

**Instanton contribution to scalar charmonium and glueball decays**

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We study instanton contributions to hadronic decays of the scalar glueball, the pseudoscalar charmonium state  $\eta_c$ , and the scalar charmonium state  $\chi_c$ . Hadronic decays of the  $\eta_c$  are of particular interest. The three main decay channels are  $K\bar{K}\pi$ ,  $\eta\pi\pi$  and  $\eta'\pi\pi$ , each with an unusually large branching ratio  $\sim 5\%$ . On the quark level, all three decays correspond to an instanton type vertex  $(\bar{c}c)(\bar{s}s)(\bar{d}d)(\bar{u}u)$ . We show that the total decay rate into three pseudoscalar mesons can be reproduced using an instanton size distribution consistent with phenomenology and lattice results. Instantons correctly reproduce the ratio  $B(\pi\pi\eta)/B(\pi\pi\eta')$  but overpredict the ratio  $B(K\bar{K}\pi)/B[\pi\pi\eta(\eta')]$ . We consider the role of scalar resonances and suggest that the decay mechanism can be studied by measuring the angular distribution of decay products.

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**I. INTRODUCTION**

The charmonium system has played an important role in shaping our knowledge of perturbative and nonperturbative QCD. The discovery of the  $J/\psi$  as a narrow resonance in  $e^+e^-$  annihilation confirmed the existence of a new quantum number, charm. The analysis of charmonium decays in  $e^+e^-$  pairs, photons and hadrons established the hypothesis that the  $J/\psi$  and  $\eta_c$  are, to a good approximation, nonrelativistic  $^3S_1$  and  $^1S_0$  bound states of heavy charm and anticharm quarks. However, nonperturbative dynamics does play an important role in the charmonium system [1,2]. For example, an analysis of the  $\psi$  spectrum led to the first determination of the gluon condensate.

The total width of charmonium is dominated by short distance physics and can be studied in perturbative QCD [3]. The only nonperturbative input in these calculations is the wave function at the origin. A systematic framework for these calculations is provided by the nonrelativistic QCD (NRQCD) factorization method [4]. NRQCD facilitates higher order calculations and relates the decays of states with different quantum numbers. QCD factorization can also be applied to transitions of the type  $\psi' \rightarrow \psi + X$  [5,6].

The study of exclusive decays of charmonium into light hadrons is much more complicated and very little work in this direction has been done. Perturbative QCD implies some helicity selection rules, for example  $\eta_c \rightarrow \rho\rho, p\bar{p}$  and  $J/\psi \rightarrow \rho\pi, \rho a_1$  [7,8], but these rules are strongly violated [9]. The  $J/\psi$  decays mostly into an odd number of Goldstone bosons. The average multiplicity is  $\sim (5-7)$ , which is consistent with the average multiplicity in  $e^+e^-$  annihilation away from the  $J/\psi$  peak. Many decay channels have been observed, but none of them stand out. Consequently, we would expect the  $\eta_c$  to decay mostly into an even number of pions with similar multiplicity. However, the measured decay rates are not in accordance with this expectation. The three main decay channels of the  $\eta_c$  are  $K\bar{K}\pi$ ,  $\eta\pi\pi$  and  $\eta'\pi\pi$ ,

each with an unusually large branching ratio of  $\sim 5\%$ . Bjorken observed that these three decays correspond to a quark vertex of the form  $(\bar{c}c)(\bar{s}s)(\bar{d}d)(\bar{u}u)$  and suggested that  $\eta_c$  decays are a “smoking gun” for instanton effects in heavy quark decays [10].

In this paper we shall try to follow up on this idea by performing a more quantitative estimate of the instanton contribution to  $\eta_c$  and  $\chi_c$  decays. The paper is organized as follows. In Sec. II we introduce the instanton induced effective Lagrangian. In the following sections we apply the effective Lagrangian to the decays of the scalar glueball, eta charm, and chi charm. We should note that this investigation should be seen as part of a larger effort to identify “direct” instanton contributions in hadronic reactions, such as deep inelastic scattering, the  $\Delta I = 1/2$  rule, or  $\eta$  production in  $pp$  scattering [11–14].

**II. EFFECTIVE LAGRANGIANS**

Instanton effects in hadronic physics have been studied extensively [15,16]. Instantons play an important role in understanding the  $U(1)_A$  anomaly and the mass of the  $\eta'$ . In addition to that, there is also evidence that instantons provide the mechanism for chiral symmetry breaking and play an important role in determining the structure of light hadrons. All of these phenomena are intimately related to the presence of chiral zero modes in the spectrum of the Dirac operator in the background field of an instanton. The situation in heavy quark systems is quite different. Fermionic zero modes are not important and the instanton contribution to the heavy quark potential is small [17].

This does not imply that instanton effects are not relevant. The nonperturbative gluon condensate plays an important role in the charmonium system [1,2], and instantons contribute to the gluon condensate. In general, the charmonium system provides a laboratory for studying nonperturbative glue in QCD. The decay of a charmonium state below the  $D\bar{D}$

threshold involves an intermediate gluonic state. Since the charmonium system is small,  $r_{c\bar{c}} \sim (vm_c)^{-1} < \Lambda_{QCD}^{-1}$ , the gluonic system is also expected to be small. For this reason charmonium decays have long been used for glueball searches.

Since charmonium decays produce a small gluonic system we expect that the  $c\bar{c}$  system mainly couples to instantons of size  $\rho \sim r_{c\bar{c}} \sim (vm_c)^{-1}$ . In this limit the instanton effects can be summarized in terms of an effective Lagrangian [18–21]:

$$\mathcal{L}_I = \int \prod_q \left[ m_q \rho - 2\pi^2 \rho^3 \bar{q}_R \left( U 1_2 U^\dagger + \frac{i}{2} t^a R^{aa'} \bar{\eta}_{\mu\nu}^{a'} \sigma^{\mu\nu} \right) q_L \right] \times \exp \left( - \frac{2\pi^2}{g} \rho^2 \bar{\eta}_{\gamma\delta}^{b'} R^{b'b} G^{b,\gamma\delta} \right) dz \frac{d_0(\rho)}{\rho^5} d\rho dU, \quad (1)$$

where  $t^a = \frac{1}{2} \lambda^a$  with  $\text{tr}[\lambda^a \lambda^b] = 2 \delta^{ab}$  are  $SU(3)$  generators,  $1_2 = \text{diag}(1, 1, 0)$ ,  $\eta_{\mu\nu}^a$  is the 't Hooft symbol and  $\sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu]$ . The instanton is characterized by  $4N_c$  collective coordinates, the instanton position  $z$ , the instanton size  $\rho$ , and the color orientation  $U \in SU(N_c)$ . We also define the rotation matrix  $R^{ab}$  by  $R^{aa'} \lambda^{a'} = U \lambda^a U^\dagger$ . For an anti-instanton we have to replace  $L \leftrightarrow R$  and  $\bar{\eta} \leftrightarrow \eta$ . The semiclassical instanton density  $d(\rho)$  is given by

$$d(\rho) = \frac{d_0(\rho)}{\rho^5} = \frac{0.466 \exp(-1.679 N_c) 1.34^{N_f}}{(N_c - 1)! (N_c - 2)!} \left( \frac{8\pi^2}{g^2} \right)^{2N_c} \times \rho^{-5} \exp \left[ - \frac{8\pi^2}{g(\rho)^2} \right], \quad (2)$$

where  $g(\rho)$  is the running coupling constant. For small  $\rho$  we have  $d(\rho) \sim \rho^{b-5}$  where  $b = (11N_c)/3 - (2N_f)/3$  is the first coefficient of the beta function.

Expanding the effective Lagrangian in powers of the external gluon field gives the leading instanton contribution to different physical matrix elements. If the instanton size is very small,  $\rho \ll m_c^{-1}$ , we can treat the charm quark mass as light and there is an effective vertex of the form  $(\bar{u}u)(\bar{d}d)(\bar{s}s)(\bar{c}c)$  which contributes to charmonium decays. Since the density of instantons grows as a large power of  $\rho$  the contribution from this regime is very small. In the realistic case  $\rho \sim (vm_c)^{-1}$  we treat the charm quark as heavy and the charm contribution to the fermion determinant is absorbed in the instanton density  $d(\rho)$ . The dominant contribution to charmonium decays then arises from expanding the gluonic part of the effective Lagrangian to second order in the field strength tensor. This provides effective vertices of the form  $(G\tilde{G})(\bar{u}\gamma_5 u)(\bar{d}\gamma_5 d)(\bar{s}\gamma_5 s)$ ,  $(G^2)(\bar{u}\gamma_5 u)(\bar{d}\gamma_5 d) \times (\bar{s}s)$ , etc.

We observe that the  $N_f=3$  fermionic Lagrangian combined with the gluonic term expanded to second order in the field strength involves an integral over the color orientation of the instanton which is of the form  $\int dU (U_{ij} U_{kl}^\dagger)^5$ . This integral gives  $(5!)^2$  terms. A more manageable result is obtained by using the vacuum dominance approximation. We

assume that the coupling of the initial charmonium or glueball state to the instanton proceeds via a matrix element of the form  $\langle 0^{++} | G^2 | 0 \rangle$  or  $\langle 0^{-+} | G\tilde{G} | 0 \rangle$ . In this case we can use

$$\langle 0^{++} | G_{\mu\nu}^a G_{\alpha\beta}^b | 0 \rangle = \frac{1}{12(N_c^2 - 1)} \delta^{ab} (\delta_{\mu\alpha} \delta_{\nu\beta} - \delta_{\mu\beta} \delta_{\nu\alpha}) \times \langle 0^{++} | G_{\rho\sigma}^{a'} G_{\rho\sigma}^{a'} | 0 \rangle \quad (3)$$

in order to simplify the color average. Equation (3) implies that the color average of the fermionic and gluonic parts of the interaction can be performed independently. The vacuum dominance approximation can be justified in the large  $N_c$  limit [22]. It is known, however, that in the scalar and pseudoscalar channel, corrections to the large  $N_c$  approximation are potentially large. In Sec. IV we will provide a perturbative estimate of the  $\eta_c$  coupling to the instanton that does not rely on the large  $N_c$  approximation.

In the limit of massless quarks the instanton ( $I$ ) and anti-instanton ( $A$ ) Lagrangian responsible for the decay of scalar and pseudoscalar charmonium decays is given by

$$\mathcal{L}_{I+A} = \int dz \frac{d_0(\rho)}{\rho^5} d\rho \frac{\pi^3 \rho^4}{(N_c^2 - 1) \alpha_s} \times \{ (G^2 - G\tilde{G}) \times \mathcal{L}_{f,I} + (G^2 + G\tilde{G}) \times \mathcal{L}_{f,A} \}. \quad (4)$$

Here,  $\mathcal{L}_{f,IA}$  is the color averaged  $N_f=3$  fermionic effective Lagrangian [15,16,21].

### III. SCALAR GLUEBALL DECAYS

Since the coupling of the charmonium state to the instanton proceeds via an intermediate gluonic system with the quantum numbers of scalar and pseudoscalar glueballs it is natural to first consider direct instanton contributions to glueball decays. This problem is of course important in its own right. Experimental glueball searches have to rely on identifying glueballs from their decay products. The successful identification of a glueball requires theoretical calculations of glueball mixing and decay properties.

We compute scalar glueball decays using the strategy outlined in the previous section. This implies, in particular, that we assume that the scalar glueball is small, and that instanton effects can be described in terms of a local effective Lagrangian. There is some evidence that this is indeed the case. A lattice calculation of the scalar glueball Bethe-Salpeter amplitude gave a size of 0.2 fm [23], and similar values have been obtained in the instanton model [24]. On the other hand, studies of finite size effects on the lattice are compatible with the assumption that scalar and tensor glueballs have sizes on the order of 0.5 fm [25,26].

In the following we compute the direct instanton contribution to the decay of the scalar  $0^{++}$  glueball state into  $\pi\pi$ ,  $K\bar{K}$ ,  $\eta\eta$  and  $\eta\eta'$ . Since the initial state is parity even only the  $G^2$  term in Eq. (4) contributes. The relevant effective interaction is given by

$$\begin{aligned}
\mathcal{L}_{I+A} = & \int dz \int d_0(\rho) \frac{d\rho}{\rho^5} \frac{1}{N_c^2 - 1} \left( \frac{\pi^3 \rho^4}{\alpha_s} \right) G^2 \left( -\frac{1}{4} \right) \left( \frac{4}{3} \pi^2 \rho^3 \right)^3 \left\{ [(\bar{u}u)(\bar{d}d)(\bar{s}s) + (\bar{u}\gamma^5 u)(\bar{d}\gamma^5 d)(\bar{s}s) + (\bar{u}\gamma^5 u)(\bar{d}d)(\bar{s}\gamma^5 s) \right. \\
& + (\bar{u}u)(\bar{d}\gamma^5 d)(\bar{s}\gamma^5 s)] + \frac{3}{8} \left[ (\bar{u}t^a u)(\bar{d}t^a d)(\bar{s}s) + (\bar{u}t^a \gamma^5 u)(\bar{d}t^a \gamma^5 d)(\bar{s}s) + (\bar{u}t^a \gamma^5 u)(\bar{d}t^a d)(\bar{s}\gamma^5 s) + (\bar{u}t^a u)(\bar{d}t^a \gamma^5 d) \right. \\
& \times (\bar{s}\gamma^5 s) - \frac{3}{4} [(\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^a \sigma_{\mu\nu} d)(\bar{s}s) + (\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^a \sigma_{\mu\nu} \gamma^5 d)(\bar{s}s) + (\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^a \sigma_{\mu\nu} d)(\bar{s}\gamma^5 s) \\
& + (\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^a \sigma_{\mu\nu} \gamma^5 d)(\bar{s}\gamma^5 s)] - \frac{9}{20} d^{abc} [(\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^b \sigma_{\mu\nu} d)(\bar{s}t^c s) + (\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^b \sigma_{\mu\nu} \gamma^5 d)(\bar{s}t^c s) \\
& + (\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^b \sigma_{\mu\nu} d)(\bar{s}t^c \gamma^5 s) + (\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^b \sigma_{\mu\nu} \gamma^5 d)(\bar{s}t^c \gamma^5 s)] + (2 \text{ cyclic permutations } u \leftrightarrow d \leftrightarrow s) \left. \right] \\
& - \frac{9}{40} d^{abc} [(\bar{u}t^a u)(\bar{d}t^b d)(\bar{s}t^c s) + (\bar{u}t^a \gamma^5 u)(\bar{d}t^b \gamma^5 d)(\bar{s}t^c s) + (\bar{u}t^a \gamma^5 u)(\bar{d}t^b d)(\bar{s}\gamma^5 t^c s) + (\bar{u}t^a u)(\bar{d}t^b \gamma^5 d)(\bar{s}t^c \gamma^5 s)] \\
& - \frac{9}{32} i f^{abc} [(\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^b \sigma_{\nu\gamma} d)(\bar{s}t^c \sigma_{\gamma\mu} s) + (\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^b \sigma_{\nu\gamma} \gamma^5 d)(\bar{s}t^c \sigma_{\gamma\mu} s) + (\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^b \sigma_{\nu\gamma} d) \\
& \times (\bar{s}t^c \sigma_{\gamma\mu} \gamma^5 s) + (\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^b \sigma_{\nu\gamma} \gamma^5 d)(\bar{s}t^c \sigma_{\gamma\mu} \gamma^5 s)] \left. \right\}. \tag{5}
\end{aligned}$$

Let us start with the process  $0^{++} \rightarrow \pi\pi$ . In practice we have Fierz rearranged Eq. (5) into structures that involve the strange quark condensate  $\bar{s}s$  as well as operators with the quantum numbers of two pions. In order to compute the coupling of these operators to the pions in the final state we have used PCAC relations

$$\langle 0 | \bar{d}\gamma^5 u | \pi^+ \rangle = \frac{i\sqrt{2}m_\pi^2 f_\pi}{m_u + m_d} \equiv K_\pi, \tag{6}$$

$$\langle 0 | \bar{s}\gamma^5 u | K^+ \rangle = \frac{i\sqrt{2}m_K^2 f_K}{m_u + m_s} \equiv K_K. \tag{7}$$

The values of the decay constants are  $f_\pi = 93$  MeV,  $f_K = 113$  MeV [27]. We also use  $Q_u \equiv \langle \bar{u}u \rangle = -(248 \text{ MeV})^3$  and  $Q_d = Q_u$  as well as  $Q_s = 0.66Q_u$  [28]. The coupling of the  $\eta'$  meson is not governed by chiral symmetry. A recent analysis of  $\eta - \eta'$  mixing and the chiral anomaly gives [29]

$$\langle 0 | \bar{u}\gamma_5 u | \eta \rangle = \langle 0 | \bar{d}\gamma_5 d | \eta \rangle = -i(358 \text{ MeV})^2 \equiv K_\eta^q, \tag{8}$$

$$\langle 0 | \bar{u}\gamma_5 u | \eta' \rangle = \langle 0 | \bar{d}\gamma_5 d | \eta' \rangle = -i(320 \text{ MeV})^2 \equiv K_{\eta'}^q,$$

$$\langle 0 | \bar{s}\gamma_5 s | \eta \rangle = i(435 \text{ MeV})^2 \equiv K_\eta^s,$$

$$\langle 0 | \bar{s}\gamma_5 s | \eta' \rangle = -i(481 \text{ MeV})^2 \equiv K_{\eta'}^s.$$

Finally, we need the coupling of the glueball state to the gluonic current. This quantity has been estimated on the lattice [23], using QCD spectral sum rules [30–32] and in the

instanton model [24]. These estimates range from 7.8 GeV<sup>3</sup> [23] to 28 GeV<sup>3</sup> [32]. We will use the result of the instanton calculation [24]

$$\langle 0^{++} | g^2 G^2 | 0 \rangle \equiv \lambda_0 = 15 \text{ GeV}^3. \tag{9}$$

We emphasize that our results for the ratio of decay rates are not affected by uncertainty in  $\lambda_0$ . We can now compute the matrix element for  $0^{++} \rightarrow \pi^+ \pi^-$ . The interaction vertex is

$$\begin{aligned}
\mathcal{L}_{I+A}^{\pi^+ \pi^-} = & \int dz \int \frac{d\rho}{\rho^5} d_0(\rho) \frac{1}{N_c^2 - 1} \left( \frac{\pi^3 \rho^4}{\alpha_s} \right) \left( \frac{4}{3} \pi^2 \rho^3 \right)^3 \\
& \times \frac{1}{4} (\alpha_s G^2) (\bar{s}s) (\bar{u}\gamma^5 d) (\bar{d}\gamma^5 u). \tag{10}
\end{aligned}$$

The integral over the position of the instanton leads to a momentum conserving delta function, while the vacuum dominance approximation allows us to write the amplitude in terms of the coupling constants introduced above. We find

$$\begin{aligned}
\langle 0^{++}(q) | \pi^+(p^+) \pi^-(p^-) \rangle = & (2\pi)^4 \delta^4(q - p^+ \\
& - p^-) \frac{A}{16\pi} \lambda_0 Q_s K_\pi^2, \tag{11}
\end{aligned}$$

where

$$A = \int \frac{d\rho}{\rho^5} d_0(\rho) \frac{1}{N_c^2 - 1} \left( \frac{\pi^3 \rho^4}{\alpha_s} \right) \left( \frac{4}{3} \pi^2 \rho^3 \right)^3. \tag{12}$$

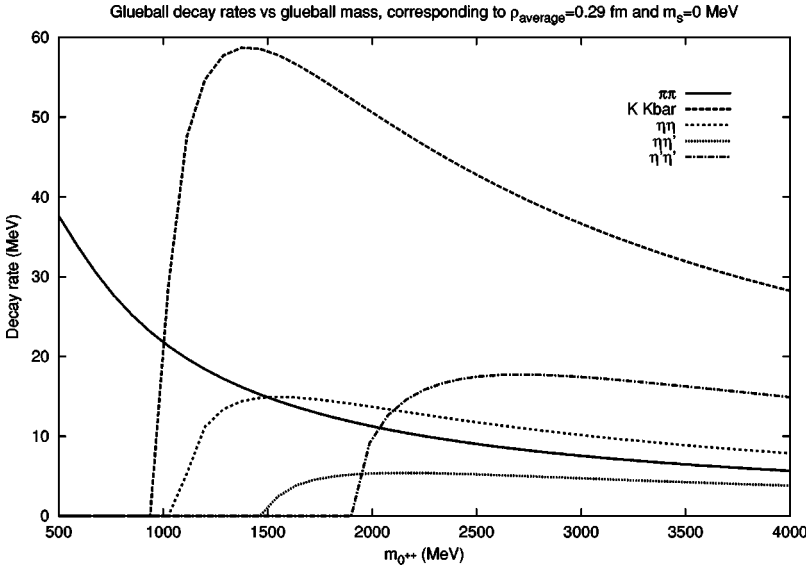


FIG. 1. Scalar glueball decay rates plotted a function of the mass of the scalar glueball. The rates shown in this figure were computed from the instanton vertex in the chiral limit. The average instanton size was taken to be  $\bar{\rho}=0.29$  fm.

The instanton density  $d_0(\rho)$  is known accurately only in the limit of small  $\rho$ . For large  $\rho$  higher loop corrections and nonperturbative effects are important. The only source of information in this regime is lattice QCD [33–36]. A very rough caricature of the lattice results is provided by the parametrization

$$\frac{d_0(\rho)}{\rho^5} = \frac{1}{2} n_0 \delta(\rho - \rho_c), \quad (13)$$

with  $n_0 \approx 1 \text{ fm}^{-4}$  and  $\rho_c \approx 0.33$  fm. This parametrization gives  $A = (379 \text{ MeV})^{-9}$ . Another way to compute  $A$  is to regularize the integral over the instanton size by replacing  $d(\rho)$  with  $d(\rho)\exp(-\alpha\rho^2)$ . The parameter  $\alpha$  can be adjusted in order to reproduce the size distribution measured on the lattice. We notice, however, that whereas the instanton density scales as  $\rho^{b-5} \sim \rho^4$ , the decay amplitude scales as  $\rho^{b+8} \sim \rho^{17}$ . This implies that the results are very sensitive to the density of large instantons. We note that when we study the decay of a small-size bound state the integral over  $\rho$  should be regularized by the overlap with the bound state

wave function. We will come back to this problem in Sec. IV below.

We begin by studying ratios of decay rates. These ratios are not sensitive to the instanton size distribution. The decay rate  $0^{++} \rightarrow \pi^+ \pi^-$  is given by

$$\Gamma_{0^{++} \rightarrow \pi^+ \pi^-} = \frac{1}{16\pi} \frac{\sqrt{m_{0^{++}}^2 - 4m_\pi^2}}{m_{0^{++}}^2} \left[ \frac{A}{16\pi} \lambda_0 Q_s K_\pi^2 \right]^2. \quad (14)$$

The decay amplitude for the process  $0^{++} \rightarrow \pi_0 \pi_0$  is equal to the  $0^{++} \rightarrow \pi^+ \pi^-$  amplitude as required by isospin symmetry. Taking into account the indistinguishability of the two  $\pi_0$  we get the total  $\pi\pi$  width

$$\Gamma_{0^{++} \rightarrow \pi\pi} = \frac{3}{32\pi} \frac{\sqrt{m_{0^{++}}^2 - 4m_\pi^2}}{m_{0^{++}}^2} \left[ \frac{A}{16\pi} \lambda_0 Q_s K_\pi^2 \right]^2. \quad (15)$$

In a similar fashion we get the decay widths for the  $K\bar{K}$ ,  $\eta\eta$ ,  $\eta\eta'$  and  $\eta'\eta'$  channels

$$\Gamma_{0^{++} \rightarrow K\bar{K}} = 2 \frac{1}{16\pi} \frac{\sqrt{m_{0^{++}}^2 - 4m_K^2}}{m_{0^{++}}^2} \left[ \frac{A}{16\pi} \lambda_0 Q_u K_K^2 \right]^2, \quad (16)$$

$$\Gamma_{0^{++} \rightarrow \eta\eta} = \frac{1}{32\pi} \frac{\sqrt{m_{0^{++}}^2 - 4m_\eta^2}}{m_{0^{++}}^2} \left[ \frac{A}{16\pi} \lambda_0 K_\eta^q 2 [Q_s K_\eta^q + (Q_u + Q_d) K_\eta^s] \right]^2, \quad (17)$$

$$\Gamma_{0^{++} \rightarrow \eta\eta'} = \frac{1}{16\pi} \frac{\sqrt{[m_{0^{++}}^2 - (m_\eta + m_{\eta'})^2][m_{0^{++}}^2 - (m_\eta - m_{\eta'})^2]}}{m_{0^{++}}^3} \times \left[ \frac{A}{16\pi} \lambda_0 [2Q_s K_\eta^q K_{\eta'}^q + (Q_u + Q_d)(K_\eta^q K_{\eta'}^s + K_\eta^s K_{\eta'}^q)] \right]^2, \quad (18)$$

$$\Gamma_{0^{++} \rightarrow \eta' \eta'} = \frac{1}{32\pi} \frac{\sqrt{m_{0^{++}}^2 - 4m_{\eta'}^2}}{m_{0^{++}}^2} \left[ \frac{A}{16\pi} \lambda_0 K_{\eta'}^q 2[Q_s K_{\eta'}^q + (Q_u + Q_d) K_{\eta'}^s] \right]^2. \quad (19)$$

Here,  $\bar{K}K$  refers to the sum of the  $K^+K^-$  and  $\bar{K}_0K_0$  final states. We note that in the chiral limit the instanton vertices responsible for  $\pi\pi$  and  $\bar{K}K$  decays are identical up to quark interchange. As a consequence, the ratio of the decay rates  $\Gamma_{0^{++} \rightarrow \pi\pi} / \Gamma_{0^{++} \rightarrow K\bar{K}}$  is given by the phase space factor multiplied by the ratio of the coupling constants

$$\begin{aligned} \frac{\Gamma_{0^{++} \rightarrow \pi\pi}}{\Gamma_{0^{++} \rightarrow K\bar{K}}} &= \frac{3}{4} \times \frac{Q_s^2 K_\pi^4}{Q_u^2 K_K^4} \times \sqrt{\frac{m_{0^{++}}^2 - 4m_\pi^2}{m_{0^{++}}^2 - 4m_K^2}} \\ &= (0.193 \pm 0.115) \sqrt{\frac{m_{0^{++}}^2 - 4m_\pi^2}{m_{0^{++}}^2 - 4m_K^2}}. \end{aligned} \quad (20)$$

The main uncertainty in this estimate comes from the value of  $m_s$ , which is not very accurately known. We have used  $m_s = (140 \pm 20)$  MeV. The ratio of  $\pi\pi$  to  $\eta\eta$  decay rates is not affected by this uncertainty,

$$\frac{\Gamma_{0^{++} \rightarrow \pi\pi}}{\Gamma_{0^{++} \rightarrow \eta\eta}} = 0.69 \sqrt{\frac{m_{0^{++}}^2 - 4m_\pi^2}{m_{0^{++}}^2 - 4m_\eta^2}}. \quad (21)$$

In Fig. 1 we show the decay rates as functions of the glueball mass. We have used  $\Lambda_{QCD} = 300$  MeV and adjusted the parameter  $\alpha$  to give the average instanton size  $\bar{\rho} = 0.29$  fm. We

observe that for glueball masses  $m_{0^{++}} > 1$  GeV the  $K\bar{K}$  phase space suppression quickly disappears and the total decay rate is dominated by the  $K\bar{K}$  final state. We also note that for  $m_{0^{++}} > 1.5$  GeV the  $\eta\eta$  rate dominates over the  $\pi\pi$  rate.

In deriving the effective instanton vertex Eq. (10) we have taken all quarks to be massless. While this is a good approximation for the up and down quarks, this is not necessarily the case for the strange quark. The  $m_s \neq 0$  contribution to the effective interaction for  $0^{++}$  decay is given by

$$\begin{aligned} \mathcal{L}_{m_s} &= \int \frac{d\rho}{\rho^5} d_0(\rho) \frac{1}{N_c^2 - 1} \frac{\pi^3 \rho^4}{\alpha_s^2} \left( \frac{4}{3} \pi^2 \rho^3 \right)^2 m_s \rho (\alpha_s G^2) \\ &\times \frac{1}{2} \left\{ (\bar{u}u)(\bar{d}d) + (\bar{u}\gamma^5 u)(\bar{d}\gamma^5 d) + \frac{3}{8} \left[ (\bar{u}t^a u)(\bar{d}t^a d) \right. \right. \\ &+ (\bar{u}\gamma^5 t^a u)(\bar{d}\gamma^5 t^a d) - \frac{3}{4} (\bar{u}\sigma_{\mu\nu} t^a u)(\bar{d}\sigma_{\mu\nu} t^a d) \\ &\left. \left. - \frac{3}{4} (\bar{u}\sigma_{\mu\nu} \gamma^5 t^a u)(\bar{d}\sigma_{\mu\nu} \gamma^5 t^a d) \right] \right\}. \end{aligned} \quad (22)$$

There is no  $m_s \neq 0$  contribution to the  $K\bar{K}$  channel. The  $m_s \neq 0$  correction to the other decay channels is

$$\begin{aligned} \Gamma_{0^{++} \rightarrow \pi\pi} &= \frac{3}{32\pi} \frac{\sqrt{m_{0^{++}}^2 - 4m_\pi^2}}{m_{0^{++}}^2} \left[ \frac{1}{16\pi} \lambda_0 K_\pi^2 (A Q_s - 2B m_s) \right]^2, \\ \Gamma_{0^{++} \rightarrow \eta\eta} &= \frac{1}{32\pi} \frac{\sqrt{m_{0^{++}}^2 - 4m_\eta^2}}{m_{0^{++}}^2} \left[ \frac{1}{16\pi} \lambda_0 2[(A Q_s - 2B m_s)(K_\eta^q)^2 + A(Q_u + Q_d)K_\eta^s K_\eta^q] \right]^2, \\ \Gamma_{0^{++} \rightarrow \eta\eta'} &= \frac{1}{16\pi} \frac{\sqrt{[m_{0^{++}}^2 - (m_\eta + m_{\eta'})^2][m_{0^{++}}^2 - (m_\eta - m_{\eta'})^2]}}{m_{0^{++}}^3} \\ &\times \left[ \frac{1}{16\pi} \lambda_0 [2(A Q_s - 2B m_s)K_\eta^q K_{\eta'}^q + A(Q_u + Q_d)(K_\eta^q K_{\eta'}^s + K_\eta^s K_{\eta'}^q)] \right]^2, \\ \Gamma_{0^{++} \rightarrow \eta' \eta'} &= \frac{1}{32\pi} \frac{\sqrt{m_{0^{++}}^2 - 4m_{\eta'}^2}}{m_{0^{++}}^2} \left[ \frac{1}{16\pi} \lambda_0 2[(A Q_s - 2B m_s)(K_{\eta'}^q)^2 + A(Q_u + Q_d)K_{\eta'}^s K_{\eta'}^q] \right]^2, \end{aligned} \quad (23)$$

where

$$B = \int \frac{d\rho}{\rho^5} d_0(\rho) \frac{1}{N_c^2 - 1} \left( \frac{\pi^3 \rho^4}{\alpha_s^2} \right) \left( \frac{4}{3} \pi^2 \rho^3 \right)^2 \rho. \quad (24)$$

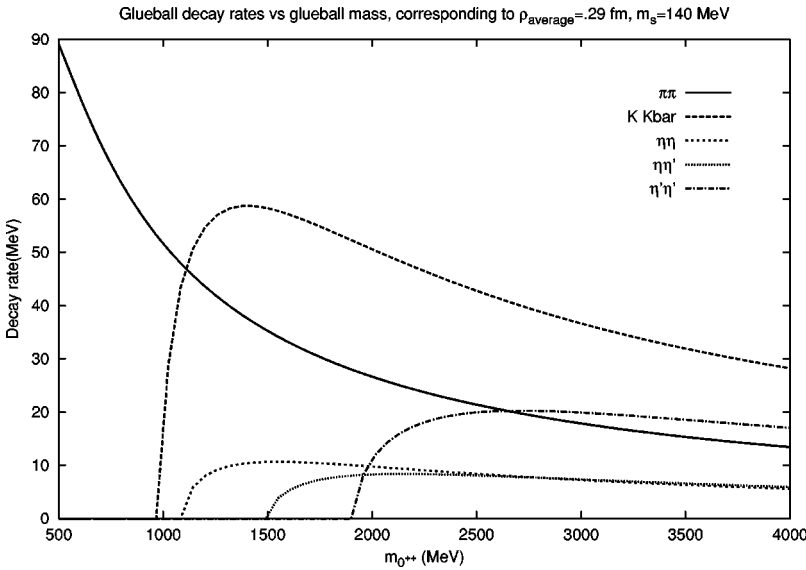


FIG. 2. Same as Fig. 1 but with  $m_s \neq 0$  corrections in the instanton vertex taken into account. The results shown in this figure correspond to  $m_s = 140$  MeV.

The decay rates with the  $m_s \neq 0$  correction to the instanton vertex taken into account are plotted in Fig. 2. We observe that effects due to the finite strange quark are not negligible. We find that the  $\pi\pi$ ,  $\eta\eta'$ , and  $\eta'\eta'$  channels are enhanced whereas the  $\eta\eta$  channel is reduced. For a typical glueball mass  $m_{0^{++}} = (1.5 - 1.7)$  GeV the ratio  $r = B(\pi\pi)/B(K\bar{K})$  changes from  $r \approx 0.25$  in the case  $m_s = 0$  to  $r \approx 0.55$  for  $m_s \neq 0$ . In Fig. 3 we show the dependence of the decay rates on the average instanton size  $\bar{\rho}$ . We observe that using the phenomenological value  $\bar{\rho} = 0.3$  fm gives a total width  $\Gamma_{0^{++}} \approx 100$  MeV. We note, however, that the decay rates are very sensitive to the value of  $\bar{\rho}$ . As a consequence, we cannot reliably predict the total decay rate. On the other hand, the ratio of the decay widths for different final states does not depend on  $\bar{\rho}$  and provides a sensitive test for the importance of direct instanton effects.

In Table I we show the masses and decay widths of scalar-

isoscalar mesons in the (1–2) GeV mass range. These states are presumably mixtures of mesons and glueballs. This means that our results cannot be directly compared to experiment without taking into account mixing effects. It will be interesting to study this problem in the context of the instanton model, but such a study is beyond the scope of this paper. It is nevertheless intriguing that the  $f_0(1710)$  decays mostly into  $K\bar{K}$ . Indeed, a number of authors have suggested that the  $f_0(1710)$  has a large glueball admixture [37–40].

#### IV. ETA CHARM DECAYS

The  $\eta_c$  is a pseudoscalar  $J^{PC} = 0^{-+}$  charmonium bound state with a mass  $m_{\eta_c} = (2979 \pm 1.8)$  MeV. The total decay width of the  $\eta_c$  is  $\Gamma_{\eta_c} = (16 \pm 3)$  MeV. In perturbation theory the total width is given by

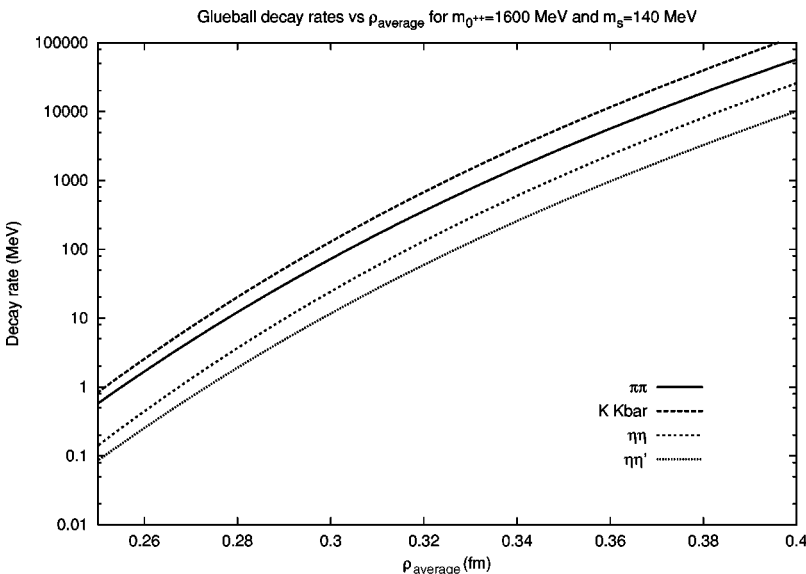


FIG. 3. Dependence of glueball decay rates on the average instanton size. The results shown in this figure correspond to the instanton vertex with  $m_s \neq 0$  terms included. The strange quark mass was taken to be  $m_s = 140$  MeV.

$$\Gamma(\eta \rightarrow 2g) = \frac{8\pi\alpha_s^2 |\psi(0)|^2}{3m_c^2} \left(1 + 4.4 \frac{\alpha_s}{\pi}\right). \quad (25)$$

Here,  $\psi(0)$  is the  $^1S_0$  ground state wave function at the origin. Using  $m_c = 1.25$  GeV and  $\alpha_s(m_c) = 0.25$  we get  $|\psi(0)| \approx 0.19$  GeV<sup>3/2</sup>, which is consistent with the expectation from phenomenological potential models. Exclusive decays cannot be reliably computed in perturbative QCD. As discussed in the Introduction, Bjorken pointed out that  $\eta_c$  decays into three pseudoscalar Goldstone bosons suggest that instanton effects are important [10]. The relevant decay channels and branching ratios are  $B(K\bar{K}\pi) = (5.5 \pm 1.7)\%$ ,  $B(\eta\pi\pi) = (4.9 \pm 1.8)\%$  and  $B(\eta'\pi\pi) = (4.1 \pm 1.7)\%$ . These three branching ratios are anomalously large

for a single exclusive channel, especially given the small multiplicity. The total decay rate into these three channels is  $(14.5 \pm 5.2)\%$  which is still a small fraction of the total width. This implies that the assumption that the three-Goldstone bosons channels are instanton dominated is consistent with our expectation that the total width is given by perturbation theory. For comparison, the next most important decay channels are  $B[2(\pi^+\pi^-)] = (1.2 \pm 0.4)\%$  and  $B(\rho\rho) = (2.6 \pm 0.9)\%$ . These channels do not receive direct instanton contributions.

The calculation proceeds along the same lines as the glueball decay calculation. Since the  $\eta_c$  is a pseudoscalar only the  $G\tilde{G}$  term in Eq. (4) contributes. The relevant interaction is

$$\begin{aligned} \mathcal{L}_{I+A} = & \int dz \int d_0(\rho) \frac{d\rho}{\rho^5} \frac{1}{N_c^2 - 1} \left(\frac{\pi^3 \rho^4}{\alpha_s}\right) G\tilde{G} \left(\frac{1}{4}\right) \left(\frac{4}{3} \pi^2 \rho^3\right)^3 \left\{ [(\bar{u}\gamma^5 u)(\bar{d}d)(\bar{s}s) + (\bar{u}u)(\bar{d}\gamma^5 d)(\bar{s}s) + (\bar{u}u)(\bar{d}d)(\bar{s}\gamma^5 s) \right. \\ & + (\bar{u}\gamma^5 u)(\bar{d}\gamma^5 d)(\bar{s}\gamma^5 s)] + \frac{3}{8} \left[ (\bar{u}t^a \gamma^5 u)(\bar{d}t^a d)(\bar{s}s) + (\bar{u}t^a u)(\bar{d}t^a \gamma^5 d)(\bar{s}s) + (\bar{u}t^a u)(\bar{d}t^a d)(\bar{s}\gamma^5 s) + (\bar{u}t^a \gamma^5 u) \right. \\ & \times (\bar{d}t^a \gamma^5 d)(\bar{s}\gamma^5 s) - \frac{3}{4} [(\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^a \sigma_{\mu\nu} d)(\bar{s}s) + (\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^a \sigma_{\mu\nu} \gamma^5 d)(\bar{s}s) + (\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^a \sigma_{\mu\nu} d)(\bar{s}\gamma^5 s) \\ & + (\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^a \sigma_{\mu\nu} \gamma^5 d)(\bar{s}\gamma^5 s)] - \frac{9}{20} d^{abc} [(\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^b \sigma_{\mu\nu} d)(\bar{s}t^c s) + (\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^b \sigma_{\mu\nu} \gamma^5 d)(\bar{s}t^c s) \\ & + (\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^b \sigma_{\mu\nu} d)(\bar{s}t^c \gamma^5 s) + (\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^b \sigma_{\mu\nu} \gamma^5 d)(\bar{s}t^c \gamma^5 s)] + (2 \text{ cyclic permutations } u \leftrightarrow d \leftrightarrow s) \left. \right] \\ & - \frac{9}{40} d^{abc} [(\bar{u}t^a \gamma^5 u)(\bar{d}t^b d)(\bar{s}t^c s) + (\bar{u}t^a u)(\bar{d}t^b \gamma^5 d)(\bar{s}t^c s) + (\bar{u}t^a u)(\bar{d}t^b d)(\bar{s}\gamma^5 t^c s) + (\bar{u}t^a \gamma^5 u)(\bar{d}t^b \gamma^5 d)(\bar{s}t^c \gamma^5 s)] \\ & - \frac{9}{32} i f^{abc} [(\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^b \sigma_{\nu\gamma} d)(\bar{s}t^c \sigma_{\gamma\mu} s) + (\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^b \sigma_{\nu\gamma} \gamma^5 d)(\bar{s}t^c \sigma_{\gamma\mu} s) + (\bar{u}t^a \sigma_{\mu\nu} u)(\bar{d}t^b \sigma_{\nu\gamma} d) \\ & \times (\bar{s}t^c \sigma_{\gamma\mu} \gamma^5 s) + (\bar{u}t^a \sigma_{\mu\nu} \gamma^5 u)(\bar{d}t^b \sigma_{\nu\gamma} \gamma^5 d)(\bar{s}t^c \sigma_{\gamma\mu} \gamma^5 s)] \left. \right\}. \quad (26) \end{aligned}$$

The strategy is the same as in the glueball case. We Fierz-rearrange the Lagrangian (26) and apply the vacuum dominance and PCAC (partial conservation of axial vector current) approximations. The coupling of the  $\eta_c$  bound state to the instanton involves the matrix element

$$\lambda_{\eta_c} = \langle \eta_c | g^2 G\tilde{G} | 0 \rangle. \quad (27)$$

We can get an estimate of this matrix element using a simple two-state mixing scheme for the  $\eta_c$  and pseudoscalar glueball. We write

$$|\eta_c\rangle = \cos(\theta)|\bar{c}c\rangle + \sin(\theta)|gg\rangle, \quad (28)$$

$$|0^{-+}\rangle = -\sin(\theta)|\bar{c}c\rangle + \cos(\theta)|gg\rangle. \quad (29)$$

The matrix element  $f_{\eta_c} = \langle 0 | 2m_c \bar{c} \gamma_5 c | \eta_c \rangle \approx 2.8$  GeV<sup>3</sup> is related to the charmonium wave function at the origin. The coupling of the topological charge density to the pseudoscalar glueball was estimated using QCD spectral sum rules,  $\lambda_{0^{-+}} = \langle 0 | g^2 G\tilde{G} | 0^{-+} \rangle \approx 22.5$  GeV<sup>3</sup> [31]. Using the two-state mixing scheme the two ‘‘off-diagonal’’ matrix elements  $f_{0^{-+}} = \langle 0 | 2m_c \bar{c} \gamma_5 c | 0^{-+} \rangle$  and  $\lambda_{\eta_c} = \langle 0 | g^2 G\tilde{G} | \eta_c \rangle$  are given in terms of one mixing angle  $\theta$ . We can estimate this mixing angle by computing the charm content of the pseudoscalar glueball using the heavy quark expansion. Using [41]

$$\bar{c}\gamma^5 c = \frac{i}{8\pi m_c} \alpha_s G \tilde{G} + O\left(\frac{1}{m_c^3}\right), \quad (30)$$

we get  $f_{0^{++}} \approx 0.14 \text{ GeV}^3$  and a mixing angle  $\theta \approx 3^\circ$ . This mixing angle corresponds to

$$\lambda_{\eta_c} \approx 1.12 \text{ GeV}^3. \quad (31)$$

The uncertainty in this estimate is hard to assess. Below we will discuss a perturbative estimate of the instanton coupling to  $\eta_c$ . In order to check the phenomenological consistency of the estimate Eq. (31) we have computed the  $\eta_c$  contribution to the  $\langle g^2 G \tilde{G}(0) g^2 G \tilde{G}(x) \rangle$  correlation function. The results are shown in Fig. 4. The contribution of the pseudo-scalar glueball is determined by the coupling constant  $\lambda_{0^{++}}$  introduced above. The couplings of the  $\eta$ ,  $\eta'$  and  $\eta(1440)$  resonances can be extracted from the decays  $J/\psi \rightarrow \gamma \eta$  [42]. We observe that the  $\eta_c$  contribution is strongly suppressed, as one would expect. We also show the  $\eta_c$  and  $0^{++}$  glueball contributions to the  $\langle \bar{c} \gamma_5 c(0) \bar{c} \gamma_5 c(x) \rangle$  correlation function. We observe that even with the small mixing matrix elements obtained from Eqs. (28)–(30) the glueball contribution starts to dominate the  $\eta_c$  correlator for  $x > 1 \text{ fm}$ .

We now proceed to the calculation of the exclusive decay rates. There are four final states that contribute to the  $K \bar{K} \pi$  channel,  $\eta_c \rightarrow K^+ K^- \pi^0$ ,  $K^0 \bar{K}^0 \pi^0$ ,  $K^+ \bar{K}^0 \pi^-$  and  $K^- \bar{K}^0 \pi^+$ . Using isospin symmetry it is sufficient to calculate only one of the amplitudes. Fierz rearranging Eq. (26) we get the interaction responsible for the  $\eta_c \rightarrow K^+ K^- \pi^0$

$$\begin{aligned} \mathcal{L}_{I+A}^{K^+ K^- \pi^0} &= \int dz \int \frac{d\rho}{\rho^5} d_0(\rho) \frac{1}{N_c^2 - 1} \left( \frac{\pi^3 \rho^4}{\alpha_s^2} \right) \left( \frac{4}{3} \pi^2 \rho^3 \right)^3 \\ &\times \frac{1}{4} (\alpha_s G \tilde{G})(\bar{s} \gamma^5 u)(\bar{u} \gamma^5 s)(\bar{d} \gamma^5 d). \end{aligned} \quad (32)$$

The decay rate is given by

$$\begin{aligned} \Gamma_{K^+ K^- \pi^0} &= \int (\text{phase space}) \times |M|^2 \\ &= \left[ \frac{1}{16\pi\sqrt{2}} A \lambda_{\eta_c} K_\pi K_K^2 \right]^2 \times (0.111 \text{ MeV}), \end{aligned} \quad (33)$$

with  $A$  given in Eq. (12). Isospin symmetry implies that the other  $K \bar{K} \pi$  decay rates are given by

$$\begin{aligned} \Gamma_{K^0 \bar{K}^0 \pi^0} &= \Gamma_{K^0 \bar{K}^0 \pi^0} = \left( \frac{1}{\sqrt{2}} \right)^2 \Gamma_{K^0 K^- \pi^+} \\ &= \left( \frac{1}{\sqrt{2}} \right)^2 \Gamma_{K^+ \bar{K}^0 \pi^-}. \end{aligned} \quad (34)$$

The total  $K \bar{K} \pi$  decay rate is

$$\Gamma_{K \bar{K} \pi} = 6 \times \left[ \frac{1}{16\pi\sqrt{2}} A \lambda_{\eta_c} K_\pi K_K^2 \right]^2 \times (0.111 \text{ MeV}). \quad (35)$$

In a similar fashion we obtain

$$\Gamma_{\eta \pi \pi} = \frac{3}{2} \times \left[ \frac{1}{16\pi} A \lambda_{\eta_c} K_\eta^s K_\pi^2 \right]^2 \times (0.135 \text{ MeV}), \quad (36)$$

$$\Gamma_{\eta' \pi \pi} = \frac{3}{2} \times \left[ \frac{1}{16\pi} A \lambda_{\eta_c} K_{\eta'}^s K_\pi^2 \right]^2 \times (0.0893 \text{ MeV}), \quad (37)$$

$$\Gamma_{K \bar{K} \eta} = 2 \times \left[ \frac{1}{16\pi} A \lambda_{\eta_c} K_\eta^q K_K^2 \right]^2 \times (0.0788 \text{ MeV}), \quad (38)$$

TABLE I. Masses, decay widths, and decay channels for scalar-isoscalar mesons with masses in the (1–2) GeV range. The data were taken from [27].

resonance	full width $\Gamma(\text{MeV})$	Mass (MeV)	decay channels
$f_0(1370)$	200–500	1200–1500	$\rho\rho$ dominant $\pi\pi, K\bar{K}, \eta\eta$ seen
$f_0(1500)$	$109 \pm 7$	$1507 \pm 5$	$\frac{\Gamma_{K\bar{K}}}{\Gamma_{\pi\pi}} = 0.19 \pm 0.07$ $\frac{\Gamma_{\eta\eta'}}{\Gamma_{\pi\pi}} = 0.095 \pm 0.026$ $\frac{\Gamma_{\eta\eta}}{\Gamma_{\pi\pi}} = 0.18 \pm 0.03$
$f_0(1710)$	$125 \pm 10$	$1713 \pm 6$	$\frac{\Gamma_{\pi\pi}}{\Gamma_{K\bar{K}}} = 0.39 \pm 0.14$ $\frac{\Gamma_{\eta\eta}}{\Gamma_{K\bar{K}}} = 0.48 \pm 0.15$



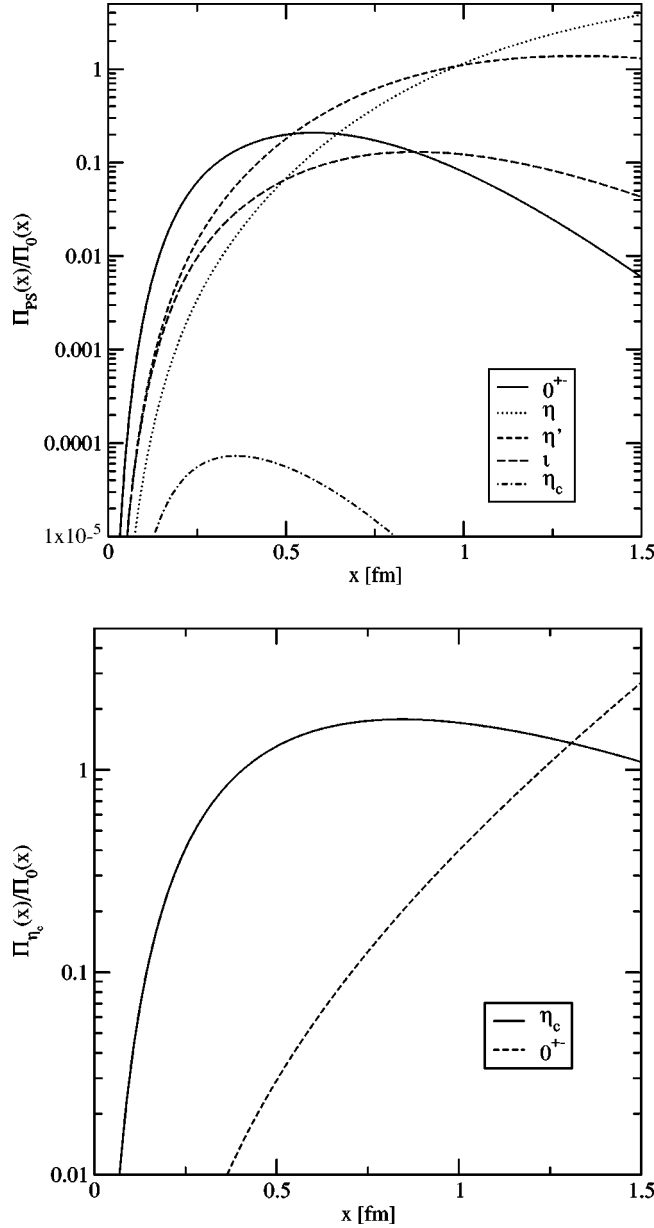


FIG. 4. Resonance contributions to the pseudoscalar glueball correlation function  $\langle g^2 G \tilde{G}(0) g^2 G \tilde{G}(x) \rangle$  and the charmonium correlator  $\langle \bar{c} \gamma_5 c(0) \bar{c} \gamma_5 c(x) \rangle$ . Both correlation functions are normalized to free field behavior. In the case of the gluonic correlation function we show the glueball contribution compared to the  $\eta$ ,  $\eta'$ ,  $\eta(1440)$  and  $\eta_c$  contribution. For the charmonium correlation function we show the  $\eta_c$  and glueball contribution.

$$\Gamma_{K\bar{K}\eta'} = 2 \times \left[ \frac{1}{16\pi} A \lambda_{\eta_c} K_{\eta'}^q K_K^2 \right]^2 \times (0.0423 \text{ MeV}), \quad (39)$$

$$\Gamma_{\eta\eta\eta} = \frac{1}{6} \times \left[ \frac{3!}{16\pi} A \lambda_{\eta_c} (K_{\eta'}^q)^2 K_{\eta}^s \right]^2 \times (0.0698 \text{ MeV}). \quad (40)$$

Here, the first factor is the product of the isospin and final state symmetrization factors. The second factor is the amplitude and the third factor is the phase-space integral.

In Fig. 5 we show the dependence of the decay rates on the average instanton size. We observe that the experimental  $K\bar{K}\pi$  rate is reproduced for  $\bar{\rho} = 0.29$  fm. This number is consistent with the phenomenological instanton size. However, given the strong dependence on the average instanton size it is clear that we cannot reliably predict the decay rate. On the other hand, the following ratios are independent of the average instanton size:

$$\frac{\Gamma_{K\bar{K}\pi}}{\Gamma_{\eta\pi\pi}} = 4 \times \left[ \frac{K_K^2}{\sqrt{2} K_{\eta}^s K_{\pi}} \right]^2 \times \left( \frac{0.111}{0.135} \right) = 4.23 \pm 1.27, \quad (41)$$

$$\frac{\Gamma_{\eta\pi\pi}}{\Gamma_{\eta'\pi\pi}} = \left( \frac{K_{\eta}^s}{K_{\eta'}^s} \right)^2 \times \left( \frac{0.135}{0.0893} \right) = 1.01, \quad (42)$$

$$\frac{\Gamma_{K\bar{K}\eta}}{\Gamma_{K\bar{K}\pi}} = \frac{1}{3} \times \left[ \frac{\sqrt{2} K_{\eta}^q}{K_{\pi}} \right]^2 \times \left( \frac{0.0788}{0.111} \right) = 0.141 \pm 0.042, \quad (43)$$

$$\frac{\Gamma_{K\bar{K}\eta}}{\Gamma_{K\bar{K}\eta'}} = \left( \frac{K_{\eta}^q}{K_{\eta'}^q} \right)^2 \times \left( \frac{0.0788}{0.0423} \right) = 2.91, \quad (44)$$

$$\begin{aligned} \frac{\Gamma_{\eta\eta\eta}}{\Gamma_{K\bar{K}\pi}} &= \frac{1}{36} \times \left[ \frac{3! \sqrt{2} (K_{\eta'}^q)^2 K_{\eta}^s}{K_{\pi} K_K^2} \right]^2 \times \left( \frac{0.0698}{0.111} \right) \\ &= 0.011 \pm 0.003, \end{aligned} \quad (45)$$

where we have only quoted the error due to the uncertainty in  $m_s$ . These numbers should be compared to the experimental results

$$\left. \frac{\Gamma_{K\bar{K}\pi}}{\Gamma_{\eta\pi\pi}} \right|_{exp} = 1.1 \pm 0.5 \quad (46)$$

$$\left. \frac{\Gamma_{\eta\pi\pi}}{\Gamma_{\eta'\pi\pi}} \right|_{exp} = 1.2 \pm 0.6. \quad (47)$$

We note that the ratio  $B(\eta\pi\pi)/B(\eta'\pi\pi)$  is compatible with our results while the ratio  $B(K\bar{K}\pi)/B(\eta\pi\pi)$  is not. This implies that either there are contributions other than instantons, or that the PCAC estimate of the ratio of coupling constants is not reliable, or that the experimental result is not reliable. The branching ratios for  $\eta\pi\pi$  and  $\eta'\pi\pi$  come from MARK II/III experiments [43,44]. We observe that our results for  $B(K\bar{K}\eta)/B(K\bar{K}\pi)$  and  $B(K\bar{K}\eta')/B(K\bar{K}\pi)$  are consistent with the experimental bounds.

Another possibility is that there is a significant contribution from a scalar resonance that decays into  $\pi\pi$ . Indeed, instantons couple strongly to the  $\sigma(600)$  resonance, and this state is not resolved in the experiments. We have therefore studied the direct instanton contribution to the decay  $\eta_c \rightarrow \sigma\eta$ . After Fierz rearrangement we get the effective vertex

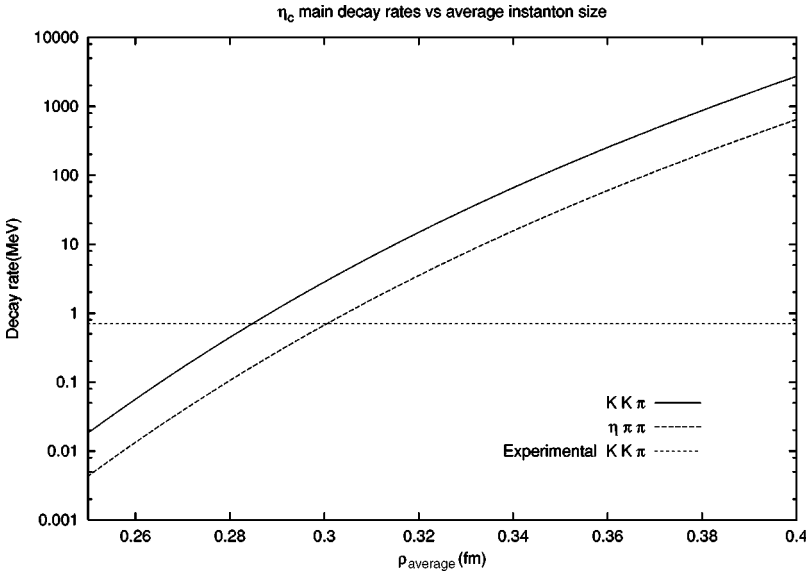


FIG. 5. Decay widths  $\eta_c \rightarrow KK\pi$  and  $\eta_c \rightarrow \eta\pi\pi$  as a function of the average instanton size  $\rho$ . The short dashed line shows the experimental  $KK\pi$  width.

$$\begin{aligned} \mathcal{L}_{\sigma\eta} = & \int dA (\alpha_s G \tilde{G}) \frac{1}{4} [(\bar{u}\gamma^5 u)(\bar{d}d)(\bar{s}s) + (\bar{u}u)(\bar{d}\gamma^5 d)(\bar{s}s) \\ & + (\bar{u}u)(\bar{d}d)(\bar{s}\gamma^5 s)] - \int dB m_s (\alpha_s G \tilde{G}) \frac{1}{2} [(\bar{u}\gamma^5 u) \\ & \times (\bar{d}d) + (\bar{u}u)(\bar{d}\gamma^5 d)], \end{aligned} \quad (48)$$

where the integrals  $A$  and  $B$  are defined in Eqs. (12), (24). The only new matrix element we need is  $f_\sigma = \langle \sigma | \bar{u}u + \bar{d}d | 0 \rangle \approx (500 \text{ MeV})^2$  [45]. We get

$$\begin{aligned} \Gamma_{\eta_c \rightarrow \sigma\eta} = & \frac{1}{16\pi m_{\eta_c}^3} \sqrt{[m_{\eta_c}^2 - (m_\sigma + m_\eta)^2][m_{\eta_c}^2 - (m_\sigma - m_\eta)^2]} \\ & \times \left[ \frac{1}{16\pi} f_\sigma \lambda_{\eta_c} [(AQ_s - 2Bm_s)K_\eta^q + AK_\eta^s Q_d] \right]^2. \end{aligned} \quad (49)$$

Compared to the direct decay  $\eta_c \rightarrow \eta\pi\pi$  the  $\eta_c \rightarrow \eta\sigma$  channel is suppressed by a factor  $\sim (2\pi^2/m_\eta^2) \cdot (Q_q f_\sigma / K_\pi^2)^2 \sim 1/100$ . Here, the first factor is due to the difference between two and three-body phase space and the second factor is the ratio of matrix elements. We conclude that the direct production of a  $\sigma$  resonance from the instanton does not give a significant contribution to  $\eta_c \rightarrow \eta(\eta')\pi\pi$ . This leaves the possibility that the  $\pi\pi$  channel is enhanced by final state interactions.

Finally, we present a perturbative estimate of the coupling of the  $\eta_c$  to the instanton. We follow the method used by Anselmino and Forte in order to estimate the instanton contribution to  $\eta_c \rightarrow p\bar{p}$  [46]. The idea is that the charmonium state annihilates into two gluons which are absorbed by the instanton. The Feynman diagram for the process is shown in Fig. 6. The amplitude is given by

$$\begin{aligned} A_{c\bar{c} \rightarrow I} = & g^2 \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} (2\pi)^4 \delta^4(p_1 + p_2 - k_1 - k_2) \\ & \times \bar{v}(p_2) \left[ \gamma_\mu \frac{\lambda^a}{2} \frac{1}{\not{p}_1 - \not{k}_1 - m_c} \gamma_\nu \frac{\lambda^b}{2} \right] \\ & \times u(p_1) A_\mu^{a,cl}(k_2) A_\nu^{b,cl}(k_1), \end{aligned} \quad (50)$$

where  $u(p)$  and  $\bar{v}(p)$  are free particle charm quark spinors and  $A_\mu^{a,cl}(k)$  is the Fourier transform of the instanton gauge potential

$$\begin{aligned} A_\mu^{a,cl}(k) = & -i \frac{4\pi^2}{g} \frac{\bar{\eta}_{\mu\nu}^a k^\nu}{k^4} \Phi(k), \quad (51) \\ \Phi(k) = & 4 \left( 1 - \frac{1}{2} K_2(k\rho)(k\rho)^2 \right). \end{aligned}$$

The amplitude for the charmonium state to couple to an instanton is obtained by folding Eq. (50) with the  $\eta_c$  wave function  $\psi(p)$ . In the nonrelativistic limit the amplitude only depends on the wave function at the origin.

The perturbative estimate of the transition rate is easily incorporated into the results obtained above by replacing the product  $A\lambda_{\eta_c}$  in Eqs. (33)–(40) according to

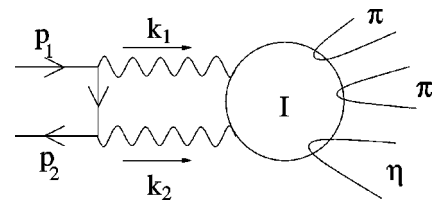


FIG. 6. The Feynman diagram corresponding to the perturbative treatment of charmonium decay.

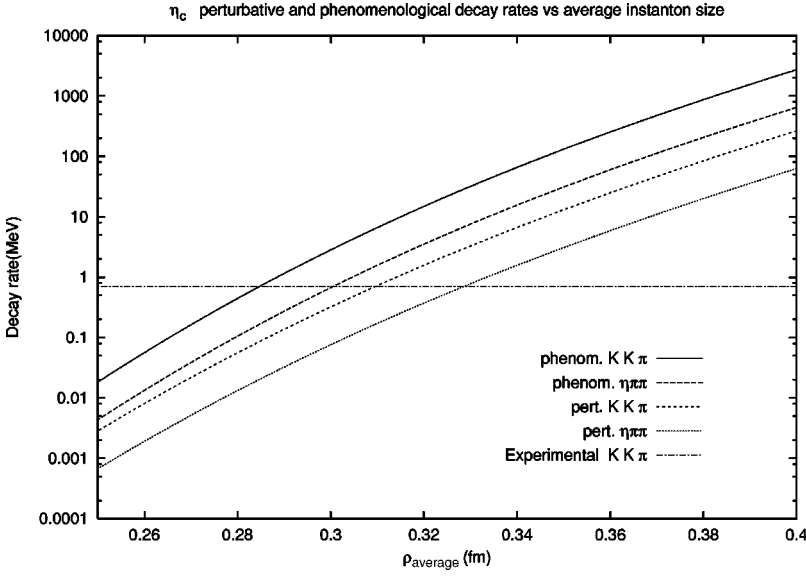


FIG. 7. Decay rates  $\Gamma(\eta_c \rightarrow K\bar{K}\pi)$  and  $\Gamma(\eta_c \rightarrow \eta\pi\pi)$  as a function of the average instanton size  $\bar{\rho}$ . We show both the results from a phenomenological and a perturbative estimate of the  $\bar{c}c$  coupling to the instanton.

$$A\lambda_{\eta_c} \rightarrow \int \frac{d\rho}{\rho^5} d_0(\rho) \left(\frac{4}{3}\pi^2\rho^3\right)^3 (4\pi) \frac{8m_c^{3/2}}{\sqrt{6}} |\psi(0)| I_{\eta_c}(\rho) \times \frac{g^2(m_c^{-1})}{g^2(\rho)}, \quad (52)$$

with

$$I_{\eta_c}(\rho) = \int d^4k \frac{\vec{k}^2 \Phi(k) \Phi(k-2p_c)}{k^4 (k-2p_c)^4 [(k-p_c)^2 + m_c^2]}. \quad (53)$$

Here,  $p_c = (m_c, 0) \simeq (M_{\eta_c}/2, 0)$  is the momentum of the charm quark in the charmonium rest frame. We note that because of the nonperturbative nature of the instanton field higher order corrections to equ. (52) are only suppressed by  $g^2(m_c^{-1})/g^2(\rho)$ .

The integral  $I_{\eta_c}$  cannot be calculated analytically. We use the parametrization

$$I_{\eta_c}(\rho) \simeq \frac{\pi^2 A_0 \rho^4 \log[1 + 1/(m_c\rho)]}{1 + B_0(m_c\rho)^4 \log[1 + 1/(m_c\rho)]}, \quad (54)$$

which incorporates the correct asymptotic behavior. We find that  $A_0 = 0.213$  and  $B_0 = 0.124$  provides a good representation of the integral. In Fig. 7 we show the results for the  $\eta_c$  decay rates as a function of the average instanton size. We observe that the results are similar to the results obtained from the phenomenological estimate Eq. (31). The effective coupling ( $A\lambda_{\eta_c}$ ) differs from the estimate Eq. (31) by about a factor of 3. The experimental  $K\bar{K}\pi$  rate is reproduced for  $\bar{\rho} = 0.31$  fm.

## V. CHI CHARM DECAYS

Another interesting consistency check on our results is provided by the study of instanton induced decays of the  $\chi_c$  into pairs of Goldstone bosons. The  $\chi_c$  is a scalar charmo-

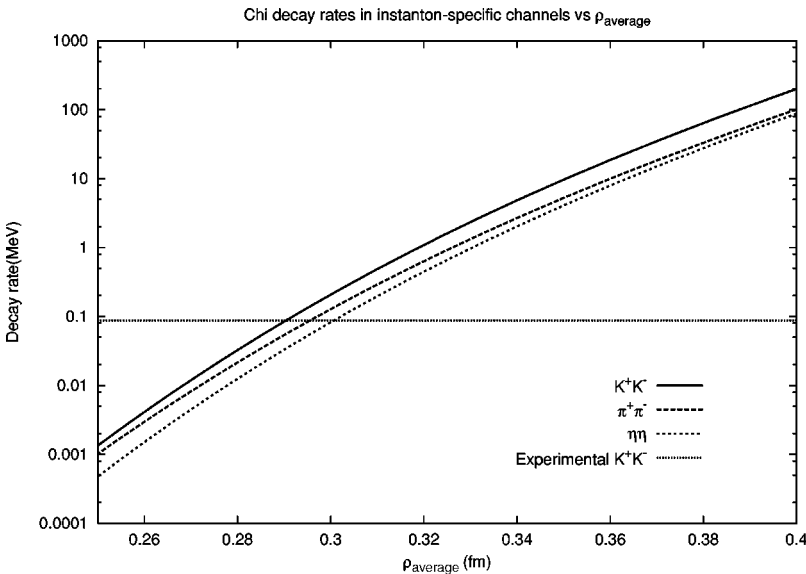


FIG. 8. Decay widths  $\chi_c \rightarrow K^+K^-$ ,  $\pi^+\pi^-$  and  $\eta\eta$  as a function of the average instanton size  $\rho$ . The short dashed line shows the experimental  $K^+K^-$  width.

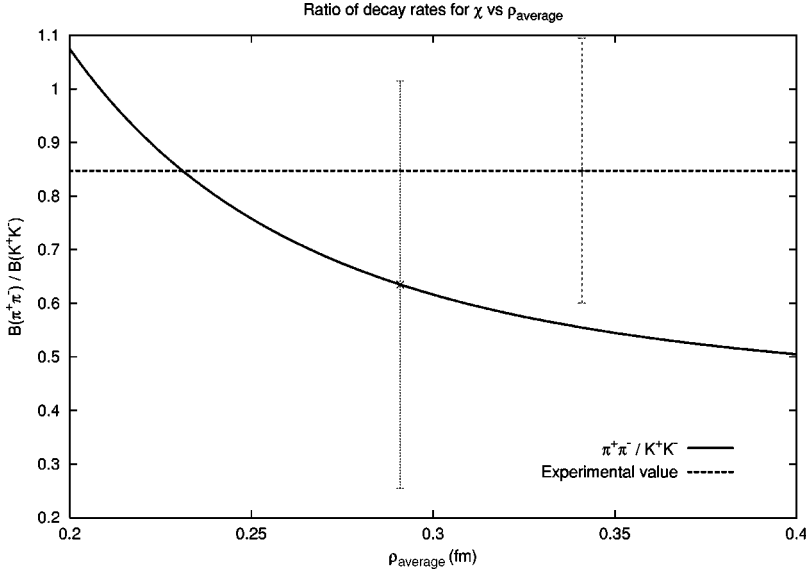


FIG. 9. Ratio  $B(\chi_c \rightarrow \pi^+ \pi^-)/B(\chi_c \rightarrow K^+ K^-)$  of decay rates as a function of the average instanton size. The dashed line shows the experimental value 0.84. We also show the experimental uncertainty, as well as the uncertainty in the instanton prediction due to the value of the strange quark mass.

onium bound state with mass  $m_{\chi_c} = 3415$  MeV and width  $\Gamma_{\chi_c} = 14.9$  MeV. In a potential model the  $\chi_c$  corresponds to the  $^3P_0$  state. In perturbation theory the total decay rate is dominated by  $\bar{c}c \rightarrow 2g$ . The main exclusive decay channels are  $\chi_c \rightarrow 2(\pi^+ \pi^-)$  and  $\chi_c \rightarrow \pi^+ \pi^- K^+ K^-$  with branching ratios  $(2.4 \pm 0.6)\%$  and  $(1.8 \pm 0.6)\%$ , respectively. It would be very interesting to know whether these final states are dominated by scalar resonances. We will concentrate on final states containing two pseudoscalar mesons. There are two channels with significant branching ratios,  $\chi_c \rightarrow \pi^+ \pi^-$  and  $\chi_c \rightarrow K^+ K^-$  with branching ratios  $(5.0 \pm 0.7) \times 10^{-3}$  and  $(5.9 \pm 0.9) \times 10^{-3}$ . We note that these branching ratios are significantly smaller than the  $\eta_c \rightarrow (3 \text{ Goldstone boson})$  branching ratios studied in the previous section. This is a simple consequence of the smallness of two-body phase space compared to three-body phase space.

The calculation of these two decay rates proceeds along the same lines as the calculation of the  $0^{++}$  glueball decays. The only new ingredient is the  $\chi_c$  coupling to the gluon field strength  $G^2$ . We observe that the total  $\chi_c$  decay rate implies that  $\langle 0 | 2m_c \bar{c}c | \chi_c \rangle = 3.1 \text{ GeV}^3 \approx \langle 0 | 2m_c \bar{c}i\gamma_5 c | \eta_c \rangle$ . This suggests that a rough estimate of the  $\chi_c$  coupling to  $G^2$  is given by

$$\lambda_{\chi_c} \equiv \langle \chi_c | g^2 G^2 | 0 \rangle \approx \lambda_{\eta_c} = 1.12 \text{ GeV}^3. \quad (55)$$

Using this result we can obtain the  $\chi_c$  decay rates by rescaling the scalar glueball decay rates Eqs. (23),(24) according to

$$\Gamma_{\chi_c \rightarrow m_1 m_2} = \Gamma_{0^{++} \rightarrow m_1 m_2} \times \left( \frac{\lambda_{\chi_c}}{\lambda_{0^{++}}} \right)^2 \Big|_{m_0^{++} \rightarrow m_{\chi_c}}, \quad (56)$$

where  $m_1, m_2$  labels the two-meson final state. In Fig. 8 we show the dependence of the  $\chi_c$  decay rates on the average instanton size  $\bar{\rho}$ . We observe that the experimental  $\pi^+ \pi^-$  decay rate is reproduced for  $\bar{\rho} = 0.29$  fm. In Fig. 9 we plot

the ratio of decay rates for  $\pi^+ \pi^-$  and  $K^+ K^-$ . Again, the experimental value is reproduced for  $\bar{\rho} \sim 0.3$  fm.

Finally, we can also estimate the  $c\bar{c}$  coupling to the instanton using the perturbative method introduced in Sec. IV. In the case of the  $\chi_c$  we use

$$\begin{aligned} \frac{1}{4\pi} \lambda_{\chi_c} A \rightarrow & \frac{1}{2\sqrt{3}\pi} \sqrt{M_\chi} R'(0) \\ & \times \int \frac{d_0(\rho)}{\rho^5} d\rho \left( \frac{4}{3} \pi^2 \rho^3 \right)^3 \frac{g^2(m_c)}{g^2(\rho)} \times I_\chi(\rho), \end{aligned} \quad (57)$$

$$\begin{aligned} \frac{1}{4\pi} \lambda_{\chi_c} B \rightarrow & \frac{1}{2\sqrt{3}\pi} \sqrt{M_\chi} R'(0) \\ & \times \int \frac{d_0(\rho)}{\rho^5} d\rho \left( \frac{4}{3} \pi^2 \rho^3 \right)^2 \rho \frac{g^2(m_c)}{g^2(\rho)} \times I_\chi(\rho), \end{aligned} \quad (58)$$

where  $R'(0) \approx 0.39 \text{ GeV}^{5/2}$  is the derivative of the  $^3P_0$  wave function at the origin and  $I_{\chi_c}$  is the loop integral

$$I_\chi(\rho) = \int d^4k \frac{\Phi(k)\Phi(|2p_c - k|)}{k^4(2p_c - k)^4} \frac{15(k - p_c)^2 + 3m_c^2 + 4\vec{k}^2}{(k - p_c)^2 + m_c^2}. \quad (59)$$

In Fig. 10 we compare the perturbative result with the phenomenological estimate. Again, the results are comparable. The experimental  $\pi^+ \pi^-$  rate is reproduced for  $\bar{\rho} = 0.29$  fm.

## VI. SUMMARY

In summary we have studied the instanton contribution to the decay of a number of ‘‘gluon rich’’ states in the (1.5–3.5) GeV range, the scalar glueball, the  $\eta_c$  and the  $\chi_c$ . In the case

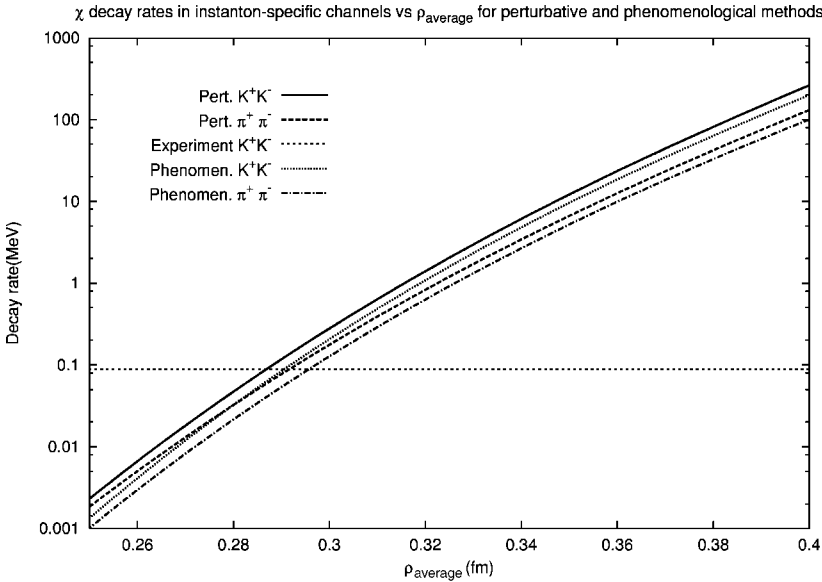


FIG. 10. Decay rates  $\Gamma(\chi_c \rightarrow \pi^+ \pi^-)$  and  $\Gamma(\chi_c \rightarrow K^+ K^-)$  as a function of the average instanton size  $\bar{\rho}$ . We show both the results from a phenomenological and a perturbative estimate of the  $\bar{c}c$  coupling to the instanton.

of charmonium, instanton induced decays are probably a small part of the total decay rate, but the final states are very distinctive. In the case of the scalar glueball, classical fields play an important role in determining the structure of the bound state and instantons may well dominate the total decay rate.

We have assumed that the gluonic system is small and that the instanton contribution to the decay can be described in terms of an effective local interaction. The meson coupling to the local operator was determined using PCAC. Using this method we find that the scalar glueball decay is dominated by the  $K\bar{K}$  final state for glueball masses  $m_{0^{++}} > 1$  GeV. In the physically interesting mass range  $1.5 \text{ GeV} < m_{0^{++}} < 1.75 \text{ GeV}$  the branching ratios satisfy  $B(\eta\eta):B(\pi\pi):B(\bar{K}K) = 1:(3.3 \pm 0.3):(5.5 \pm 0.5)$ .

Our main focus in this work are  $\eta_c$  decays into three pseudoscalar Goldstone bosons. We find that the experimental decay rate  $\Gamma(\eta_c \rightarrow K\bar{K}\pi)$  can be reproduced for an average instanton size  $\bar{\rho} = 0.31$ , consistent with phenomenological determinations and lattice results. This in itself is quite remarkable, since the phenomenological determination is based on properties of the QCD vacuum. We found that the decay rates are very sensitive to the instanton size distribution. On the one hand, this implies that it is very hard to compute absolute rates. On the other hand, this means that  $\eta_c$  decays may provide a useful constraint on the size distribution.

The ratio of decay rates  $B(\eta' \pi\pi):B(\eta\pi\pi):B(K\bar{K}\pi) = 1:1:(4.2 \pm 1.3)$  is insensitive to the average instanton size. While the ratio  $B(\eta' \pi\pi):B(\eta\pi\pi) = 1:1$  is consistent with experiment, the ratio  $B(\eta\pi\pi):B(K\bar{K}) = 1:(4.2 \pm 1.3)$  is at best marginally consistent with the experimental value  $1.1 \pm 0.5$ . We have also studied  $\chi_c$  decays into two pseudoscalars. We find that the absolute decay rates can be reproduced for  $\bar{\rho} = 0.29$  fm. Instantons are compatible with the measured ratio  $B(K^+ K^-):B(\pi^+ \pi^-) = 1.2$

There are many questions that remain to be answered. On the experimental side it would be useful if additional data for the channels  $\eta_c \rightarrow \eta' \pi\pi, \eta\pi\pi$  were collected. One important question is whether  $(\pi\pi)$  resonances are important. It should also be possible to identify the smaller decay channels  $\eta_c \rightarrow K\bar{K}\eta, K\bar{K}\eta'$ . In addition to that, it is interesting to study the distribution of the final state mesons in all three-meson channels. Instantons predict that the production mechanism is completely isotropic and that the final state mesons are distributed according to three-body phase space.

In addition to that, there are a number of important theoretical issues that remain to be resolved. In the limit in which the scalar glueball is light the decay  $0^{++} \rightarrow \pi\pi(\bar{K}K)$  can be studied using effective Lagrangians based on broken scale invariance [47–49]. Our calculation based on direct instanton effects is valid in the opposite limit. Nevertheless, the instanton liquid model respects Ward identities based on broken scale invariance [16] and one should be able to recover the low energy theorem. In the case  $0^{++} \rightarrow \pi\pi(\bar{K}K)$  one should also be able to study the validity of the PCAC approximation in more detail. This could be done, for example, using numerical simulations of the instanton liquid. Finally we need to address the question how to properly compute the overlap of the initial  $\bar{c}c$  system with the instanton. This, of course, is a more general problem that also affects calculations of electroweak baryon number violation in high energy  $p\bar{p}$  collisions [50,51] and QCD multiparticle production in hadronic collisions [52].

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