L0+}$, leads to the angular condition in its known form [16,17],

$$(2\eta+1)G^{+}_{L++}+\sqrt{8\eta}G^{+}_{L0+}+G^{+}_{L+-}-G^{+}_{L00}=0, \tag{4.7}$$

where $\eta=Q^2/4m_d^2$. For the record, we have removed an overall factor, $1/2(1+\eta)$.

Recently, Bakker and Ji [18] obtained two constraints on the deuteron helicity amplitudes by noting that there were five amplitudes, and that all five could be derived from three independent form factors. Both constraints they called angular conditions. They appeared differently in different frames; their Drell-Yan-West frame results can be most directly compared to our present results. The constraint they call ''AC1'' is, for momenta in the x-z plane, just $G^{+}_{L0+}+G^{+}_{L+0}=0$. In the present paper this follows from light-front time reversal invariance. Their constraint ''AC2'' is then precisely the same as the angular condition here.

C. A consequence of the angular condition for deuterons

Perturbative QCD predicts, as we shall review below, that the hadron helicity conserving amplitude G^{+}_{L00} is the leading amplitude at high Q and that

$$G^{+}_{L+0}=\frac{a\Lambda_{\rm QCD}}{Q}G^{+}_{L00}$$
$$G^{+}_{L+-}=\left(\frac{b\Lambda_{\rm QCD}}{Q}\right)^2 G^{+}_{L00} \tag{4.8}$$

to leading order in $1/Q$. No statement is initially made about the size of a and b.

One may go further, following Chung *et al.* [19] or Brodsky and Hiller [20] (who interestingly mention the work of Carlson and Gross [21] in this regard), to argue that the scale of QCD is given by $\Lambda_{\rm QCD}$ and that we can implement this in the light-front frame by saying that

$$a,b=O(1). \tag{4.9}$$

A consequence of this, written in terms of the deuteron charge, magnetic and quadrupole form factors [22], is that to good approximation one gets the ''universal ratios'' [20],

$$G_C:G_Q:G_M=\left(\frac{2}{3}\eta-1\right):1:-2. \tag{4.10}$$

This agrees with the leading power of Q^2 result [21] that $G_C=(2/3)\eta G_Q$, but goes beyond it and also gives a prediction for G_M.

We have so far in this section used only three light-front helicity amplitudes. There are more that are not zero, and we find a difficulty when we discuss a fourth. Amplitude G^{+}_{L++} is related to the others by the angular condition quoted above. Also, the perturbative QCD arguments that give the scaling behavior of the other helicity amplitudes give for G^{+}_{L++} at very high Q^2,

$$G^{+}_{L++}=\left(\frac{c\Lambda_{\rm QCD}}{Q}\right)^2 G^{+}_{L00}. \tag{4.11}$$

(Helicity is conserved, but other spin dependent rules [21] dictate a two power asymptotic suppression of G^{+}_{L++}. This is also consistent with a naturalness condition discussed in Ref. [18].)

The angular condition to leading order now reads

$$1+\sqrt{2}\,\frac{a\Lambda_{\rm QCD}}{m_d}-\frac{1}{2}\left(\frac{c\Lambda_{\rm QCD}}{m_d}\right)^2=0. \tag{4.12}$$

The hypothesis that $\Lambda_{\rm QCD}$ sets the scale of the subleading amplitudes would suggest that c as well as a is of $\mathcal{O}(1)$. Given the angular condition result just above, this cannot be right; at least one of a and c must be $O(m_d/\Lambda_{\rm QCD})\approx 20$. Hence the hypothesis is not generally workable, and one needs to consider thinking the same about the next-to-leading corrections in the ''universal ratios'' expression, Eq. (4.10).

D. The angular condition for N-Δ transitions

The $\gamma^* N\to\Delta(1232)$ transition is an important reaction that involves final and initial states with different spins and

masses. This makes working out the angular condition more involved technically, but not unduly so, as we shall demonstrate.

There is one angular condition,

$$\begin{aligned}0 &= d^{3/2}_{\lambda',3/2}(-\theta')G^{+}_{L\lambda'\lambda}d^{1/2}_{1/2,\lambda}(\theta)\\ &= G^{+}_{L,-3/2,1/2}(-d^{3/2}_{3/2,3/2}d^{1/2}_{1/2,-1/2}+d^{3/2}_{-3/2,3/2}d^{1/2}_{1/2,1/2})\\ &\quad + G^{+}_{L,-1/2,1/2}(d^{3/2}_{1/2,3/2}d^{1/2}_{1/2,-1/2}+d^{3/2}_{-1/2,3/2}d^{1/2}_{1/2,1/2})\\ &\quad + G^{+}_{L,1/2,1/2}(-d^{3/2}_{-1/2,3/2}d^{1/2}_{1/2,-1/2}+d^{3/2}_{1/2,3/2}d^{1/2}_{1/2,1/2})\\ &\quad + G^{+}_{L,3/2,1/2}(d^{3/2}_{-3/2,3/2}d^{1/2}_{1/2,-1/2}+d^{3/2}_{3/2,3/2}d^{1/2}_{1/2,1/2}),\end{aligned} \tag{4.13}$$

using the light-front parity. Explicit substitution for the d-functions yields

$$\begin{aligned}0 &= -\cos^2\frac{\theta'}{2}\sin\frac{\theta'}{2}\cos\frac{\theta}{2}\Bigg\{G^{+}_{L,-3/2,1/2}\Bigg(-\tan\frac{\theta}{2}\cot\frac{\theta'}{2}\\ &\quad +\tan^2\frac{\theta'}{2}\Bigg)-\sqrt{3}G^{+}_{L,-1/2,1/2}\Bigg(\tan\frac{\theta}{2}+\tan\frac{\theta'}{2}\Bigg)\\ &\quad +\sqrt{3}G^{+}_{L,1/2,1/2}\Bigg(1-\tan\frac{\theta}{2}\tan\frac{\theta'}{2}\Bigg)\\ &\quad -G^{+}_{L,3/2,1/2}\Bigg(\cot\frac{\theta'}{2}+\tan\frac{\theta}{2}\tan^2\frac{\theta'}{2}\Bigg)\Bigg\}.\end{aligned} \tag{4.14}$$

Finally, removing the overall factors and substituting for the trigonometric functions gives the angular condition for the N-Δ transition,

$$\begin{aligned}0 &= [(M-m)(M^2-m^2)+mQ^2]G^{+}_{L,-3/2,1/2}\\ &\quad +\sqrt{3}MQ(M-m)G^{+}_{L,-1/2,1/2}+\sqrt{3}MQ^2G^{+}_{L,1/2,1/2}\\ &\quad +Q[Q^2-m(M-m)]G^{+}_{L,3/2,1/2},\end{aligned} \tag{4.15}$$

where m is the nucleon mass and M is the Δ mass. For the record, we have removed another overall factor, $Q_+(Q_+-Q_-)/(2mM^2Q^2)$.

The asymptotic scaling rules, cited in the next section, say that $G^{+}_{L,1/2,1/2}$ goes like $1/Q^4$ at high Q, that $G^{+}_{L,3/2,1/2}$ and $G^{+}_{L,-1/2,1/2}$ go like $1/Q^5$, and that $G^{+}_{L,-3/2,1/2}$ goes like $1/Q^6$. If we write

$$G^{+}_{L,3/2,1/2}=\frac{b\Lambda_{\rm QCD}}{Q}G^{+}_{L,1/2,1/2} \tag{4.16}$$

modulo logarithms at high Q, then the leading Q part of the angular condition says

$$\sqrt{3}+\frac{b\Lambda_{\rm QCD}}{M}=0. \tag{4.17}$$

E. Equivalence of leading powers in Breit and light-front frames

The idea of ''good currents'' and ''bad currents'' is native to the light-front frame. In analyzing the power law scaling behavior at high Q^2, for a given helicity amplitude, it is often thought to be safest to stay in the light-front frame and use only ''good currents.'' We shall here derive the Breit frame helicity amplitude scaling behaviors from their light-front counterparts. Note here $q^+=0$ both in the light-front frame and the Breit frame that we discuss in this section. All the $q^+=0$ frames are related to each other only by the kinematical operators that make the light-front time τ intact. We will find, nicely enough, that the scaling behaviors are the same as one would have found using the Breit frame only. That is, one can get the correct leading power scaling behavior from a Breit frame analysis alone.

For a light-front helicity amplitude, the scaling behavior at high Q is [23]

$$G^{+}_{L\lambda'\lambda}\propto(m/Q)^{2(n-1)+|\lambda'-\lambda_{min}|+|\lambda-\lambda_{min}|}, \tag{4.18}$$

where n is the number of quarks in the state, m is a mass scale, and λ_{min} is the minimum helicity of the incoming or outgoing state (i.e., 0 or 1/2 for bosons or fermions, respectively).

Regarding the Breit frame, we have thus far given its helicity amplitudes in terms of G^{ν}_{B} where ν is a Lorentz index. It is usual to substitute a photon helicity index for the Lorentz index, using (for incoming photons)

$$G^{\lambda_\gamma}_{B\mu'\mu}=\epsilon_\nu(q,\lambda_\gamma)G^{\nu}_{B\mu'\mu}, \tag{4.19}$$

with polarizations [in (t,x,y,z)-type notation]

$$\begin{aligned}\epsilon_\pm &= \epsilon(q,\lambda_\gamma=\pm)=(0,\pm\cos\theta,-i,\pm\sin\theta)/\sqrt{2}\\ \epsilon_0 &= \epsilon(q,\lambda_\gamma=0)=(\csc\theta,\cos\theta,0,-\cos^2\theta\csc\theta),\end{aligned} \tag{4.20}$$

where θ is the angle between $\vec{q}$ and the negative z direction, as shown in Fig. 1. One can work out that

$$\eta\equiv(1,0,0,-1)=\sin\theta\left[\epsilon_0-\frac{1}{\sqrt{2}}(\epsilon_+-\epsilon_-)\right], \tag{4.21}$$

so that

$$G^{\nu=+}_{B\ldots}=\sin\theta\left[G^{0}_{B\ldots}-\frac{1}{\sqrt{2}}(G^{+}_{B\ldots}-G^{-}_{B\ldots})\right], \tag{4.22}$$

where the superscripts on G_B will in the rest of this section refer to photon helicity unless explicitly stated otherwise. With the general relation, Eq. (2.6), this gives directly the expression we use to obtain the scaling behavior of the Breit amplitudes,

$$G^0_{B\mu'\mu} - \frac{1}{\sqrt{2}}(G^+_{B\mu'\mu} - G^-_{B\mu'\mu}) = \csc\theta d^{j'}_{\lambda',\mu'}(-\theta') G^+_{L\lambda'\lambda} d^j_{\mu\lambda}(\theta). \quad (4.23)$$

We can select terms on the left-hand side by choice of μ' and μ, since Breit amplitudes are non-zero only for $\lambda_\gamma = \mu' + \mu$.

The d functions can be written in terms of sines and cosines of half angles, so we record that at high Q,

$$\sin\frac{\theta}{2} = \frac{m}{Q} + \mathcal{O}\left(\frac{m}{Q}\right)^3 \quad \text{and} \quad \cos\frac{\theta}{2} = 1 + \mathcal{O}\left(\frac{m}{Q}\right)^2$$
$$\sin\frac{\theta'}{2} = -\frac{M}{Q} + \mathcal{O}\left(\frac{m}{Q}\right)^3 \quad \text{and} \quad \cos\frac{\theta'}{2} = 1 + \mathcal{O}\left(\frac{m}{Q}\right)^2, \quad (4.24)$$

where m inside the $\mathcal{O}$ symbol is a generic mass scale. Equation (4.24) can be derived from Eqs. (2.7) and (2.8), or equivalently from Eqs. (2.17) and (2.27). The d functions can be expanded as [24]

$$d^j_{\mu\lambda}(\theta) = a_1\left(\cos\frac{\theta}{2}\right)^{2j-|\mu-\lambda|}\left(\sin\frac{\theta}{2}\right)^{|\mu-\lambda|} + a_2\left(\cos\frac{\theta}{2}\right)^{2j-|\mu-\lambda|-2}\left(\sin\frac{\theta}{2}\right)^{|\mu-\lambda|+2} + \ldots, \quad (4.25)$$

where $a_1, a_2, \ldots$ are numerical coefficients with $a_1 \neq 0$. Thus for large Q,

$$d^j_{\mu\lambda}(\theta) \propto \left(\frac{m}{Q}\right)^{|\mu-\lambda|} + \mathcal{O}\left(\frac{m}{Q}\right)^{|\mu-\lambda|+2}. \quad (4.26)$$

On the right-hand side of Eq. (4.23), there is no term that falls slower than the term that has $\lambda' = \lambda = \lambda_{min}$, and $G^+_{L,\lambda_{min},\lambda_{min}} \propto (m/Q)^{2(n-1)}$. Thus the Breit amplitude leading falloff at high Q is

$$G^{\lambda_\gamma}_{B\mu'\mu} \propto \left(\frac{m}{Q}\right)^{-1+2(n-1)+|\mu'-\lambda_{min}|+|\mu-\lambda_{min}|}. \quad (4.27)$$

These are the same results one can get by directly analyzing amplitudes in the Breit frame for various photon helicities [25].

By way of example, we will give the Breit and light-front frame helicity amplitudes for elastic electron-nucleon scattering. In terms of the standard Dirac, Pauli, and Sachs form factors one may work out

$$G^+_{L++} \equiv {}_L\left\langle p', \frac{1}{2}\middle| J^+ \middle| p, \frac{1}{2}\right\rangle_L = 2p^+ F_1(q^2),$$
$$G^+_{L-+} \equiv {}_L\left\langle p', -\frac{1}{2}\middle| J^+ \middle| p, \frac{1}{2}\right\rangle_L = 2p^+ \frac{Q}{2m} F_2(q^2), \quad (4.28)$$

for the light-front states and

$$G^+_{B++} \equiv {}_B\left\langle p', \frac{1}{2}\middle| \epsilon_+ \cdot J \middle| p, \frac{1}{2}\right\rangle_B = \sqrt{2} Q G_M(q^2),$$
$$G^0_{B-+} \equiv {}_B\left\langle p', -\frac{1}{2}\middle| \epsilon_0 \cdot J \middle| p, \frac{1}{2}\right\rangle_B = 2m G_E(q^2), \quad (4.29)$$

for the Breit frame states. The scaling rules predict that G_M, G_E, and F_1 scale as $1/Q^4$ and that F_2 scales as $1/Q^6$, consistent with the relations

$$G_M = F_1 + F_2,$$
$$G_E = F_1 + \frac{q^2}{4m^2} F_2. \quad (4.30)$$

(There is recent data [26] and commentary [27] on the helicity flip scaling results.)

V. SUMMARY AND CONCLUSION

The purpose of the present paper has been largely kinematical. We have examined the relationship between the helicity amplitudes in the Breit and light-front frames. One particular result has been a clear view of where the angular condition comes from. The angular condition is a constraint on light-front helicity amplitudes. It follows from applying angular momentum conservation in the Breit frame, where the application of angular momentum conservation to the helicity amplitudes is elementary. One consequence is that an amplitude must be zero if it requires the photon to have more than one unit magnitude of helicity, and this statement cast in terms of light-front amplitudes is the angular condition [14,16].

Another set of constraints follows from parity and time-reversal invariance. Neither of these symmetries can be used directly on the light front because the light front has a preferred spatial direction. However, each of them can be modified to give a valid symmetry operation (at least for strong and electromagnetic interactions) for the light front [15]. To define a light-front parity, choose the x-z plane to contain all the momenta and then consider mirror reflection in the x-z plane. This reflection leaves the momenta unchanged but reverses the helicities [12]. Technically, it is the same as ordinary parity followed by a 180° rotation about the y axis, and helicity amplitude relations that follow from it were given in Sec. III.

Similarly one defines light-front time reversal as ordinary time reversal followed by the 180° rotation about the y axis. Time reversal invariance implies that the amplitudes are always real for momenta in the x-z plane, with additional relations possible for elastic scattering, as detailed also in Sec. III.

The general angular condition appears compactly in terms of d functions, the representations of the rotation operators. It can be rewritten in terms of masses and momentum transfer. We gave the translations for two cases. For electron-deuteron

elastic scattering, the result is well known [16]. Nonetheless, it does have an unchronicled (we believe) consequence, which is that the mass scale associated with the asymptotic power-law falloff of non-leading amplitudes must generally be of order of the nucleon or deuteron mass. It had been hoped that the light front was a favored frame where the non-leading amplitudes would have small numerical coefficients: asymtotically of order $\Lambda_{\rm QCD}/Q$, to an appropriate power, times the leading amplitude. As the angular condition contradicts this, it also takes away the motivation that underlay the suggestion of the universal behavior for spin-1 form factors.

Additionally, we gave the angular condition explicitly for the N-Δ electromagnetic transition. We believe this result is also new. The angular condition here fixes precisely the leading power of one subleading amplitude.

The power law scaling of the helicity amplitudes can be analyzed, with a definite power of $1/Q$ given in terms of the number of constituents in the wave function and in terms of the helicities of the incoming and outgoing states, and this can be done in either the light-front frame or in the Breit frame. A preference for one or the other is sometimes given. Our final ''application'' was to obtain the Breit frame scaling from the light-front scaling using the transformation between them given earlier in Sec. II, and to find that the result is the same as one obtains doing the analysis directly in the Breit frame.

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