# Next-to-leading order QCD corrections to $A_{TT}$ for prompt photon production

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We present a next-to-leading order QCD calculation of the cross section for isolated large- $p_T$  prompt photon production in collisions of transversely polarized protons. We devise a simple method of dealing with the phase space integrals in dimensional regularization in the presence of the  $\cos(2\Phi)$  azimuthal-angular dependence occurring for transverse polarization. Our results allow us to calculate the double-spin asymmetry  $A_{TT}^{\gamma}$  for this process at next-to-leading order accuracy, which may be used at BNL RHIC to measure the transversity parton distributions of the proton.

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## I. INTRODUCTION

The partonic structure of spin-1/2 targets at the leadingtwist level is characterized entirely by the unpolarized, longitudinally polarized, and transversely polarized distribution functions f,  $\Delta f$ , and  $\delta f$ , respectively [1]. By virtue of the factorization theorem [2], these nonperturbative parton densities can be probed universally in a multitude of inelastic scattering processes, for which it is possible to separate ("factorize") the long-distance physics relating to nucleon structure from a partonic short-distance scattering that is amenable to QCD perturbation theory. Combined experimental and theoretical efforts have led to an improved understanding of the spin structure of longitudinally polarized nucleons,  $\Delta f$ , in the past years. In contrast, the "transversity" distributions  $\delta f$ , first introduced in [3], remain the quantities about which we have the least knowledge.

Current and future experiments are designed to further unravel the spin structure of both longitudinally and transversely polarized nucleons. Information will soon be gathered for the first time from polarized proton-proton collisions at the BNL Relativistic Heavy Ion Collider (RHIC) [4]. Collisions of transversely polarized protons will be studied, and the potential of RHIC in accessing transversity  $\delta f$  in transverse double-spin asymmetries A<sub>TT</sub> was recently examined in [5] for high transverse momentum  $p_T$  prompt photon and jet production. Several other studies of A<sub>TT</sub> for these reactions have been presented in the past [6-8], as well as for the Drell-Yan process [3,9-11]. With the exception of the latter reaction [10,11], all of these calculations were performed at the lowest order (LO) approximation only. As is well known, next-to-leading order (NLO) QCD corrections are generally indispensable in order to arrive at a firmer theoretical prediction for hadronic cross sections and spin asymmetries. Only with their knowledge can one reliably confront theory with experimental data and achieve the goal of extracting information on the partonic spin structure of nucleons.

In this paper we extend the results of [5] for isolated high $p_T$  prompt photon production,  $pp \rightarrow \gamma X$ , to the NLO of QCD. Apart from the motivation given above, also interesting new technical questions arise beyond the NLO in case of transverse polarization. Unlike for longitudinally polarized cross sections where the spin vectors are aligned with momentum, transverse spin vectors specify extra spacial directions, giving rise to nontrivial dependence of the cross section on the azimuthal angle of the observed photon. As is well known [3], for  $A_{TT}$  this dependence is always of the form  $\cos(2\Phi)$ , if the z axis is defined by the direction of the initial protons in their center-of-mass system (c.m.s.), and the spin vectors are taken to point in the  $\pm x$  direction. Integration over the photon's azimuthal angle is therefore not appropriate. On the other hand, standard techniques developed in the literature for performing NLO phase-space integrations usually rely on the choice of particular reference frames that are related in complicated ways to the one just specified. This makes it difficult to fix  $\Phi$  in the higher order phase space integration. The problem actually becomes more severe if dimensional regularization techniques are used for dealing with the collinear and infrared singularities, as is customary. Even for the kinematically rather simple Drell-Yan process the NLO calculation for the cross section with transverse polarization is quite more complicated as for the unpolarized or longitudinally polarized cases [10]. In this paper, we will present a new general technique which facilitates NLO calculations with transverse polarization by conveniently projecting on the azimuthal dependence of the matrix elements in a covariant way. This method then allows us to carry out phase space integrals with standard tools known from unpolarized calculations.

After presenting our technique and verifying that it recovers the known result for the transversely polarized NLO Drell-Yan cross section, we apply it to high- $p_T$  prompt photon production. We also present some first numerical calculations of the cross sections and the transverse spin asymmetry

try for this process at NLO. Here we of course have to rely on some model for the transversity densities, for which we make use of the Soffer inequality [12]. As in experiment, we impose an isolation cut on the photon. We find a moderate size of the NLO corrections and the expected reduced scale dependence of the cross section at NLO.

## **II. CALCULATION OF THE NLO CORRECTIONS**

#### **A. Preliminaries**

The transversity density  $\delta f(x,\mu)$  is defined [1,3,7,13] as the difference of probabilities for finding a parton of flavor *f* at scale  $\mu$  and light-cone momentum fraction *x* with its spin aligned ( $\uparrow\uparrow$ ) or anti-aligned ( $\downarrow\uparrow$ ) to that of the transversely polarized nucleon:

$$\delta f(x,\mu) \equiv f_{\uparrow\uparrow}(x,\mu) - f_{\downarrow\uparrow}(x,\mu) \tag{1}$$

(an arrow always denotes transverse polarization in the following). The unpolarized densities are recovered by taking the sum in Eq. (1). When the transverse polarization is described as a superposition of helicity eigenstates,  $\delta f$  reveals its helicity-flip, chirally odd, nature [1,7]. As a result, there is no leading-twist transversity gluon density, since helicity changes by two units cannot be absorbed by a spin-1/2 target [1,7,14].<sup>1</sup> The property of helicity conservation in QCD hard scattering processes implies that there have to be two soft hadronic pieces in the process that each flip chirality, in order to give sensitivity to transversity. One possibility, which we are going to consider in the following, is to have two transversely polarized hadrons in the initial-state and to measure double-spin asymmetries

$$A_{TT} = \frac{\frac{1}{2} [d\sigma(\uparrow\uparrow) - d\sigma(\uparrow\downarrow)]}{\frac{1}{2} [d\sigma(\uparrow\uparrow) + d\sigma(\uparrow\downarrow)]} \equiv \frac{d\,\delta\sigma}{d\sigma}.$$
 (2)

Here  $d\delta\sigma$  denotes the transversely polarized cross section.  $A_{TT}$  is expected to be rather small for most processes [6,8,5], since gluonic contributions are absent in the numerator while in the denominator they often play a dominant role.<sup>2</sup> Nevertheless, the LO study [5] suggests that the asymmetry for prompt photon production should be measurable at RHIC, provided the transversity densities are not too small.

According to the factorization theorem [2] the fully differential transversely polarized single-inclusive cross section  $A+B\rightarrow\gamma+X$  for the production of a prompt photon with transverse momentum  $p_T$ , azimuthal angle  $\Phi$  with respect to the initial spin axis, and pseudorapidity  $\eta$  reads

$$\frac{d^{3}\delta\sigma}{dp_{T}d\eta d\Phi} = \frac{p_{T}}{\pi S} \sum_{a,b} \int_{VW}^{V} \frac{dv}{v(1-v)} \int_{VW/v}^{1} \frac{dw}{w} \,\delta f_{a}(x_{a},\mu_{F})$$
$$\times \delta f_{b}(x_{b},\mu_{F}) \left[ \frac{d\delta\hat{\sigma}_{ab\to\gamma}^{(0)}(v)}{dv d\Phi} \,\delta(1-w) + \frac{\alpha_{s}(\mu_{R})}{\pi} \frac{d\delta\hat{\sigma}_{ab\to\gamma}^{(1)}(s,v,w,\mu_{R},\mu_{F})}{dv dw d\Phi} \right], \quad (3)$$

with hadron-level variables

$$V \equiv 1 + \frac{T}{S}, \quad W \equiv \frac{-U}{S+T}, \quad S \equiv (P_A + P_B)^2,$$
$$T \equiv (P_A - P_{\gamma})^2, \quad U \equiv (P_B - P_{\gamma})^2, \quad (4)$$

in obvious notation of the momenta, and corresponding partonic ones

$$v \equiv 1 + \frac{t}{s}, \quad w \equiv \frac{-u}{s+t}, \quad s \equiv (p_a + p_b)^2,$$
  
 $t \equiv (p_a - p_\gamma)^2, \quad u \equiv (p_b - p_\gamma)^2.$  (5)

Neglecting all masses, one has the relations

$$s = x_a x_b S$$
,  $t = x_a T$ ,  $u = x_b U$ ,  $x_a = \frac{VW}{v w}$ ,  $x_b = \frac{1 - V}{1 - v}$ .  
(6)

The  $d\delta \hat{\sigma}_{ab \to \gamma}^{(i)}$  are the LO (i=0) and NLO (i=1) contributions in the partonic cross sections for the reactions  $ab \to \gamma X$ .  $\mu_R$  and  $\mu_F$  are the renormalization and factorization scales.

#### B. Projection technique for azimuthal dependence

Let us consider the scattering in the hadronic c.m.s. frame, assuming both initial spin vectors to be in  $\pm x$  direction. Then, on general grounds, for a parity-conserving theory with vector couplings, the  $\Phi$  dependence of the cross section is constrained to be of the form  $\cos(2\Phi)$ :

$$\frac{d^3 \delta \sigma}{dp_T d \eta d\Phi} \equiv \cos(2\Phi) \left\langle \frac{d^2 \delta \sigma}{dp_T d \eta} \right\rangle. \tag{7}$$

We may obtain  $\langle d^2 \delta \sigma / dp_T d\eta \rangle$  by integrating the cross section over  $\Phi$  with a cos(2 $\Phi$ ) weight:

$$\left\langle \frac{d^2 \delta \sigma}{dp_T d\eta} \right\rangle = \frac{1}{\pi} \int_0^{2\pi} d\Phi \cos(2\Phi) \frac{d^3 \delta \sigma}{dp_T d\eta d\Phi}.$$
 (8)

For the lowest order contribution to prompt-photon production in Eq. (3) one has only the channel  $q\bar{q} \rightarrow \gamma g$ . Polarization for, say, the initial quark may be projected out by

$$u(p_{a},s_{a})\overline{u}(p_{a},s_{a}) = \frac{1}{2}p_{a}[1+\gamma_{5}k_{a}], \qquad (9)$$

<sup>&</sup>lt;sup>1</sup>We note that a gluon density does contribute beyond leading twist [14,15], where it will lead to terms in  $A_{TT}$  strongly suppressed by inverse powers of the photon  $p_T$ . An estimate of such effects could follow the lines in [15].

<sup>&</sup>lt;sup>2</sup>The only exception is the Drell-Yan process, which however suffers from rather low rates.

where  $p_a$  and  $s_a$  are the quark's momentum and transverse spin vector, and  $u(p_a, s_a)$  its Dirac spinor. One readily finds for the LO process

$$\left\langle \frac{d\,\delta\hat{\sigma}_{q\bar{q}\to\gamma g}^{(0)}(v)}{dv} \right\rangle = \frac{2\,C_F}{N_C}\,\frac{\alpha\alpha_s}{s}e_q^2\,,\tag{10}$$

where  $C_F = 4/3$ ,  $N_C = 3$  and  $e_q$  is the fractional quark charge.

As discussed in the Introduction, in the NLO calculation one wants to make as much use as possible of calculational techniques established for the unpolarized case. For a single inclusive cross section such as prompt photon production, the appropriate methods were developed in [16]. They involve integration over azimuthal angles. We therefore would like to follow a projection analogous to Eq. (8); however, we should formulate it in a covariant way. To this effect, we first note that the factor  $\cos(2\Phi)/\pi$  in the cross section actually results from the covariant expression

$$\mathcal{F}(p_{\gamma}, s_a, s_b) = \frac{s}{\pi t u} \bigg[ 2(p_{\gamma} \cdot s_a)(p_{\gamma} \cdot s_b) + \frac{t u}{s}(s_a \cdot s_b) \bigg],$$
(11)

which reduces to  $\cos(2\Phi)/\pi$  in the hadronic c.m.s. frame. We may, therefore, use  $\mathcal{F}(p_{\gamma}, s_a, s_b)$  instead of the explicit  $\cos(2\Phi)/\pi$ .

Even though employing  $\mathcal{F}(p_{\gamma}, s_a, s_b)$  becomes a real advantage only at NLO, let us illustrate its use in case of the LO cross section for the partonic reaction  $q\bar{q} \rightarrow \gamma g$ . We there have

$$\frac{d\delta^2 \hat{\sigma}_{q\bar{q}\to\gamma g}^{(0)}}{dt d\Phi} = \frac{1}{32\pi^2 s^2} \delta |M(q\bar{q}\to\gamma g)|^2, \qquad (12)$$

where  $\delta |M|^2$  is the squared invariant matrix element for the reaction with transverse polarization and reads

$$\delta |M(q\bar{q} \to \gamma g)|^{2} = (ee_{q}g)^{2} \frac{4C_{F}}{N_{C}} \frac{s}{tu} \bigg[ 2(p_{\gamma} \cdot s_{a})(p_{\gamma} \cdot s_{b}) + \frac{tu}{s}(s_{a} \cdot s_{b}) \bigg].$$
(13)

One recognizes the factor  $\mathcal{F}(p_{\gamma}, s_a, s_b)$  emerging in  $\delta |M|^2$ . We now multiply  $\delta |M|^2$  by  $\mathcal{F}(p_{\gamma}, s_a, s_b)$ , equivalent to the multiplication by  $\cos(2\Phi)/\pi$  in Eq. (8). The resulting expression may then be integrated over the full azimuthal phase space without producing a vanishing result, unlike the case of  $\delta |M|^2$  itself. This integration may again be performed in a covariant way by noting first that the dependence of  $\mathcal{F}(p_{\gamma}, s_a, s_b) \delta |M|^2$  on the spin vectors comes as  $(p_{\gamma} \cdot s_a)^2 (p_{\gamma} \cdot s_b)^2$ ,  $(p_{\gamma} \cdot s_a) (p_{\gamma} \cdot s_b) (s_a \cdot s_b)$ , and  $(s_a \cdot s_b)^2$ . The first two of these terms correspond to contractions with the tensors  $p_{\gamma}^{\mu} p_{\gamma}^{\nu} p_{\gamma}^{\rho} p_{\gamma}^{\sigma}$  and  $p_{\gamma}^{\mu} p_{\gamma}^{\nu}$ , respectively. Expanding these tensors into all possible tensors made up of the metric tensor and the incoming partonic momenta, one finds, straightforwardly,

$$\int d\Omega_{\gamma}(p_{\gamma} \cdot s_{a})^{2}(p_{\gamma} \cdot s_{b})^{2}$$

$$= \int d\Omega_{\gamma} \frac{t^{2}u^{2}}{8s^{2}} [2(s_{a} \cdot s_{b})^{2} + s_{a}^{2}s_{b}^{2}]$$

$$= \int d\Omega_{\gamma} \frac{3t^{2}u^{2}}{8s^{2}},$$

$$\int d\Omega_{\gamma}(p_{\gamma} \cdot s_{a})(p_{\gamma} \cdot s_{b})(s_{a} \cdot s_{b})$$

$$= -\int d\Omega_{\gamma} \frac{tu}{2s}(s_{a} \cdot s_{b})^{2}$$

$$= -\int d\Omega_{\gamma} \frac{tu}{2s},$$
(14)

where  $\int d\Omega_{\gamma}$  denotes integration over the photon phase space, and where we have chosen both spin vectors to point in the same direction. We also recall that  $s_i \cdot p_a = s_i \cdot p_b$ = 0 (*i*=*a*,*b*) and  $s_a^2 = s_b^2 = -1$ . We emphasize that after the replacements (14) the whole invariant phase space over  $p_{\gamma}$ remains to be integrated, including the (now trivial) azimuthal part, as indicated by the  $\int d\Omega_{\gamma}$  on the right hand side. This is the virtue of our method that becomes particularly convenient at NLO. It is crucial here that the other observed ("fixed") quantities, transverse momentum  $p_T$  and rapidity  $\eta$ , are determined entirely by scalar products  $(p_a \cdot p_{\gamma})$  and  $(p_b \cdot p_{\gamma})$ . This allows the above tensor decomposition with tensors only made up of  $p_a$  and  $p_b$  and of course the metric tensor.

Inserting all results, and including the azimuthal part of the  $d\Omega_{\gamma}$  integration, we find

$$\langle \delta | M(q\bar{q} \to \gamma g) |^2 \rangle = (ee_q g)^2 \frac{4C_F}{N_C},$$
 (15)

and hence, using Eq. (12), we recover Eq. (10).

In the NLO calculation, one has  $2\rightarrow 3$  reactions  $ab \rightarrow \gamma c d$ . For an inclusive photon spectrum, one integrates over the full phase spaces  $d\Omega_c$  and  $d\Omega_d$  of particles c and d, respectively. The momentum of particle d may be fixed by momentum conservation, and the integration is trivial. One then ends up with

$$\int d\Omega_{\gamma} \int d\Omega_{c} \mathcal{F}(p_{\gamma}, s_{a}, s_{b}) \,\delta |M(ab \to \gamma cd)|^{2}.$$
(16)

Besides scalar products of the  $s_i(i=a,b)$  with  $p_{\gamma}$ , the integrand may contain terms  $\alpha(s_a \cdot p_c)(s_b \cdot p_c)$  and  $\alpha(s_i \cdot p_c)$ . As

before, we may expand the ensuing tensor and vector integrals in terms of the available tensors. As far as the integration over  $d\Omega_c$  is concerned, such tensors may be made up of the metric tensor,  $p_a$ ,  $p_b$ , and  $p_{\gamma}$ . It is also important to keep in mind that in the NLO calculation we will need to use dimensional regularization due to the presence of singularities in the phase space integrations. We find, in  $d=4-2\varepsilon$ dimensions,

$$\int d\Omega_{c}(p_{c} \cdot s_{a})(p_{c} \cdot s_{b}) = \int d\Omega_{c} \left\{ \frac{tu}{s} \left[ \frac{1}{2} \mathcal{A} - \mathcal{B} \right](s_{a} \cdot s_{b}) \right. \\ \left. + \left[ (1 - \varepsilon) \mathcal{A} - \mathcal{B} \right](p_{\gamma} \cdot s_{a}) \right. \\ \left. \times (p_{\gamma} \cdot s_{b}) \right\},$$
$$\int d\Omega_{c}(p_{c} \cdot s_{i}) = \int d\Omega_{c} \mathcal{C} \cdot (p_{\gamma} \cdot s_{i}), \qquad (17)$$

where

$$\mathcal{A} = \frac{2}{(1-2\varepsilon)} \mathcal{C}^2,$$
$$\mathcal{B} = \frac{1}{(1-2\varepsilon)} \frac{t_c u_c}{t u},$$
(18)

$$\mathcal{C} = -\frac{ss_{\gamma c} - tu_c - t_c u}{2tu},$$

with

$$t_c \equiv (p_a - p_c)^2, \quad u_c \equiv (p_b - p_c)^2, \quad s_{\gamma c} \equiv (p_{\gamma} + p_c)^2.$$
(19)

After scalar products involving  $p_c$  with the  $s_i$  have been eliminated in this way, only those with  $(p_{\gamma} \cdot s_i)$  remain. As in our LO example, when we apply the factor  $\mathcal{F}(p_{\gamma}, s_a, s_b)$ , these terms enter as  $(p_{\gamma} \cdot s_a)^2 (p_{\gamma} \cdot s_b)^2$ and  $(p_{\gamma} \cdot s_a) (p_{\gamma} \cdot s_b)$ . We then may use Eq. (14) after appropriate modification to  $d=4-2\varepsilon$  dimensions:

$$\int d\Omega_{\gamma} (p_{\gamma} \cdot s_{a})^{2} (p_{\gamma} \cdot s_{b})^{2}$$

$$= \int d\Omega_{\gamma} \frac{t^{2} u^{2}}{4(1-\varepsilon)(2-\varepsilon)s^{2}} [2(s_{a} \cdot s_{b})^{2} + s_{a}^{2} s_{b}^{2}],$$

$$\int d\Omega_{\gamma} (p_{\gamma} \cdot s_{a}) (p_{\gamma} \cdot s_{b}) (s_{a} \cdot s_{b})$$

$$= -\int d\Omega_{\gamma} \frac{tu}{2(1-\varepsilon)s} (s_{a} \cdot s_{b})^{2}.$$
(20)

After this step, there are no scalar products involving the  $s_i$  left in the squared matrix element (except the trivial  $s_a \cdot s_b = -1$ ). We may now integrate over all phase space, employing techniques familiar from the corresponding calculations in the unpolarized and longitudinally polarized cases. As a

check, we have applied our method to the Drell-Yan transversity cross section and recovered the known NLO result [11] in a straightforward manner. For the interested reader, we list some details of this calculation in the Appendix.

# C. Details of the NLO calculation for prompt photon production

From here on, all steps in the calculation are fairly standard, albeit still involved and lengthy. Since many of them have been documented in previous papers [16–19], we only give a brief summary here. We emphasize that the general method we have employed is to perform the integrations over the phase space of the unobserved particles in the 2  $\rightarrow$ 3 contributions analytically. We have also simultaneously calculated the unpolarized cross section and found agreement with the expressions available in the literature [17,18].

At NLO, there are two subprocesses that contribute for transverse polarization:

$$q\bar{q} \rightarrow \gamma X,$$
  
 $qq \rightarrow \gamma X.$  (21)

The first one of course was already present at LO, where X = g. At NLO, one has virtual corrections to the Born cross section (X=g), but also  $2\rightarrow 3$  real emission diagrams, with  $X = gg + q\bar{q} + q'\bar{q}'$ . For the second subprocess, X = qq. All contributions are treated as discussed in the previous subsection, i.e., we project on their  $\cos(2\Phi)$  dependence by multiplying with the function  $\mathcal{F}(p_{\gamma}, s_a, s_b)$  in Eq. (11) and integrating over the azimuthal phase space using Eqs. (17) and (20).

Owing to the presence of ultraviolet, infrared, and collinear singularities at intermediate stages of the calculation, it is necessary to introduce a regularization. Our choice is dimensional regularization, that is, the calculation is performed in  $d=4-2\varepsilon$  space-time dimensions. Subtractions of singularities are made in the modified minimal subtraction ( $\overline{\text{MS}}$ ) scheme throughout.

Projection on a definite polarization state for the initial partons involves the Dirac matrix  $\gamma_5$ , as is evident from Eq. (9). It is well known that dimensional regularization becomes a somewhat subtle issue if  $\gamma_5$  enters the calculation, the reason being that  $\gamma_5$  is a genuinely four-dimensional object with no natural extension to  $d \neq 4$  dimensions. Extending the relation  $\{\gamma_5, \gamma^{\mu}\} = 0$  to d dimensions leads to algebraic inconsistencies in Dirac traces with an odd number of  $\gamma_5$  [20]. Owing to the chirally odd nature of transversity, in our calculation all Dirac traces contain two  $\gamma_5$  matrices, and there should be no problem using a naive, totally anticommuting  $\gamma_5$  in d dimensions. Nevertheless, we also did the calculation using the widely used "'t Hooft-Veltman-Breitenlohner-Maison (HVBM) scheme" [21] for  $\gamma_5$ , which is known to be fully consistent. It is mainly characterized by splitting the d-dimensional metric tensor into a four-dimensional and a (d-4)-dimensional one. In the four-dimensional subspace,  $\gamma_5$  continues to anti-commute with the other Dirac matrices; however, it commutes with them in the (d-4)-dimensional

one. The HVBM scheme thus leads to a higher complexity of the algebra<sup>3</sup> and of phase space integrals. We found the same final answers for both  $\gamma_5$  prescriptions in all our calculations.

Ultraviolet poles in the virtual diagrams are removed by the renormalization of the strong coupling constant at a scale  $\mu_R$ . Infrared singularities cancel in the sum between virtual and real-emission diagrams. After this cancellation, only collinear poles are left. These result for example from a parton in the initial state splitting collinearly into a pair of partons, corresponding to a long-distance contribution in the partonic cross section. From the factorization theorem it follows that such contributions need to be factored, at a factorization scale  $\mu_F$ , into the parton distribution functions. A similar situation occurs in the final state. The high- $p_T$  photon may result from collinear radiation off a quark, which again is singular. This singularity is absorbed into a "quark-tophoton" fragmentation function [17,18] that describes photon production in jet fragmentation and hence by itself contains long-distance information. The fragmentation contribution has not been written down in Eq. (3). It has a structure similar to Eq. (3), but with an extra integration over the fragmentation function. Its size also depends on the experimental selection of prompt photon events, as we will discuss below.

The subtraction of initial-state collinear singularities is particularly simple in case of transversity since there is no gluon transversity and only  $q \rightarrow qg$  collinear splittings can occur. Only the process  $q\bar{q} \rightarrow \gamma gg$  has such poles. Their cancellation is effected by adding a "counterterm" that has the structure (for radiation off the initial quark)

$$-\frac{\alpha_s}{\pi} \int_0^1 dx \,\delta H_{qq}(x,\mu_F) \frac{d \,\delta \hat{\sigma}_{q\bar{q}\to\gamma g}^{(0)}(xs,xt,u,\varepsilon)}{dv} \\ \times \,\delta[x(s+t)+u], \tag{22}$$

where, in the MS scheme,

$$\delta H_{qq}(z,\mu_F) \equiv \left(-\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi\right) \delta P_{qq}(z) \left(\frac{s}{\mu_F^2}\right)^{\varepsilon},$$
(23)

with the LO transversity splitting function [23]

$$\delta P_{qq}(z) = C_F \left[ \frac{2z}{(1-z)_+} + \frac{3}{2} \,\delta(1-z) \right]. \tag{24}$$

Here the "plus" distribution is defined in the usual way. As indicated in Eq. (22), the  $2\rightarrow 2$  cross section in the integrand needs to be evaluated in *d* dimensions. The result, which turns out to be the same in the anticommuting  $\gamma_5$  and the HVBM schemes, is given by

$$\left\langle \frac{d\,\delta\hat{\sigma}_{q\bar{q}\to\gamma g}^{(0)}(s,t,u,\varepsilon)}{dt} \right\rangle = \frac{2C_F}{N_C} \frac{\alpha \alpha_s}{s^2} e_q^2 \frac{\mu^{2\varepsilon}}{\Gamma(1-\varepsilon)} \left(\frac{4\pi\mu^2 s}{tu}\right)^{\varepsilon} \times \frac{2(1-\varepsilon+\varepsilon^2)}{(1-\varepsilon)(2-\varepsilon)} \left(1-\varepsilon-\frac{\varepsilon^2 s^2}{2tu}\right).$$
(25)

Needless to say that we have applied also here our "projector"  $\mathcal{F}(p_{\gamma}, s_a, s_b)$  of Eq. (11) and performed the integration over the scalar products involving spin vectors according to Eq. (20).

In the final-state collinear case, an expression very similar to Eq. (22) is to be used, involving now the unpolarized quark-to-photon splitting function

$$P_{\gamma q}(z) = \frac{1 + (1 - z)^2}{z}$$
(26)

and the  $2\rightarrow 2$  "pure-QCD" transversity cross sections in *d* dimensions, given by

$$\left\langle \frac{d\delta\hat{\sigma}_{q\bar{q}\to q'\bar{q}'}^{(0)}(s,t,u,\varepsilon)}{dt} \right\rangle = \frac{C_F}{2N_C} \frac{\alpha_s^2}{s^2} \frac{\mu^{2\varepsilon}}{\Gamma(1-\varepsilon)} \left(\frac{4\pi\mu^2 s}{tu}\right)^{\varepsilon} \times (2+\varepsilon) \frac{tu}{s^2},$$
$$\left\langle \frac{d\delta\hat{\sigma}_{q\bar{q}\to q\bar{q}}^{(0)}(s,t,u,\varepsilon)}{dt} \right\rangle = \frac{C_F}{2N_C} \frac{\alpha_s^2}{s^2} \frac{\mu^{2\varepsilon}}{\Gamma(1-\varepsilon)} \left(\frac{4\pi\mu^2 s}{tu}\right)^{\varepsilon} \times \left[ (2+\varepsilon) \frac{tu}{s^2} - \frac{(2-\varepsilon)}{N_C} \frac{u}{s} \right],$$
(27)

$$\left\langle \frac{d\delta\hat{\sigma}_{qq \to qq}^{(0)}(s,t,u,\varepsilon)}{dt} \right\rangle = \frac{C_F}{2N_C^2} \frac{\alpha_s^2}{s^2} \frac{\mu^{2\varepsilon}}{\Gamma(1-\varepsilon)} \left(\frac{4\pi\mu^2 s}{tu}\right)^{\varepsilon} \times (2-\varepsilon).$$

In these expressions, we have neglected contributions  $\propto \mathcal{O}(\varepsilon^2)$ , which do not contribute. Then, the results for a fully anticommuting  $\gamma_5$  and for the HVBM prescription are again the same.

Before coming to our final results, we would like to make two more comments on the use of our "projector" on the azimuthal-angular dependence, Eq. (11). In an NLO calculation, carried out in *d* dimensions, we could have a projector that by itself contains terms  $\propto \varepsilon$ . Indeed, some of the Born cross sections, when evaluated in *d* dimensions, suggest a projector of the form

$$\mathcal{F}_{\varepsilon}(p_{\gamma}, s_{a}, s_{b}) = \frac{s}{\pi t u} \bigg[ 2(p_{\gamma} \cdot s_{a})(p_{\gamma} \cdot s_{b}) + (1 - a\varepsilon) \\ \times \frac{t u}{s}(s_{a} \cdot s_{b}) \bigg], \qquad (28)$$

<sup>&</sup>lt;sup>3</sup>We use the program TRACER [22] to perform Dirac traces in d dimensions.

with some constant *a*. Clearly, the final answer of the calculation must not depend on *a* because our projection is a physical operation which could be done in experiment. We have used the above projector with an arbitrary *a* and checked that indeed no answer depends on *a*. Also, we have integrated all squared matrix elements over the spin vectors *without* using any projector at all. This amounts to integrating  $\cos(2\Phi)$  over all  $0 \le \Phi \le 2\pi$ , and, as expected, we get zero in the final answer. It should be stressed, however, that individual pieces in the calculation (the virtual, the  $2 \rightarrow 3$ , and the factorization part) do not by themselves integrate to zero, but only their sum does. In this way, we have a very powerful check on the correctness of our calculation.

# D. Final results for inclusive and isolated photon cross sections

For both subprocesses, the final results for the NLO corrections can be cast into the following form:

$$\left\langle s \frac{d\delta\hat{\sigma}_{ab \to \gamma X}^{(1)}(s, v, w, \mu_R, \mu_F)}{dv dw} \right\rangle = \frac{\alpha \alpha_s(\mu_R)}{\pi^2} \left[ \left( A_0 \delta(1-w) + B_0 \frac{1}{(1-w)_+} + C_0 \right) \ln \frac{\mu_F^2}{s} + C_1 \mathcal{I}^{\text{final}}(1-v+vw) + A_2 \delta(1-w) \ln \frac{\mu_R^2}{s} + A \delta(1-w) + B \frac{1}{(1-w)_+} + C + D \left( \frac{\ln(1-w)}{1-w} \right)_+ + E \ln w + F \ln v + G \ln(1-v) + H \ln(1-w) + I \ln(1-vw) + J \ln(1-v+vw) + K \frac{\ln w}{1-w} + L \frac{\ln \frac{1-v}{1-vw}}{1-w} + M \frac{\ln(1-v+vw)}{1-w} \right],$$
(29)

where all coefficients are functions of v and w, except those multiplying the distributions  $\delta(1-w)$ ,  $1/(1-w)_+$ ,  $[\ln(1-w)/(1-w)]_+$  which may be written as functions just of v. Terms with distributions are present only for the subprocess  $q\bar{q} \rightarrow \gamma X$ . The coefficients in Eq. (29) are too lengthy to be given here but are available upon request.

Let us now specify the function  $\mathcal{I}^{\text{final}}(z=1-v+vw)$ . It results from the configurations where the photon is collinear with a final-state quark or antiquark. As we discussed earlier, these will lead to final-state collinear singularities that are absorbed, at the factorization scale<sup>4</sup>  $\mu_F$ , into photon fragmentation functions. The actual form of  $\mathcal{I}^{\text{final}}$  depends on the kind of photon signal under consideration. Let us first consider the fully inclusive cross section. In this case, one just counts all photon candidates in the kinematical bin, without imposing any constraint on additional particles in the event. This is the simplest cross section and the one usually measured in fixed-target experiments. In the theoretical calculation, final-state singularities arise and there is a need to introduce a fragmentation contribution, as discussed earlier.

At collider energies, the background from pions decaying into photon pairs is so severe that so-called isolation cuts are imposed on the photon. The basic idea is that photons that have little hadronic energy around them are less likely to result from  $\pi^0$  decay. The standard procedure is to define a "cone" around the photon by  $\sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \leq R$ , where typically  $R \approx 0.4 \dots 0.7$ , and to demand that the hadronic transverse energy in the cone be smaller than  $\tau p_T$ , where  $\tau$ is a parameter of order 0.1. For the theoretical calculation, isolation implies a strong reduction of the size of the fragmentation contribution because photons produced by fragmentation are always accompanied by a certain amount of hadronic energy. A slightly refined type of isolation has been proposed in [24]. Again a cone is defined, centered on the photon, within which the hadronic transverse energy must not exceed the limit  $\tau p_T$ . However, one chooses a larger  $\tau$  $\sim 1$  and then further restricts the hadronic energy by demanding that for any  $r \leq R$  the hadronic energy inside a cone of opening r be smaller than roughly  $\tau(r/R)^2 p_T$ . In other words, the closer hadronic energy is deposited to the photon, the smaller it has to be in order for the event to pass the isolation cut. This isolation method has not yet been used in any experiment, but it is possible that it will become the choice for the PHENIX experiment at RHIC [25]. On the theoretical side, it has the advantage that it "eliminates" any kind of fragmentation contribution [24] because fragmentation is assumed to be a (mainly) collinear process, and no hadronic activity is allowed exactly parallel to the photon.

We recall from the previous section that we have performed an analytical integration over the full phase space of

<sup>&</sup>lt;sup>4</sup>We could also choose a final-state factorization scale  $\mu'_F \neq \mu_F$  here.

the unobserved particles in the final state. This seems at first sight to preclude the implementation of an isolation cut "afterwards." However, as was shown in [26,27], it is possible to impose the isolation cut in an approximate, but accurate, analytical way by introducing certain "subtraction cross sections." The approximation is based on assuming the isolation cone to be rather narrow. In this case, dependence on the cone opening can be shown to be of the form  $a \ln(R)+b$  $+\mathcal{O}(R^2)$ . a and b are straightforwardly determined and yield a very accurate description of isolation even at R = 0.7. Analytical calculations [26,27] are therefore as capable to describe the isolated prompt-photon cross section as NLO computations in which phase space integrals are performed numerically employing Monte Carlo techniques [28,24,27].

For the cases of the fully-inclusive ("incl.") cross section, the standard isolation ("std."), and for the isolation proposed in [24] ("smooth") the function  $\mathcal{I}^{\text{final}}(z=1-v+vw)$  takes the following forms:

$$\mathcal{I}^{\text{final}}(z) = \begin{cases} P_{\gamma q}(z) \ln\left(\frac{\mu_F^2}{s}\right) & \text{incl.,} \\ P_{\gamma q}(z) \ln\left(\frac{\mu_F^2}{s}\right) + \Theta(1 - z[1 + \tau]) \left[ P_{\gamma q}(z) \ln\left(\frac{(1 - z)^2 p_T^2 R^2}{\mu_F^2}\right) + z \right] & \text{std.,} \\ P_{\gamma q}(z) \ln\left(\frac{(1 - z)^3 p_T^2 R^2}{s \tau z}\right) & \text{smooth.} \end{cases}$$
(30)

One can see the presence of the quark-to-photon splitting function  $P_{\gamma q}$  of Eq. (26), as is expected for contributions resulting from near-collinear photon emission in the final state. It also becomes clear that for the standard isolation the dependence on the final-state factorization scale is reduced and disappears altogether for the isolation of [24]. This is in line with our remarks above about the size of the fragmentation contribution in these cases.

### **III. NUMERICAL RESULTS**

In this section, we present a first numerical application of our analytical results. We focus on the main features of the NLO corrections and describe their impact on the cross section  $d\delta\sigma/dp_T$  and the spin asymmetry  $A_{TT}^{\gamma}$ . Our predictions will apply for prompt photon measurements with the PHENIX detector at RHIC. This implies that the pseudorapidity region  $|\eta| \leq 0.35$  is covered, and only half of the photon's azimuthal angle. Using Eq. (7) we restore the  $\cos(2\Phi)$  dependence of the cross section. We take the two quadrants in  $\Phi$  covered by the PHENIX detector to be  $-\pi/4 < \Phi < \pi/4$  and  $3\pi/4 < \Phi < 5\pi/4$  and integrate over these. This gives  $(\int_{-\pi/4}^{\pi/4} + \int_{3\pi/4}^{5\pi/4})\cos(2\Phi)d\Phi=2$ . We consider photons isolated according to the isolation of [24] discussed above, using R = 0.4 and  $\tau=1$ .

Before we can perform numerical studies of  $A_{TT}^{\gamma}$  we have to model the  $\delta f$  we will use. Nothing is known experimentally about transversity so far. The only guidance is provided by the Soffer inequality [12]

$$2|\delta q(x)| \leq q(x) + \Delta q(x) \tag{31}$$

which gives an upper bound for each  $\delta f$ . As in [5] we utilize this inequality by saturating the bound at some low input scale  $\mu_0 \approx 0.6$  GeV using the NLO (LO) GRV [29] and GRSV ("standard scenario") [30] densities  $q(x,\mu_0)$  and  $\Delta q(x,\mu_0)$ , respectively. For  $\mu > \mu_0$  the transversity densities  $\delta f(x,\mu)$  are then obtained by solving the evolution equations with the LO [7,23] or NLO [11,31] kernels. Obviously, the sign to be used when saturating the inequality is at our disposal; we choose all signs to be positive. We refer the reader to [5] for more details on our model distributions. We note that we will always perform the NLO (LO) calculations using NLO (LO) parton distribution functions and the two-loop (one-loop) expression for  $\alpha_s$ .

Figure 1 shows our results for the transversely polarized prompt photon production cross sections at NLO and LO for two different c.m.s. energies. The lower part of the figure displays the so-called "*K* factor"

$$K = \frac{d\,\delta\sigma^{\rm NLO}}{d\,\delta\sigma^{\rm LO}}.\tag{32}$$

One can see that NLO corrections are somewhat smaller for  $\sqrt{S} = 500$  GeV and increase with  $p_T$ . As we have mentioned in the Introduction, one reason why it is generally important to know NLO corrections is that they should considerably reduce the dependence of the cross sections on the unphysical factorization and renormalization scales. In this sense, the K factor has actually limited significance since it is likely to be rather scale dependent through the presence of the LO cross section in its denominator. The improvement in scale dependence when going from LO to NLO is, therefore, a better measure of the impact of the NLO corrections. The shaded bands in the upper panel of Fig. 1 indicate the uncertainties from varying the scales in the range  $p_T/2 \leq \mu_R = \mu_F$  $\leq 2p_T$ . The solid and dashed lines are always for the choice where all scales are set to  $p_T$ , and so is the K factor underneath. One can see that the scale dependence indeed becomes much weaker at NLO.



FIG. 1. Predictions for the transversely polarized prompt photon production cross sections at LO and NLO, for  $\sqrt{S} = 200$  and 500 GeV. The LO results have been scaled by a factor of 0.01. The shaded bands represent the theoretical uncertainty if  $\mu_F(=\mu_R)$  is varied in the range  $p_T/2 \le \mu_F \le 2p_T$ . The lower panel shows the ratios of the NLO and LO results for both c.m.s. energies.

Figure 2 shows the spin asymmetry  $A_{TT}^{\gamma}$  which is perhaps the main quantity of interest here, calculated at LO<sup>5</sup> (dashed lines) and NLO (solid lines). We have again chosen all scales to be  $p_T$ . Due to a larger K factor for the unpolarized cross section, the asymmetry is smaller at NLO than at LO. We also display in Fig. 2 the statistical errors expected in experiment. They may be estimated by the formula [4]

$$\delta A_{TT}^{\gamma} \simeq \frac{1}{P^2 \sqrt{\mathcal{L}\sigma_{\rm bin}}},\tag{33}$$

where *P* is the transverse polarization of each beam,  $\mathcal{L}$  the integrated luminosity of the collisions, and  $\sigma_{\text{bin}}$  the unpolarized cross section integrated over the  $p_T$  bin for which the error is to be determined. We have used P=0.7 and  $\mathcal{L} = 320(800)/\text{pb}$  for  $\sqrt{S} = 200(500)$  GeV.

### **IV. CONCLUSIONS**

We have presented in this paper the complete NLO QCD corrections for the partonic hard-scattering cross sections rel-



FIG. 2. Predictions for the transverse spin asymmetry  $A_{TT}^{\gamma}$  for isolated prompt photon production in LO and NLO for  $\sqrt{S} = 200$  and 500 GeV. The "error bars" indicate the expected statistical accuracy for bins in  $p_T$  (see text).

evant for the spin asymmetry  $A_{TT}^{\gamma}$  for high- $p_T$  prompt photon production in transversely polarized proton-proton collisions. This asymmetry could be a tool to determine the transversity content of the nucleon at RHIC.

Our calculation is based on a largely analytical evaluation of the NLO partonic cross sections. We have presented a simple technique for treating, in an NLO calculation, the azimuthal-angle dependence introduced by the transverse spin vectors. We will apply this technique to other  $A_{TT}$  in the future, such as for inclusive pion and jet production [32].

We found that at RHIC energies the NLO corrections to the polarized cross section are somewhat smaller than those in the unpolarized case. The transversely polarized cross section shows a significant reduction of scale dependence when going from LO to NLO.

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### APPENDIX: NLO TRANSVERSITY DRELL-YAN CROSS SECTION WITH PROJECTION TECHNIQUE

In this appendix we briefly report the results we find for the NLO corrections to the Drell-Yan "coefficient function"  $\delta C^{\text{DY}}$  when using our projection method of Sec. II B. For details on the kinematics for the process, see [10,11]. We use

<sup>&</sup>lt;sup>5</sup>We note that our LO asymmetries are larger than those reported in [5]. This is due to an error in the numerical computation in [5]. Our LO curves in Fig. 2 correct this mistake.

a fully anticommuting  $\gamma_5$  and choose the scales  $\mu_F = \mu_R = Q$  everywhere, with Q the dilepton mass. The LO cross section and the virtual corrections at NLO rely on the underlying  $2\rightarrow 2$  reaction  $q\bar{q}\rightarrow l^+l^-$ . The real-emission NLO 2  $\rightarrow 3$  process is  $q\bar{q}\rightarrow l^+l^-g$ . We apply our projector, Eq. (11), to the squared matrix elements for each of these processes and integrate over the appropriate phase spaces. For the  $2\rightarrow 3$  process this gives

$$\delta C_{2 \to 3}^{\rm DY} = \frac{\alpha_s}{2\pi} \frac{C_F(4\pi)^{2\varepsilon}}{\Gamma(1-2\varepsilon)} \left[ \left( \frac{2}{\varepsilon^2} + \frac{13}{3\varepsilon} - \frac{\pi^2}{3} - \frac{29}{18} \right) \delta(1-z) + \left( -\frac{4}{\varepsilon} - \frac{26}{3} \right) \frac{z}{(1-z)_+} + 8z \left( \frac{\ln(1-z)}{1-z} \right)_+ - 4z \frac{\ln z}{1-z} - 6z \frac{\ln^2 z}{1-z} + 4(1-z) \right], \quad (A1)$$

where  $z = Q^2/s$ . For the virtual contributions we get

$$\delta C_{\text{virt.}}^{\text{DY}} = \frac{\alpha_s}{2\pi} \frac{C_F (4\pi)^{2\varepsilon}}{\Gamma(1-2\varepsilon)} \left[ -\frac{2}{\varepsilon^2} - \frac{22}{3\varepsilon} + \pi^2 - \frac{116}{9} \right] \delta(1-z),$$
(A2)

and for the MS collinear-factorization term

$$\delta C_{\text{fact.}}^{\text{DY}} = \frac{\alpha_s}{2\pi} \frac{C_F (4\pi)^{2\varepsilon}}{\Gamma(1-2\varepsilon)} \left[ \left(\frac{3}{\varepsilon} + \frac{13}{2}\right) \delta(1-z) + \left(\frac{4}{\varepsilon} + \frac{26}{3}\right) \frac{z}{(1-z)_+} \right].$$
(A3)

Adding all terms, the poles cancel, and one obtains the NLO  $\overline{\text{MS}}$  coefficient function:

$$\delta C^{\rm DY}(z) = \frac{\alpha_s}{2\pi} C_F \left[ \left( \frac{2}{3} \pi^2 - 8 \right) \delta(1-z) + 8z \left( \frac{\ln(1-z)}{1-z} \right)_+ -4z \frac{\ln z}{1-z} - 6z \frac{\ln^2 z}{1-z} + 4(1-z) \right]$$
(A4)

in agreement with [11].

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