Neutrinoless double beta decay from singlet neutrinos in extra dimensions

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(Received 3 February 2003; published 2 June 2003)

We study the model-building conditions under which a sizable $0\nu\beta\beta$-decay signal to the recently reported level of 0.4 eV is due to Kaluza-Klein singlet neutrinos in theories with large extra dimensions. Our analysis is based on 5-dimensional singlet-neutrino models compactified on an S^1/Z_2 orbifold, where the standard-model fields are localized on a 3-brane. We show that a successful interpretation of a positive signal within the above minimal 5-dimensional framework would require a non-vanishing shift of the 3-brane from the orbifold fixed points by an amount smaller than the typical scale $(100\ \text{MeV})^{-1}$ characterizing the Fermi nuclear momentum. The resulting 5-dimensional models predict a sizable effective Majorana-neutrino mass that could be several orders of magnitude larger than the light neutrino masses. Most interestingly, the brane-shifted models with only one bulk sterile neutrino also predict novel trigonometric textures leading to mass scenarios with hierarchical active neutrinos and large ν_μ-ν_τ and ν_e-ν_μ mixings that can fully explain the current atmospheric and solar neutrino data.

DOI: 10.1103/PhysRevD.67.113001 PACS number(s): 14.60.Pq, 11.10.Kk

I. INTRODUCTION

Recently, realizations of phenomenologically viable theories with large compact dimensions of TeV size [1] have enriched dramatically our perspectives in searching for physics beyond the standard model (SM). Among the possible higher-dimensional realizations, sterile neutrinos propagating in large extra dimensions [2–9] may provide interesting alternatives for generating the observed light neutrino masses. On the other hand, detailed experimental studies of neutrino properties may even shed light on the geometry and/or shape of the new dimensions. In this context, one of the most sensitive experimental approaches to neutrino masses and their properties is the search for neutrinoless double beta decay [10]. Neutrinoless double beta decay, denoted in short as $0\nu\beta\beta$, corresponds to two single beta decays [11,12] occurring simultaneously in one nucleus, thereby converting a nucleus (Z,A) into a nucleus $(Z+2,A)$: i.e.,

$$^{A}_{Z}X \rightarrow ^{A}_{Z+2}X + 2e^{-}.$$

This process violates lepton number by two units and hence its observation would signal physics beyond the SM. To a very good approximation, the half-life for a $0\nu\beta\beta$ decay mediated by light neutrinos is given by

$$[T_{1/2}^{0\nu\beta\beta}]^{-1} = \frac{|\langle m\rangle|^2}{m_e^2}|\mathcal{M}_{0\nu\beta\beta}|^2 G_{01}, \tag{1.1}$$

where $\langle m\rangle$ denotes the effective neutrino Majorana mass, m_e is the electron mass and $\mathcal{M}_{0\nu\beta\beta}$ and G_{01} denote the appropriate nuclear matrix element and the phase space factor, respectively. For details, see [10–12] and our discussion in Sec. IV.

Most recently, the Heidelberg-Moscow Collaboration has reanalyzed its experimental data [13], using appropriate statistical methods as well as new information from the form of the contributing background. They found an excess of $0\nu\beta\beta$ events, with statistical significance 2.2–3.1σ, depending on the method used. From this result, a half-life of $1.5^{+16.8}_{-0.7}\times 10^{25}$ years at 95% confidence level (C.L.) for ^{76}Ge is deduced, which implies an absolute value for the effective Majorana-neutrino mass:

$$|\langle m\rangle| = 0.39^{+0.45}_{-0.34}\ \text{eV} \quad (95\%\ \text{C.L.}), \tag{1.2}$$

allowing an uncertainty of the nuclear matrix element values of $\pm 50\%$.

The above experimental result (1.2), combined with information from solar and atmospheric neutrino data, restricts the admissible forms of the light-neutrino mass hierarchies in 4-dimensional models with 3 left-handed (active) neutrinos. The allowed scenarios contain either degenerate neutrinos or neutrinos that have an inverse mass hierarchy [14]. Evidently, a successful interpretation of a positive $0\nu\beta\beta$ signal of the appropriate size mentioned above imposes certain constraints on the structure of a theory. Here, we study these constraints on the model building of minimal 5-dimensional theories compactified on a S^1/Z_2 orbifold. Within the framework of theories with large extra dimensions, previous studies on neutrinoless double beta decays were performed within the context of higher-dimensional models that utilize the shining mechanism from a distant brane [15] and of theories with wrapped geometric space [16]. In Ref. [15] the $0\nu\beta\beta$ decay is accompanied with emission of Majorons, whereas the prediction in [16] falls short by two orders of magnitude to account for the observable excess in Eq. (1.2).

In this paper we consider an even more minimal higher-dimensional framework of lepton-number violation, in which only one 5-dimensional (bulk) sterile neutrino is added to the field content of the SM. In this minimal model, the SM fields are localized on a 4-dimensional Minkowski subspace, also termed 3-brane. The violation of the lepton number may occur in three distinct ways: (i) by adding lepton-number violating bilinears of the Majorana type in the Lagrangian; (ii) by generating lepton-number-violating mass terms through the Scherk-Schwartz mechanism [17]; (iii) by simultaneously coupling the Z_2-even and Z_2-odd two-component spinors of the 5-dimensional sterile neutrino to the same left-handed charged lepton state. As we will see in Sec. II, the last case (iii) is only possible if the 3-brane describing our observable world is shifted from the S^1/Z_2 orbifold fixed point. Here, we should also note that after integration of the extra dimension, the 5-dimensional orbifold model predicts an infinite tower of Kaluza-Klein (KK) neutrinos, for which the cases (i) and (ii) become fully equivalent.

One salient feature of the S^1/Z_2 orbifold compactification is that the KK neutrinos group themselves into approximately degenerate pairs of opposite *CP* parities. As a result, the lepton-number-violating KK-neutrino effects cancel each other and so the predicted $0\nu\beta\beta$ decay turns out to be exceedingly small to account for the recent observable excess. The latter appears to be a major obstacle in theories with large extra dimensions and imposes by itself constraints on the model building of higher-dimensional theories. A minimal way that avoids the above disastrous *CP*-parity cancellation effects on the $0\nu\beta\beta$ decay amplitude would be to arrange the opposite *CP*-parity KK neutrinos to couple to the $W^{\pm}$ bosons with unequal strength. Within the minimal 5-dimensional orbifold model outlined above, such a realization can be accomplished only if the 3-brane is displaced from one of the S^1/Z_2 orbifold fixed points. In our phenomenological bottom-up approach, the amount of brane-shifting is not arbitrary but dictated by the requirement that the model can accommodate the result (1.2) for the effective Majorana-neutrino mass. In particular, we will see in Sec. IV how the resulting brane-shifted 5-dimensional models can predict a sizable effective Majorana-neutrino mass that could be several orders of magnitude larger than the light neutrino masses and hence than the difference of their squares as required from neutrino oscillation data.

Another important constraint on the structure of higher-dimensional neutrino theories arises from their ability to explain the solar and atmospheric neutrino data by means of neutrino oscillations. In particular, orbifold models with one bulk neutrino, as those considered earlier in the literature [2,4,7–9], seem to prefer the small mixing angle (SMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution [18] which is highly disfavored by recent neutrino data analyses. Alternatively, if all neutrino data are to be explained by oscillations of active neutrinos with a small admixture of sterile KK component, then the compactification scale has to be much higher than the brane-Dirac mass terms. After integrating out the bulk neutrino of the model, the effective light-neutrino mass matrix has a rather restricted form; it is effectively of rank 1. As a result, two out of the three active neutrinos are massless. This is rather undesirable, since only one neutrino-mass difference can be formed in this case, so accommodating all neutrino oscillation data proves rather problematic [7–9]. However, the earlier studies have not included the possibility of a shifted brane. As was mentioned above, brane-shifting gives rise to sizable lepton-number violation. Hence, the tree-level rank-1 form of the effective neutrino mass matrix can be significantly modified through lepton-number violating Yukawa terms. As we will see in Sec. V, the resulting neutrino mass matrix has sufficiently rich structure to enable adequate description of the neutrino data.

Our paper is organized as follows: Sec. II describes the low-energy structure of the 5-dimensional orbifold models. Technical details are relegated to the Appendixes. In Sec. III, we study the renormalization-group (RG) effects of the neutrino Yukawa couplings and their possible impact on the $0\nu\beta\beta$ decay amplitude. In Sec. IV we give estimates of the effective Majorana-neutrino mass, which are predicted in these models presented in Sec. II. In Sec. V, we discuss the compatibility of such models with solar and atmospheric neutrino data. Finally, we draw our conclusions in Sec. VI.

II. MINIMAL HIGHER-DIMENSIONAL NEUTRINO MODELS

In this section we will describe the basic low-energy structure of minimal higher-dimensional extensions of the SM that include singlet neutrinos. In particular, we assume that singlet neutrinos being neutral under the $SU(2)_L \otimes U(1)_Y$ gauge group can freely propagate in a higher-dimensional space of $[1+(3+\delta)]$ dimensions, the so-called bulk, whereas all SM particles are localized in a $(1+3)$-dimensional subspace, known as 3-brane or simply brane. However, even singlet neutrinos themselves may live in a subspace of an even higher-dimensional space of $[1+(3+n_g)]$ dimensions, with $\delta \leqslant n_g$, in which gravity propagates.

We shall restrict our study to 5-dimensional models, i.e. the case $\delta=1$, where the singlet neutrinos are compactified on a S^1/Z_2 orbifold. Specifically, the leptonic sector of our 5-dimensional model consists of the SM lepton fields:

$$L(x)=\begin{pmatrix} \nu_l(x) \\ l_L(x) \end{pmatrix}, \quad l_R(x), \tag{2.1}$$

with $l=e,\mu,\tau$, and one 5-dimensional (bulk) singlet neutrino:

$$N(x,y)=\begin{pmatrix} \xi(x,y) \\ \bar{\eta}(x,y) \end{pmatrix}, \tag{2.2}$$

where y denotes the additional compact dimension, and ξ and η are 5-dimensional two-component spinors. The SM leptons are localized at the one of the two fixed points of the S^1/Z_2 orbifold, e.g. $y=0$. For generality, we will assume that the brane is shifted from the orbifold fixed point to $y=a$.

As usual, we impose the periodic boundary condition $N(x,y)=N(x,y+2\pi R)$ with respect to y dimension on the singlet neutrino field. In addition, the action of S^1/Z_2 orbifolding on the 5-dimensional spinors ξ and η entails the additional identifications:

$$\xi(x,y)=\xi(x,-y), \quad \eta(x,y)=-\eta(x,-y). \tag{2.3}$$

In other words, the spinors ξ and η are symmetric and antisymmetric under a y reflection, respectively.

With the above definitions, the most generic effective 4-dimensional Lagrangian of such a model is given by [2,5][1]

$$\mathcal{L}_{\rm eff}=\int_0^{2\pi R}dy\Bigg\{\overline{N}(i\gamma^\mu\partial_\mu+\gamma_5\partial_y)N -\frac{1}{2}(MN^TC^{(5)-1}N+\text{H.c.}) +\delta(y-a)\Bigg[\frac{h_1^l}{(M_F)^{\delta/2}}L\tilde{\Phi}^*\xi+\frac{h_2^l}{(M_F)^{\delta/2}}L\tilde{\Phi}^*\eta+\text{H.c.}\Bigg] +\delta(y-a)\mathcal{L}_{\rm SM}\Bigg\}, \tag{2.4}$$

where $\tilde{\Phi}=i\sigma_2\Phi^*$ is the hypercharge-conjugate of the SM Higgs doublet Φ, with hypercharge $Y(\Phi)=1$, and $\mathcal{L}_{\rm SM}$ denotes the SM Lagrangian which is restricted on a brane at $y=a$ [2]. In addition, M_F is the fundamental n_g-dimensional Planck scale and $\delta=1$ for sterile neutrinos propagating in 5 dimensions. Notice that the mass term $m_D\overline{N}N$ is not allowed in Eq. (2.4), as a result of the Z_2 discrete symmetry. Finally, in writing Eq. (2.4), we have used the following conventions:

$$\gamma^\mu=\begin{pmatrix}0 & \sigma^\mu\\ \bar{\sigma}^\mu & 0\end{pmatrix},\qquad \gamma_5=\begin{pmatrix}-\mathbf{1}_2 & 0\\ 0 & \mathbf{1}_2\end{pmatrix},$$

$$C^{(5)}=-\gamma_1\gamma_3=\begin{pmatrix}-i\sigma_2 & 0\\ 0 & -i\sigma_2\end{pmatrix}, \tag{2.5}$$

with $\sigma^\mu=(\mathbf{1}_2,\boldsymbol{\sigma})$ and $\bar{\sigma}^\mu=(\mathbf{1}_2,-\boldsymbol{\sigma})$, where $\sigma_{1,2,3}$ are the usual Pauli matrices.

We now proceed with the compactification of the y dimension of the S^1/Z_2 orbifold model. Because of their symmetric and antisymmetric properties (2.3) under y reflection, the two-component spinors ξ and η can be expanded in a Fourier series of cosine and sine harmonics:

$$\xi(x,y)=\frac{1}{\sqrt{2\pi R}}\xi_0(x)+\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\xi_n(x)\cos\left(\frac{ny}{R}\right), \tag{2.6}$$

$$\eta(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\eta_n(x)\sin\left(\frac{ny}{R}\right), \tag{2.7}$$

where the chiral spinors $\xi_n(x)$ and $\eta_n(x)$ form an infinite tower of KK modes.

After substituting Eq. (2.6) into Eq. (2.4) and integrating out the y coordinate, we obtain the effective 4-dimensional Lagrangian

$$\mathcal{L}_{\rm eff}=\mathcal{L}_{\rm SM}+\bar{\xi}_0(i\bar{\sigma}^\mu\partial_\mu)\xi_0+\left(\bar{h}_1^{l(0)}L\tilde{\Phi}^*\xi_0-\frac{1}{2}M\xi_0\xi_0+\text{H.c.}\right) +\sum_{n=1}^{\infty}\Bigg[\bar{\xi}_n(i\bar{\sigma}^\mu\partial_\mu)\xi_n+\bar{\eta}_n(i\bar{\sigma}^\mu\partial_\mu)\eta_n +\frac{n}{R}(\xi_n\eta_n+\bar{\xi}_n\bar{\eta}_n)-\frac{1}{2}M(\xi_n\xi_n+\bar{\eta}_n\bar{\eta}_n+\text{H.c.}) +\sqrt{2}(\bar{h}_1^{l(n)}L\tilde{\Phi}^*\xi_n+\bar{h}_2^{l(n)}L\tilde{\Phi}^*\eta_n+\text{H.c.})\Bigg], \tag{2.8}$$

where

$$\bar{h}_1^{l(n)}=\frac{h_1^l}{(2\pi M_FR)^{\delta/2}}\cos\left(\frac{na}{R}\right)=\left(\frac{M_F}{M_{\rm P}}\right)^{\delta/n_g}h_1^l\cos\left(\frac{na}{R}\right), \tag{2.9}$$

$$\bar{h}_2^{l(n)}=\frac{h_2^l}{(2\pi M_FR)^{\delta/2}}\sin\left(\frac{na}{R}\right)=\left(\frac{M_F}{M_{\rm P}}\right)^{\delta/n_g}h_2^l\sin\left(\frac{na}{R}\right). \tag{2.10}$$

In deriving the last step on the right-hand sides (RHS's) of Eqs. (2.9) and (2.10), we have employed the basic relation among the Planck mass $M_{\rm P}$, the corresponding n_g-dimensional Planck mass M_F and the compactification radii R (all taken to be of equal size):

$$M_{\rm P}=(2\pi M_FR)^{n_g/2}M_F. \tag{2.11}$$

From Eqs. (2.9) and (2.10), we see that the reduced 4-dimensional Yukawa couplings $\bar{h}_{1,2}^{(n)}$ can be suppressed by many orders of magnitude [2,3] if there is a large hierarchy between $M_{\rm P}$ and the quantum gravity scale M_F. Thus, if gravity and bulk neutrinos have the same number of extra dimensions, i.e. $\delta=n_g$, the 4-dimensional Yukawa couplings $\bar{h}_1^{(n)}$ and $\bar{h}_2^{(n)}$ are naturally suppressed by a huge factor $M_F/M_{\rm P}\sim10^{-15}$, for $M_F\approx10$ TeV. From Eq. (2.2), we observe that ξ and $\bar{\eta}$ belong to the same multiplet and hence have the same lepton number. It then follows from Eq. (2.8) that the simultaneous presence of $\bar{h}_1^{(n)}$ and $\bar{h}_2^{(n)}$ in an amplitude gives rise to lepton number violation by two units.

We should note that the above large suppression factor can be also obtained in a 5-dimensional neutrino model ($\delta=1$), where gravity propagates in a 6-dimensional space with compactification radii R_1 and R_2 of unequal size ($n_g=2$). In this case, one has to use the general toroidal compactification condition:

$$M_{\rm P}=(2\pi M_F)^{n_g/2}(R_1R_2\ldots R_{n_g})^{1/2}M_F. \tag{2.12}$$

Note that Eq. (2.12) reduces to Eq. (2.11) if all compactification radii are equal. With the help of Eq. (2.12), we find, for $n_g=2$,

$$\frac{h_{1,2}^l}{(2\pi M_FR_1)^{1/2}}=(2\pi M_FR_2)^{1/2}\frac{M_F}{M_{\rm P}}h_{1,2}^l. \tag{2.13}$$

[1]Further non-covariant extensions to this model have been considered in [8].

We easily see that if $R_2 \sim 1/M_F$, the original Yukawa couplings $h^l_{1,2}$ undergo the same large degree of suppression by a factor M_F/M_P.

If the brane were located at one of the two orbifold fixed points, e.g. at $y=0$, the operator $L\tilde{\Phi}^*\eta$ would be absent as a consequence of the Z_2 discrete symmetry. However, if the brane is shifted by an amount $a \neq 0$, the above operator is no longer absent. In fact, as we will see in Sec. IV, the coexistence of the two operators $L\tilde{\Phi}^*\xi$ and $L\tilde{\Phi}^*\eta$ breaks the lepton number leading to observable effects in neutrinoless double beta decay experiments.

Let us now introduce the weak basis for the KK-Weyl spinors

$$\chi_{\pm n} = \frac{1}{\sqrt{2}}(\xi_n \pm \eta_n), \tag{2.14}$$

which enables us to express the effective kinetic term of the neutrino sector as follows:

$$\mathcal{L}_{\rm kin} = \bar{\chi} i \bar{\sigma}^\mu \partial_\mu \chi - \left(\frac{1}{2}\chi^T \mathcal{M}\chi + \text{H.c.}\right), \tag{2.15}$$

where $\chi^T = (\nu_l\,,\ \xi_0\,,\ \chi_1\,,\ \chi_{-1}\,, \dots,\ \chi_n\,,\ \chi_{-n}\,, \dots)$ and

$$\mathcal{M}^{\rm KK} = \begin{pmatrix} 0 & m & m & m & m & m & \cdots \\ m & M & 0 & 0 & 0 & 0 & \cdots \\ m & 0 & M+\frac{1}{R} & 0 & 0 & 0 & \cdots \\ m & 0 & 0 & M-\frac{1}{R} & 0 & 0 & \cdots \\ m & 0 & 0 & 0 & M+\frac{2}{R} & 0 & \cdots \\ m & 0 & 0 & 0 & 0 & M-\frac{2}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{2.16}$$

with $m = v\bar{h}_1/\sqrt{2}$. In a three-generation model, m and $\bar{h}_1$ are both 3-vectors in the flavor space, i.e. $\bar{h}_1 = (\bar{h}^e_1, \bar{h}^\mu_1, \bar{h}^\tau_1)^T$. We will discuss intergenerational mixing effects in more detail in Sec. V. Here, we assume for simplicity that $\bar{h}_1 = \bar{h}^e_1$.

Following [2], we rearrange the singlet KK-Weyl spinors ξ_0 and $\chi^\pm_n$, such that the smallest diagonal entry of the KK neutrino mass matrix $\mathcal{M}^{\rm KK}$ in Eq. (2.16) is $|\varepsilon| = \min(|M - k/R|) \leq 1/(2R)$ for a given value $k = k_0$. In this newly defined basis, the effective kinetic Lagrangian (2.15) becomes

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\bar{\Psi}_\nu (i\partial\!\!\!/ - \mathcal{M}^{\rm KK}_\nu)\Psi_\nu, \tag{2.17}$$

where Ψ_ν is the reordered (4-component) Majorana-spinor vector

$$\Psi^T_\nu = \left[\begin{pmatrix}\nu_l \\ \bar{\nu}_l\end{pmatrix}, \begin{pmatrix}\chi_{k_0} \\ \bar{\chi}_{k_0}\end{pmatrix}, \begin{pmatrix}\chi_{k_0+1} \\ \bar{\chi}_{k_0+1}\end{pmatrix}, \begin{pmatrix}\chi_{k_0-1} \\ \bar{\chi}_{k_0-1}\end{pmatrix}, \dots, \begin{pmatrix}\chi_{k_0+n} \\ \bar{\chi}_{k_0+n}\end{pmatrix}, \begin{pmatrix}\chi_{k_0-n} \\ \bar{\chi}_{k_0-n}\end{pmatrix}, \dots\right] \tag{2.18}$$

and $\mathcal{M}^{\rm KK}_\nu$ the corresponding KK neutrino mass matrix

$$\mathcal{M}^{\rm KK}_\nu = \begin{pmatrix} 0 & m & m & m & m & m & \cdots \\ m & \varepsilon & 0 & 0 & 0 & 0 & \cdots \\ m & 0 & \varepsilon+\frac{1}{R} & 0 & 0 & 0 & \cdots \\ m & 0 & 0 & \varepsilon-\frac{1}{R} & 0 & 0 & \cdots \\ m & 0 & 0 & 0 & \varepsilon+\frac{2}{R} & 0 & \cdots \\ m & 0 & 0 & 0 & 0 & \varepsilon-\frac{2}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{2.19}$$

The eigenvalues of $\mathcal{M}^{\rm KK}_\nu$ can be computed from the characteristic eigenvalue equation $\det(\mathcal{M}^{\rm KK}_\nu - \lambda\mathbf{1}) = 0$, which is analytically given by

$$\prod_{n=0}^{\infty}\left[(\lambda-\varepsilon)^2 - \frac{n^2}{R^2}\right]\left[1 + \frac{\varepsilon}{\lambda-\varepsilon} - m^2\sum_{n=-\infty}^{\infty}\frac{1}{(\lambda-\varepsilon)^2 - \frac{n^2}{R^2}}\right] = 0. \tag{2.20}$$

Since it can be shown that $\lambda - \varepsilon = \pm n/R$ is never an exact solution to the characteristic equation, only the second factor in Eq. (2.20) can vanish. Employing complex contour integration techniques, the summation in the second factor in Eq. (2.20) can be performed exactly, leading to an equivalent transcendental equation

$$\lambda = \pi m^2 R \cot[\pi R(\lambda - \varepsilon)]. \tag{2.21}$$

As was already discussed in [2], if $\varepsilon = 0$, Eq. (2.21) implies that the mass spectrum consists of massive KK Majorana neutrinos degenerate in pairs with opposite *CP* parities. If $\varepsilon = 1/(2R)$, the KK mass spectrum contains a massless state, which is predominantly left-handed if $mR < 1$, while the remaining massive KK states form degenerate pairs with opposite *CP* parities, exactly as in the $\varepsilon = 0$ case. However, if $\varepsilon \neq 0, 1/(2R)$, the lepton number gets broken.[2] In this case, there is no massless state in the spectrum, and the above

[2]Alternatively, the lepton number may also be broken through the Scherk-Schwarz mechanism, where the Scherk-Schwarz rotation angle will induce terms very similar to those depending on ε [2,19].

exact degeneracy among the massive Majorana neutrinos becomes only approximate, with a mass splitting of order 2ε for each would-be ($\varepsilon \to 0$) degenerate KK pair.

We now consider an orbifold model, in which the $y=0$ brane is displaced from the orbifold fixed points by an amount a. Under certain restrictions in type I string theory [2,20], such an operation can be performed respecting the Z_2 invariance of the original higher-dimensional action. In particular, one can take explicitly account of this last property by considering the following replacements in the effective Lagrangian (2.4):

$$\xi\delta(y-a)\to\frac{1}{2}\xi[\delta(y-a)+\delta(y+a-2\pi R)],$$

$$\eta\delta(y-a)\to\frac{1}{2}\eta[\delta(y-a)-\delta(y+a-2\pi R)], \tag{2.22}$$

with $0\leqslant a<\pi R$ and $0\leqslant y\leqslant 2\pi R$. It is obvious that a Z_2-invariant implementation of brane-shifted couplings requires the existence of two branes at least, placed at $y=a$ and $y=2\pi R-a$. In addition, we assume that a is a rational number in units of πR, i.e.

$$a=\frac{r}{q}\pi R, \tag{2.23}$$

where r,q are natural numbers. This last assumption has been introduced for technical reasons. It enables us to carry out analytically the infinite summations over KK states (see also our discussion below).

Proceeding as above, the effective KK neutrino mass matrix $\mathcal{M}_\nu^{\rm KK}$ for the orbifold model with a shifted brane can be written down in an analogous form

$$\mathcal{M}_\nu^{\rm KK}=\begin{pmatrix} 0 & m^{(0)} & m^{(1)} & m^{(-1)} & m^{(2)} & m^{(-2)} & \cdots \\ m^{(0)} & \varepsilon & 0 & 0 & 0 & 0 & \cdots \\ m^{(1)} & 0 & \varepsilon+\frac{1}{R} & 0 & 0 & 0 & \cdots \\ m^{(-1)} & 0 & 0 & \varepsilon-\frac{1}{R} & 0 & 0 & \cdots \\ m^{(2)} & 0 & 0 & 0 & \varepsilon+\frac{2}{R} & 0 & \cdots \\ m^{(-2)} & 0 & 0 & 0 & 0 & \varepsilon-\frac{2}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{2.24}$$

where

$$m^{(n)}=\frac{v}{\sqrt{2}}\left[\bar{h}_1\cos\left(\frac{(n-k_0)a}{R}\right)+\bar{h}_2\sin\left(\frac{(n-k_0)a}{R}\right)\right]$$

$$=m\cos\left(\frac{na}{R}-\phi_h\right), \tag{2.25}$$

with $m=v\sqrt{(\bar{h}_1^2+\bar{h}_2^2)/2}$ and $\phi_h=\tan^{-1}(\bar{h}_2/\bar{h}_1)+k_0a/R$. As before, we consider an one-generation model with $\bar{h}_1=\bar{h}_1^e$ and $\bar{h}_2=\bar{h}_2^e$, which renders the analytic determination of the eigenvalue equation tractable. We will relax this assumption in Sec. V, when discussing the compatibility of this model with neutrino oscillation data. Thus, for our one-generation brane-shifted model, the characteristic eigenvalue equation reads

$$\prod_{n=0}^{\infty}\left[(\lambda-\varepsilon)^2-\frac{n^2}{R^2}\right]\left[1+\frac{\varepsilon}{\lambda-\varepsilon}-\frac{1}{\lambda-\varepsilon}\sum_{n=-\infty}^{\infty}\frac{m^{(n)2}}{\lambda-\varepsilon-\frac{n}{R}}\right]$$

$$=0, \tag{2.26}$$

which is equivalent to

$$\lambda=\sum_{n=-\infty}^{\infty}\frac{m^{(n)2}}{\lambda-\varepsilon-\frac{n}{R}}. \tag{2.27}$$

As opposed to the $a=0$ case, complex contour integration techniques are not directly applicable in evaluating the infinite sum in Eq. (2.27). The preventive reason is that the function $m^{(n)}$, analytically continued to the complex n plane, is not bounded from above as $n\to\pm i\infty$, as it had to be, because of its dependence on $\cos(na/R)$. However, as has been mentioned above and discussed further in Appendix A, this difficulty may be circumvented by assuming that a is a rational number in units of πR, as stated in Eq. (2.23). Under this technical assumption, we carry out in Appendix A the infinite sum in Eq. (2.27) analytically and derive the eigenvalue equation for the simplest class of cases, where $a=\pi R/q$ with $r=1$ and q an integer larger than 1, i.e. $q\geqslant 2$. More precisely, we find[3]

$$\lambda=\pi m^2R\left\{\cos^2[\phi_h-a(\lambda-\varepsilon)]\cot[\pi R(\lambda-\varepsilon)]\right.$$

$$\left.-\frac{1}{2}\sin[2\phi_h-2a(\lambda-\varepsilon)]\right\}. \tag{2.28}$$

Observe that unless $\varepsilon=1/(2R)$, $a=\pi R/2$ and $\phi_h=\pi/4$, the mass spectrum consists of massive non-degenerate KK neutrinos. However, it can be shown from Eq. (2.28) that this tree-level mass splitting between a pair of KK Majorana neutrinos is generally small for $m_{(n)}\gg 1/R$. In particular, this tree-level mass splitting is almost independent of a and subleading so as to play any relevant role in our calculations.

At this stage, it is important to comment on taking the limit $a=\pi R/q\to 0$ in Eq. (2.28), or equivalently $q\to\infty$. This limit is not the eigenvalue equation (2.21) which is valid for $a=0$, because of the presence of the extra non-vanishing term that depends on $\sin(2\phi_h)$ in Eq. (2.28). This apparent

[3]The so-derived formula generalizes the one presented in [2] to include brane-shifting and arbitrary Yukawa-coupling effects.

paradox can be resolved by noticing that the existence of this would-be anomalous term is ensured only if the brane-shifting a is much larger than the fundamental quantum gravity scale M_F, i.e. $a \gg 1/M_F$. Since M_F represents a natural ultraviolet cutoff of the theory, we expect the onset of new physics above the scale M_F, most likely of stringy nature, effectively implying that the KK-Yukawa mass terms $m^{(n)}$ are exponentially suppressed or zero for KK numbers $n \gtrsim M_F R$. As we will explicitly demonstrate in Sec. IV [see our discussion in Eq. (4.18)], such a truncation of the KK sum at M_F effectively results in a modification of the eigenvalue equation (2.28) to

$$\lambda = m^2 R\{\pi \cos^2[\phi_h - a(\lambda-\varepsilon)]\cot[\pi R(\lambda-\varepsilon)] - \mathrm{Si}(2aM_F)\sin[2\phi_h - 2a(\lambda-\varepsilon)]\}. \tag{2.29}$$

In the above, $\mathrm{Si}(x) = \int_0^x dt(\sin t/t)$ is the integral-sine function. For any finite value of its argument, $\mathrm{Si}(x)$ can be expanded as

$$\mathrm{Si}(x) = \sum_{n=1}^{+\infty} \frac{(-1)^{(n-1)} x^{(2n-1)}}{(2n-1)(2n-1)!}. \tag{2.30}$$

For small x, it is $\mathrm{Si}(x) \approx x$, while $\mathrm{Si}(x) = \pi/2$ for $x \to \infty$. Clearly, as long as $a \gg 1/M_F$, the eigenvalue equations (2.28) and (2.29) are almost identical, since $\mathrm{Si}(2aM_F) = \pi/2$ to a very good approximation. On the other hand, the limit $a \to 0$ does now smoothly go over to Eq. (2.21), as it should be.

Finally, in addition to the aforementioned tree-level mass splitting, one-loop radiative effects may also contribute to further increase the mass difference between two nearly degenerate KK Majorana neutrinos, if $\bar{h}_1$ and $\bar{h}_2$ do not vanish simultaneously. The one-loop generated mass splitting, however, is expected to be small [5] of order $\bar{h}_1 \bar{h}_2 m_{(n)}/(8\pi^2) \sim 10^{-2} \times (M_F/M_P)^2 \times m_{(n)} \lesssim 10^{-2} \times \Delta m_{(n)}$, where $m_{(n)} \approx n/R \leqslant M_F$ is the approximate mass of the nth KK pair of nearly degenerate Majorana neutrinos, and $\Delta m_{(n)} = m_{(n+1)} - m_{(n)} \approx 1/R$ is the mass difference between two adjacent KK Majorana pairs. Although such a radiatively-induced mass splitting may play a significant role for leptogenesis [5], its effect on the double beta decay amplitude is negligible. Therefore, we neglect radiative effects on the KK mass spectrum throughout the paper.

III. RG EVOLUTION OF NEUTRINO YUKAWA COUPLINGS

The RG evolution of the Yukawa couplings in the standard 4-dimensional scenario involving sterile neutrinos has been discussed in [21]. Here, we derive the corresponding RG equations for the higher-dimensional case. Since the RG evolution equations for $\bar{h}_1^l$ and $\bar{h}_2^l$ will be similar, we concentrate only on the former ($\equiv \bar{h}$). In such a higher-dimensional scenario, the presence of the KK sterile states alters the RG running. The triangle and self-energy diagrams that contribute to the running remain the same as in the SM, except that in the higher dimensional context, wherever there are internal ξ_n lines, there is a multiplicative factor $t_\delta = (\mu R)^\delta X_\delta$, with $X_\delta = 2\pi^{\delta/2}/\delta\Gamma(\delta/2)$. The RG equation for the Yukawa coupling $\bar{h}$ is given by

$$16\pi^2 \frac{d\bar{h}}{d\ln\mu} = \frac{3}{2}[t_\delta(\bar{h}\bar{h}^\dagger)\bar{h} - \bar{h}(h_e^\dagger h_e)] + \bar{h}\,\mathrm{Tr}(3h_u^\dagger h_u + 3h_d^\dagger h_d + h_e^\dagger h_e + t_\delta \bar{h}^\dagger \bar{h}) - \bar{h}\left(\frac{9}{4}g_w^2 + \frac{3}{4}g'^2\right), \tag{3.1}$$

where g_w and g' are the $SU(2)_L$ and $U(1)_Y$ gauge-coupling constants, respectively. Note that for $\delta = 0$ (1), it is $X_\delta = 1$ (2). Also, for $\delta = 0$, $t_\delta = 1$, the standard RG equation is reproduced [21].

We now observe that the four-dimensional Yukawa coupling ($\bar{h}$) is suppressed with respect to the higher-dimensional coupling (h) by means of the relation: $\bar{h} = (M_F/M_P)^{\delta/n_g} h$. Thus, even if we consider $h(1/R) \sim 1$, the four-dimensional $\bar{h}$ is suppressed by many orders of magnitude. From Eq. (3.1), it is also obvious that unless t_δ is large enough to be comparable with $(M_P^2/M_F^2)^{\delta/n_g}$, the contributions from the top-quark Yukawa coupling or the gauge couplings dominate the running, and hence there is no power-law behavior at lower energies.

On the contrary, if we go to a very high energy such that we can ignore h_t, then the terms multiplying t_δ dominate. In such a case, ignoring the gauge contribution, we can write

$$16\pi^2 \frac{d\bar{h}}{d\ln\mu} \sim \frac{5}{2} t_\delta \bar{h}^3. \tag{3.2}$$

Integrating Eq. (3.2) from the scale $\mu_0 \equiv R^{-1}$ to μ, we obtain

$$\frac{1}{\bar{h}^2(1/R)} - \frac{1}{\bar{h}^2(\mu)} \simeq \frac{5X_\delta}{16\pi^2\delta}(\mu R)^\delta. \tag{3.3}$$

In terms of the Yukawa fine structure constant $\alpha(\mu) = h^2(\mu)/(4\pi)$ of the original 5-dimensional Yukawa coupling (h) and for the simple case $\delta = n_g$, Eq. (3.3) takes on the form

$$\frac{1}{\alpha(\mu)} \simeq \frac{1}{\alpha(1/R)} - \frac{5X_\delta}{4\pi\delta}\left(\frac{\mu}{M_F}\right)^\delta. \tag{3.4}$$

Clearly, $\alpha(\mu) \to \infty$, for a critical scale

$$\mu_{\mathrm{critical}} = M_F\left(\frac{4\pi\delta}{5X_\delta \alpha(1/R)}\right)^{1/\delta}. \tag{3.5}$$

Interestingly enough, Eq. (3.5) implies that the power-law behavior sets in not just above the compactification scale R^{-1}, as was naively expected [2], but well above the quantum gravity scale M_F. On the other hand, requiring that $\alpha(M_F) \leqslant 1$ in Eq. (3.4) implies that $\alpha(1/R) < 0.55$ for $\delta = 1$. This last condition assures that our theory remains perturbative up to the quantum gravity scale M_F. From our discussion above, it is obvious that power-law effects on the Yukawa neutrino couplings can be safely neglected in our analysis.

IV. EFFECTIVE NEUTRINO-MASS ESTIMATES

In this section we calculate the $0\nu\beta\beta$ observable $\langle m\rangle$ in orbifold 5-dimensional models. This quantity determines the size of the neutrinoless double beta decay amplitude, which is induced by W-boson exchange graphs. To this end, it is important to know the interactions of the $W^{\pm}$ bosons to the charged leptons $l=e,\mu,\tau$ and the KK-neutrino mass eigenstates $n_{(n)}$. Adopting the conventions of [22], the effective charged current Lagrangian is given by

$$\mathcal{L}_{\rm int}^{W^{\pm}} = -\frac{g_w}{\sqrt{2}} W^{-\mu} \sum_{l=e,\mu,\tau} \left(B_{l\nu_l} \bar{l} \gamma_\mu P_L \nu_l + \sum_{n=-\infty}^{+\infty} B_{l,n} \bar{l} \gamma_\mu P_L n_{(n)} \right) + \text{H.c.}, \tag{4.1}$$

where g_w is the weak coupling constant, $P_L=(1-\gamma_5)/2$ is the left-handed chirality projector, and B is an infinite dimensional mixing matrix. The matrix B satisfies the following crucial identities:

$$B_{l\nu_l} B^*_{l'\nu_l} + \sum_{n=-\infty}^{+\infty} B_{l,n} B^*_{l',n} = \delta_{ll'}, \tag{4.2}$$

$$B_{l\nu_l} m_{\nu_l} B_{l'\nu_l} + \sum_{n=-\infty}^{+\infty} B_{l,n} m_{(n)} B_{l',n} = 0. \tag{4.3}$$

Equation (4.2) reflects the unitarity properties of the charged lepton weak space, and Eq. (4.3) holds true, as a result of the absence of the Majorana mass terms $\nu_l \nu_{l'}$ from the effective Lagrangian in the flavor basis. For the models under discussion, the KK neutrino masses $m_{(n)}$ can be determined exactly by the solutions of the corresponding transcendental equations. To a good approximation, however, these solutions for large n simplify to[4]

$$m_{(n)} \approx \frac{n}{R} + \varepsilon. \tag{4.4}$$

This last expression proves to be a good approximation in our estimates.

According to Eq. (1.1), the $0\nu\beta\beta$-decay amplitude $\mathcal{T}_{0\nu\beta\beta}$ is given by [11]

$$\mathcal{T}_{0\nu\beta\beta} = \frac{\langle m\rangle}{m_e} \mathcal{M}_{\rm GTF}(m_\nu), \tag{4.5}$$

where $\mathcal{M}_{\rm GTF}=\mathcal{M}_{\rm GT}-\mathcal{M}_{\rm F}$ is the difference of the nuclear matrix elements for the so-called Gamow-Teller and Fermi transitions. Note that this difference of nuclear matrix elements sensitively depends on the mass of the exchanged KK neutrino in a $0\nu\beta\beta$ decay. Especially if the exchanged KK-neutrino mass $m_{(n)}$ is comparable or larger than the characteristic Fermi nuclear momentum $q_F \approx 100$ MeV, the nuclear matrix element $\mathcal{M}_{\rm GTF}$ decreases as $1/m^2_{(n)}$. The general expression for the effective Majorana-neutrino mass $\langle m\rangle$ in (4.5) is given by

$$\langle m\rangle = \frac{1}{\mathcal{M}_{\rm GTF}(m_\nu)} \sum_{n=-\infty}^{\infty} B^2_{e,n} m_{(n)} \times [\mathcal{M}_{\rm GTF}(m_{(n)}) - \mathcal{M}_{\rm GTF}(m_\nu)]. \tag{4.6}$$

In the above, the first term describes the genuine higher-dimensional effect of KK-neutrino exchanges, while the second term is the standard contribution of the light neutrino ν, rewritten by virtue of Eq. (4.3). Note that the dependence of the nuclear matrix element $\mathcal{M}_{\rm GTF}$ on the KK-neutrino masses $m_{(n)}$ has been allocated to $\langle m\rangle$ in Eq. (4.6). The latter generally leads to predictions for $\langle m\rangle$ that depend on the double beta emitter isotope used in experiment. However, the difference in the predictions is too small for the higher-dimensional singlet-neutrino models to be able to operate as a smoking gun for different $0\nu\beta\beta$-decay experiments.

A. Factorization Ansatz for analytic estimates

To obtain analytic estimates that will help us to gain a better insight into the dynamical properties of Eq. (4.6), it proves useful to approximate the $0\nu\beta\beta$-decay amplitude $\mathcal{T}_{0\nu\beta\beta}$ in Eq. (4.5) by means of the factorizable ansatz [23]:

$$\mathcal{T}_{0\nu\beta\beta} \approx \frac{\langle m\rangle_{\rm SA}}{m_e} \mathcal{M}_{\rm GTF}(m_\nu) + \frac{m_p^2}{m_e} \langle m^{-1}\rangle \mathcal{M}_{\rm GTF}(m_p), \tag{4.7}$$

where m_p is the proton mass, and $\mathcal{M}_{\rm GTF}(m_\nu)$ and $\mathcal{M}_{\rm GTF}(m_p)$ are the values of the nuclear matrix element $\mathcal{M}_{\rm GTF}$ at m_ν and m_p, respectively. In Eq. (4.7), the $0\nu\beta\beta$ matrix element has been written as a sum of two terms. The first term, which is the dominant one, accounts for effects coming from KK neutrinos lighter than the characteristic Fermi nuclear momentum $q_F \approx 100$ MeV. In this kinematic region, the nuclear matrix element $\mathcal{M}_{\rm GTF}$ is almost independent of the KK neutrino mass $m_{(n)}$. The second term in Eq. (4.7) is due to KK neutrinos much heavier than q_F. This is generically a subdominant contribution to $\mathcal{T}_{0\nu\beta\beta}$, since $\mathcal{M}_{\rm GTF}(m_p) \ll \mathcal{M}_{\rm GTF}(m_\nu)$.

The quantity $\langle m\rangle_{\rm SA}$ is an approximation of the effective Majorana-neutrino mass $\langle m\rangle$, which is obtained by approximating the nuclear matrix elements $\mathcal{M}_{\rm GTF}(m_{(n)})$ entering $\langle m\rangle$ in Eq. (4.6) by a step function at $|m_{(n)}| = q_F$:

[4]For $|n|>\varepsilon$ and $n<0$, the KK mass eigenvalues $m_{(n)}$ are negative. This corresponds to a neutrino with positive physical mass $|m_{(n)}|$ and negative CP parity. One can always take account of the negative CP parity by redefining the mixing matrix elements $B_{l,-n}$ as $B_{l,-n} \to iB_{l,-n}$, for $n>\varepsilon R>0$. Although we will allow negative neutrino masses in our calculations, we should stress that both approaches are fully equivalent leading to the same analytic results.

$$\mathcal{M}_{\rm GTF}(m_{(n)}) = \begin{cases} \mathcal{M}_{\rm GTF}(m_\nu) & \text{for } |m_{(n)}| \leqslant q_F, \\ 0 & \text{for } |m_{(n)}| > q_F. \end{cases} \tag{4.8}$$

In what follows, we refer to such an approach to the nuclear matrix elements as the step approximation (SA). The effective neutrino mass in the SA reads

$$\langle m\rangle_{\rm SA} = B_{e\nu}^2 m_\nu + \sum_{n=-[(q_F+\varepsilon)R]}^{[(q_F-\varepsilon)R]} B_{e,n}^2 m_{(n)}$$
$$= -\sum_{n=[(q_F-\varepsilon)R]}^{+\infty} B_{e,n}^2 m_{(n)} - \sum_{n=[(q_F+\varepsilon)R]}^{+\infty} B_{e,-n}^2 m_{(-n)}, \tag{4.9}$$

where we used Eq. (4.3) to arrive at the last equality for the effective neutrino mass. Notice that $\langle m\rangle$ is not zero, simply because the sum over the KK neutrino states is truncated to those with a mass $|m_{(n)}|,|m_{(-n)}| \leqslant q_F$.

Correspondingly, the effects of the heavier KK neutrinos, with masses $m_{(n)} \gtrsim q_F$, have been taken into account in the factorizable Ansatz (4.7) by means of the inverse effective neutrino mass $\langle m^{-1}\rangle$. This newly introduced quantity is given by

$$\langle m^{-1}\rangle = \sum_{n=[(q_F-\varepsilon)R]}^{+\infty} B_{e,n}^2 m_{(n)}^{-1} + \sum_{n=[(q_F+\varepsilon)R]}^{+\infty} B_{e,-n}^2 m_{(-n)}^{-1}. \tag{4.10}$$

The factorizable form (4.5) of the matrix element constitutes a good approximation except for the isolated region where $|m_{(n)}| \approx q_F \approx 100$ MeV. Nevertheless, the effect of the KK neutrinos on the effective neutrino mass is cumulative [6] due to a sum of an infinite number of states, since each KK state has either a tiny Majorana mass or a very suppressed mixing with the electron neutrinos. Therefore, we expect that excluding this isolated region of KK-neutrino contributions around q_F will not alter quantitatively our results in a relevant way.

We will now rely on Eq. (4.9) to estimate the effective neutrino mass $\langle m\rangle_{\rm SA}$ in different settings of 5-dimensional orbifold models discussed in Sec. II. To begin with, let us consider a simple orbifold model, with $\varepsilon \neq 0$ and $\varepsilon \neq 1/(2R)$. In addition, we consider the case $a=0$; namely, we take the brane to be located at the one of the two orbifold fixed points. Like the neutrino masses, the mixing-matrix elements $B_{e\nu}$ and $B_{e,n}$ can also be computed exactly [2]:

$$B_{e\nu} = \frac{1}{\mathcal{N}}, \quad B_{e,n} = \frac{1}{\mathcal{N}_{(n)}}, \tag{4.11}$$

where the squares of the normalization factors $\mathcal{N}$ and $\mathcal{N}_{(n)}$ are given by

$$\mathcal{N}^2 = 1 + \sum_{n=-\infty}^{+\infty} \frac{m^2}{\left(\varepsilon - m_\nu + \frac{n}{R}\right)^2},$$

$$\mathcal{N}_{(n)}^2 = 1 + \sum_{k=-\infty}^{+\infty} \frac{m^2}{\left(\varepsilon - m_{(n)} + \frac{k}{R}\right)^2}. \tag{4.12}$$

Applying complex integration methods for convergent infinite sums, the squared normalization factor $\mathcal{N}^2$ can be calculated to give

$$\mathcal{N}^2 = 1 + \frac{\pi^2 m^2 R^2}{\sin^2[\pi R(m_\nu - \varepsilon)]} = 1 + \pi^2 m^2 R^2 + \frac{m_\nu^2}{m^2}. \tag{4.13}$$

In obtaining the last equality in Eq. (4.13), we used the eigenvalue equation (2.21) for $\lambda = m_\nu$. From Eqs. (4.11) and (4.13), we immediately see that if $mR \ll 1$ and $m_\nu \ll m$, it is $B_{e\nu} \approx 1$ and hence the lightest neutrino state is predominantly left handed. For the calculation of the effective neutrino mass, we need

$$\mathcal{N}_{(n)}^2 = 1 + \frac{\pi^2 m^2 R^2}{\sin^2[\pi R(m_{(n)} - \varepsilon)]} = 1 + \pi^2 m^2 R^2 + \frac{m_{(n)}^2}{m^2}$$
$$\approx \frac{\left(\frac{n}{R} + \varepsilon\right)^2}{m^2}, \tag{4.14}$$

where the last approximate equality in Eq. (4.14) corresponds to a large n. In Appendix B, we show that the KK neutrino masses derived from Eq. (2.21) and the mixing-matrix elements given in Eqs. (4.11) satisfy the sum rules given by the identities (4.2) and (4.3).

Based on Eq. (4.9), we will now perform an estimate of the effective neutrino mass in the simple orbifold model mentioned above. Plugging the value of $B_{e,n} = 1/\mathcal{N}_{(n)}$ into Eq. (4.9), we may estimate the effective neutrino mass in the SA through the following steps:

$$\langle m\rangle_{\rm SA} = -m^2 \sum_{n=[q_F R]}^{\infty} \left(\frac{1}{\varepsilon + \frac{n}{R}} + \frac{1}{\varepsilon - \frac{n}{R}} \right) + \mathcal{O}\left(\frac{\varepsilon m^2 R}{q_F}\right)$$
$$\approx m^2 R \int_{q_F R}^{+\infty} dn \left(\frac{1}{n - \varepsilon R} - \frac{1}{n + \varepsilon R} \right)$$
$$= -m^2 R \ln\left(\frac{q_F - \varepsilon}{q_F + \varepsilon}\right)$$
$$= \mathcal{O}\left(\frac{\varepsilon m^2 R}{q_F}\right). \tag{4.15}$$

In arriving at the last equality in Eq. (4.15), we approximated the sum over the KK states by an integral, and used the fact that $\varepsilon/q_F \ll 1$. Since $2\varepsilon R \lesssim 1$, we can estimate that for $m = 10$ eV, $\langle m\rangle_{\rm SA} \lesssim 10^{-6}$ eV, which is undetectably small.

The above large suppression of the effective neutrino mass $\langle m\rangle_{\rm SA}$ is a consequence of the very drastic cancellations due to KK neutrinos with opposite *CP* parities. However, we might be able to overcome this difficulty by arranging the opposite *CP*-parity KK neutrinos to couple to the electron and *W* boson with unequal strength. In fact, this is what happens in orbifold models automatically, if the $y=0$ brane is shifted to $y=a\neq 0$. In this case, the mixing-matrix elements $B_{e\nu}$ and $B_{e,n}$ are given by the inverse of $\mathcal{N}$ and $\mathcal{N}_{(n)}$, respectively; but now for the shifted brane, $\mathcal{N}_{(n)}$ is given by

$$\mathcal{N}_{(n)}^2 = 1 + m^2 \sum_{k=-\infty}^{+\infty} \frac{\cos^2\left(\frac{ka}{R} - \phi_h\right)}{\left(\varepsilon - m_{(n)} + \frac{k}{R}\right)^2} \approx \frac{\left(\frac{n}{R} + \varepsilon\right)^2}{m^2 \cos^2\left(\frac{na}{R} - \phi_h\right)}, \tag{4.16}$$

where the second approximate equality in Eq. (4.16) corresponds to large n.

By analogy to Eq. (4.15), we may compute the effective Majorana-neutrino mass for the brane-shifted scenario ($a \neq 0$) as follows:

$$\begin{aligned}\langle m\rangle_{\rm SA} &\approx -m^2 R \int_{q_F R}^{+\infty} dn \left(\frac{\cos^2\left(\frac{na}{R} - \phi_h\right)}{n + \varepsilon R} - \frac{\cos^2\left(\frac{na}{R} + \phi_h\right)}{n - \varepsilon R} \right) \\ &= -\sin(2\phi_h) m^2 R \int_{q_F R}^{M_F R} \frac{dn}{n} \sin\left(\frac{2na}{R}\right) + \mathcal{O}\left(\frac{\varepsilon m^2 R}{q_F}\right).\end{aligned} \tag{4.17}$$

In the second step, we have truncated the upper limit of the integral at the fundamental quantum gravity scale M_F. The scale M_F represents a natural ultra-violet cut-off of the problem, beyond of which the onset of string-threshold effects are expected to occur. The last result in Eq. (4.17) can now be expressed in terms of the integral-sine function $\mathrm{Si}(x) = \int_0^x dt(\sin t/t)$. Thus, the effective neutrino mass can be given by

$$\langle m\rangle_{\rm SA} \approx -\sin(2\phi_h) m^2 R [\mathrm{Si}(2aM_F) - \mathrm{Si}(2aq_F)] + \mathcal{O}\left(\frac{\varepsilon m^2 R}{q_F}\right). \tag{4.18}$$

Notice that for a fixed given value of M_F, the analytic expression (4.18) for the effective neutrino mass goes smoothly to Eq. (4.15) in the limit $a\to 0$, as it should be. In order that the prediction for neutrinoless double beta decay effects is at the level reported recently [13], we only need to have $\phi_h \sim \pm\pi/4$ and $1/M_F \ll a \lesssim 1/(2q_F)$, i.e. the brane is slightly displaced from its origin. For instance, if $a \approx 1/(3q_F)$, $m = 10$ eV and $1/R = 300$ eV, we find that $\langle m\rangle_{\rm SA}$ is exactly at the observable level, i.e. $\langle m\rangle_{\rm SA} \sim 0.4$ eV.

It is now interesting to give an estimate of the inverse effective neutrino mass $\langle m^{-1}\rangle$ in the orbifold model with a shifted brane ($a\neq 0$). The quantity $\langle m^{-1}\rangle$ can be approximately calculated as follows:

$$\begin{aligned}\langle m^{-1}\rangle &\approx m^2 R^3 \int_{q_F R}^{+\infty} dn \left(\frac{\cos^2\left(\frac{na}{R} - \phi_h\right)}{(n + \varepsilon R)^3} - \frac{\cos^2\left(\frac{na}{R} + \phi_h\right)}{(n - \varepsilon R)^3} \right) \\ &= \sin(2\phi_h) m^2 R^3 \int_{q_F R}^{+\infty} \frac{dn}{n^3} \sin\left(\frac{2na}{R}\right) \\ &\quad + \frac{3}{2}\cos(2\phi_h) m^2 \varepsilon R^4 \int_{q_F R}^{+\infty} \frac{dn}{n^4} \sin\left(\frac{2na}{R}\right) - \frac{\varepsilon m^2 R}{2q_F^3}.\end{aligned} \tag{4.19}$$

The RHS of the last equality in Eq. (4.19) can be written down in a lengthy expression in terms of the integral-sine, integral-cosine and known trigonometric functions. For example, for $\phi_h = \pi/4$, $\langle m^{-1}\rangle$ is given by

$$\langle m^{-1}\rangle \approx 2m^2 R\left[a^2\left(\mathrm{Si}(2aq_F) - \frac{\pi}{2}\right) - \frac{1}{4q_F^2}\sin(2aq_F) - \frac{a}{2q_F}\cos(2aq_F)\right] - \frac{\varepsilon m^2 R}{2q_F^3}. \tag{4.20}$$

For the specific model considered above, with $m = 10$ eV, $1/R = 300$ eV and $a = 1/(3q_F)$, we find that $\langle m^{-1}\rangle \lesssim 10^{-5}$ TeV^{-1}. Hence, the above exercise shows that the contribution from $\langle m^{-1}\rangle$ to the double beta decay amplitude (4.5) is subdominant; it gets even more suppressed for $a \ll 1/q_F$.

B. Numerical evaluation

To obtain realistic predictions for the double beta decay observable $\langle m\rangle$, one has to take into account the dependence of $\mathcal{M}_{\rm GTF}$ on the KK neutrino masses $m_{(n)}$. To properly implement this $m_{(n)}$ dependence in our extractions of the effective Majorana mass $\langle m\rangle$ from the different nuclei, we have used the general formula (4.6), where the infinite sum over n has been truncated at $|n_{\rm max}| = M_F R$, namely at the quantum gravity scale M_F. Notice that the general formula for $\langle m\rangle$ in Eq. (4.6) includes the contributions from the KK neutrinos heavier than q_F, described by the inverse effective neutrino mass $\langle m^{-1}\rangle$ in Eq. (4.20).

TABLE I. QRPA estimates of the relevant combination of nuclear matrix elements, $\mathcal{M}_{\rm GTF}=\mathcal{M}_{\rm GT}-\mathcal{M}_{\rm F}$, as a function of the KK neutrino mass $m_{(n)}$.

$m_{(n)}$ (MeV)	$\mathcal{M}_{\rm GTF}(m_{(n)})$			
	^{76}Ge	^{82}Se	^{100}Mo	^{116}Cd
$\leqslant 1$	4.33	4.03	4.86	3.29
10	4.34	4.04	4.81	3.29
10^2	3.08	2.82	3.31	2.18
10^3	1.40×10^{-1}	1.25×10^{-1}	1.60×10^{-1}	9.34×10^{-2}
10^4	1.39×10^{-3}	1.24×10^{-3}	1.60×10^{-3}	9.26×10^{-4}
10^5	1.39×10^{-5}	1.24×10^{-5}	1.60×10^{-5}	9.26×10^{-6}
10^6	1.39×10^{-7}	1.24×10^{-7}	1.60×10^{-7}	9.26×10^{-8}
10^7	1.39×10^{-9}	1.24×10^{-9}	1.60×10^{-9}	9.26×10^{-10}
$m_{(n)}$ (MeV)	$\mathcal{M}_{\rm GTF}(m_{(n)})$			
	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
$\leqslant 1$	4.50	3.89	1.83	5.30
10	4.52	3.91	1.88	5.45
10^2	3.19	2.79	1.48	4.24
10^3	1.46×10^{-1}	1.29×10^{-1}	7.07×10^{-2}	2.02×10^{-1}
10^4	1.46×10^{-3}	1.28×10^{-3}	7.04×10^{-4}	2.02×10^{-3}
10^5	1.46×10^{-5}	1.28×10^{-5}	7.05×10^{-6}	2.02×10^{-5}
10^6	1.46×10^{-7}	1.28×10^{-7}	7.05×10^{-8}	2.02×10^{-7}
10^7	1.46×10^{-9}	1.28×10^{-9}	7.05×10^{-10}	2.02×10^{-9}

In Table I we present numerical values for the difference of the nuclear matrix elements, $\mathcal{M}_{\rm GTF}=\mathcal{M}_{\rm GT}-\mathcal{M}_{\rm F}$, as a function of the KK neutrino mass $m_{(n)}$. Our estimates are obtained within the so-called quasiparticle random phase approximation (QRPA) [24,25]. Here, we should note that the numerical values for the nuclear matrix element of ^{100}Mo exhibit some instability due to its sensitive dependence on the particle–particle coupling g_{PP} within the context of the QRPA. In addition, we should remark that in our numerical evaluation of $\langle m\rangle$, the nuclear matrix elements $\mathcal{M}_{\rm GTF}$ have been interpolated between the values given in Table I.

In Table II we show numerical values for the effective Majorana-neutrino mass $\langle m\rangle$ as derived for different nuclei in a 5-dimensional brane-shifted model, with $m=10$ eV, $1/R=300$ eV, $\varepsilon=1/(4R)$, $\phi_h=-\pi/4$ and $M_F=1$ TeV. In addition, we have varied discretely the brane-shifting scale $1/a$ from 0.05 GeV up to values much larger than M_F. The first column in Table II gives the predictions obtained in the SA for the nuclear matrix elements. The SA is closely related to our approximative method followed above, leading to results that are in a very good agreement with Eq. (4.18). Remarkably enough, even the change of sign of $\langle m\rangle_{\rm SA}$ at $1/a\approx0.1$ GeV in Table II can be determined sufficiently accurately by analyzing the multiplicative expression $\pi/2-{\rm Si}(2aq_F)$ in Eq. (4.18), which oscillates around $\pi/2$ [26], for $1/a\lesssim0.1$ GeV. Analogous remarks can be made for the inverse effective neutrino mass $\langle m^{-1}\rangle$ in Eq. (4.20).

As can be seen from Table II, the deviation between the SA and the one based on the general formula (4.6) is rather significant if a is close to $1/q_F$ due to the non-trivial nuclear matrix element effects mentioned above and due to heavier KK-neutrino effects coming from $\langle m^{-1}\rangle$. However, for smaller values of a, i.e. for $a\lesssim1/(3q_F)$, the agreement between the effective neutrino mass computed in the SA and the general formula (4.6) is fairly good. In this kinematic regime, the inverse effective neutrino mass $\langle m^{-1}\rangle$ becomes rather suppressed according to our discussion in Eq. (4.20). Our numerical estimates in the last column of Table II offer firm support of this last observation. Thus, the main contribution to $\langle m\rangle$ originates from KK neutrinos much lighter than q_F. Consequently, within the 5-dimensional brane-shifted model, we have numerically established a sizable value for $\langle m\rangle$ in the presently explorable range 0.05–0.84 eV. Finally, for very small values of a, i.e. for $a\ll1/M_F$, we recover the undetectably small result (4.15) for the unshifted brane $a=0$.

C. $\langle m\rangle$ and the neutrino mass scale

Apart from explaining the recent excess in $0\nu\beta\beta$ decays, the 5-dimensional model with a small but non-vanishing shifted brane exhibits another very important property. The

TABLE II. Numerical estimates of $\langle m\rangle$ for different nuclei in a 5-dimensional brane-shifted model, with $m=10$ eV, $1/R=300$ eV, $\varepsilon=1/(4R)$, $\phi_h=-\pi/4$ and $M_F=1$ TeV. The first column exhibits the numerical values for $\langle m\rangle$ in the step approximation (SA) for the nuclear matrix elements, while the last column shows the results for the inverse effective neutrino mass $\langle m^{-1}\rangle$.

$1/a$					$\langle m\rangle$ (eV)					$\langle m^{-1}\rangle$
(GeV)	SA	^{76}Ge	^{82}Se	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd	(TeV^{-1})
0.05	-0.062	0.009	0.010	0.016	0.012	0.009	0.008	-0.004	-0.004	6.2×10^{-6}
0.1	-0.012	0.052	0.054	0.061	0.062	0.052	0.050	0.025	0.026	-3.6×10^{-6}
0.2	0.208	0.096	0.100	0.109	0.114	0.097	0.094	0.058	0.061	-1.3×10^{-5}
0.3	0.307	0.123	0.128	0.136	0.143	0.124	0.121	0.082	0.086	-1.2×10^{-5}
1	0.457	0.271	0.275	0.280	0.287	0.272	0.269	0.241	0.243	-5.7×10^{-6}
10	0.516	0.493	0.493	0.494	0.495	0.493	0.493	0.489	0.489	-6.6×10^{-7}
10^2	0.515				0.513					-6.7×10^{-8}
10^3					0.535					-6.7×10^{-8}
10^4					0.066					-6.9×10^{-10}
10^{10}					$\lesssim10^{-6}$					0

effective Majorana-neutrino mass $\langle m\rangle$ can be several orders of magnitude larger than the light neutrino mass m_ν, for certain choices of the parameters ε and ϕ_h. To understand this phenomenon, let us first consider the eigenvalue equation (2.27) for $\lambda=m_\nu$, written in the form

$$m_\nu+\sum_{n=-\infty}^{\infty}\frac{m^{(n)2}}{\varepsilon+\frac{n}{R}-m_\nu}=0. \tag{4.21}$$

Notice that Eq. (4.21) constitutes an excellent and very practical approximation of the neutrino-mass–mixing sum rule, when the small m_ν dependence in the infinite sum over the KK neutrino states is neglected and the approximate formulas (4.4) and (4.16) for the KK masses $m_{(n)}$ and mixing-matrix elements $B_{e,n}$, along with $B_{e\nu}=1$, are substituted in Eq. (4.3). Then, the infinite sum over KK neutrino states can be performed with the help of Eq. (2.28) for rational values of a in πR units. Especially for $a=\pi R/q$ with q being an integer much larger than 1, i.e. for $1/M_F\ll a\lesssim 1/q_F$, the light neutrino mass m_ν is given by

$$m_\nu\approx-\pi m^2R\left[\cos^2\phi_h\cot(\pi R\varepsilon)+\frac{1}{2}\sin(2\phi_h)\right]. \tag{4.22}$$

It is now easy to see that the light neutrino mass m_ν can be very suppressed for specific values of ϕ_h and ε. For instance, one obvious choice would be $\phi_h\approx-\pi/4$ and $\varepsilon\approx1/(4R)$. On the other hand, the effective neutrino mass $\langle m\rangle_{\rm SA}$ is determined by the second sine-dependent term in Eq. (4.22) [cf. (4.18)], which is induced by brane-shifting effects. Unlike the suppressed light neutrino mass m_ν, the effective neutrino mass $\langle m\rangle$ can be sizable in the observable range 0.05–0.84 eV. This loss of correlation between the quantities $\langle m\rangle$ and m_ν is a rather unique feature of our higher-dimensional brane-shifted scenario. As we will discuss in the next section, the above decorrelation property plays a key role in our model building of 5-dimensional brane-shifted scenarios that could explain the neutrino oscillation data.

V. ATMOSPHERIC AND SOLAR NEUTRINO DATA

Atmospheric and solar neutrino data [27–29], together with information from laboratory experiments, such as the CHOOZ experiment [30], are very crucial for a given higher-dimensional singlet-neutrino model to qualify as viable. In particular, the latest SNO results [28] appear to disfavor large components of sterile neutrinos, indicating a preference among the different solutions to the solar and atmospheric neutrino puzzles for those involving transitions between almost active neutrinos.[5] To account for this experimental indication, we assume that the compactification scale $1/R$ and the lepton-number-violating bulk parameter ε are much larger than the KK Dirac mass terms $m^{(n)}$ in Eq. (2.25).

In the following, we shall explicitly demonstrate that our 5-dimensional brane-shifted model with only one bulk neutrino is able to fully explain the neutrino oscillation data. Specifically, we will show that the preferred solar large mixing angle (LMA) and atmospheric solutions, which both require large ν_e-ν_μ and ν_μ-ν_τ mixings, can be realized within our 5-dimensional model. These particular solutions are allowed, only if the differences of the squares of the light neutrino masses lie in the ranges:

[5]A recent study [31] seems to suggest that the active neutrino component in the solar neutrinos has to be larger than 86% at 1 σ C.L. A loophole may exist for atmospheric neutrinos, see [32].

$$1.8\times10^{-3}<\Delta m^2_{\rm atm}\ [\mathrm{eV}^2]<4.0\times10^{-3},$$

$$2.0\times10^{-5}<\Delta m^2_{\odot}\ [\mathrm{eV}^2]<2.0\times10^{-4},\qquad(5.1)$$

with $\Delta m^2_{\rm atm}=m^2_{\nu_3}-m^2_{\nu_2}$ and $\Delta m^2_{\odot}=m^2_{\nu_2}-m^2_{\nu_1}$. According to the usual conventions, the physical light neutrino masses m_{ν_1}, m_{ν_2} and m_{ν_3} are labeled in increasing hierarchical order, i.e. $m_{\nu_1}\leqslant m_{\nu_2}\leqslant m_{\nu_3}$.

To start with, let us consider the weak basis in which the charged lepton mass matrix is diagonal. Then, in the three-generation brane-shifted model, the KK-Dirac Yukawa terms are given by the 3-vectors

$$\mathbf{m}^{(n)}=\begin{pmatrix} m^e\cos\left(\dfrac{na}{R}-\phi_e\right)\\ m^\mu\cos\left(\dfrac{na}{R}-\phi_\mu\right)\\ m^\tau\cos\left(\dfrac{na}{R}-\phi_\tau\right)\end{pmatrix},\qquad(5.2)$$

where

$$m^l=\frac{v}{\sqrt{2}}\sqrt{(\overline{h_1^l})^2+(\overline{h_2^l})^2},\qquad \phi_l=\tan^{-1}\left(\frac{\overline{h_2^l}}{\overline{h_1^l}}\right)+\frac{k_0a}{R},\qquad(5.3)$$

with $l=e,\mu,\tau$. Given our assumption that $\varepsilon,1/R\gg m^l$, the KK neutrinos can now be integrated out. Analogously with Eq. (2.27), the effective light neutrino mass matrix $\mathcal{M}^\nu$ can be computed by

$$\mathcal{M}^\nu=-\sum_{n=-\infty}^{+\infty}\frac{\mathbf{m}^{(n)}\mathbf{m}^{(n)T}}{\dfrac{n}{R}+\varepsilon}.\qquad(5.4)$$

Following the same line of steps as in Appendix A, one is able to analytically carry out the infinite sum in Eq. (5.4) for the phenomenologically interesting case of $a=\pi R/q$, with q being an integer much larger than 1. In this limit, we obtain the novel trigonometric mass texture:

$$\mathcal{M}^\nu_{ll'}=-\pi R m^l m^{l'}\Big[\cos\phi_l\cos\phi_{l'}\cot(\pi R\varepsilon)+\frac{1}{2}\sin(\phi_l+\phi_{l'})\Big],\qquad(5.5)$$

with $l,l'=e,\mu,\tau$. The effective neutrino mass matrix (5.5) consists of two terms: (i) the cosine-dependent term that arises from the lepton-number-violating bulk mass M (or equivalently ε) and (ii) the sine-dependent term which is due to lepton-number violation in the effective Yukawa couplings and is caused by slightly shifting the brane from the orbifold fixed points. The occurrence of the second brane-shifting mass term is always ensured as long as $a\gg1/M_F$. Without the presence of this brane-shifting-induced term, the effective neutrino mass matrix (5.5) is of rank 1, leading to two massless neutrinos. This last fact is very undesirable, as it would be very difficult to explain both solar and atmospheric neutrino data with only one non-trivial difference of neutrino masses in the frequently discussed scenario without brane shifting.

As has been discussed in Sec. IV, however, even a small amount of brane shifting may induce sizable lepton-number-violating Yukawa interactions. The latter generate brane-shifting mass terms that break the rank-1 structure of the effective neutrino mass matrix $\mathcal{M}^\nu$. The resulting $\mathcal{M}^\nu$ in Eq. (5.5) exhibits a novel trigonometric structure that can predict hierarchical neutrinos with large $\nu_\mu-\nu_\tau$ and $\nu_\mu-\nu_e$ mixings to explain the atmospheric and solar neutrino anomalies, along with a small $\nu_e-\nu_\tau$ mixing as required by the CHOOZ experiment [30]. At this point, it is important to stress that the effective neutrino mass $\langle m\rangle$ entering the $0\nu\beta\beta$-decay amplitude gets fully decoupled from the neutrino-mass matrix element $\mathcal{M}^\nu_{ee}$. According to our discussions in Sec. IV [cf. Eq. (4.18)], the effective neutrino mass for the three-generation case is given by

$$\langle m\rangle\approx-\frac{1}{2}\sin(2\phi_e)\pi(m^e)^2R\neq\mathcal{M}^\nu_{ee}.\qquad(5.6)$$

It is important to recall again that unlike $\mathcal{M}^\nu_{ee}$, KK neutrinos heavier than the Fermi nuclear momentum q_F do not contribute significantly to $\langle m\rangle$, leading to the loss of correlation between $\langle m\rangle$ and $\mathcal{M}^\nu_{ee}$. The latter is a distinctive feature of the KK-neutrino dynamics. This de-correlation between $\langle m\rangle$ and $\mathcal{M}^\nu_{ee}$ permit us to consider the interesting case $|\langle m\rangle|\gg|\mathcal{M}^\nu_{ll'}|$, for all $l,l'=e,\mu,\tau$. Such a realization enables us to accommodate a sizable positive signal of $0\nu\beta\beta$ decays together with the present neutrino oscillation data.

To realize the aforementioned hierarchy $|\langle m\rangle|\gg|\mathcal{M}^\nu_{ll'}|$, we assume that all phases ϕ_l are close to $-\pi/4$. For concreteness, we adopt the following scheme of phases:

$$\phi_l=-\frac{\pi}{4}+\delta_l,\quad \pi R\varepsilon=\frac{\pi}{4}-\delta_\varepsilon,\qquad(5.7)$$

where $\delta_l,\delta_\varepsilon\ll1$. Our choice of phases has been motivated by the fact that the above-described decorrelation between $\langle m\rangle$ and $\mathcal{M}^\nu_{ee}$ becomes fully operative in this case. To implement the CHOOZ constraint in our model building, we require that $\mathcal{M}^\nu_{e\tau}=\mathcal{M}^\nu_{\tau e}=0$. This last constraint implies that

$$2\delta_\varepsilon=-\delta_e-\delta_\tau.\qquad(5.8)$$

Moreover, without loss of generality within our phase scheme, we may take $\delta_\mu=0$. Under these assumptions, the light neutrino-mass matrix takes on the simple form

$$\mathcal{M}^\nu=\frac{\pi R}{2}\begin{pmatrix} m^{e2}(\delta_\tau-\delta_e) & m^e m^\mu\delta_\tau & 0\\ m^\mu m^e\delta_\tau & m^{\mu2}(\delta_e+\delta_\tau) & m^\mu m^\tau\delta_e\\ 0 & m^\tau m^\mu\delta_e & m^{\tau2}(\delta_e-\delta_\tau)\end{pmatrix}.\qquad(5.9)$$

Let us now consider the following numerical example:

$$\delta_\tau = \delta, \quad \delta_e = 2\delta,$$

$$\frac{m^\mu}{m^e} \approx 1.468, \quad \frac{m^\tau}{m^e} \approx 2.542. \tag{5.10}$$

This leads to the neutrino mass matrix

$$\mathcal{M}^\nu = \delta \frac{\pi m^{e2} R}{2} \begin{pmatrix} -1 & 1.47 & 0 \\ 1.47 & 6.46 & 7.46 \\ 0 & 7.46 & 6.46 \end{pmatrix}. \tag{5.11}$$

Notice that all elements of the neutrino-mass matrix $\mathcal{M}^\nu$ in Eq. (5.11) can be suppressed by choosing a small value for the factorizable parameter δ. In our numerical example, the neutrino mass matrix (5.11) can be diagonalized through ν_μ-ν_τ and ν_e-ν_μ mixing angles close to $\pi/4$, whereas the ν_e-ν_τ mixing angle is small, below 0.1. In addition, its mass eigenvalues are approximately given by

$$(\mathcal{M}^\nu)_{\rm diag} \approx \delta \pi m^{e2} R(0,1,7). \tag{5.12}$$

Assuming that $m^e = 10$ eV and $1/R = 300$ eV for a successful interpretation of the recent excess in $0\nu\beta\beta$ decays, then it should be $\delta = (6–9) \times 10^{-3}$ to accommodate the neutrino oscillation data through the LMA solution. In particular, we obtain the neutrino-mass differences:

$$\Delta m^2_{\rm atm} \approx (2-4) \times 10^{-3} \ \ {\rm eV}^2, \quad \Delta m^2_\odot \approx (4-8) \times 10^{-5} \ \ {\rm eV}^2. \tag{5.13}$$

These results are fully compatible with the currently preferred atmospheric and solar LMA solutions to the neutrino anomalies.

In our demonstrative analysis carried out in this section, we have not attempted to fit the results of the Liquid Scintillator Neutrino Detector (LSND) as well [33]. In principle, our brane-shifted 5-dimensional models are capable of accommodating the LSND results through active-sterile neutrino transitions. In this case, however, the lowest-lying KK singlet neutrinos should be relatively light. As a result, they cannot be integrated out from the light neutrino spectrum, thereby leading to a much more involved effective neutrino-mass matrix. A complete study of this issue, including possible constraints from the cooling of supernova SN 1987A [8,34], is beyond the scope of the present paper and may be given elsewhere.

VI. CONCLUSIONS

We have studied the model-building constraints derived from the requirement that KK singlet neutrinos in theories with large extra dimensions can give rise to a sizable $0\nu\beta\beta$-decay signal to the level of 0.4 eV reported recently. Our analysis has been focused on 5-dimensional S^1/Z_2 orbifold models with one sterile (singlet) neutrino in the bulk, while the SM fields are considered to be localized on a 3-brane. In our model building, we have also allowed the 3-brane to be displaced from the S^1/Z_2 orbifold fixed points. Within this minimal 5-dimensional brane-shifted framework, lepton-number violation can be introduced through Majorana-like bilinears, which may or may not arise from the Scherk–Schwarz mechanism, and through lepton-number-violating Yukawa couplings. However, lepton-number-violating Yukawa couplings can be admitted in the theory, only if the 3-brane is shifted from the S^1/Z_2 orbifold fixed points. Apart from a possible stringy origin [20], brane-shifting might also be regarded as an effective result owing to a non-trivial 5-dimensional profile of the Higgs particle [35] and/or other SM fields [36,37] that live in different locations of a 3-brane with non-zero thickness which is centered at one of the S^1/Z_2 orbifold fixed points.

One major difficulty of the higher-dimensional theories is their generic prediction of a KK neutrino spectrum of approximately degenerate states with opposite *CP* parities that lead to exceedingly suppressed values for the effective Majorana-neutrino mass $\langle m \rangle$. Nevertheless, we have shown that within the 5-dimensional brane-shifted framework, the KK neutrinos can couple to the $W^\pm$ bosons with unequal strength, thus avoiding the disastrous *CP*-parity cancellations in the $0\nu\beta\beta$-decay amplitude. In particular, the brane-shifting parameter a can be determined from the requirement that the effective Majorana mass $\langle m \rangle$ is in the observable range [13]: 0.05–0.84 eV. In this way, we have found that $1/a$ has to be larger than the typical Fermi nuclear momentum $q_F = 100$ MeV and smaller than the quantum gravity scale M_F, or equivalently $1/M_F \lesssim a \lesssim 1/q_F$.

An important prediction of our 5-dimensional brane-shifted model is that the effective Majorana-neutrino mass $\langle m \rangle$ and the scale of light neutrino masses can be completely de-correlated for certain natural choices of the Majorana-like bilinear term ε and the original 5-dimensional Yukawa couplings h_1^l and h_2^l in Eq. (2.4). For example, if $\varepsilon \approx 1/(4R)$ and $h_1^l \approx -h_2^l$, we obtain light-neutrino masses that can be several orders of magnitude smaller than $\langle m \rangle$. Nevertheless, it is worth mentioning that if future data did not substantiate the presently reported $0\nu\beta\beta$ excess, the above model-building conditions would then need to be modified. Such a possible modification would not jeopardize, though, the viability of our brane-shifted scenario. Indeed, if the upper limit on the effective neutrino mass became even lower and lower, this would imply that the above decorrelation property is less and less necessary.

Another important prediction of the 5-dimensional brane-shifted model with *only one* bulk sterile neutrino is that the emerging effective light-neutrino mass matrix does no longer possess the rank-1 form, as opposed to the brane-unshifted $a=0$ case. As we have shown in Sec. V, the above properties of the brane-shifted models are sufficient to explain, even with only one neutrino in the bulk, the present solar and atmospheric neutrino data by means of oscillations of hierarchical neutrinos with large ν_e-ν_μ and maximal ν_μ-ν_τ mixings. In particular, neutrino-mass textures can be constructed that utilize the currently preferred LMA solution, where the ν_e-ν_τ mixing is small in agreement with the CHOOZ experiment.

Although a sizable $0\nu\beta\beta$-decay signal can be predicted within our brane-shifted 5-dimensional models, the above-described decorrelation property between $\langle m\rangle$ and the actual light neutrino masses suggests, however, that it is rather unlikely that such a signal be accompanied by a corresponding signal in tritium beta-decay experiments. For example, the KATRIN project [38] has a sensitivity to active neutrino masses larger than 0.35 eV at 95% C.L., and so it can only probe the existence of light neutrinos much heavier than those considered in our 5-dimensional models. Finally, the brane-shifted models under study also have the potential to accommodate the LSND results by virtue of active-sterile neutrino oscillations. In this case, the lowest-lying KK-neutrino states will contribute to the effective light neutrino-mass matrix, giving rise to more involved mass textures. In this context, it would be very interesting to investigate the question whether a simple higher-dimensional model accounting for all the observed neutrino anomalies can be established. We plan to address this interesting question in the near future.

ACKNOWLEDGMENT

We thank Martin Hirsch for discussions on QRPA computations and Antonio Delgado for comments on the geometric breaking of lepton number violation in higher-dimensional theories.

APPENDIX A: EIGENVALUE EQUATION

Starting from Eq. (2.27), we will derive here the transcendental eigenvalue equation (2.28) for the simplest class of brane-shiftings with $a=\pi R/q$, where $r=1$ and q is an integer larger than 1, i.e. $q\geqslant 2$. Then, the eigenvalue equation (2.27) can be equivalently written as

$$\lambda=\sum_{l=0}^{q-1}\sum_{k=-\infty}^{\infty}\frac{m^{(qk+l)2}}{\lambda-\varepsilon-\dfrac{qk+l}{R}}=\sum_{l=0}^{q-1}m^{(l)2}\sum_{k=-\infty}^{\infty}\frac{1}{\lambda-\varepsilon-\dfrac{qk+l}{R}},\qquad (A1)$$

where we have used the periodicity property $(m^{(l)})^2=(m^{(qk+l)})^2$ in the second step of Eq. (A1). In fact, it is this last periodicity property of the KK-Yukawa terms that we wish to exploit here to carry out analytically the infinite sums in Eq. (A1), which has forced us to introduce the technical constraint (2.23), namely that $a/(\pi R)$ is a rational number. Now, the individual l-dependent infinite sums over k in Eq. (A1) can be performed independently, using complex contour integration techniques. In this way, we obtain

$$\lambda=\frac{1}{q}\pi m^2R\sum_{l=0}^{q-1}\cos^2\left(\phi_h-\frac{l\pi}{q}\right)\cot\left[\frac{1}{q}\pi R(\lambda-\varepsilon)-\frac{l\pi}{q}\right].\qquad (A2)$$

Our next task is to carry out the summation over l in Eq. (A2). For this purpose, we express the RHS of Eq. (A2) entirely in terms of sine and cosine functions by factoring out the common divisor, i.e.

$$\lambda=\frac{\pi m^2R}{q\prod_{l=0}^{q-1}\sin\left(\dfrac{\theta}{q}-\dfrac{l\pi}{q}\right)}\sum_{l=0}^{q-1}\cos^2\left(\phi_h-\frac{l\pi}{q}\right)\times\cos\left(\frac{\theta}{q}-\frac{l\pi}{q}\right)\prod_{\substack{m=0\\(m\neq l)}}^{q-1}\sin\left(\frac{\theta}{q}-\frac{m\pi}{q}\right),\qquad (A3)$$

with $\theta=\pi R(\lambda-\varepsilon)$. To further evaluate Eq. (A3), we exploit the following trigonometric identities:[6]

$$\prod_{l=0}^{q-1}\sin\left(\frac{\theta}{q}-\frac{l\pi}{q}\right)=\frac{(-1)^{q-1}}{2^{q-1}}\sin\theta,\qquad (A4)$$

$$\sum_{l=0}^{q-1}\cos\left(\frac{\theta}{q}-\frac{l\pi}{q}\right)\prod_{\substack{m=0\\(m\neq l)}}^{q-1}\sin\left(\frac{\theta}{q}-\frac{m\pi}{q}\right)=\frac{(-1)^{q-1}}{2^{q-1}}q\cos\theta,\qquad (A5)$$

$$\sum_{l=0}^{q-1}\cos\left(2\phi_h-\frac{2l\pi}{q}\right)\cos\left(\frac{\theta}{q}-\frac{l\pi}{q}\right)\prod_{\substack{m=0\\(m\neq l)}}^{q-1}\sin\left(\frac{\theta}{q}-\frac{m\pi}{q}\right)=\frac{(-1)^{q-1}}{2^{q-1}}q\cos\left(2\phi_h+\frac{q-2}{q}\theta\right).\qquad (A6)$$

With the help of Eqs. (A4) and (A5), we arrive at the transcendental eigenvalue equation

$$\lambda=\frac{\pi m^2R}{2}\left\{\cot[\pi R(\lambda-\varepsilon)]+\frac{\cos\left[2\phi_h+\dfrac{q-2}{q}\pi R(\lambda-\varepsilon)\right]}{\sin[\pi R(\lambda-\varepsilon)]}\right\}.\qquad (A7)$$

If we replace q with $\pi R/a$ in Eq. (A7), we arrive after simple trigonometric algebra at the transcendental eigenvalue equation (2.28). Although we focused our attention on the simplest class with $a=\pi R/q$, we should remark that our methodology described above can apply equally well to the most general case where the brane-shifting a is any rational number r/q in πR units.

[6] The proof of these identities is rather lengthy and relies on the particular properties of the q roots of the unity, i.e. the roots of the equation $z^q=1$. Specifically, we used the basic property of the unit roots that their sum and the sum of their products are zero, while their total product is $(-1)^{q-1}$.

APPENDIX B: SUM RULES

In this appendix we will show that the KK-neutrino masses determined by the roots of Eq. (2.21) and the mixing-matrix elements given in Eq. (4.11) satisfy the sum rules (4.2) and (4.3). For simplicity, we consider the case $a=0$. However, our considerations carry over very analogously to the case $a=\pi R/q\neq 0$, where q is an integer larger than 1.

Let us first consider (4.2) for $l=l'=e$. We will then prove that

$$|B_{e\nu}|^2+\lim_{N\to\infty}\sum_{n=-N}^{N}|B_{e,n}|^2=1. \tag{B1}$$

Our proof will rely on Cauchy's integral theorem. Thus, the LHS of Eq. (B1) can be expressed in terms of a complex integral as follows:

$$\begin{aligned}|B_{e\nu}|^2&+\lim_{N\to\infty}\sum_{n=-N}^{N}|B_{e,n}|^2\\&=\frac{1}{2\pi i}\lim_{N\to\infty}\oint_{C_N}dz\left(\frac{1}{z-m_\nu}+\sum_{n=-N}^{N}\frac{1}{z-m_{(n)}}\right)\\&\quad\times\frac{1}{1+\pi^2m^2R^2/\sin^2[\pi R(z-\varepsilon)]}\\&=\frac{1}{2\pi i}\lim_{N\to\infty}\oint_{C_N}dz\,\frac{1}{z-\pi m^2R\cot[\pi R(z-\varepsilon)]}.\end{aligned}\tag{B2}$$

In deriving the second equality in Eq. (B2), we have noticed that for z in the vicinity of the pole, e.g. for $z\approx m_{(n)}$, it is

$$\begin{aligned}z-\pi m^2R\cot[\pi R(z-\varepsilon)]&\\ \approx(z-m_{(n)})&\left\{1+\frac{\pi^2m^2R^2}{\sin^2[\pi R(z-\varepsilon)]}\right\}.\end{aligned}\tag{B3}$$

Such a substitution is only valid under complex integration, provided there are no singularities of the complex function $\cot[\pi R(z-\varepsilon)]$ on the contour C_N. For this purpose, we choose our contours to be circles represented in the complex plane as

$$z_N=\frac{\left(N+\frac{1}{2}\right)e^{i\theta}}{R}+\varepsilon. \tag{B4}$$

Then, it can be shown that on the complex contours $z=z_N$, $|\cot\pi R(z_N-\varepsilon)|$ is bounded from above by a constant independent of N. Thus, on C_N the last integral in Eq. (B2) may be successively computed as

$$\begin{aligned}&\frac{1}{2\pi i}\lim_{N\to\infty}\oint_{C_N}dz\,\frac{1}{z-\pi m^2R\cot[\pi R(z-\varepsilon)]}\\&=\frac{1}{2\pi i}\lim_{N\to\infty}\int_0^{2\pi}d\theta\,\frac{i(z_N-\varepsilon)}{z_N-\pi m^2R\cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right]}\\&=1+\frac{1}{2\pi}\lim_{N\to\infty}\int_0^{2\pi}d\theta\,\frac{\pi m^2R\cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right]-\varepsilon}{z_N-\pi m^2R\cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right]}.\end{aligned}\tag{B5}$$

The second term in the last equality of Eq. (B5) vanishes in the limit $N\to\infty$ or equivalently when z_N is taken to infinity in a discrete manner as prescribed by Eq. (B4). Thus, the complex integral in the last equality of Eq. (B2) is exactly 1, which proves the unitarity sum rule (B1).

In the remainder of the appendix, we will prove the neutrino-mass-mixing sum rule:

$$B_{e\nu}^2m_\nu+\lim_{N\to\infty}\sum_{n=-N}^{N}B_{e,n}^2m_{(n)}=0. \tag{B6}$$

In our proof, we will follow a path very analogous to the one outlined above for showing Eq. (B1). Thus, the LHS of Eq. (B6) may be expressed in terms of a complex integral as follows:

$$\begin{aligned}B_{e\nu}^2m_\nu&+\lim_{N\to\infty}\sum_{n=-N}^{N}B_{e,n}^2m_{(n)}\\&=\frac{1}{2\pi i}\lim_{N\to\infty}\oint_{C_N}dz\,\frac{z}{z-\pi m^2R\cot[\pi R(z-\varepsilon)]}.\end{aligned}\tag{B7}$$

Evaluating the complex integral on the contours C_N defined by Eq. (B4) yields

$$\begin{aligned}&\frac{1}{2\pi i}\lim_{N\to\infty}\oint_{C_N}dz\,\frac{z}{z-\pi m^2R\cot[\pi R(z-\varepsilon)]}\\&=\frac{1}{2\pi i}\lim_{N\to\infty}\int_0^{2\pi}d\theta\,\frac{i(z_N-\varepsilon)\pi m^2R\cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right]}{z_N-\pi m^2R\cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right]}\\&=\frac{1}{2}m^2R\lim_{N\to\infty}\left\{\int_0^{2\pi}d\theta\cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right]+\mathcal{O}(1/z_N)\right\}.\end{aligned}\tag{B8}$$

Similar to the second term in the last equality of Eq. (B8), which goes to zero for $N\to\infty$, the first term vanishes as well after integration over θ. This can be readily seen by exploiting, respectively, the periodic and antisymmetric properties of the integrand with respect to θ and its argument:

$$\begin{aligned}\int_0^{2\pi} d\theta \cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right] &= \int_0^{\pi} d\theta \cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right] + \int_{\pi}^{2\pi} d\theta \cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right] \\ &= \int_0^{\pi} d\theta \cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right] + \int_0^{\pi} d\theta \cot\left[\pi\left(N+\frac{1}{2}\right)e^{i(\theta+\pi)}\right] \\ &= \int_0^{\pi} d\theta \cot\left[\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right] + \int_0^{\pi} d\theta \cot\left[-\pi\left(N+\frac{1}{2}\right)e^{i\theta}\right] \\ &= 0. \end{aligned} \tag{B9}$$

Consequently, the complex integral on the RHS of Eq. (B7) vanishes identically, q.e.d.

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