Final state interactions in nonleptonic hyperon decays

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We investigate the importance of final state interactions in weak nonleptonic hyperon decays within a relativistic chiral unitary approach based on coupled channels. The effective potentials for meson-baryon scattering are derived from a chiral effective Lagrangian and iterated in a Bethe-Salpeter equation, which generates the low-lying baryon resonances dynamically. The inclusion of final state interactions decreases the discrepancy between theory and experiment for both s and p waves. Our study indicates that contributions from higher-order terms of the weak effective Lagrangian may play an important role in these decays.

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I. INTRODUCTION

Nonleptonic hyperon decays have been a topic of interest for more than three decades and a convincing theoretical description is still missing. There are seven such transitions and after omitting final state interactions their matrix elements are usually described in terms of two amplitudes—the parity violating s wave and the parity conserving p wave. A convenient framework to study these decays is provided by chiral perturbation theory (ChPT), whereby the amplitudes are expanded in terms of small four-momenta and the current quark masses m_q of the light quarks $q=u,d,s$. At lowest order in this expansion the amplitudes are given in terms of two unknown coupling constants of the effective Lagrangian: the so-called low-energy constants (LECs). It is well known that with just these two LECs it is not possible to obtain a reasonable description of both s and p waves [1]. If, e.g., one employs values that provide a reasonable fit to the s waves, a poor description for the p waves is obtained. A good p wave representation, on the other hand, yields a poor s wave fit. Different authors have tried to overcome this problem by going beyond leading order. In [2] the leading chiral logarithms to these decays were calculated neglecting local counterterms, but the resulting s wave predictions no longer agreed with the data, and the corrections to the p waves were even larger.

The decays were reinvestigated by Jenkins [3] within the framework of heavy baryon ChPT, explicitly taking into account spin-3/2 decuplet loops. Again, no local counterterms were included and only the nonanalytic pieces of the meson loops were retained. She found significant cancellations between the octet and decuplet components in the loops, restoring agreement between theory and experiment for the s waves, although for the p waves the chiral corrections did not provide a good description of the data. Hence, the inability to fit s and p waves simultaneously still remained even after including the lowest nonanalytic contributions.

A complete calculation at the one-loop level and including local counterterms was performed in [4]. Because of the proliferation of new unknown LECs at higher chiral orders, a fit to both s and p waves is possible, but not unique, so that the theory lacks predictive power. However, it remains unclear whether the values of the coupling constants have been chosen appropriately.

Another intriguing possibility was examined by Le Yaouanc *et al.*, who assert that a reasonable fit for both s and p waves can be provided by adding pole contributions from $SU(6)$ $(70,1^-)$ states to the s waves [5]. Their calculations were performed in a simple constituent quark model and appear to be able to provide a resolution to the s and p wave dilemma. This approach has been studied also within the framework of ChPT in [6]. To this end, the spin-$1/2^-$ octet from the $(70,1^-)$ states and the octet of Roper-like $1/2^+$ fields have been included in the effective field theory. In ChPT the inclusion of the Roper octet, which is in the same mass range as the $1/2^-$ octet, was necessary to improve the agreement with the experimental data, since the two lowest-order couplings for the p wave amplitudes tend to cancel, thus enhancing the contributions from terms of higher chiral order.[1] Integrating out the resonances generates counterterms of the Lagrangian at next-to-leading order and by fitting the weak couplings of the resonances one obtains satisfactory agreement with experiment. Thus the inclusion of spin-1/2 resonances in nonleptonic hyperon decays provides an estimate of the importance of higher-order counterterms.

There is, however, an independent way of investigating the importance of low-lying resonances in nonleptonic hyperon decays, whereby the resonances are produced dynamically within a relativistic chiral unitary approach based on coupled channels. In this framework the hyperon decays via the weak interaction into a Goldstone boson (π,K,η) and a ground state baryon which can then undergo final state interactions. These can be accurately described by deriving effective potentials for meson-baryon scattering from the chiral effective Lagrangian and subsequently iterating them in a Bethe-Salpeter equation (BSE). The approach has been successfully applied to both meson-baryon scattering processes and photoproduction of pseudoscalar mesons and the perti-

[1]In [5], on the other hand, the inclusion of the $(70,1^-)$ states was sufficient to obtain a satisfactory fit to both s and p waves, since in the quark model the expressions for the p waves include additional explicit $SU(3)$ symmetry breaking corrections of second chiral order, in which case a much improved fit to the p waves is possible.

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nent resonances have been observed [7–11]. In the purely mesonic sector, work along these lines has been successfully applied, e.g., to radiative ϕ decays [12,13]. Here, we extend this method to the weak sector by appending to the weak hyperon decays at the tree level strong final state interactions, which accounts for the exchange of resonances without including them explicitly. The chiral parameters of the effective potentials are chosen in such a way that they reproduce the experimental phase shifts of pion-nucleon scattering in the energy range we are interested in. Once these parameters have been fixed, the only remaining unknown LECs are the two weak couplings of the leading ground state baryon Lagrangian. The resulting weak matrix elements acquire imaginary components and can no longer be described in terms of just two but rather three independent amplitudes. Our model can help to clarify the role of low-lying baryon resonances for nonleptonic hyperon decays and can answer the question whether omission of final state interactions was justified in previous work.

The work is organized as follows. In the next section, we introduce the effective Lagrangian of the strong and weak interactions and derive the tree level results. The coupled channel method is explained in Sec. III. In Sec. IV, after having matched the chiral parameters of the strong interactions to the experimental phase shifts of pion-nucleon scattering, we implement final state interactions in nonleptonic hyperon decays and discuss the role of resonances. Section V contains our summary and conclusions.

II. NONLEPTONIC HYPERON DECAYS

There exist seven experimentally accessible nonleptonic hyperon decays: $\Sigma^+\to n\pi^+$, $\Sigma^+\to p\pi^0$, $\Sigma^-\to n\pi^-$, $\Lambda\to p\pi^-$, $\Lambda\to n\pi^0$, $\Xi^-\to\Lambda\pi^-$, and $\Xi^0\to\Lambda\pi^0$, and the matrix elements of these decays can each be expressed in terms of a parity violating s wave amplitude A_{ij} and a parity conserving p wave amplitude B_{ij}:

$$\mathcal{A}(B_i\to B_j\pi)=\bar{u}_{B_j}\{A_{ij}+B_{ij}\gamma_5\}u_{B_i}. \tag{1}$$

The underlying strangeness-changing Hamiltonian transforms under $SU(3)\times SU(3)$ as $(8_L,1_R)\oplus(27_L,1_R)$ and, experimentally, the octet piece dominates over the 27-plet by a factor of 20 or so. Consequently, we will neglect the 27-plet in what follows. Isospin symmetry of the strong interactions then implies the relations

$$\begin{gathered}\mathcal{A}(\Lambda\to p\pi^-)+\sqrt{2}\mathcal{A}(\Lambda\to n\pi^0)=0,\\ \mathcal{A}(\Xi^-\to\Lambda\pi^-)+\sqrt{2}\mathcal{A}(\Xi^0\to\Lambda\pi^0)=0,\\ \sqrt{2}\mathcal{A}(\Sigma^+\to p\pi^0)+\mathcal{A}(\Sigma^-\to n\pi^-)-\mathcal{A}(\Sigma^+\to n\pi^+)=0,\end{gathered} \tag{2}$$

which hold for both s and p waves. We choose $\Sigma^+\to n\pi^+$, $\Sigma^-\to n\pi^-$, $\Lambda\to p\pi^-$, and $\Xi^-\to\Lambda\pi^-$ as the four independent decay amplitudes which are not related by isospin. With our sign conventions one defines the amplitudes s and p as [14]

$$s=A, \quad p=\frac{|\mathbf{p}_f|}{E_f+m_f}B, \tag{3}$$

which are related to the decay width Γ and the decay parameters α,β,γ by

$$\Gamma=\frac{|\mathbf{p}_f|(E_f+m_f)}{4\pi m_i}(|s|^2+|p|^2), \quad \alpha=-\frac{2\,\mathrm{Re}(s^*p)}{|s|^2+|p|^2},$$
$$\beta=-\frac{2\,\mathrm{Im}(s^*p)}{|s|^2+|p|^2}, \quad \phi=\arcsin\left(\frac{\beta}{\sqrt{1-\alpha^2}}\right), \tag{4}$$

where $\mathbf{p}_f$, E_f, and m_f are the three-momentum, energy, and mass of the final baryon, respectively, and m_i is the mass of the incoming baryon. Only three of the four observables in Eq. (4) are independent, and if one omits final state interactions, the amplitudes s and p are real and β and ϕ vanish. Previous fits to nonleptonic hyperon decays have always been performed under this assumption, and the imaginary components of the amplitudes were neglected. In our case, since we include final state interactions, it is more convenient to work directly with the real observables Γ, α, β, and γ which are quoted in experiments [14].

Our starting point is the relativistic effective chiral strong interaction Lagrangian for the pseudoscalar bosons coupled to the lowest-lying $1/2^+$ baryon octet, which reads at lowest chiral order

$$\mathcal{L}_{\phi B}^{(1)}=i\langle\bar{B}\gamma_\mu[D^\mu,B]\rangle-\mathring{M}\langle\bar{B}B\rangle-\frac{1}{2}D\langle\bar{B}\gamma_\mu\gamma_5\{u^\mu,B\}\rangle$$
$$-\frac{1}{2}F\langle\bar{B}\gamma_\mu\gamma_5[u^\mu,B]\rangle, \tag{5}$$

where the superscript denotes the chiral order, $\mathring{M}$ is the octet baryon mass in the chiral limit, and $\langle\dots\rangle$ denotes the trace in flavor space. The values of D and F are extracted phenomenologically from the semileptonic hyperon decays and a fit to data delivers $D=0.80\pm0.01$, $F=0.46\pm0.01$ [15]. The pseudoscalar Goldstone fields ($\phi=\pi,K,\eta$) are collected in the 3×3 unimodular, unitary matrix $U(x)$,

$$U(\phi)=u^2(\phi)=\exp\{2i\phi/f_\pi\}, \quad u_\mu=iu^\dagger\nabla_\mu Uu^\dagger, \tag{6}$$

where $f_\pi\simeq92.4$ MeV is the pion decay constant,

$$\phi=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & \pi^+ & K^+\\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & K^0\\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta\end{pmatrix} \tag{7}$$

represents the contraction of the pseudoscalar fields with the Gell-Mann matrices, and B is the standard $SU(3)$ matrix representation of the low-lying spin-1/2 baryons (N,Λ,Σ,Ξ).

The leading strong effective Lagrangian is in general not sufficient to obtain an appropriate description of meson-baryon scattering and one needs to go beyond leading order [8,11]. At next-to-leading order the terms relevant for meson-baryon scattering are

$$\begin{aligned}\mathcal{L}_{\phi B}^{(2)}=&\, b_D\langle\bar{B}\{\chi_+,B\}\rangle+b_F\langle\bar{B}[\chi_+,B]\rangle+b_0\langle\bar{B}B\rangle\langle\chi_+\rangle+d_1\langle\bar{B}\{v\cdot u,[v\cdot u,B]\}\rangle+d_2\langle\bar{B}[v\cdot u,[v\cdot u,B]]\rangle\\&+d_3\langle\bar{B}v\cdot u\rangle\langle v\cdot uB\rangle+d_4\langle\bar{B}B\rangle\langle(v\cdot u)(v\cdot u)\rangle+g_1\langle\bar{B}\{\mathbf{u},[\mathbf{u},B]\}\rangle+g_2\langle\bar{B}[\mathbf{u},[\mathbf{u},B]]\rangle+g_3\langle\bar{B}\mathbf{u}\rangle\langle\mathbf{u}B\rangle+g_4\langle\bar{B}B\rangle\langle\mathbf{uu}\rangle\\&+ih_1\langle\bar{B}\sigma_{\mu\nu}\{[u^\mu,u^\nu],B\}\rangle+ih_2\langle\bar{B}\sigma_{\mu\nu}[[u^\mu,u^\nu],B]\rangle+ih_3\langle\bar{B}u^\mu\rangle\sigma_{\mu\nu}\langle u^\nu B\rangle,\end{aligned}\tag{8}$$

where we made use of a Cayley-Hamilton identity, in order to eliminate $\langle\bar{B}\{v\cdot u,\{v\cdot u,B\}\}\rangle$ and $\langle\bar{B}\{\mathbf{u},\{\mathbf{u},B\}\}\rangle$, and v is a four-velocity with $v^2=1$. We work in the isospin limit $m_u=m_d=\hat{m}$ and explicit chiral symmetry breaking is induced via $\chi_+=u^\dagger\chi u^\dagger+u\chi^\dagger u$, with $\chi=\mathrm{diag}(m_\pi^2,m_\pi^2,2m_K^2-m_\pi^2)$. Instead of the counterterm structures $\langle\bar{B}\mathbf{u}\cdot\mathbf{u}B\rangle$ and $\langle\bar{B}(v\cdot u)(v\cdot u)B\rangle$, one may employ $\langle\bar{B}u_\mu u^\mu B\rangle$, $\langle\bar{B}u^\mu u^\nu\gamma_\mu[D_\nu,B]\rangle$, and $\langle\bar{B}u^\mu u^\nu[D_\mu,[D_\nu,B]]\rangle$. The contact terms of the strong effective Lagrangian, however, will be used to derive the effective potentials of meson-baryon scattering for on-shell particles, in which case the latter two terms reduce to $\langle\bar{B}u_0^2B\rangle$ modulo higher-order corrections which are beyond the accuracy of the present investigation. One can thus choose to work with the two combinations $(v\cdot u)^2=u_0^2$ and $\mathbf{u}\cdot\mathbf{u}=u_0^2-u_\mu\cdot u^\mu$ for the mesonic fields (see also [8]).

The LECs b_D and b_F are responsible for the splitting of the baryon octet masses at leading order in symmetry breaking,

$$\begin{aligned}M_\Sigma-M_N&=4(b_D-b_F)(m_K^2-m_\pi^2),\\M_\Xi-M_N&=-8b_F(m_K^2-m_\pi^2),\\M_\Sigma-M_\Lambda&=\frac{16}{3}b_D(m_K^2-m_\pi^2).\end{aligned}\tag{9}$$

Since the three baryon mass differences are represented in terms of two parameters, there is a corresponding sum rule—the Gell-Mann–Okubo mass relation for the baryon octet [16]:

$$M_\Sigma-M_N=\frac{1}{2}(M_\Xi-M_N)+\frac{3}{4}(M_\Sigma-M_\Lambda),\tag{10}$$

which experimentally has only a 3% deviation. A least-squares fit to the mass differences (9) yields $b_D=0.066\ \mathrm{GeV}^{-1}$ and $b_F=-0.213\ \mathrm{GeV}^{-1}$, but we will allow for small variations of these values in the fitting procedure to the experimental phase shifts, in order to account for higher-order contributions to the baryon masses. The b_0 term, on the other hand, cannot be determined from the masses alone. One needs further information, which is provided by the pion-nucleon σ term and reads at leading order

$$\sigma_{\pi N}=\hat{m}\langle N|\bar{u}u+\bar{d}d|N\rangle=-2m_\pi^2(b_D+b_F+2b_0).\tag{11}$$

Employing the empirical value of [17], $\sigma_{\pi N}=45\pm8$ MeV, one obtains $b_0=-0.52\pm0.10\ \mathrm{GeV}^{-1}$. Recently, this value has been questioned [18], and the authors of this work arrive at a value $\sigma_{\pi N}=60\pm7$ MeV which would translate into a value of $b_0=-0.71\pm0.09\ \mathrm{GeV}^{-1}$. In particular, the latter value yields a large strangeness content of the proton; however, both values may change if loop effects are included [19]. Due to these uncertainties in the value of b_0, any fitted value which lies in the range $-0.80\ \mathrm{GeV}^{-1}<b_0<-0.20\ \mathrm{GeV}^{-1}$ is still acceptable. The situation is less clear for the derivative terms d_i, g_i, and h_i. In the present investigation, they will be constrained in the next section by fitting them to the phase shifts of pion-nucleon scattering.

Turning to the weak component of the meson-baryon Lagrangian, the form of the lowest-order Lagrangian is

$$\mathcal{L}_{\phi B}^{W}=d\langle\bar{B}\{h_+,B\}\rangle+f\langle\bar{B}[h_+,B]\rangle,\tag{12}$$

where we have defined

$$h_+=u^\dagger hu+u^\dagger h^\dagger u,\tag{13}$$

which transforms as a matter field, with $h_b^a=\delta_2^a\delta_b^3$ being the weak transition matrix. The weak LECs d,f are the only weak counterterms considered in most previous calculations [1–3], and use of this Lagrangian does not provide a simultaneously satisfactory fit to s and p waves at both the tree and the one-loop levels. As mentioned in the Introduction, a fit to the decay amplitudes has been shown to be possible by neglecting final state interactions and under the inclusion of higher chiral orders of the weak Lagrangian due to the proliferation of new unknown LECs [4]. However, such a fit is not unique, and we refrain from performing one here. In this work, we will rather employ the weak Lagrangian at the tree level and then append final state interactions of the meson-baryon system within the coupled channel model which is described in Sec. III. This will help to understand the role of final state interactions and the pertinent resonances in nonleptonic hyperon decays.

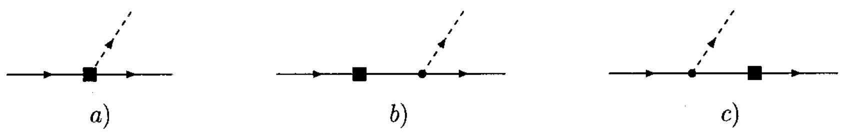

FIG. 1. Shown are the diagrams that contribute at the tree level. (a) contributes to the s waves, whereas (b) and (c) contribute to the p waves. Solid and dashed lines denote baryons and pions, respectively. Solid squares and circles are vertices of the weak and strong interactions, respectively.

Tree level results

We first calculate the tree level diagrams for the nonleptonic hyperon decays which are depicted in Fig. 1. The s wave decay amplitudes $A^{(tr)}$ at the tree level are given by

$$A^{(tr)}_{\Sigma^+ n}=0, \quad A^{(tr)}_{\Sigma^- n}=\frac{1}{\sqrt{2}f_\pi}(d-f),$$

$$A^{(tr)}_{\Lambda p}=-\frac{1}{2\sqrt{3}f_\pi}(d+3f),$$

$$A^{(tr)}_{\Xi^- \Lambda}=-\frac{1}{2\sqrt{3}f_\pi}(d-3f). \tag{14}$$

For the p wave amplitudes $B^{(tr)}$ one finds at the tree level

$$B^{(tr)}_{\Sigma^+ n}=-\frac{M_\Sigma+M_N}{\sqrt{2}f_\pi}\left(\frac{D[d-f]}{M_\Sigma-M_N}+\frac{D[d+3f]}{3[M_\Lambda-M_N]}\right),$$

$$B^{(tr)}_{\Sigma^- n}=-\frac{M_\Sigma+M_N}{\sqrt{2}f_\pi}\left(\frac{F[d-f]}{M_\Sigma-M_N}+\frac{D[d+3f]}{3[M_\Lambda-M_N]}\right),$$

$$B^{(tr)}_{\Lambda p}=\frac{M_\Lambda+M_N}{2\sqrt{3}f_\pi}\left(\frac{[D+F][d+3f]}{M_\Lambda-M_N}+\frac{2D[d-f]}{M_\Sigma-M_N}\right),$$

$$B^{(tr)}_{\Xi^- \Lambda}=-\frac{M_\Xi+M_\Lambda}{2\sqrt{3}f_\pi}\left(\frac{[D-F][d-3f]}{M_\Xi-M_\Lambda}+\frac{2D[d+f]}{M_\Xi-M_\Sigma}\right). \tag{15}$$

As mentioned in the Introduction, a decent fit to both s and p waves in terms of the weak couplings d and f is not possible, and we postpone the discussion of the numerical results of such a tree level fit to Sec. IV. The next step in our approach consists of appending final state interactions for the outgoing meson-baryon system.

III. FINAL STATE INTERACTIONS

Final state interactions are expected to be moderate for nonleptonic hyperon decays, and they have therefore been omitted in previous work. This assumption is based on Watson's theorem, which states that the phase of the amplitudes is given by the strong baryon-meson phase shifts, if CP is conserved. Aside from the πN system, these phase shifts are not known precisely, but are estimated to be about 10°. The main purpose of this work is to critically reexamine this assumption by explicitly including final state interactions within a coupled channel approach. This is achieved by taking into account the rescattering of the meson and baryon in both s and p waves as depicted in Fig. 2. This procedure implies that the initial baryon decays into an intermediate meson-baryon pair which then rescatters into the final states of the decay.

In order to describe the rescattering process, we derive the effective meson-baryon potentials from the strong effective Lagrangian of Sec. II. In our model the effective potentials for the meson-baryon scattering process $B_a\phi_i \to B_b\phi_j$ are obtained by the diagrams shown in Fig. 3 [11].

At leading order the scattering amplitude is derived from the effective Lagrangian $\mathcal{L}^{(1)}_{\phi B}$ and reads in the center-of-mass frame

$$V=N_a N_b \chi_b^\dagger[g(s,\vartheta)+ih(s,\vartheta)(\mathbf{p}'\times\mathbf{p})\cdot\boldsymbol{\sigma}]\chi_a, \tag{16}$$

where χ_i are Pauli spinors and the normalization constants are given by $N_i=\sqrt{M_i+E_i}$ with M_i and E_i being the physical mass and energy of the baryon i, respectively. Furthermore, s is the invariant energy squared and ϑ is the center-of-mass angle between the incoming and outgoing baryons. The functions g and h are given by

$$g(s,\vartheta)=A(s)\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{N_a^2N_b^2}\right) +B(s)\left(\sqrt{s}-M_a+\frac{\sqrt{s}+M_a}{N_a^2N_b^2}\mathbf{p}'\cdot\mathbf{p}\right),$$

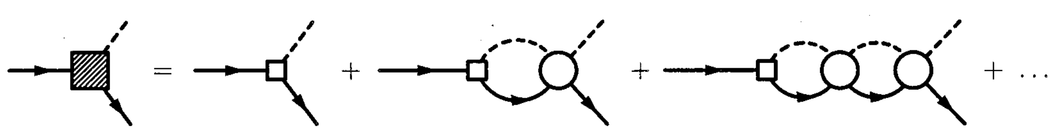

FIG. 2. The full weak decay amplitude (shaded square) is the sum of the tree level weak vertex (empty square) and diagrams which include strong rescattering processes of a meson-baryon pair to all orders (empty circles).

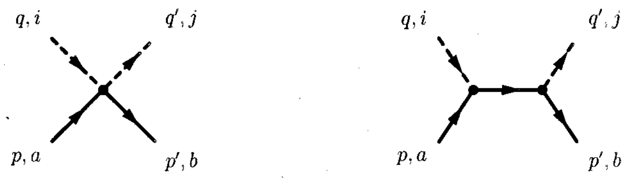

FIG. 3. Contact interaction and s channel Born term for meson-baryon scattering. Solid and dashed lines denote the baryons and pseudoscalar mesons, respectively. p, p', q, q' and a, b, i, j are the four-momenta and flavor indices of the particles, respectively.

$$h(s,\vartheta) = -A(s)\frac{1}{N_a^2 N_b^2} + B(s)\frac{\sqrt{s}+M_a}{N_a^2 N_b^2}, \tag{17}$$

with

$$\begin{aligned} A(s) = &-\frac{1}{4f_\pi^2}(M_a - M_b)\sum_k f_{kji}f_{kab} + \frac{1}{2f_\pi^2}\sum_k (Dd_{jbk} \\ &+ Ff_{jbk})(Dd_{iak} + Ff_{iak})(M_k + M_b)\frac{s - M_a^2}{s - M_k^2}, \\ B(s) = &-\frac{1}{2f_\pi^2}\sum_k f_{kji}f_{kab} - \frac{1}{2f_\pi^2}\sum_k (Dd_{jbk} + Ff_{jbk}) \\ &\times (Dd_{iak} + Ff_{iak})\frac{s + M_a M_b + M_a M_k + M_k M_b}{s - M_k^2}. \end{aligned} \tag{18}$$

We employed the physical values of the baryon masses instead of the common octet mass $\mathring{M}$ which is consistent at the order we are working, and use of the physical masses is also mandatory, in order to reproduce the correct threshold positions of the different channels involved. The structure constants are defined by $f_{ijk} = \langle [\lambda_i, \lambda_j]\lambda_k^\dagger \rangle$ and $d_{ijk} = \langle \{\lambda_i, \lambda_j\}\lambda_k^\dagger \rangle$ with λ_i being the generators of the $SU(3)$ algebra normalized according to $\langle \lambda_i \lambda_j^\dagger \rangle = \delta_{ij}$.

The partial wave amplitude that contributes to the s waves reads

$$V_{0+}^{(1)} = N_a N_b (A(s) + B(s)[\sqrt{s} - M_a]), \tag{19}$$

while the p wave contribution is

$$V_{1-}^{(1)} = \frac{|\mathbf{p}||\mathbf{p}'|}{N_a N_b}(-A(s) + B(s)[\sqrt{s} + M_a]), \tag{20}$$

and the remaining partial wave amplitude V_{1+} does not contribute in nonleptonic hyperon decays due to angular momentum conservation. The superscript denotes the chiral order of the effective meson-baryon Lagrangian used.

Inclusion of the meson-baryon Lagrangian at second chiral order, Eq. (8), yields the additional contributions

$$V_{0+}^{(2)} = N_a N_b \left(M - N\frac{|\mathbf{p}|^2|\mathbf{p}'|^2}{3N_a^2 N_b^2} + L\left[\frac{q_0|\mathbf{p}'|^2}{N_b^2} + \frac{q_0'|\mathbf{p}|^2}{N_a^2} + \frac{2|\mathbf{p}|^2|\mathbf{p}'|^2}{3N_a^2 N_b^2} \right] \right) \tag{21}$$

and

$$V_{1-}^{(2)} = N_a N_b |\mathbf{p}||\mathbf{p}'| \left(\frac{1}{3}N - M\frac{1}{N_a^2 N_b^2} - L\left[\frac{q_0}{N_a^2} + \frac{q_0'}{N_b^2} + \frac{2}{3} \right] \right) \tag{22}$$

with

$$\begin{aligned} M = &-\frac{1}{2f_\pi^2} b_D(\langle \lambda_b^\dagger \{\{\lambda_i, \{\lambda_j^\dagger, \chi\}\}, \lambda_a\}\rangle + \langle \lambda_b^\dagger \{\{\lambda_j^\dagger, \{\lambda_i, \chi\}\}, \lambda_a\}\rangle) - \frac{1}{2f_\pi^2} b_F(\langle \lambda_b^\dagger [\{\lambda_i, \{\lambda_j^\dagger, \chi\}\}, \lambda_a]\rangle + \langle \lambda_b^\dagger [\{\lambda_j^\dagger, \{\lambda_i, \chi\}\}, \lambda_a]\rangle) \\ &- \frac{2}{f_\pi^2} b_0 \delta_{ab} \langle \{\lambda_i, \lambda_j^\dagger\}\chi\rangle + \frac{2}{f_\pi^2}(v\cdot q)(v\cdot q')\left(d_1 \sum_k [f_{jka}d_{ikb} + f_{iak}d_{jbk}] + d_2 \sum_k [f_{jka}f_{ikb} + f_{iak}f_{jbk}] \right. \\ &\left. + d_3[\delta_{ib}\delta_{ja} + \delta_{i^\dagger a}\delta_{j^\dagger b}] + 2d_4 \delta_{ab}\delta_{ij} \right), \\ N = &\frac{2}{f_\pi^2}\left(g_1 \sum_k [f_{jka}d_{ikb} + f_{iak}d_{jbk}] + g_2 \sum_k [f_{jka}f_{ikb} + f_{iak}f_{jbk}] + g_3[\delta_{ib}\delta_{ja} + \delta_{i^\dagger a}\delta_{j^\dagger b}] + 2g_4 \delta_{ab}\delta_{ij} \right), \\ L = &\frac{2}{f_\pi^2}\left(2h_1 \sum_k f_{kji}d_{kab} + 2h_2 \sum_k f_{kji}f_{kab} + h_3[\delta_{ib}\delta_{ja} - \delta_{i^\dagger a}\delta_{j^\dagger b}] \right), \end{aligned} \tag{23}$$

where we have introduced the notation $\langle \lambda_i \lambda_j \rangle = \delta_{i^\dagger j}$.

After deriving the effective potentials for meson-baryon scattering for both s and p waves, they are iterated in a BSE which couples the different physical channels. The following channels contribute via final state interactions to nonleptonic hyperon decays: $\pi N, \eta N, K\Lambda, K\Sigma$ for $S=0$, $I=1/2, \pi N, K\Sigma$ for $S=0$, $I=3/2$, and $\pi\Lambda, \pi\Sigma, \bar{K}N$, $\eta\Sigma$, $K\Xi$ for $S=-1$, $I=1$. In the Bethe-Salpeter formalism the total strong T matrix can be written in channel space in the matrix form $T=(1+V\cdot G)^{-1}V$ [10,11]. In a similar fashion, the total weak decay amplitude is given by

$$A=[1+(V_{0+}^{(1)}+V_{0+}^{(2)})\cdot G]^{-1}A^{(tr)},$$

$$B=[1+(V_{1-}^{(1)}+V_{1-}^{(2)})\cdot G]^{-1}B^{(tr)}, \tag{24}$$

where G is the finite part of the scalar loop integral $\tilde{G}$,

$$\tilde{G}(q^2)=\int \frac{d^d l}{(2\pi)^d}\frac{i}{[(q-l)^2-M_B^2+i\epsilon][l^2-m_\phi^2+i\epsilon]}, \tag{25}$$

and is understood to be a diagonal matrix in channel space. One obtains, e.g., in dimensional regularization

$$\begin{aligned} G(q^2)=\frac{1}{32\pi^2 q^2}\Bigg\{ & q^2\left[\ln\left(\frac{m_\phi^2}{\mu^2}\right)+\ln\left(\frac{M_B^2}{\mu^2}\right)-2\right] \\ & +(m_\phi^2-M_B^2)\ln\left(\frac{m_\phi^2}{M_B^2}\right) \\ & -8\sqrt{q^2}|\boldsymbol{q}_{cm}|\operatorname{arctanh}\left(\frac{2\sqrt{q^2}|\boldsymbol{q}_{cm}|}{(m_\phi+M_B)^2-q^2}\right)\Bigg\}, \end{aligned} \tag{26}$$

where μ is the regularization scale. Note that the expression for G is regularization scale dependent and μ is treated as a so-called finite range parameter [7,8,11]. Along with the unknown coupling constants of the strong effective Lagrangian of Eq. (8), it is used in this approach to reproduce the experimental phase shifts of pion-nucleon scattering at the relevant energies. In Eq. (24) the s and p wave amplitudes have been summarized in column vectors $A^{(tr)}$ and $B^{(tr)}$.

The imaginary part of the matrix T^{-1} of the strong interactions is identical with the imaginary piece of the fundamental scalar loop integral $\tilde{G}$ above threshold,

$$\operatorname{Im} T^{-1} = -\frac{|\boldsymbol{q}_{cm}|}{8\pi\sqrt{s}} \tag{27}$$

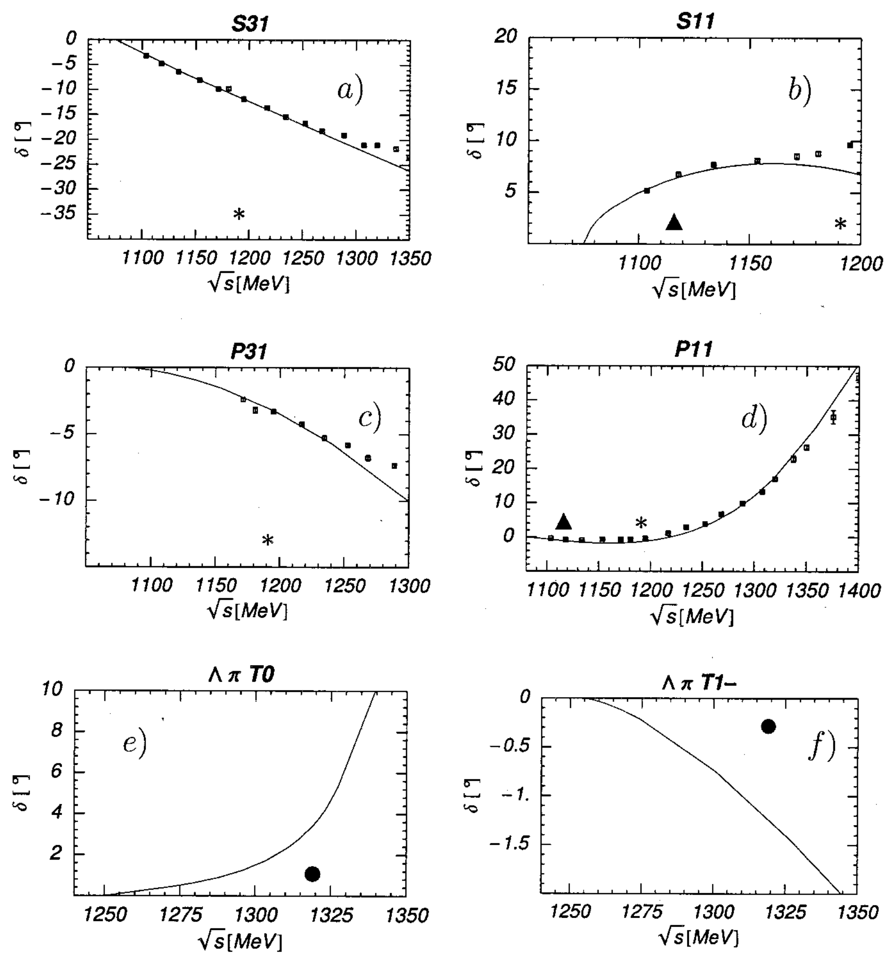

FIG. 4. Shown are fits to the phase shifts of π-N [diagrams (a) to (d)] and π-Λ [(e),(f)] partial wave amplitudes. The masses of the Λ,Σ,Ξ are denoted by a triangle, asterisk, and circle, respectively.

TABLE I. Experimental values [14] and results of our fit including final state interactions for all decay channels.

	$\Gamma_{\rm expt}$ (μeV)	$\alpha_{\rm expt}$	$\phi_{\rm expt}$ (deg)	Γ (μeV)	α	ϕ (deg)
$\Lambda\to p\pi^-$	1.597 ± 0.012	0.642 ± 0.013	-6.5 ± 3.5	1.70	0.28	-1.0
$\Lambda\to n\pi^0$	0.895 ± 0.007	0.65 ± 0.05	—	0.85	0.28	-1.0
$\Sigma^+\to n\pi^+$	3.966 ± 0.013	0.068 ± 0.013	167 ± 20	0.82	-0.86	59.2
$\Sigma^+\to p\pi^0$	4.233 ± 0.014	-0.980 ± 0.016	36 ± 34	2.55	-0.60	-1.4
$\Sigma^-\to n\pi^-$	4.450 ± 0.033	-0.068 ± 0.008	10 ± 15	2.83	0.0	0.22
$\Xi^0\to\Lambda\pi^0$	2.270 ± 0.070	-0.411 ± 0.022	21 ± 12	2.30	-0.27	1.76
$\Xi^-\to\Lambda\pi^-$	4.016 ± 0.037	-0.456 ± 0.014	4 ± 4	4.61	-0.27	1.76

with $\boldsymbol{q}_{cm}$ being the three-momentum in the center-of-mass frame of the channel under consideration. Equation (27) is the restriction that unitarity imposes on the T matrix for each partial wave. The BSE (24) amounts to a summation of a bubble chain as depicted in Fig. 2, and isospin invariance of the strong interactions guarantees that the solutions of the BSE in (24) satisfy the isospin relations in Eq. (2).

IV. NUMERICAL RESULTS

In this section, we compare the numerical results of our Bethe-Salpeter approach in Eq. (24) for the observables Γ,α, and ϕ with the experimental data, while values for β can be deduced directly by employing Eq. (4). The unknown coupling coefficients of the strong effective Lagrangian are constrained by reproducing the phase shifts of pion-nucleon partial wave amplitudes. The results are shown in Fig. 4. We are clearly able to obtain a good fit to the phase shifts in the energy range we are working in. The fit also yields predictions for π-Λ scattering for which—to our knowledge—experimental data are not available. Our results for the s wave π-Λ phase shifts are slightly above those of [20]. However, the authors of [20] made use of the leading meson-baryon Lagrangian only, and omission of the contact terms from the Lagrangian of second chiral order reduces our result for this phase shift, bringing it into better agreement with [20].

For the fit presented in Fig. 4 we employed the values (all in units of GeV^{-1}) $b_D=0.066$, $b_F=-0.185$, $b_0=-0.254$, $d_1=-0.6$, $d_2=0.14$, $d_3=0$, $d_4=-0.37$, $g_1=0.5$, $g_2=0.5$, $g_3=0$, $g_4=0.2$, $h_1=0.25$, $h_2=0.25$, $h_3=0$, and the regularization scale $\mu=1.1$ GeV in all channels. The fit is, of course, not unique, since small changes in some of the chiral parameters yield equally good representations of the phase shifts. Nevertheless, it is sufficient to work with a particular choice of parameters that ensures the accurate inclusion of the final state interactions. After constraining the coefficients of the strong effective Lagrangian we are left with the two weak parameters d and f. Performing a fit for d and f to the decay observables Γ, α, and ϕ of the seven nonleptonic hyperon decays yields the values $d=0.31\times10^{-7}$ GeV and $f=-0.38\times10^{-7}$ GeV.

The results for this fit are shown in Table I and compared with the experimental values. Despite the inclusion of final state interactions, we are clearly not able to obtain a good fit for the decays, in particular for $\Sigma^+\to n\pi^+$. These numbers must be compared with the results from a pure tree level calculation, Eqs. (14) and (15), which are summarized in Table II for two different choices of d and f. We first employ $d=0.17\times10^{-7}$ GeV and $f=-0.4\times10^{-7}$ GeV, which are obtained from a tree level fit to the s waves omitting final state interactions. Second, we use the values $d=0.31\times10^{-7}$ GeV and $f=-0.38\times10^{-7}$ GeV from the above mentioned fit of the BSE approach. Both results are given in Table II, and comparison with Table I shows that the inclusion of final state interactions provides an improved overall fit, e.g., it yields the correct sign for α in Ξ decays, which is not the case for the tree level fit. We observe that final state interactions have sizable effects for nonleptonic hyperon decays, i.e., much larger than the anticipated 10% level, if the same values for d and f are employed. Furthermore, the inclusion of final state interactions allows for a prediction of the decay parameter ϕ, which vanishes in the tree level approximation.

V. CONCLUSIONS

In this work, we have investigated the importance of final state interactions in nonleptonic hyperon decays within a coupled channel approach. First, the tree level contributions to the decays were calculated from the lowest-order weak Lagrangian. In a second step, we considered final state interactions of the decay particles, which are modeled in this approach by calculating the effective meson-baryon scatter-

TABLE II. Given are the tree level contributions using two sets of parameters. (1) $d=0.17\times10^{-7}$ GeV and $f=-0.4\times10^{-7}$ GeV from a tree level fit to the s waves. (2) Results using the values of d and f from our fit including final state interactions, $d=0.31\times10^{-7}$ GeV and $f=-0.38\times10^{-7}$ GeV. There is no contribution to ϕ at the tree level.

	Γ_1 (μeV)	α_1	Γ_2 (μeV)	α_2
$\Lambda\to p\pi^-$	1.66	0.69	0.97	0.39
$\Lambda\to n\pi^0$	0.83	0.69	0.48	0.39
$\Sigma^+\to n\pi^+$	0.04	0	0.56	0
$\Sigma^+\to p\pi^0$	2.46	-0.53	3.6	-0.53
$\Sigma^-\to n\pi^-$	4.71	-0.36	6.67	0.01
$\Xi^0\to\Lambda\pi^0$	1.70	0.13	1.95	-0.31
$\Xi^-\to\Lambda\pi^-$	3.41	0.13	3.90	-0.32

ing potentials and iterating them in a Bethe-Salpeter equation. The effective potentials are obtained from the strong effective Lagrangian at next-to-leading order. The inclusion of the strong counterterms of second chiral order is necessary, in order to obtain better agreement with experimental phase shifts from meson-baryon scattering and to ensure a proper treatment of the final state interactions. In fact, we are able to reproduce accurately the experimental phase shifts of pion-nucleon scattering in the energy range we are working in.

The iteration of the effective potential in the BSE produces dynamically the pertinent low-lying baryon resonances. Thus, without including the resonances explicitly, we can independently check their importance in nonleptonic hyperon decays. This allows for a critical reexamination of previous work in which resonant states have been taken into account explicitly. In this work, we were able to show that the generation of resonances by means of final state interactions and omission of higher-order weak LECs does not enable a reasonable fit to nonleptonic hyperon decays in terms of just the lowest-order weak parameters d and f, although it decreases the discrepancy with experiment. In [6], on the other hand, the resonances were integrated out, producing weak counterterms of higher chiral orders which were parametrized by the weak decay parameters of the resonances, and a fit to the decay amplitudes was possible. This indicates that in [6] the weak couplings that were obtained from a fit to the decay amplitudes may have been overestimated. But it also underlines the importance of higher-order weak counterterms in order to obtain a reasonable fit to both s and p waves. This observation is reinforced by the investigation in the quark model [5], since contributions from higher resonances are included in this approach, whereas they were omitted both in [6] and in the present study. In ChPT the effects of higher resonances are hidden in contributions to higher-order coupling coefficients of the weak Lagrangian. Furthermore, we have shown that final state interactions yield sizable effects in nonleptonic hyperon decays, definitely larger than the expected 10% level, and should not be omitted.

We can therefore conclude that a reasonable fit to the nonleptonic hyperon decays is not possible in ChPT without the inclusion of higher-order weak counterterms. In the BSE approach contributions from low-lying resonances are sizable and yield a slight improvement, but are definitely not capable of accounting for the discrepancy with experiment.

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