$\pi\pi$ scattering S wave from the data on the reaction $\pi^- p \to \pi^0\pi^0 n$

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The results of the recent unpolarized target experiments on the reaction $\pi^- p \to \pi^0\pi^0 n$ performed at KEK, BNL, IHEP, and CERN are analyzed in detail. In practice, the unpolarized data allow information on $\pi\pi$ scattering to be obtained under the assumption of the dominance of the one-pion exchange mechanism. In this way, for the $I=0\,\pi\pi S$ wave phase shift δ_0^0 and inelasticity η_0^0 a few sets of values are obtained up to about 1.64 GeV. These sets correspond to the ambiguous solutions found for the partial wave amplitudes. The difficulties emerging when using the physical solutions for the $\pi^0\pi^0$ S and D wave amplitudes extracted with partial wave analyses are discussed. Attention is drawn to the fact that, for the $\pi^0\pi^0$ invariant mass m above 1 GeV, the other solutions, in principle, are found to be preferred to the physical ones. To clarify the situation and further study the $f_0(980)$ resonance, thorough experimental investigations of the reaction $\pi^- p \to \pi^0\pi^0 n$ in the m region near the $K\bar{K}$ threshold are required.

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I. INTRODUCTION

The reactions $\pi N \to \pi\pi N$ so far are the major source of information on the processes $\pi\pi \to \pi\pi$. At high energies and small values of the momentum-transfer squared from the incident π to the outgoing $\pi\pi$ system, $0 < -t < 0.2$ GeV^2, the reactions $\pi N \to \pi\pi N$ are dominated by the one-pion exchange mechanism. In treating the data on these reactions the partial wave analysis method is used. As a rule, a few possible solutions for the partial wave amplitudes of the final $\pi\pi$ system are obtained. In some cases, the preferred solution is selected from additional physical arguments. Generally, to obtain reliable and unambiguous results in a wide region of m, high statistics, polarized targets, and precise measurements of the $\pi N \to \pi\pi N$ reaction cross section at different energies are needed. Detailed reviews and comprehensive discussions of the experimental results on the reactions $\pi N \to \pi\pi N$ and $\pi\pi$ scattering in the region $2m_\pi < m < 2$ GeV available by 1999 were presented in Refs. [1,2].

In this work we analyze the recent data on the intensities and relative phases of the S and D partial waves of the $\pi^0\pi^0$ system produced in the reaction $\pi^- p \to \pi^0\pi^0 n$. The data were obtained in unpolarized target experiments with incident π^- energies of 8.9, 18.3, 38, and 100 GeV performed at KEK [3], BNL [4], IHEP [5], and CERN [6], respectively. Our main goal is to obtain information on the $\pi\pi$ S wave phase shift δ_0^0 and inelasticity η_0^0 in the channel with isospin $I=0$, which would be complementary to the previous "canonical" data extracted from the 17.2 GeV experiments on the reactions $\pi^- p \to \pi^+\pi^- n$ [7–12]. It is well known from the earlier analyses [1,2,7–12] that in practice the unpolarized data allow information on $\pi\pi$ scattering to be obtained under the assumption of the dominance of the one-pion exchange (OPE) mechanism; here too we continue the discussion of the possible consequences of this approach. Furthermore, there exists the above problem of the ambiguous solutions for the partial wave amplitudes. In this connection we especially emphasize the strong likeness of the physical solutions selected in all four experiments on $\pi^0\pi^0$ production and also the common difficulties that emerge in interpreting these solutions and in their comparison with the $\pi^+\pi^-$ data. It turns out, in particular, that some of the physical solutions found lead to considerable violations of the unitarity condition for the $\pi\pi$ scattering amplitude in question. In addition, we conclude that the data on $\pi^0\pi^0$ production are indicative of a noticeably smaller value of the $f_2(1270) \to \pi\pi$ decay branching ratio in comparison with the Particle Data Group (PDG) data [13]. In connection with the considerable interest in the light scalar meson sector (see for reviews Refs. [1,2,13–15]), we suggest performing especially careful measurements of the reaction $\pi^- p \to \pi^0\pi^0 n$ in the m region from 0.9 to 1.1 GeV, i.e., near the $K\bar{K}$ threshold. This would allow the $f_0(980)$ coupling constant to the $K\bar{K}$ channel to be determined more reliably and also resolve the long-standing question [16] of a possible ambiguity in the behavior of the phase shift δ_0^0 above the $K\bar{K}$ threshold.

The paper is organized as follows. In Sec. II, the KEK results [3] are analyzed. In Ref. [3] the data on the phase shift δ_0^0 were obtained in the m region from 0.36 to 1 GeV. The δ_0^0 values found by us in a different way in the interval $0.68 \leqslant m \leqslant 1$ GeV agree with the KEK data [3] within experimental uncertainties. We also present new results for δ_0^0 and η_0^0 in the region $1 < m < 1.64$ GeV. In Sec. III, an extrapolation of the S and D wave mass distributions obtained in the BNL experiment [4] from the physical region of the reaction $\pi^- p \to \pi^0\pi^0 n$ to the pion pole ($t = m_\pi^2$) is performed. Considering the different solutions found in Ref. [4] for these distributions, we obtain a few sets of values for δ_0^0 and η_0^0 in the m region from 0.32 to 1.52 GeV. In Sec. IV, the GAMS results on the reaction $\pi^- p \to \pi^0\pi^0 n$ [5,6] are discussed. The difficulties encountered while analyzing the $\pi^0\pi^0$ data [3–6] are summarized and discussed in Sec. V. In

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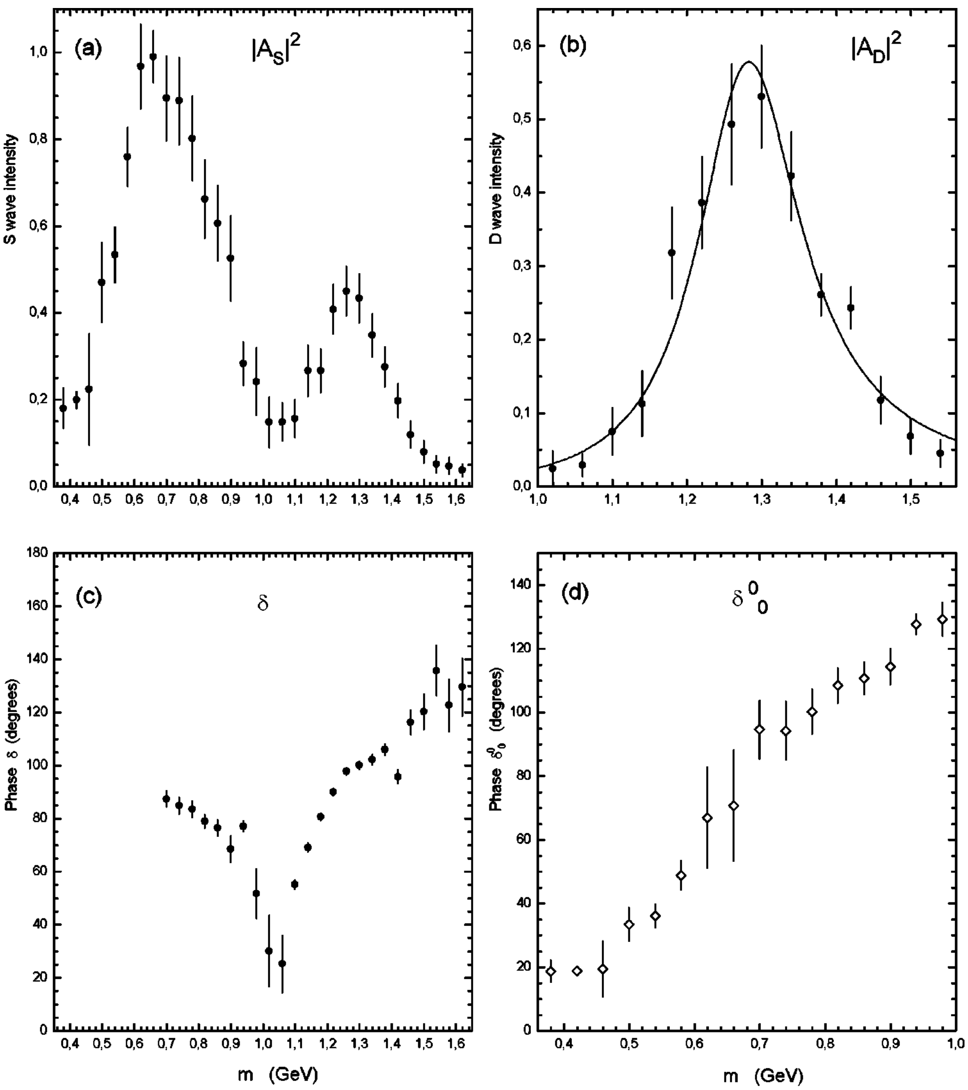

FIG. 1. (a)–(c) The KEK data on the reaction $\pi^+\pi^-\to\pi^0\pi^0$ [3]. (a) The normalized S wave intensity $|A_S|^2$. (b) The normalized D wave intensity $|A_D|^2$; the curve is the fit using Eq. (1) with the parameters of $f_2(1270)$ presented in Eq. (2). (c) The relative phase δ between the amplitudes A_S and A_D. (d) The $I=0$ $\pi\pi S$ wave phase shift δ_0^0 obtained from the data on $|A_S|^2$ alone under the assumption that η_0^0 is unity.

Sec. VI, we formulate briefly a few concrete suggestions for further studying the reaction $\pi^-p\to\pi^0\pi^0n$ which, one can hope, will be used to clarify the experimental situation.

II. ANALYSIS OF THE KEK DATA

In the KEK experiment [3], the data on the intensities and relative phases of the S and D partial waves for the reaction $\pi^+\pi^-\to\pi^0\pi^0$ were obtained in the m interval from 0.36 to 1.64 GeV. They were extracted from the $\pi^-p\to\pi^0\pi^0n$ data by using the linear Chew-Low extrapolation and partial wave analysis. Because the absolute $\pi^0\pi^0$ production cross section was not determined in the experiment, the S and D wave intensities $|A_S|^2$ and $|A_D|^2$ were initially presented in arbitrary (identical) units [3]. Any alternative solution for $|A_S|^2$ and $|A_D|^2$ was not discussed in Ref. [3]. The S and D wave intensities are related to the phase shifts δ_0^I and δ_2^I and inelasticities η_0^I and η_2^I in the conventional way: $|A_S|^2\sim|a_0^0-a_0^2|^2$, where $a_0^I=[\eta_0^I\exp(2i\delta_0^I)-1]/2i$, and $|A_D|^2\sim|a_2^0-a_2^2|^2$, where $a_2^I=[\eta_2^I\exp(2i\delta_2^I)-1]/2i$. To find δ_0^0 below the $K\bar{K}$ threshold, it was assumed [3] that in this region $\eta_0^0=\eta_0^2=1$, and consequently $|A_S|^2\sim\sin^2(\delta_0^0-\delta_0^2)$. As is well known from a large number of previous experiments, the phase shift δ_0^0 smoothly goes through 90° in the region $0.7<m<0.9$ GeV and the phase shift δ_0^2 is negative, smooth, and small (see, for example, [2,7,8,17]). Therefore, to extract the phase shift difference $\delta_0^0-\delta_0^2$ from the unnormalized data, the following normalization condition was accepted in Ref. [3]: the maximum value of $|A_S|^2$ is equal to 1. The KEK data for the S and D partial wave intensities normalized in this way are shown in Fig. 1, together with the data on the relative phase $\delta=\phi_S-\phi_D$ between the amplitudes $A_S=|A_S|\exp(i\phi_S)$ and $A_D=|A_D|\exp(i\phi_D)$. The values of the $I=2$ $\pi\pi$ S wave phase shift δ_0^2 used in Ref. [3] were given by the parametrization $\delta_0^2=-0.87q$ [with $q=(m^2/4-m_\pi^2)^{1/2}$ in GeV and δ_0^2 in radians], and in this way the data for δ_0^0 were obtained in the m region from 0.36 to 1 GeV. In the following, for δ_0^2 we shall use the fit to the data from Refs. [17,18]

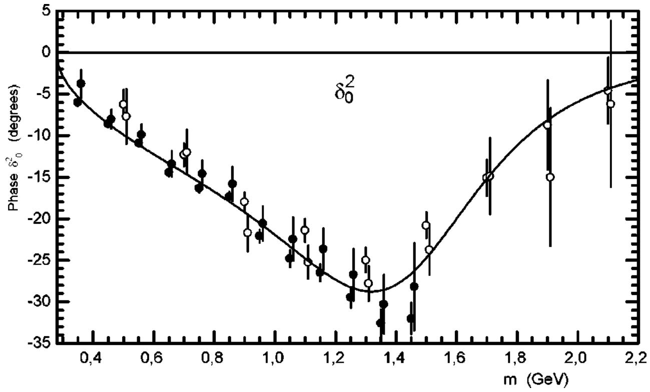

FIG. 2. The $I=2\,\pi\pi S$ wave phase shift δ_0^2. The data are from Refs. [17] (solid circles) and [18] (open circles). The curve corresponds to the fit described in the text.

which is shown in Fig. 2.[1] Using the fit and the data for $|A_S|^2$ shown in Fig. 1(a) we also determined the values of δ_0^0 for $m<1$ GeV. They are plotted in Fig. 1(d). The resulting values are in excellent agreement with those obtained in Ref. [3].

We now determine δ_0^0 and η_0^0 simultaneously by using the available data on the relative phase δ and the intensity $|A_S|^2$ [see Fig. 1(c) and 1(d)] in the m region from 0.68 to 1.64 GeV. In order to estimate the phase ϕ_D we neglect the tiny amplitude a_2^2 [17,18] (which is quite reasonable because the experimental errors of $|A_D|^2$, as is seen from Fig. 1(b), are not too small) and assume that the D wave amplitude is dominated by the $f_2(1270)$ resonance contribution and can be written in the form

$$A_D=\frac{m_{f_2}B_{f_2\pi\pi}\Gamma}{m_{f_2}^2-m^2-im_{f_2}\Gamma}, \tag{1}$$

where $\Gamma=(m_{f_2}/m)\Gamma_{f_2}(q/q_{f_2})^5D(q_{f_2}R_{f_2})/D(qR_{f_2})$, $D(x)=9+3x^2+x^4$, $q_{f_2}=(m_{f_2}^2/4-m_\pi^2)^{1/2}$, R_{f_2} is the interaction radius, and m_{f_2}, Γ_{f_2}, and $B_{f_2\pi\pi}$ are the mass, width, and $\pi\pi$ decay branching ratio of the $f_2(1270)$. The fitted curve in Fig. 1(b) corresponds to the following values of the $f_2(1270)$ resonance parameters:

$$m_{f_2}=1.283\pm0.008\ \text{GeV},\quad \Gamma_{f_2}=0.170\pm0.014\ \text{GeV},$$

$$R_{f_2}=3.59\pm0.71\ \text{GeV}^{-1},\quad B_{f_2\pi\pi}=0.760\pm0.034. \tag{2}$$

Thus, the phase ϕ_D is defined by that of the Breit-Wigner amplitude (1). To express the parameters δ_0^0 and η_0^0 in terms of the known values δ, $|A_S|^2$, δ_0^2, and ϕ_D, it is convenient to represent the amplitude A_S in the form (see footnote 1)

$$A_S=e^{2i\delta_0^2}\left(\frac{\eta_0^0e^{2i(\delta_0^0-\delta_0^2)}-1}{2i}\right)=e^{2i\delta_0^2}\tilde{A}_S=e^{i(2\delta_0^2+\phi)}|\tilde{A}_S|, \tag{3}$$

where ϕ is the phase of the amplitude $\tilde{A}_S$. The distinctive feature of the amplitude $\tilde{A}_S$ is that in its Argand diagram the relations between $\delta_0^0-\delta_0^2$, ϕ, η_0^0, and $|\tilde{A}_S|$ formally look like the relations between the corresponding parameters of any unitary partial wave amplitude with definite isospin I; for example, the phase ϕ is confined within the range from 0° to 180° because $\mathrm{Im}(\tilde{A}_S)>0$. Thus, we have

$$\phi=\delta-2\,\delta_0^2+\phi_D,\quad \eta_0^0=\sqrt{1-4|A_S|\sin\phi+4|A_S|^2}, \tag{4}$$

$$\sin 2(\delta_0^0-\delta_0^2)=\frac{2|A_S|\cos\phi}{\eta_0^0},$$

$$\cos 2(\delta_0^0-\delta_0^2)=\frac{1-2|A_S|\sin\phi}{\eta_0^0}. \tag{5}$$

Since the interference between the S and D partial waves is defined by the product $|A_S||A_D|\cos\delta$ and $\cos\delta$ determines δ only up to the sign, two solutions always exist: the solution with $\delta>0$ and the other one with $\delta<0$. Moreover, if $\cos\delta$ is close to 1 (and $|\delta|\approx0$) in some region of m then in this region a transition from one solution to the other is possible. The KEK data [3] presented in Fig. 1(c) show that the phase δ changes most rapidly near the $K\bar{K}$ threshold [which is one of the evident manifestations of the $f_0(980)$ resonance] and that just near 1 GeV $\cos\delta\approx1$. Thus, in principle, we have four possible variants: (i) $\delta>0$ for $m<1$ GeV and $\delta<0$ for $m>1$ GeV, (ii) $\delta>0$ for all m, (iii) $\delta<0$ for all m, and (iv) $\delta<0$ for $m<1$ GeV and $\delta>0$ for $m>1$ GeV. However, the variants (iii) and (iv) with $\delta<0$ for $m<1$ GeV can be rejected at once. Estimating δ for $m<1$ GeV by using the relation $\delta=\delta_0^0+\delta_0^2-\phi_D$, one can easily verify that δ must be positive in this region with the conventional definition of the signs of the phase shifts δ_0^0, δ_0^2, and ϕ_D [see Fig. 1(d), 2, and Eq. (1)]. So, we shall consider only the variants (i) and (ii).

Figure 3 shows the values of δ_0^0 and η_0^0 extracted from the KEK data [see Fig. 1(a), 1(b), and 1(c)] in the region $0.68<m<1.64$ GeV by using Eqs. (4) and (5) for the two above mentioned variants of the δ phase behavior. The values of δ_0^0 in the region $0.36<m<1$ GeV obtained above from the data on $|A_S|^2$ with $\eta_0^0=1$ [see Fig. 1(d)] are also shown in Fig. 3(a) and 3(b) for comparison and completeness. As is seen, for example, from Fig. 3(a), the sets of δ_0^0 values found in

[1]This fit was obtained with the parametrization $\delta_0^2=-aq/(1+bm^2+cm^4+dm^6)$, with $a=55.21\pm3.18$ deg/GeV, $b=0.853\pm0.254\ \text{GeV}^{-2}$, $c=-0.959\pm0.247\ \text{GeV}^{-4}$, and $d=0.314\pm0.070\ \text{GeV}^{-6}$. In addition, lacking reliable data on the deviation of η_0^2 from 1, we set $\eta_0^2=1$ for all considered values of m. When more detailed data on η_0^2 are available, it will be interesting to take into account possible inelastic effects. It is not unreasonable to think that such effects may appear in the $I=2\,\pi\pi$ channel only above the nominal $\rho\rho$ threshold (1.54 GeV), but not above the $K\bar{K}$ threshold as in the $I=0\ \pi\pi$ channel.

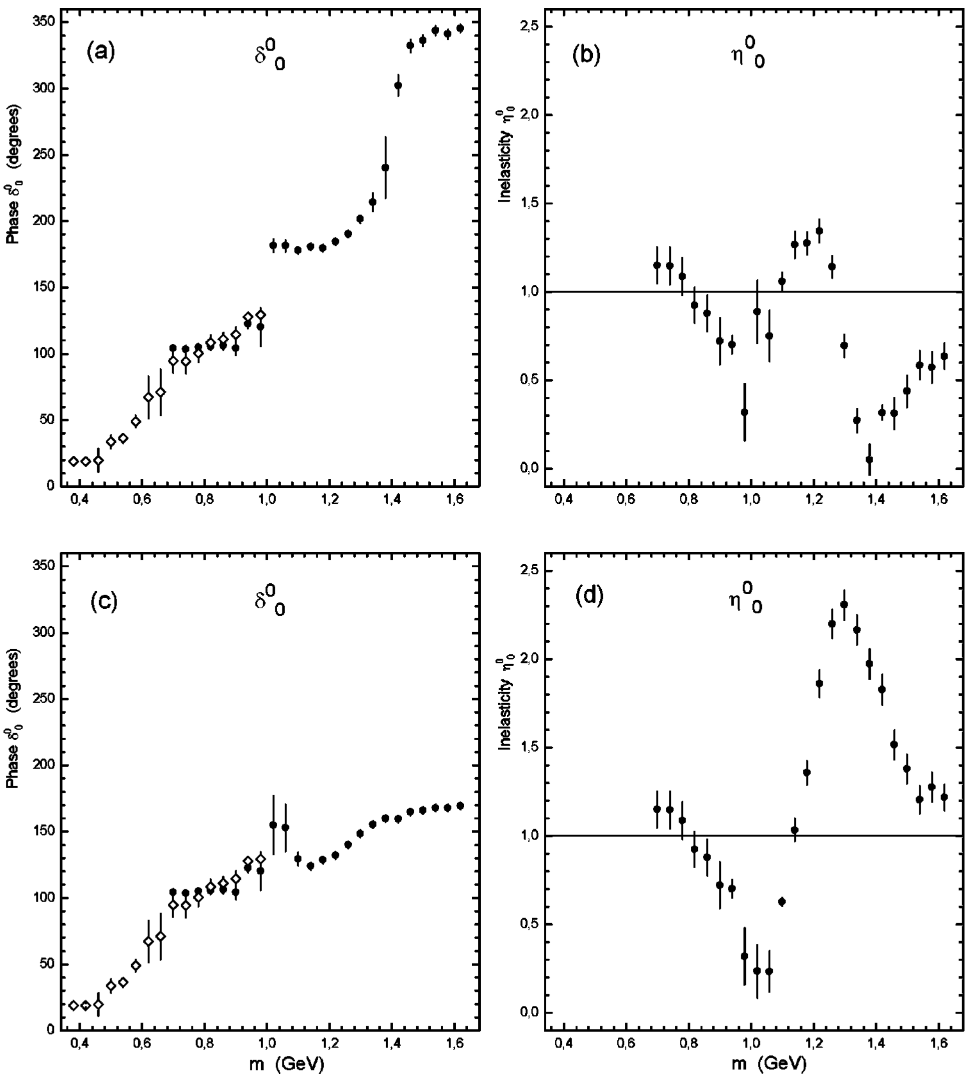

FIG. 3. The solid circles show the values of the phase shift δ_0^0 (a) and inelasticity η_0^0 (b) extracted from the KEK data [3] on the reaction $\pi^+\pi^-\rightarrow\pi^0\pi^0$ in the case that the m dependence of the phase δ corresponds to variant (i) described in the text. (c),(d) The same for variant (ii). The open diamonds show the values of δ_0^0 corresponding to Fig. 1(d).

the region $0.68<m<1$ GeV in two different ways are in quite reasonable agreement with each other. In obtaining δ_0^0 and η_0^0 in the general case, the Argand diagram of the amplitude $\tilde{A}_S$ was built for each variant. After this the values of $2(\delta_0^0-\delta_0^2)$ obtained from Eqs. (4) and (5) were finally defined by the requirement that those of δ_0^0 be smoothly connected as a function of m. That a strong violation of unitarity takes place in variant (ii) for $m>1.16$ GeV [see Fig. 3(d)] can easily be understood from the relation $\phi=\delta-2\,\delta_0^2+\phi_D$ [see Eq. (4)]. The fact is that the values of ϕ in this case fall into the range from 180° to 360°, which is forbidden, as ϕ is the phase of the formally unitary amplitude $\tilde{A}_S$. Furthermore, in variant (ii), the phase sift δ_0^0 for $m>1$ GeV [see Fig. 3(c)] is in rather poor agreement with the $\pi^+\pi^-$ production data [7–12] according to which, for example, at $m\approx1.3$ GeV δ_0^0 has to be close to 270°. Thus, variant (ii) with $\delta>0$ for all m can be rejected. As for variant (i), there is a set of specific features which, to our knowledge, are missing from the $\pi^+\pi^-$ data [7–12]. As is seen from Fig. 3(a) and 3(b), in this case we have noticeable differences of η_0^0 from unity for $m<1$ GeV, its approximate equality to 1 for $1<m<1.12$ GeV, violation of unitarity near 1.2 GeV, and sharp jumps of δ_0^0 and η_0^0 with further increasing m. There is little doubt that these features are artifacts of the partial wave analysis of the $\pi^-p\rightarrow\pi^0\pi^0n$ data.

Another difficulty is that the accepted normalization for $|A_S|^2$ leads to $B_{f_2\pi\pi}=0.760\pm0.034$ [see Eq. (2)], while according to the PDG data [13] $B_{f_2\pi\pi}=0.847\pm^{0.024}_{0.013}$. These two values agree with one another only within their double errors. Hence, in principle, one may conclude that the $\pi^0\pi^0$ data [3] indicate that the absolute cross section of the $f_2(1270)$ resonance formation through the OPE mechanism in the reaction $\pi^-p\rightarrow\pi^0\pi^0n$ might turn out to be approximately 20% smaller, at least at $m\approx m_{f_2}$, than that expected from the PDG data [13]. Alternatively, the KEK data [3] might be normalized with use of the known value $\max|A_D|^2=(1+\eta_2^0)^2/4=B^2_{f_2\pi\pi}$ with $B_{f_2\pi\pi}$ from Ref. [13]. However, in this case, the resulting values of $|A_S|^2$ in the most interesting region of the lightest scalar resonance $\sigma(600)$ [3,13,19] would be approximately 25% higher than the unitarity limit for $|A_S|^2$.

We shall see in the next sections that the other experimental data for the reaction $\pi^-p\rightarrow\pi^0\pi^0n$ lead to very similar difficulties.

III. ANALYSIS OF THE BNL DATA

In the BNL experiment [4], high statistics on the reaction $\pi^- p \to \pi^0\pi^0 n$ (about 8.5×10^5 events) were accumulated and a detailed partial wave analysis of the $\pi^0\pi^0$ angular distributions was performed. This analysis has been done for ten sequential intervals in $-t$ covering the region $0<-t<1.5\ \text{GeV}^2$ and over the m range from 0.32 to 2.2 GeV scanned with a 0.04-GeV-wide step. As a result, two solutions for the unnormalized intensities of the S and D_0 partial waves and four (because of a sign ambiguity) for their relative phase were obtained. The above quantities were denoted in Ref. [4] by $|S|^2$, $|D_0|^2$, and $\varphi_{S\text{-}D_0}$, respectively; in so doing, D_0 denotes the D wave with $L_z=0$, where L_z is a projection of the $\pi^0\pi^0$ relative orbital angular momentum on the z axis in the Gottfried-Jackson reference frame [4]. In the following, we shall use these notations, too. One of the solutions for the S and D_0 wave intensities, which is characterized by a large magnitude of $|S|^2$ and a small one of $|D_0|^2$ for $m<1$ GeV, and which is smoothly continued to the higher mass region, where the D_0 wave is dominated by the $f_2(1270)$ resonance contribution, was selected in Ref. [4] as the physical solution. Together with the intensities $|S|^2$ and $|D_0|^2$, the physical solution also includes two corresponding sets of the $\varphi_{S\text{-}D_0}$ phase values which differ only in sign. Note that the other solution intersects with the physical one at $m\approx1$ GeV. We agree with the physical arguments given in Ref. [4] based on which the other solution can be rejected in the region $m<1$ GeV. However, for $m>1$ GeV we shall analyze the two solutions and also the cases with transitions of the phase φ_{S-D_0} from one solution to the other one.

For the analysis we take the BNL data [4] on $|S|^2$, $|D_0|^2$, and $\varphi_{S\text{-}D_0}$ pertaining to five intervals of $-t$, $0.01<-t<0.03\ \text{GeV}^2$, $0.03<-t<0.06\ \text{GeV}^2$, $0.06<-t<0.1\ \text{GeV}^2$, $0.1<-t<0.15\ \text{GeV}^2$, and $0.15<-t<0.2\ \text{GeV}^2$, and to the region $0.32<m<1.6$ GeV. Note that data on $\varphi_{S\text{-}D_0}$ are available only for $m>0.8$ GeV. To obtain the values of the quantities $|A_S|^2$, $|A_D|^2$, and δ (see Sec. II) as functions of m, characterizing the reaction $\pi^+\pi^-\to\pi^0\pi^0$ on the mass shell, we parametrize the t dependence of $|S|^2$, $|D_0|^2$, and $\varphi_{S\text{-}D_0}$ by means of the following expressions:

$$|S|^2=\frac{m^2}{q}|A_S|^2\frac{-t\exp[b_S(t-m_\pi^2)]}{(t-m_\pi^2)^2},$$

$$|D_0|^2=5\frac{m^2}{q}|A_D|^2\frac{-t\exp[b_{D_0}(t-m_\pi^2)]}{(t-m_\pi^2)^2}, \tag{6}$$

$$\varphi_{S\text{-}D_0}=\delta+\alpha(t/m_\pi^2-1), \tag{7}$$

and, in each 0.04 GeV mass bin, thus determine the unnormalized intensities $|A_S|^2$ and $|A_D|^2$, the phase δ, and also the slopes b_S, b_{D_0}, and α by fitting to the BNL data on the t and m distributions by the formulas (6) and (7).[2] In so doing, for $|S|^2$, $|D_0|^2$, and $\varphi_{S\text{-}D_0}$ in each $-t$ bin we take into account the physical solution for $m<1$ GeV and the physical and other ones for $m>1$ GeV. Unfortunately, the absolute value of the $\pi^- p\to\pi^0\pi^0 n$ reaction cross section has not been determined in the BNL experiment [4]. Therefore, to normalize the extrapolated intensities $|A_S|^2$ and $|A_D|^2$ we proceed in the same way as in Sec. II. The extrapolated and normalized data corresponding to the physical and other solutions are plotted in Fig. 4(a), 4(c), and 4(e) with solid and open symbols, respectively. It is interesting to note that as a result of the extrapolation two branches of the $\varphi_{S\text{-}D_0}$ phase values for the other solution (i.e., the branch with $\varphi_{S\text{-}D_0}>0$ for all m and that with $\varphi_{S-D_0}<0$ for all m) interweave with each other in the region $m>1.24$ GeV [see Fig. 4(e)] in such a way that there arise two new branches of the extrapolated phase δ, which are characterized by a smooth dependence on m and which, for example, can be considered as either intersecting or osculating near 1.26 GeV.

As in Sec. II, we begin with the determination of the phase shift δ_0^0 for $m<1$ GeV from the data on $|A_S|^2$ [see Fig. 4(a)] assuming that $\eta_0^0=1$ in this region. The resulting phase shift values are shown in Fig. 5 by open circles. Note that two points in the region $m\approx m_K$ disturbed by the $K_S^0\to\pi^0\pi^0$ events [4] are omitted. Then, having the data on $|A_S|^2$, $|A_D|^2$, and δ for $m>0.8$ GeV (see Fig. 4), we determine the values of δ_0^0 and η_0^0 with use of the general formulas (3), (4), and (5). In extracting the information on δ_0^0 and η_0^0, the phase ϕ_D was defined by fitting to the data on $|A_D|^2$ [see Fig. 4(c)] with use of Eq. (1), and the parameters of $f_2(1270)$ were found to be [see also the curves on Fig. 4(c)], for the physical solution,

[2]Such two-parametric fits to the off-shell partial wave intensities have been widely used in the literature to obtain suitably extrapolated data (see, for example, Refs. [9,17,20,21]). However, the determination of the phase δ by use of direct extrapolation of the data on the phase $\varphi_{S\text{-}D_0}$ [see Eq. (7)] may provoke a question. If data on the S-D_0 interference contribution, as such, had been presented in Ref. [4], the problem would not have arisen. The fit of such data to the function $-2ta\exp[b(t-m_\pi^2)]/(t-m_\pi^2)^2$, analogous to those in Eq. (6), and the identification of the fitted parameter a with $\sqrt{5}(m^2/q)|A_S||A_D|\cos\delta$, would allow $|\delta|$ to be determined in the proper way. Because such data are not available, an indirect test of the results obtained with Eq. (7) was carried out. Using the data [4] on $|S|^2$, $|D_0|^2$, and $\varphi_{S\text{-}D_0}$, we constructed the quantity $2|S||D_0|\cos\varphi_{S\text{-}D_0}$ and found with the above extrapolation the on-shell S-D interference contribution. Then, knowing $|A_S|^2$ and $|A_D|^2$ independently, we determined δ as a function of m. The δ phase values obtained in the two ways are in very close agreement with each other. Certainly, owing to the forced double recounting of the errors of the input data with the indirect test, the errors of δ turn out to be larger than those obtained from the fit by Eq. (7). On the other hand, when the values of δ are determined by using Eq. (7), their errors in practice are not different from the errors of the input data for $\varphi_{S\text{-}D_0}$. All the aforesaid allowed us to prefer the determination of the phase δ by use of Eq. (7).

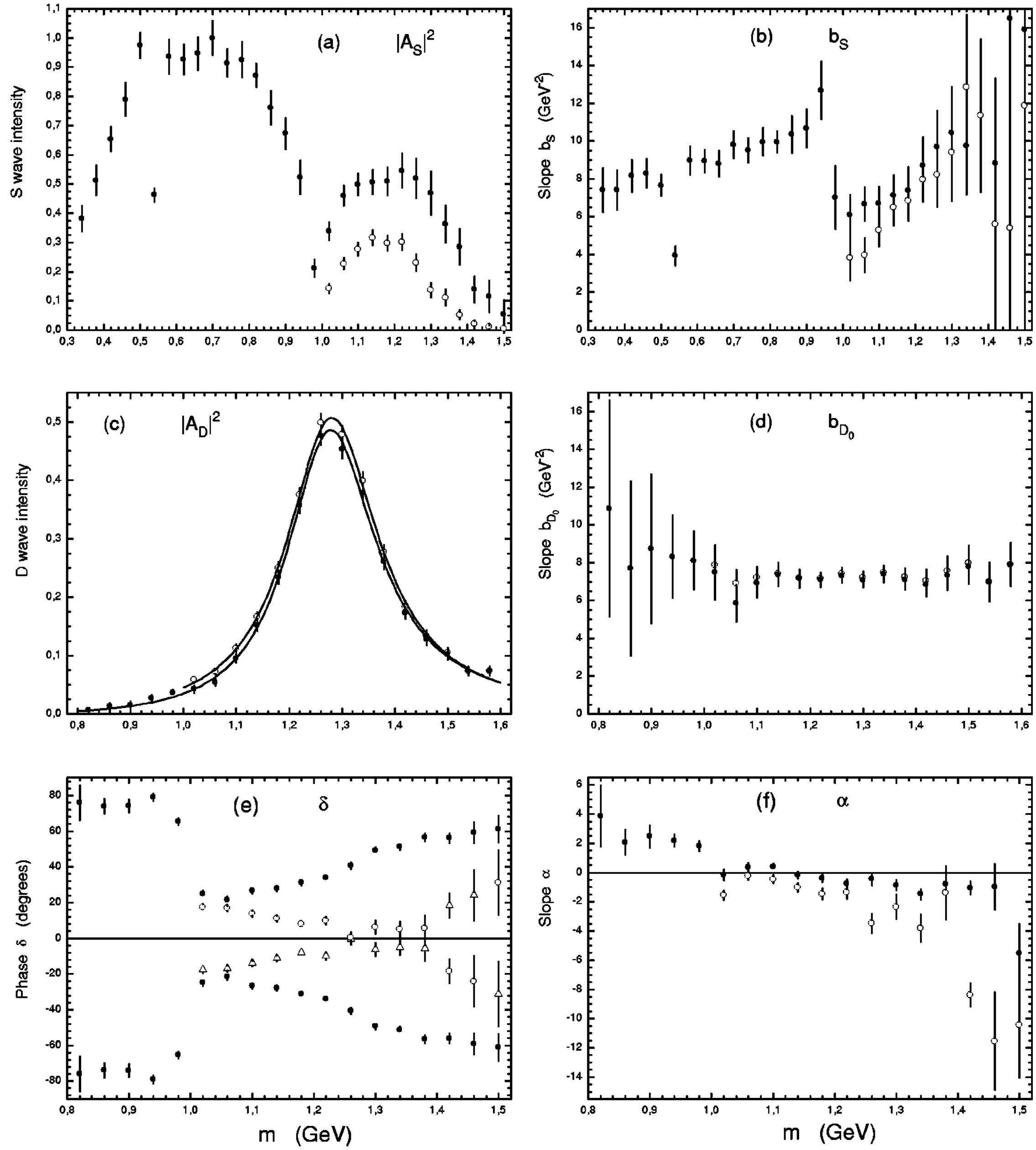

FIG. 4. The results of the extrapolation of the BNL data [4]. (a) The extrapolated and normalized S wave intensity. (c) The extrapolated and normalized D wave intensity. (e) The extrapolated relative phase δ between the S and D wave amplitudes. The slopes b_S (b), b_{D_0} (d), and α (f) as functions of m. The solid circles correspond to the physical solution. The open circles [and also the open triangles in plot (e) for δ] correspond to the other solution. The lower and upper curves in plot (c) are the fits using Eq. (1) with the parameters of $f_2(1270)$ presented in Eqs. (8) and (9), respectively.

$$m_{f_2}=1.279\pm0.002\ \text{GeV},\quad \Gamma_{f_2}=0.205\pm0.005\ \text{GeV},$$

$$R_{f_2}=3.96\pm0.24\ \text{GeV}^{-1},\quad B_{f_2\pi\pi}=0.697\pm0.008, \tag{8}$$

and, for the other solution,

$$m_{f_2}=1.281\pm0.002\ \text{GeV},\quad \Gamma_{f_2}=0.211\pm0.005\ \text{GeV},$$

$$R_{f_2}=4.65\pm0.33\quad \text{GeV}^{-1},\quad B_{f_2\pi\pi}=0.712\pm0.007. \tag{9}$$

Our results obtained for the previously selected solutions among all the possible ones, which are shown in Fig. 4, are plotted in Fig. 5 with solid circles. Strictly speaking, the selection is reduced to rejection of the physical solution with $\delta<0$ for $m<1$ GeV [see Fig. 4(e)], since a simple estimate $\delta_0^0=\delta-\delta_0^2+\phi_D$ (see Sec. II) yields $\delta_0^0\approx-(25\text{–}40)^\circ$ for this solution in the region of 0.8–1 GeV, which is, certainly, unsatisfactory. In its turn, Fig. 5(a) and 5(b) show that the physical solution with $\delta>0$ for all m can also be rejected due to strong violation of the unitarity condition for $m>1.2$ GeV. Figures 5(c) and 5(d) correspond to the physical

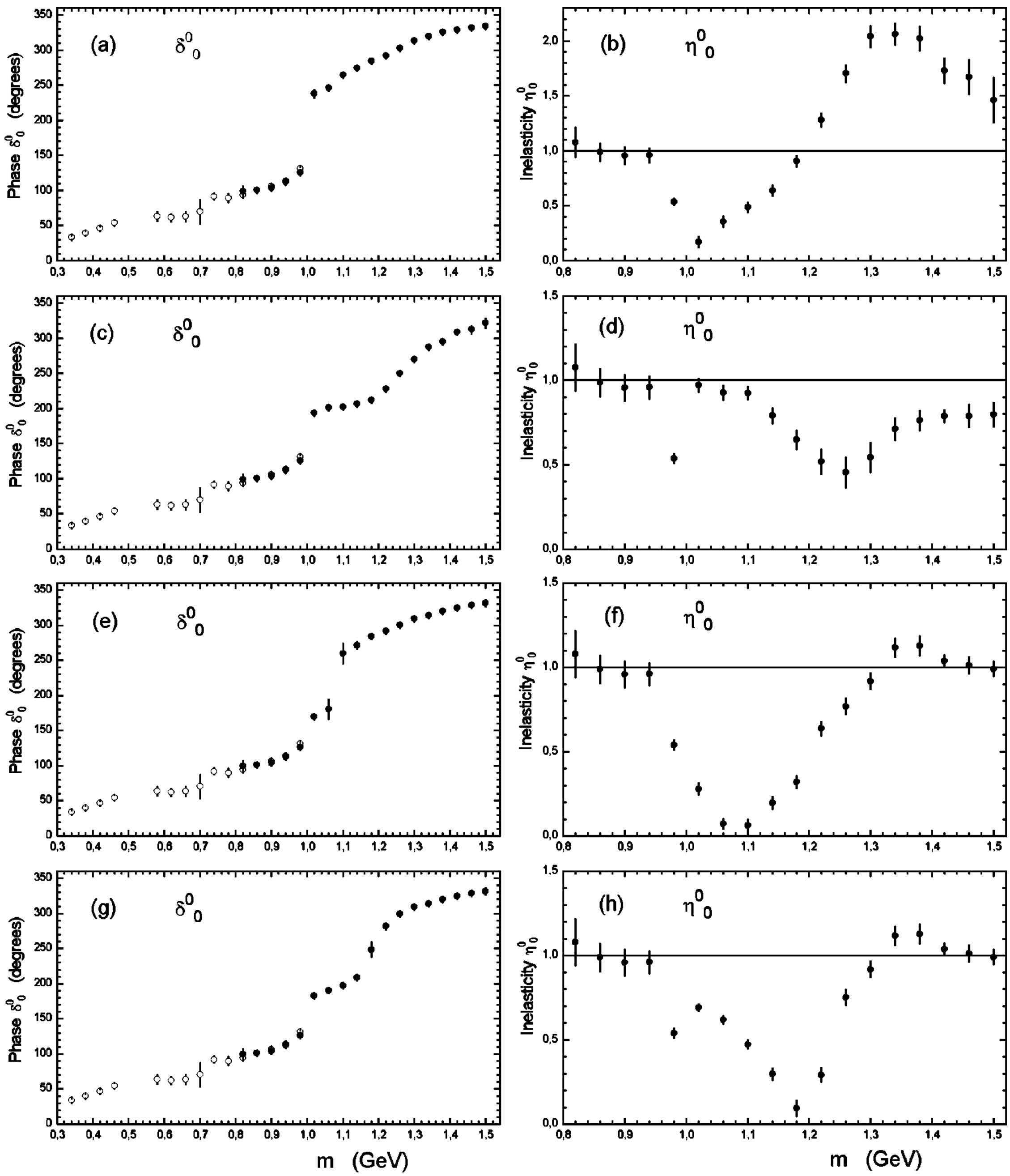

FIG. 5. The phase shift δ_0^0 and inelasticity η_0^0 extracted from the BNL data [4]. Plots (a) and (b) correspond to the physical solution (see Fig. 4) with $\delta>0$ for all m. Plots (c) and (d) correspond to the physical solution with a transition of the phase δ at $m\approx1$ GeV from the branch pertaining to its positive values to that with its negative ones [see Fig. 4(e)]. Plots (e) and (f) correspond to the combination of the physical solution with $\delta>0$ for $m<1$ GeV and the other solution for $m>1$ GeV with $\delta>0$ in the region $1<m<1.28$ GeV and with $\delta<0$ in the region $1.28<m<1.52$ GeV [see Fig. 4(a) and 4(e)]. Plots (g) and (h) correspond to the combination of the physical solution with $\delta>0$ for $m<1$ GeV and the other solution for $m>1$ GeV with $\delta<0$ in the region $1<m<1.52$ GeV [see Fig. 4(a) and 4(e)]. The open circles show the values of the phase shift δ_0^0 obtained from the data on $|A_S|^2$ for $m<1$ GeV [see Fig. 4(a)] alone under the assumption that η_0^0 is unity.

solution for $|A_S|^2$, $|A_D|^2$, and δ with the transition of the phase δ at $m\approx1$ GeV from the branch with $\delta>0$ to that with $\delta<0$ [see Fig. 4(e)]. Such a physical solution consists with unitarity but corresponds to the weak coupling between the $\pi\pi$ and $K\bar{K}$ channels near the $K\bar{K}$ threshold. Indeed, for this solution η_0^0 is close to unity in the region $1<m<1.15$ GeV. However, the latter disagrees with the data obtained from the reactions $\pi^-p\to\pi^+\pi^-n$, $\pi^+p\to\pi^+\pi^-\Delta^{++}$, and $\pi N\to K\bar{K}(N,\Delta)$ (see, for example, Refs. [7,9,10,16,22,23]). Figures 5(e) and 5(f) correspond to the combination of the physical solution with $\delta>0$ for $m<1$ GeV and the other one for $m>1$ GeV with $\delta>0$ in the region $1<m<1.28$ and $\delta<0$ in the region $1.28<m<1.52$ GeV [see Fig. 4(a) and 4(e)]. Finally, Fig. 5(g) and

5(h) correspond to a similar combination of the physical solution and the other one for which $\delta<0$ for m from 1 to 1.52 GeV [see also Fig. 4(e)]. Certainly, there are two more variants which differ from the last two ones only by the sign of δ for $m>1.28$ GeV [see Fig. 4(e)]. These variants lead, however, to appreciable violations of the unitarity condition for $m>1.32$ GeV, and therefore are of little interest. Thus, one can conclude that just the variant presented in Fig. 5(e) and 5(f) is, in many respects, in qualitative agreement with the results of the previous partial wave analyses of the $\pi^+\pi^-$ data [1,7,9,11]. As indicated above, this variant corresponds to the positive relative phase $\delta=\phi_S-\phi_D$ up to the $f_2(1270)$ resonance and the negative one above it. An important point is that this behavior of δ as a function of m is strongly confirmed by the pioneering data from the polarized target experiment for the reaction $\pi^-p\to\pi^+\pi^-n$ at 17.2 GeV [11].

Comparing Fig. 5 with Fig. 3, we just note that the BNL data lead to obviously higher values of the phase shift δ_0^0 for $m<0.5$ GeV than the KEK data.

Finally, as is seen from Eqs. (8) and (9), the BNL data indicate that the branching ratio $B_{f_2\pi\pi}$ can amount to approximately 84% of the PDG value [13]. The possible consequences of a similar discrepancy have already been discussed in connection with the KEK data at the end of Sec. II (recall that the relevant ratio for the KEK data has been found to be approximately 90%). Here we add only the evident remark that, in the case when the absolute normalization of the produced events is known, the value of $B^2_{f_2\pi\pi}$ defines the height of the resonance mass distribution, whereas the product $m_{f_2}\Gamma_{f_2}B^2_{f_2\pi\pi}$ is responsible for the integrated cross section.

IV. DISCUSSION OF THE GAMS DATA

The highest statistics on the reaction $\pi^-p\to\pi^0\pi^0n$ were accumulated by the GAMS Collaboration in two experiments at 38 GeV [5] and 100 GeV [6]. However, the small $-t$ region from 0 to 0.2 GeV^2 were examined in [5,6] very sparingly. But the data averaged over the small $-t$ region were presented for $|S|^2$ and $\varphi_{S\text{-}D_0}$ in Ref. [5] and for $|S|^2$, $|D_0|^2$, and $\varphi_{S\text{-}D_0}$ in Ref. [6]. Such "global" data, of course, do not permit to perform a proper extrapolation of the mass distributions measured from the physical region to the pion pole. Nevertheless, we discuss some typical features of the GAMS data. The physical solution for $|S|^2$ and $\varphi_{S\text{-}D_0}$ and the other solution only for $|S|^2$ were presented in Ref. [5] in the region $0.8<m<1.6$ GeV. The $\varphi_{S\text{-}D_0}$ phase was found to be positive in the full mass range [5] (about the existing ambiguous solution with $\varphi_{S\text{-}D_0}<0$ the readers, probably, have to guess by themselves). In general, the available GAMS data [5] are very similar to the corresponding BNL data [4]. For example, in the case of the physical solution, $|S|^2$ and $\varphi_{S\text{-}D_0}$ from Ref. [5] behave as functions of m in the same way as the extrapolated quantities $|A_S|^2$ and δ shown in Fig. 4(a) and 4(e) by solid circles. However, it is such a physical solution for $|A_S|^2$ and δ (with $\delta>0$ for all m) that leads to strong violation of the unitarity condition for $m>1.2$ GeV [see Fig. 5(b)]. In analyzing the $\pi^0\pi^0$ system produced in the reaction $\pi^-p\to\pi^0\pi^0n$ at 100 GeV, the only solution for $|S|^2$, $|D_0|^2$, and $\varphi_{S\text{-}D_0}$ was selected and presented by the GAMS Collaboration in Ref. [6]. Unfortunately, this unique solution is very close to the above physical one obtained in the GAMS 38 GeV $\pi^-p\to\pi^0\pi^0n$ experiment [5].

It is of first importance that the GAMS Collaboration measured the absolute cross section of the $f_2(1270)$ resonance formation in the D_0 wave in the region $0<-t<0.2$ GeV^2. According to Ref. [24], at 38 GeV, $\sigma_{D_0}(\pi^-p\to f_2(1270)n\to\pi^0\pi^0n)=2.3\pm0.2$ μb. More recently, this value was used, in particular, to normalize the 100 GeV data [6]. Although the cross section value obtained is approximately 1.5–2 times greater than in a set of previous $\pi^0\pi^0$ production experiments [24,25], nevertheless, it is 1.57 times smaller than the estimate based on the OPE model [the experimental underestimation of the $\pi^-p\to f_2(1270)n\to\pi^0\pi^0n$ reaction cross section is an old story, all details of which can be found in Ref. [25]]. By using this model with the PDG values of m_{f_2}, Γ_{f_2}, and $B_{f_2\pi\pi}$ [13] we estimate

$$\begin{aligned}\sigma_{D_0}&(\pi^-p\to f_2(1270)n\to\pi^0\pi^0n)\\&\approx\sigma^{OPE}(\pi^-p\to f_2(1270)n\to\pi^0\pi^0n)\\&\approx\frac{g^2_{\pi^-pn}}{4\pi}\frac{5\pi}{m_p^2P^2_{\pi^-}}m_{f_2}\Gamma_{f_2}\frac{2}{9}B^2_{f_2\pi\pi}\\&\times\int_{-0.2\ \text{GeV}^2}^{0}\frac{-t\exp[b_{f_2}(t-m_\pi^2)]}{(t-m_\pi^2)^2}dt\\&\approx3.6\ \ \mu\text{b},\end{aligned}\tag{10}$$

where $P_{\pi^-}=38$ GeV, $g^2_{\pi^-pn}/4\pi\approx2\times14.3$, and $b_{f_2}\approx7.5$ GeV$^{-2}+2\times0.8$ GeV$^{-2}\ln(38/18.3)\approx8.68$ GeV^{-2} [in estimating the slope b_{f_2}, its Regge energy dependence and the results for the slope b_{D_0} in the $f_2(1270)$ mass region presented in Fig. 4(d) were taken into account]. Note that this estimate is in good agreement with the result of the extrapolation of the available data on the reaction $\pi^-p\to f_2(1270)n\to\pi^+\pi^-n$ at 17.2 GeV [8], 100 GeV, and 175 GeV [26] to the GAMS energy (see Ref. [25] for details). Thus, the GAMS data [24] indicate that the value of $B_{f_2\pi\pi}$ can amount to about 80% of that given by the PDG [13].

V. DISCUSSION OF DIFFICULTIES

We now briefly summarize the common difficulties encountered in analyzing the data obtained in four recent experiments on the reaction $\pi^-p\to\pi^0\pi^0n$. First, the physical solutions selected by using the partial wave analyses of the $\pi^0\pi^0$ production data lead to values of δ_0^0 and η_0^0 which are incompatible with the known results obtained from the $\pi^+\pi^-$ data, at least for $m>1$ GeV. Some of these solutions lead to strong violations of the unitarity condition. On the other hand, among the other solutions one can point out, in principle, the preferred ones. Secondly, it is astonishing that the data of the four recent experiments on $\pi^0\pi^0$ production include indications that the value of $B_{f_2\pi\pi}$ might be distinctly smaller than the currently accepted one. This difficulty is rather serious and highly interesting. Let us recall that the $\pi\pi$ production experiments on unpolarized targets, in particular, those under discussion here, do not permit the contributions of the π and a_1 exchange mechanisms to be separated in principle, even with huge statistics, because these contributions to the observed unpolarized cross section are incoherent [27]; in other words, there is no model-independent way to do this in the physical region. Therefore, in our opinion, the difficulty with $B_{f_2\pi\pi}$ may present itself as further evidence that, in practice, partial wave analyses of the unpolarized data allow one to determine the intensities and the relative phases of the S, D, etc., $\pi\pi$ partial waves only approximately, in fact, with any extrapolation method. "The degree of proximity" is associated with the poorly known relative magnitude of the nonleading a_1 exchange contribution. With high statistical accuracy of the unpolarized data the presence of the a_1 exchange mechanism can manifest itself in the events responsible for $|S|^2$ just in the form of the above difficulty. In fact, this statement follows naturally from the analysis of the unnormalized KEK and BNL data (see the end of Sec. II and also Sec. V for details). As for the GAMS data [24], they appear to point merely to the general problem involving accurate measurement of the $\pi^- p \to \pi^0\pi^0 n$ reaction cross section.

Because the a_1 exchange certainly exists, we suggest using in future the more suitable notation for the S-D_0 interference contribution to be extracted in the unpolarized target experiments, instead of the commonly used simplified one of the form $|S||D_0|\cos\varphi_{S\text{-}D_0}$. It includes mention of the coherence factor (see, for example, Ref. [28]) and is more adequate for the measured quantity. Experimentally, the S and D_0 wave intensities $|S|^2$ and $|D_0|^2$ and the S-D_0 interference contribution $\xi|S||D_0|\cos\tilde{\varphi}$ are measured simultaneously. In fact, $|S| \equiv [|S_\pi|^2 + |S_{a_1}|^2]^{1/2}$, $|D_0| \equiv [|D_{0\pi}|^2 + |D_{0a_1}|^2]^{1/2}$, and the coherence factor $\xi (0 \leqslant \xi \leqslant 1)$ and the phase $\tilde{\varphi}$ have the forms

$$\xi = \left| \sum_{i=\pi,a_1} S_i D_{0i}^* \right| \Bigg/ \left[\left(\sum_{i=\pi,a_1} \left| S_i \right|^2 \right) \left(\sum_{i=\pi,a_1} \left| D_{0i} \right|^2 \right) \right]^{1/2}, \quad (11)$$

$$\tilde{\varphi} = \arctan \left[\left(\sum_{i=\pi,a_1} \left| S_i \right| \left| D_{0i} \right| \sin\varphi_i \right) \Bigg/ \times \left(\sum_{i=\pi,a_1} \left| S_i \right| \left| D_{0i} \right| \cos\varphi_i \right) \right], \quad (12)$$

where $S_\pi(D_{0\pi})$ and $S_{a_1}(D_{0a_1})$ are the $S(D_0)$ wave production amplitudes caused by the π and a_1 exchange mechanisms, respectively (at high energies the π and a_1 exchanges contribute to the $\pi N \to \pi\pi N$ reaction amplitudes with and without nucleon helicity flip, respectively), and φ_i is the relative phase between the amplitudes S_i and D_{0i}. Let us consider the case when the amplitude D_{0a_1} is negligible. Then, denoting $\tilde{\varphi} = \varphi_\pi$ by $\varphi_{S\text{-}D_0}$, one can see that the real interference contribution differs from that presented with the simplified notation $|S||D_0|\cos\varphi_{S\text{-}D_0}$ by the coherence factor $\xi = 1/\sqrt{1+|S_{a_1}|^2/|S_\pi|^2}$. If we put $\xi=1$ for all m, we shall always deal with the effectively underestimated values of $|\cos\varphi_{S\text{-}D_0}|$.

More discussion both of the additional assumptions needed in analyzing the unpolarized data and of the a_1 exchange contribution can be found in [1,2,11,12,27,29,30].

VI. CONCLUSION

Using the simplest method we have extracted the values of the $I=0$ $\pi\pi S$ wave phase shift δ_0^0 and inelasticity η_0^0 from the current data on the reaction $\pi^- p \to \pi^0\pi^0 n$. Considering the ambiguous solutions found with the partial wave analysis we have shown that the so-called other solutions, in principle, are found to be preferred to the physical ones in the mass region above 1 GeV. It seems clear that a new set of precise experiments on the reaction $\pi^- p \to \pi^0\pi^0 n$ is needed both for more precise definition of the $\pi^0\pi^0$ production mechanism and to obtain more detailed information on $\pi\pi$ scattering and light scalar resonances in the $\pi\pi$ channel. Let us formulate in this connection several concrete suggestions, leaving aside the general wish to investigate the reaction $\pi^- p \to \pi^0\pi^0 n$ on a polarized target.

(1) Detailed data on the m and t distributions for the $\pi^0\pi^0 S$ and D_0 partial waves, especially in the region $0 < -t < 0.2$ GeV^2 where the OPE mechanism dominates, and measurements of the absolute value of the $\pi^- p \to \pi^0\pi^0 n$ reaction cross section at different energies, for example, at KEK, BNL, IHEP, and CERN, would be highly desirable. The relative accuracy of new measurements must be comparable with (or better than) that given by the PDG [13] for $B^2_{f_2\pi\pi}$. This would allow one to perform an accurate description of the $f_2(1270)$ formation differential cross section within the OPE model and to test how well the S wave $\pi^0\pi^0$ production cross section at its absolute maximum (which is located in the region $0.6 < m < 0.8$ GeV) agrees with the OPE model prediction under the standard normalization condition according to which $|A_S|^2 = 1$ (i.e., $\delta_0^0 - \delta_0^2 = 90°$ and $\eta_0^0 = \eta_0^2 = 1$) at the absolute maximum point. An excess of the experimental values over the model expectations would be good evidence, obtained from the unpolarized target data, for the presence of the a_1 exchange contribution to the S wave $\pi^0\pi^0$ production cross section in the region of its absolute maximum. Alternatively, if the maximal experimental value of the S wave cross section turns out to be less than in the OPE model, then it will completely disturb existing ideas about the δ_0^0 phase shift for $m < 1$ GeV, which seems to be highly unlikely.

(2) We suggest performing in the low $-t$ region especially careful measurements of $\pi^0\pi^0$ production in the S wave for m from 0.9 to 1.1 GeV, i.e., in the region of the well known interference minimum in $|S|^2$ located near the $K\bar{K}$ threshold. This would allow one to obtain important additional information on the $f_0(980)$ resonance coupling constant to the $K\bar{K}$ channel, $g_{f_0K\bar{K}}$, and to resolve the long-

standing question [16] concerning a possible ambiguity in the behavior of the phase shift δ_0^0 above the $K\bar{K}$ threshold which arises at $g^2_{f_0 K^+K^-}/4\pi > 4\pi m_K^2 \approx 3.1$ GeV^2. Furthermore, the magnitude of the S wave intensity in the immediate region of the minimum (if it lies below the $K\bar{K}$ threshold) can be used to obtain a very strong upper limit on the a_1 exchange contribution at small $-t$ in this region of m.

(3) As a rule, the assumption of phase coherence between the D_0 and D_- amplitudes is one of those used to select the physical solution; see, for example, Refs. [4,6,9,31]. Here D_- denotes the D wave with $|L_z|=1$, in the Gottfried-Jackson reference frame, which is produced via unnatural parity exchanges in the t channel of the reaction $\pi N \to \pi\pi N$. In this connection we would like to call attention to a new curious circumstance. According to the GAMS measurements [6], the ratio $|D_-|^2/|D_0|^2$ in the $f_2(1270)$ mass region at 100 GeV is half as large as that at 38 GeV. This fact may testify to compensation of the πP and $a_2 P$ Regge cut contributions (P denotes the Pomeron exchange) to the D_- wave production amplitude with increasing energy, i.e., to violation of phase coherence.

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[1] P. Minkovski and W. Ochs, Eur. Phys. J. C **9**, 283 (1999).
[2] V.V. Vereshchagin, K.N. Mukhin, and O.O. Patarakin, Usp. Fiz. Nauk **170**, 353 (2000) [Phys. Usp. **43**, 315 (2000)].
[3] K. Takamatsu *et al.*, in *Proceedings of the International Workshop on e^+e^- Collisions from ϕ to J/Ψ*, Novosibirsk, 1999, edited by G.V. Fedotovich and S.I. Redin (BINP, Novosibirsk, 2000), p. 163; K. Takamatsu *et al.*, Nucl. Phys. **A675**, 312c (2000); Prog. Theor. Phys. **102**, E52 (2001).
[4] J. Gunter *et al.*, Phys. Rev. D **64**, 072003 (2001).
[5] D. Alde *et al.*, Z. Phys. C **66**, 375 (1995).
[6] D. Alde *et al.*, Eur. Phys. J. A **3**, 361 (1998); Yad. Fiz. **62**, 446 (1999).
[7] B. Hyams *et al.*, Nucl. Phys. **B64**, 134 (1973).
[8] G. Grayer *et al.*, Nucl. Phys. **B75**, 189 (1974).
[9] P. Estabrooks and A.D. Martin, Nucl. Phys. **B79**, 301 (1974); **B95**, 322 (1975).
[10] B. Hyams *et al.*, Nucl. Phys. **B100**, 205 (1975).
[11] H. Becker *et al.*, Nucl. Phys. **B151**, 46 (1979).
[12] R. Kamiński, L. Leśniak and K. Rybicki, Z. Phys. C **74**, 79 (1997).
[13] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[14] N.N. Achasov, Usp. Fiz. Nauk **168**, 1257 (1998) [Phys. Usp. **41**, 1149 (1998)]; Nucl. Phys. **A675**, 279c (2000); Yad. Fiz. **65**, 573 (2002) [Phys. At. Nucl. **65**, 546 (2002)].
[15] N.N. Achasov, in *Hadron Spectroscopy*, edited by D. Amelin and A.M. Zaitsev, AIP Conf. Proc. 619 (AIP, Melville, NY, 2002), p. 112; W. Ochs, *ibid.*, p. 167; T. Barnes, *ibid.*, p. 447; E. Klempt, *ibid.*, p. 463.
[16] N.N. Achasov, S.A. Devyanin, and G.N. Shestakov, Phys. Lett. **102B**, 196 (1981); Yad. Fiz. **32**, 1098 (1980) [Sov. J. Nucl. Phys. **32**, 566 (1980)].
[17] W. Hoogland *et al.*, Nucl. Phys. **B126**, 109 (1977).
[18] N.B. Durusoy *et al.*, Phys. Lett. **45B**, 517 (1973).
[19] N.N. Achasov and G.N. Shestakov, Phys. Rev. D **49**, 5779 (1994); M. Harada, F. Sannino, and J. Schechter, *ibid.* **54**, 1991 (1996); S. Ishida *et al.*, Prog. Theor. Phys. **98**, 1005 (1997).
[20] P. Estabrooks *et al.*, in *π-π Scattering—1973*, edited by P. K. Williams and V. Hagopian, AIP Conf. Proc. 13 (AIP, New York, 1973), p. 37.
[21] N.N. Biswas *et al.*, Phys. Rev. Lett. **47**, 1378 (1981); N.M. Cason *et al.*, *ibid.* **48**, 1316 (1982); Phys. Rev. D **7**, 1586 (1983).
[22] S.D. Protopopescu *et al.*, Phys. Rev. D **7**, 1279 (1973).
[23] A.D. Martin, E.N. Ozmutlu and E.J. Squires, Nucl. Phys. **B121**, 514 (1977); N.N. Achasov, S.A. Devyanin, and G.N. Shestakov, Usp. Fiz. Nauk **142**, 361 (1984) [Sov. Phys. Usp. **27**, 161 (1984)]; Z. Phys. C **22**, 53 (1984).
[24] A.A. Kondashov and Yu.D. Prokoshkin, Nuovo Cimento Soc. Ital. Fis., A **107**, 1903 (1994); Yu.D. Prokoshkin, Yad. Fiz. **62**, 396 (1999) [Phys. At. Nucl. **62**, 356 (1999)].
[25] N.N. Achasov and G.N. Shestakov, Mod. Phys. Lett. A **8**, 2343 (1993).
[26] C. Bromberg *et al.*, Nucl. Phys. **B232**, 189 (1984).
[27] N.N. Achasov and G.N. Shestakov, Phys. Rev. D **58**, 054011 (1998); Yad. Fiz. **62**, 548 (1999) [Phys. At. Nucl. **62**, 505 (1999)].
[28] S. Hagopian *et al.*, Phys. Rev. Lett. **25**, 1050 (1970); G.S. Adams *et al.*, Phys. Rev. D **4**, 653 (1971); N.N. Achasov and G.N. Shestakov, Fiz. Elem. Chastits At. Yadra **9**, 48 (1978).
[29] M. Svec, Phys. Rev. D **47**, 2132 (1993); **53**, 2343 (1996).
[30] R. Kamiński, L. Leśniak, and K. Rybicki, EPJdirect C **4**, 4 (2002).
[31] A.D. Martin and C. Michael, Nucl. Phys. **B84**, 83 (1975).