PHYSICAL REVIEW D **67**, 111901(R) (2003)

Charge symmetry violation corrections to determination of the Weinberg angle in neutrino reactions

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(Received 6 March 2003; published 23 June 2003)

We show that the correction to the Paschos-Wolfenstein relation associated with charge symmetry violation in the valence quark distributions is essentially model independent. It is proportional to a ratio of quark momenta that is independent of Q^2. This result provides a natural explanation of the surprisingly good agreement found between our earlier estimates within several different models. When applied to the recent NuTeV measurement, this effect significantly reduces the discrepancy with other determinations of the Weinberg angle.

DOI: 10.1103/PhysRevD.67.111901 PACS number(s): 11.30.Hv, 12.15.Mm, 12.38.Qk, 13.15.+g

In 1973, Paschos and Wolfenstein [1] derived an expression using the ratio of neutral-current and charge-changing neutrino interactions on isoscalar targets. This ratio is

$$R^- \equiv \frac{\frac{1}{\rho_0^2}(\langle \sigma_{NC}^{\nu N_0}\rangle - \langle \sigma_{NC}^{\bar{\nu} N_0}\rangle)}{\langle \sigma_{CC}^{\nu N_0}\rangle - \langle \sigma_{CC}^{\bar{\nu} N_0}\rangle} = \frac{1}{2} - \sin^2\theta_W. \tag{1}$$

In Eq. (1), $\langle \sigma_{NC}^{\nu N_0}\rangle$ and $\langle \sigma_{CC}^{\nu N_0}\rangle$ are, respectively, the neutral-current and charged-current inclusive, total cross sections for neutrinos on an isoscalar target. The quantity $\rho_0 \equiv M_W/(M_Z \cos\theta_W)$ is one in the standard model. From this ratio, one can obtain an independent measurement of the Weinberg angle ($\sin^2\theta_W$).

The NuTeV group recently measured neutrino charged-current and neutral-current cross sections on iron [2]. From the ratios of these cross sections for neutrinos and anti-neutrinos they extracted $\sin^2\theta_W = 0.2277 \pm 0.0013$ (stat) ± 0.0009 (syst). This value is three standard deviations above the measured fit to other electroweak processes, $\sin^2\theta_W = 0.2227 \pm 0.00037$ [3]. The discrepancy between the NuTeV measurement and the determination of the Weinberg angle from electromagnetic measurements is surprisingly large, and it may be evidence of physics beyond the standard model.

As the NuTeV experiment did not strictly involve the Paschos-Wolfenstein relation, Eq. (1), there are a number of additional corrections that need to be considered, such as differences in shadowing for photons, $W^\pm$ and Z^0's [4], asymmetries in s and $\bar{s}$ distributions [5] and so on—Ref. [6] summarizes corrections to the NuTeV result from within and outside the standard model. In addition, Eq. (1) is valid only for an isoscalar target and it is based upon the assumption of charge symmetry. There is thus a premium on knowing the corrections as accurately as possible.

Let us first review the corrections due to the fact that $N \neq Z$ for the iron target. The corrections take the form [6]

$$\Delta R_I = -\left[3\Delta_u^2 + \Delta_d^2 + \frac{4\alpha_s}{9\pi}(\overline{g_L^2} - \overline{g_R^2})\right]\left(\frac{N-Z}{A}\right)\left[\frac{U_\mathrm{v} - D_\mathrm{v}}{U_\mathrm{v} + D_\mathrm{v}}\right] \tag{2}$$

where

$$\Delta_q^2 \equiv (g_L^q)^2 - (g_R^q)^2, \quad 3\Delta_u^2 + \Delta_d^2 = 1 - \frac{7}{3}\sin^2\theta_W,$$

$$\overline{g_L^2} - \overline{g_R^2} = (g_L^u)^2 + (g_L^d)^2 - (g_R^u)^2 - (g_R^d)^2 = \frac{1}{2} - \sin^2\theta_W,$$

$$Q_\mathrm{v} \equiv \int_0^1 x q_\mathrm{v}(x) dx. \tag{3}$$

The additional QCD radiative term in Eq. (2) was calculated by Davidson *et al.*, Ref. [6]; it is quite small at the Q^2 for the NuTeV experiment. The final term in Eq. (2) involves the ratio of momentum carried by up and down valence quarks. Since both the numerator and the denominator involve the same moments of QCD non-singlet parton distributions, they evolve identically, so this ratio can be evaluated at any convenient value of Q^2. Using the CTEQ3D parton distributions [7] in Eq. (2), one obtains $\delta R_I = -0.0126$. The NuTeV group has emphasized [2,8] that they do not actually measure the Paschos-Wolfenstein ratio, but instead combine separate measurements of ratios of neutral to charged-current cross sections for neutrinos and anti-neutrinos with a full Monte Carlo simulation of their experiment. Using their simulation, the NuTeV group reports an isoscalar correction of -0.0080. This represents a 36% reduction from the Paschos-Wolfenstein correction, and the NuTeV group cited a very small error for this correction [9]. Kulagin [10] claimed that the uncertainty in this correction is likely to be considerably

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larger. The largest uncertainty in Eq. (2) is the momentum carried by up and down valence quarks, and according to Davidson *et al.* [6], these quantities are known rather accurately.

Davidson *et al.* [6] noted that, although charge symmetry violating (CSV) corrections are likely to be small, these effects could in principle generate a substantial correction to the NuTeV result. Recently, we calculated CSV contributions to the NuTeV experiment arising from the small difference of u and d quark masses [11]. Following earlier work on CSV in parton distributions [12,13], our method involved calculating CSV distributions at a low momentum scale, and using QCD evolution to generate the CSV distributions at the Q^2 values appropriate for the NuTeV experiment. We obtained a CSV correction to the NuTeV result $\Delta R_{CSV} \sim -0.0015$. The NuTeV group also reported an estimate of the CSV parton distributions, using a rather different procedure [8]; they obtained a much smaller correction than ours, $\Delta R_{CSV} \sim +0.0001$. The large discrepancy between these two results suggested that the CSV correction might be strongly dependent on the starting scale, Q_0^2, the phenomenological valence parton distribution chosen, or other details of the calculation.

Here, we will demonstrate that one can obtain firm predictions for the CSV corrections, and that the CSV contributions to the Paschos-Wolfenstein ratio are essentially model independent. The charge symmetry violating contribution to the Paschos-Wolfenstein ratio has the form

$$\Delta R_{CSV} = \left[3\Delta_u^2 + \Delta_d^2 + \frac{4\alpha_s}{9\pi}(\overline{g_L^2} - \overline{g_R^2})\right]\left[\frac{\delta U_{\rm v} - \delta D_{\rm v}}{2(U_{\rm v} + D_{\rm v})}\right] \quad (4)$$

where

$$\delta Q_{\rm v} = \int_0^1 x\, \delta q_{\rm v}(x)\, dx$$

$$\delta d_{\rm v}(x) = d_{\rm v}^p(x) - u_{\rm v}^n(x), \quad \delta u_{\rm v}(x) = u_{\rm v}^p(x) - d_{\rm v}^n(x). \quad (5)$$

The denominator in the final term in Eq. (4) gives the total momentum carried by up and down valence quarks, while the numerator gives the charge symmetry violating momentum difference, e.g., $\delta U_{\rm v}$ is the total momentum carried by up quarks in the proton minus the momentum of down quarks in the neutron. As for the isoscalar correction, this ratio is completely independent of Q^2 and can be evaluated at any convenient value of Q^2.

In our paper [11] we used an analytic approximation to the charge symmetry violating valence parton distributions that was initially proposed by Sather [12]. His equations were

$$\delta d_{\rm v}(x) = -\frac{\delta M}{M}\frac{d}{dx}[x d_{\rm v}(x)] - \frac{\delta m}{M}\frac{d}{dx} d_{\rm v}(x)$$

$$\delta u_{\rm v}(x) = \frac{\delta M}{M}\left(-\frac{d}{dx}[x u_{\rm v}(x)] + \frac{d}{dx} u_{\rm v}(x)\right). \quad (6)$$

In Eq. (6), M is the average nucleon mass, $\delta M = 1.3$ MeV is the neutron-proton mass difference, and $\delta m = m_d - m_u \sim 4$ MeV is the down-up quark mass difference. Equation (6) is valid for a low scale, Q_0^2, appropriate for a (valence dominated) quark or bag model [14].

Sather's approximation allows us to evaluate directly the relevant integrals of the CSV distributions. For $\delta D_{\rm v}$, we obtain

$$\begin{aligned}\delta D_{\rm v} &= \int_0^1 x\left[-\frac{\delta M}{M}\frac{d}{dx}(x d_{\rm v}(x)) - \frac{\delta m}{M}\frac{d}{dx} d_{\rm v}(x)\right] dx \\ &= \frac{\delta M}{M}\int_0^1 x d_{\rm v}(x)\, dx + \frac{\delta m}{M}\int_0^1 d_{\rm v}(x)\, dx \\ &= \frac{\delta M}{M} D_{\rm v} + \frac{\delta m}{M}. \end{aligned} \quad (7)$$

The second line of Eq. (7) is obtained by integrating by parts, using the fact that there is one down valence quark in the proton. In an analogous fashion, the integral of the up quark CSV distribution is

$$\begin{aligned}\delta U_{\rm v} &= \frac{\delta M}{M}\int_0^1 x\left(-\frac{d}{dx}[x u_{\rm v}(x)] + \frac{d}{dx} u_{\rm v}(x)\right) dx \\ &= \frac{\delta M}{M}\left(\int_0^1 x u_{\rm v}(x)\, dx - \int_0^1 u_{\rm v}(x)\, dx\right) \\ &= \frac{\delta M}{M}(U_{\rm v} - 2). \end{aligned} \quad (8)$$

Using Sather's approximation relating CSV distributions to valence quark distributions, Eqs. (7) and (8) show that the CSV correction to the Paschos-Wolfenstein ratio depends only on the fraction of the nucleon momentum carried by up and down valence quarks. At no point do we have to calculate specific CSV distributions. At the bag model scale, $Q_0^2 \approx 0.5\ \mathrm{GeV}^2$, the momentum fraction carried by down valence quarks, $D_{\rm v}$, is between 0.2 and 0.33, and the total momentum fraction carried by valence quarks is $U_{\rm v} + D_{\rm v} \sim .80$. From Eqs. (7) and (8) this gives $\delta D_{\rm v} \approx 0.00463$, $\delta U_{\rm v} \approx -0.00203$. Consequently, evaluated at the quark model scale, the CSV correction to the Paschos-Wolfenstein ratio is

$$\Delta R_{CSV} \approx [3\Delta_u^2 + \Delta_d^2]\frac{\delta U_{\rm v} - \delta D_{\rm v}}{2(U_{\rm v} + D_{\rm v})} \approx -0.00203. \quad (9)$$

Once the CSV correction has been calculated at some quark model scale, Q_0^2, the ratio appearing in Eq. (4) is independent of Q^2 because both the numerator and the denominator involve the same moment of non-singlet distributions [in Eq. (9) we have dropped the QCD radiative correction, which is very small at the Q^2 appropriate to the NuTeV measurements].

We stress that both Eqs. (7) and (8) are only weakly dependent on the choice of quark model scale—through the momentum fractions $D_{\rm v}$ and $U_{\rm v}$, which are slowly varying functions of Q_0^2, and are not the dominant terms in those equations. This, together with the Q^2 independence of the Paschos-Wolfenstein ratio, Eq. (4), under QCD evolution, explains why our previous results, obtained with different models at different Q^2 values [11], were so similar. For example, the result of Eq. (9) $\Delta R_{CSV}=-0.00203$, at $Q_0^2=0.5\ \mathrm{GeV}^2$, is virtually identical with results using the Rodionov CSV distribution (-0.0020) and the Sather CSV distribution (-0.0021), at $Q^2=10$ and $12.6\ \mathrm{GeV}^2$, respectively. Using Eqs. (7) and (8), we also calculated a CSV distribution using the CTEQ4LQ phenomenological parton distribution [15] at $Q^2=0.49\ \mathrm{GeV}^2$, evolved this to $20\ \mathrm{GeV}^2$, and obtained $\Delta R_{CSV}=-0.0021$ [16].

Cao and Signal [5] point out some limitations of Sather's approximation, Eq. (6). However, we have compared $\delta U_{\mathrm{v}}-\delta D_{\mathrm{v}}$ obtained by Sather [12], and by Rodionov *et al.* [13], who did not use Sather's approximation, and they differ by only a few percent.

As noted earlier, the NuTeV group [2,8] do not measure the Paschos-Wolfenstein ratio, but combine separate measurements of neutrinos and anti-neutrinos with a Monte Carlo simulation of their experiment. They have produced functionals giving the sensitivity of their observables to various effects, including parton charge symmetry violation. These are summarized in a single integral

$$\Delta\mathcal{E}=\int_0^1 F[\mathcal{E},\delta;x]\,\delta(x)\,dx. \qquad (10)$$

Equation (10) gives the change in the extracted quantity $\mathcal{E}$ resulting from the symmetry violating quantity $\delta(x)$. The functionals appropriate for the observable $\sin^2\theta_W$ and the parton CSV distributions were provided in Ref. [8].

In our previous paper we found that including the NuTeV functional with evolved distributions decreased the CSV correction by about 33% from the Paschos-Wolfenstein result. This is very similar to the 36% reduction obtained by NuTeV for the isoscalar correction. After applying this reduction, the CSV correction to the NuTeV experiment is -0.0015. When the NuTeV measurement is adjusted accordingly, the disagreement between the NuTeV and electromagnetic results for $\sin^2\theta_W$ is reduced from 0.0050 to 0.0035—a 30% decrease in that discrepancy.

In conclusion, we have a robust prediction for the CSV contribution to the Paschos-Wolfenstein ratio, and also to the NuTeV measurement of the Weinberg angle. The Sather approximation allows us to write integrals of $x\,\delta q_{\mathrm{v}}$ in terms of the total momentum carried by valence quarks. These integrals can be calculated without ever specifying the CSV distributions. The Paschos-Wolfenstein ratio involves ratios of integrals that behave identically under QCD evolution, so these ratios are independent of Q^2. Despite the fact that parton charge symmetry violation has not been directly measured experimentally, and that parton CSV effects are predicted to be quite small, we have strong theoretical arguments regarding both the sign and magnitude of these corrections. The CSV effects should make a significant contribution to the value for the Weinberg angle extracted from the NuTeV neutrino measurements.

This work was supported in part by the Australian Research Council. One of the authors (J.T.L.) was supported in part by National Science Foundation research contract PHY-0070368. The authors wish to thank G.P. Zeller and K. McFarland of the NuTeV Collaboration, and W. Melnitchouk, for useful discussions regarding these issues.

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