

Noncommutative $U(1)$ instantons in eight dimensional Yang-Mills theory

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We study the noncommutative version of the extended ADHM construction in eight dimensional $U(1)$ Yang-Mills theory. This construction gives rise to the solutions of the BPS equations in Yang-Mills theory, and these solutions preserve at least $3/16$ of supersymmetries. In a wide subspace of the extended ADHM data, we show that the integer k which appears in the extended ADHM construction should be interpreted as the $D4$ -brane charge rather than the $D0$ -brane charge by explicitly calculating the topological charges in the case that the noncommutativity parameter is anti-self-dual. We also find the relationship with the solution generating technique and show that the integer k can be interpreted as the charge of the $D0$ -brane bound to the $D8$ -brane with the B field in the case that the noncommutativity parameter is self-dual.

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I. INTRODUCTION

Noncommutative geometry has played an important role in the study of string or M theory [1]. In particular, D -branes with a constant Neveu-Schwarz (NS) B field are of interest in the context of understanding the nonperturbative aspects of string theory. The effective world-volume field theory on D -branes with a B field turns out to be the noncommutative Yang-Mills theory [2], which has an interesting feature that the singularity of the instanton moduli space is naturally resolved [3].

Four dimensional $U(N)$, k instantons are realized as k $D0$ -branes within N $D4$ -branes in type IIA string theory. When we turn on a constant B field which preserves $1/4$ of supersymmetries, the moduli space of the noncommutative instantons is resolved and the $D0$ -branes cannot escape from the $D4$ -branes. From the viewpoint of $D0$ -brane theory, the Higgs branch of the moduli space coincides with the moduli space of the noncommutative instantons and the B field corresponds to the Fayet-Iliopoulos parameters.

The instanton solutions of Yang-Mills theory are constructed by the well-known Atiyah-Drinfeld-Hitchin-Manin (ADHM) method. There is the one-to-one correspondence between the moduli space of the instantons and that of the ADHM data in the commutative case [4,5]. On the other hand, most of the noncommutative instantons in four dimensions have been obtained by modifying the ADHM construction. See, e.g. [3,6–23] and references therein. In particular, it has been proven that the instanton number is generally an integer in the noncommutative $U(N)$ gauge theory by Sako [17].

It is also of interest to generalize the above system to higher dimensions in the context of both D -brane dynamics and the world-volume theories. The systems of the $D0$ - $D6$ and the $D0$ - $D8$ with a B field have been investigated by several authors [24–37]. Especially we consider the system of the $D0$ -brane and the $D8$ -brane with a B field. These studies reduce to finding the solutions of the higher dimensional analogue of the “self-duality” equations which are the

first order linear relations amongst the components of the field strength [36,38–40]. It has been shown that there are many kinds of the BPS equations which preserve $1/16, 2/16, \dots, 6/16$ of supersymmetries and these equations are related to the subgroup $SO(7), SO(6), \dots, SO(2)$ of the eight dimensional rotation group $SO(8)$.

In this paper, we focus on the case that is related to the $SO(5) = Sp(2)$ symmetry. In this case the configuration of the gauge field preserves at least $3/16$ of supersymmetries. It is known that there is the extended ADHM construction which gives rise to the solutions of the $3/16$ BPS equations in eight dimensions [41]. We consider the noncommutative $U(1)$ gauge theory and study the noncommutative version of this extended ADHM construction [30]. It is worth constructing the simple solutions explicitly and investigating their properties such as the topological charges since little about the noncommutative version of the extended ADHM construction is known until now. This subject has been studied in some references [30,31,34].

This paper is organized as follows. In Sec. II, we review the Bogomol’nyi-Prasad-Sommerfield (BPS) equations and the extended ADHM construction in eight dimensions. In Sec. III, we briefly review the Yang-Mills theory on the noncommutative space. As in the four dimensional case, it is an important difference whether the noncommutativity parameter is anti-self-dual or self-dual. In Sec. IV we consider the case that the noncommutativity parameter is anti-self-dual. In a wide subspace of the extended ADHM data, we show that the integer k which appears in the extended ADHM construction should be interpreted as the $D4$ -brane charge rather than the $D0$ -brane charge by explicitly calculating the topological charges. In Sec. V, we consider the case that the noncommutativity parameter is self-dual. We find the relationship with the solution generating technique and show that the integer k can be interpreted as the charge of the $D0$ -brane bound to the $D8$ -brane with a B field. The final section is devoted to the conclusion.

II. EXTENDED ADHM CONSTRUCTION OF EIGHT DIMENSIONAL INSTANTONS

In this section, we review the BPS equations and the extended ADHM construction of the instantons in eight dimen-

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sions. The instantons in higher dimensions are defined as the solutions of the BPS equations. This definition is the natural generalization of the four dimensional $U(N)$, k instantons which are constructed by the ADHM construction with the gauge group $U(N)$ and the ADHM parameter k . These instantons have the D -brane interpretation as the bound states of k $D0$ -branes and N $D4$ -branes.

Therefore, in the following, we consider the extended ADHM construction with the gauge group $U(N)$ and the extended ADHM parameter k since these situations are expected to correspond to the systems of k $D0$ -branes and N $D8$ -branes.

A. BPS equations in eight dimensions

In this subsection, we briefly review the BPS equations in the eight dimensional Yang-Mills theory. These equations were studied in Refs. [38–40], and systematically classified by the authors of [36]. The BPS equations are the higher dimensional analogue of the “self-duality” equations, which are the linear relations amongst the components of the field strength

$$\frac{1}{2}T_{\mu\nu\rho\sigma}F_{\rho\sigma}=F_{\mu\nu} \quad (\mu, \nu, \rho, \sigma=1, \dots, 8), \quad (2.1)$$

with the constant 4-form tensor $T_{\mu\nu\rho\sigma}$. These equations are the natural generalizations of the four dimensional self-duality equations:

$$\frac{1}{2}\epsilon_{abcd}F_{cd}=F_{ab} \quad (a, b, c, d=1, \dots, 4). \quad (2.2)$$

When the equations (2.1) hold, the equations of motion, $D_\mu F_{\mu\nu}=0$, are automatically satisfied due to the Jacobi identity, and the lower bound of the action is reached. This bound is obtained as in the four dimensional case by the identity

$$-\frac{1}{4}F_{\mu\nu}F_{\mu\nu}=-\frac{1}{16}\left(F_{\mu\nu}-\frac{1}{2}T_{\mu\nu\rho\sigma}F_{\rho\sigma}\right)^2 -\frac{1}{8}T_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}, \quad (2.3)$$

where the gauge field is taken to be anti-Hermitian. This identity was shown by the authors of [36].

It was also shown in Ref. [36] that there are many kinds of the BPS equations which preserve $1/16, 2/16, \dots, 6/16$ of supersymmetries and these equations are related to the subgroup $SO(7), SO(6) \dots, SO(2)$ of the eight dimensional rotation group $SO(8)$. Especially in this paper we concentrate on the case that is related to the $SO(5)=Sp(2)$ symmetry. In this case the configuration of the gauge field preserves at least $3/16$ of supersymmetries.

B. Extended ADHM construction on \mathbf{R}^8

The ADHM construction is a powerful tool to construct the Yang-Mills instantons in four dimensions [4,5]. Espe-

cially, it is well known that the instanton moduli space and the ADHM moduli space completely coincide in four dimensions. It is also known that there exists the extended ADHM construction which gives rise to solutions of the $3/16$ BPS equations in eight dimensions. This extended ADHM construction was investigated by several authors [30,31,33–35,41].

In this subsection, we review this extended ADHM construction of the eight dimensional “self-dual” instantons associated with the $Sp(2)$ group given in [39,41]. This construction of the instantons is the slight extension of the four dimensional ADHM construction. When we take $\mathbf{B}=0$, or $\mathbf{B}'=0$ which are defined in the following, we reproduce the four dimensional ADHM construction.

In order to treat the eight dimensional space, it is useful to regard eight coordinates of the \mathbf{R}^8 as two quaternionic coordinates:

$$\mathbf{x}=\sum_{\mu=1}^8\tilde{\sigma}_\mu x^\mu=\begin{pmatrix} z_2 & z_1 \\ -\bar{z}_1 & \bar{z}_2 \end{pmatrix},$$

$$\mathbf{x}'=\sum_{\mu=1}^8\tilde{\sigma}'_\mu x^\mu=\begin{pmatrix} z_4 & z_3 \\ -\bar{z}_3 & \bar{z}_4 \end{pmatrix}. \quad (2.4)$$

Here we defined the eight vector matrices using the Pauli matrices τ_i ($i=1,2,3$) by

$$\tilde{\sigma}_\mu=(i\tau_1,0,i\tau_2,0,i\tau_3,0,\mathbf{1}_2,0),$$

$$\tilde{\sigma}'_\mu=(0,i\tau_1,0,i\tau_2,0,i\tau_3,0,\mathbf{1}_2), \quad (2.5)$$

and the four complex coordinates by

$$z_1=x^3+ix^1, \quad z_2=x^7+ix^5, \quad z_3=x^4+ix^2, \quad z_4=x^8+ix^6. \quad (2.6)$$

The four dimensional ADHM construction gives rise to the instantons through the zero mode of a zero dimensional massless Dirac-like operator. This construction can be easily extended to the eight dimensional case. At first we define the Dirac-like operator

$$D_z=\mathbf{A}+\vec{\mathbf{B}}\cdot\vec{\mathbf{X}}, \quad (2.7)$$

using the $(N+2k)\times 2k$ matrices \mathbf{A} , \mathbf{B} and \mathbf{B}' . Here we also define $\vec{\mathbf{B}}=(\mathbf{B},\mathbf{B}')$ and $\vec{\mathbf{X}}=(\mathbf{x},\mathbf{x}')$. Then we can construct the $U(N)$ gauge field as

$$A_\mu=\psi^\dagger\partial_\mu\psi, \quad (2.8)$$

where the $(N+2k)\times N$ matrix ψ is the solution of the following Dirac-like equation

$$D_z^\dagger\psi=0, \quad (2.9)$$

and is normalized as $\psi^\dagger\psi=\mathbf{1}_{N\times N}$.

We can see that the gauge field (2.8) gives the “self-dual” field strength as

$$F_{\mu\nu} = 2\psi^\dagger \left(\partial_{[\mu} D_z \frac{1}{D_z^\dagger D_z} \partial_{\nu]} D_z^\dagger \right) \psi = 2\psi^\dagger \vec{\mathbf{B}} \bar{N}_{\mu\nu} \frac{1}{D_z^\dagger D_z} \vec{\mathbf{B}}^\dagger \psi, \quad (2.10)$$

where we used the completeness relation

$$\mathbf{1}_{N+2k} = \psi\psi^\dagger + D_z \frac{1}{D_z^\dagger D_z} D_z^\dagger, \quad (2.11)$$

and defined the ‘‘self-dual’’ tensor $\bar{N}_{\mu\nu}$. This tensor is explicitly written as

$$\bar{N}_{\mu\nu} = \frac{1}{2} (\Sigma_\mu \Sigma_\nu^\dagger - \Sigma_\nu \Sigma_\mu^\dagger), \quad (2.12)$$

by using the following quantity:

$$\Sigma_\mu \equiv \partial_\mu \vec{\mathbf{X}} = \begin{pmatrix} \tilde{\sigma}_\mu \\ \tilde{\sigma}'_\mu \end{pmatrix}, \quad (2.13)$$

and satisfies the ‘‘self-duality’’ equation,

$$\frac{1}{2} T_{\mu\nu\rho\sigma} \bar{N}_{\rho\sigma} = \bar{N}_{\mu\nu}, \quad (2.14)$$

where $T_{\mu\nu\rho\sigma}$ is the $Sp(2)$ invariant tensor.

The identity (2.14) is surely identical to the ‘‘self-duality’’ equation (2.1). Therefore we can automatically construct the solutions of the 3/16 BPS equations through the extended ADHM construction given above. In our choice of the complex coordinates (2.6), we can write down the general form of the field strength which satisfies the ‘‘self-duality’’ equation (2.14) as

$$\begin{aligned} F = & F_{z_1 \bar{z}_1} (dz_1 \wedge d\bar{z}_1 + dz_2 \wedge d\bar{z}_2) + F_{z_3 \bar{z}_3} (dz_3 \wedge d\bar{z}_3 \\ & + dz_4 \wedge d\bar{z}_4) + F_{z_1 \bar{z}_3} (dz_1 \wedge d\bar{z}_3 + dz_2 \wedge d\bar{z}_4) \\ & + F_{z_1 \bar{z}_3} (d\bar{z}_1 \wedge dz_3 + d\bar{z}_2 \wedge dz_4) \\ & + F_{z_1 \bar{z}_4} (dz_1 \wedge dz_4 + dz_3 \wedge dz_2) \\ & + F_{z_1 \bar{z}_4} (d\bar{z}_1 \wedge d\bar{z}_4 + d\bar{z}_3 \wedge d\bar{z}_2) \\ & + F_{z_1 \bar{z}_2} dz_1 \wedge dz_2 + F_{z_1 \bar{z}_2} d\bar{z}_1 \wedge d\bar{z}_2 \\ & + F_{z_3 \bar{z}_4} dz_3 \wedge dz_4 + F_{z_3 \bar{z}_4} d\bar{z}_3 \wedge d\bar{z}_4. \end{aligned} \quad (2.15)$$

C. Extended ADHM equations

It is crucial that the $D_z^\dagger D_z$ is invertible and commutes with Σ_μ . This is a necessary condition to obtain the ‘‘self-dual’’ gauge field strength on the \mathbf{R}^8 . This condition corresponds to the extended ADHM equations in eight dimensions both for the commutative and the noncommutative case.

Before writing this condition, we notice that there are equivalence relations between different sets of \mathbf{A} , \mathbf{B} , and \mathbf{B}' ,

$$\mathbf{A} \sim U\mathbf{A}M, \quad \mathbf{B} \sim U\mathbf{B}M, \quad \mathbf{B}' \sim U\mathbf{B}'M, \quad (2.16)$$

where $U \in U(N+2k)$ and $M \in GL(2k, \mathbf{C})$. The gauge field is invariant under this transformation. Using these relations, we can reduce \mathbf{A} , \mathbf{B} , and \mathbf{B}' in the form

$$\mathbf{A} = \begin{pmatrix} A_2 & A_1 \\ -A_1^\dagger & A_2^\dagger \\ I & J \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \mathbf{1}_k & 0 \\ 0 & \mathbf{1}_k \\ 0 & 0 \end{pmatrix},$$

$$\mathbf{B}' = \begin{pmatrix} B_2 & B_1 \\ -B_1^\dagger & B_2^\dagger \\ K & L \end{pmatrix}. \quad (2.17)$$

Here A_i and B_i ($i=1,2$) are $k \times k$ matrices, and I, J, K , and L are $N \times k$ matrices. In this representation, we can write down the condition that the $D_z^\dagger D_z$ commutes with Σ_μ [30].

In this way, the extended ADHM equations in the commutative case can be obtained as

$$\mu_{\mathbf{R}}^1 = \mu_{\mathbf{C}}^1 = \mu_{\mathbf{C}}^2 = \mu_{\mathbf{C}}^{2'} = \mu_{\mathbf{R}}^3 = \mu_{\mathbf{C}}^3 = 0, \quad (2.18)$$

where several quantities are defined by

$$\begin{aligned} \mu_{\mathbf{R}}^1 &= [A_2^\dagger, A_2] - [A_1^\dagger, A_1] + I^\dagger I - J^\dagger J, \\ \mu_{\mathbf{C}}^1 &= [A_2^\dagger, A_1] + I^\dagger J, \\ \mu_{\mathbf{C}}^2 &= [A_2^\dagger, B_2] - [B_1^\dagger, A_1] + I^\dagger K - L^\dagger J, \\ \mu_{\mathbf{C}}^{2'} &= [A_2^\dagger, B_1] + [B_2^\dagger, A_1] + I^\dagger L + K^\dagger J, \\ \mu_{\mathbf{R}}^3 &= [B_2^\dagger, B_2] - [B_1^\dagger, B_1] + K^\dagger K - L^\dagger L, \\ \mu_{\mathbf{C}}^3 &= [B_2^\dagger, B_1] + K^\dagger L. \end{aligned} \quad (2.19)$$

There are two real and four complex equations, which are related to the adjoint representation $\mathbf{10}$ of $Sp(2)$. Therefore the moduli space of the eight dimensional instantons we consider here is expected to have the structure of the $Sp(2)$ holonomy. Examples of the $Sp(2)$ holonomy manifold are given in Refs. [42,43].

III. YANG-MILLS THEORY ON NONCOMMUTATIVE SPACE

In this section, we briefly review the Yang-Mills theory on the noncommutative space. A field theory on the noncommutative space is defined by deforming the ring of functions on it. More concretely, the product of functions f and g is replaced with the Moyal star product that is defined by

$$(f \star g)(x) \equiv e^{(i/2)\theta^{\mu\nu} \partial_\mu \partial'_\nu} f(x) g(x') \Big|_{x=x'}. \quad (3.1)$$

This equation implies that

$$[x^\mu, x^\nu] = x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu}. \quad (3.2)$$

This commutation relation characterizes the noncommutative space we treat in this paper. The constant $\theta^{\mu\nu}$ is called the noncommutativity parameter.

There is another equivalent description of the noncommutative space, which is called the operator formalism and is useful for explicit calculations. These two descriptions are related via the Moyal-Weyl correspondence. In the operator formalism, we regard the coordinates as operators. In this section, we denote the hat on the operators in order to emphasize that they are operators. The commutation relation between coordinates becomes as follows:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}. \quad (3.3)$$

This relation is represented by using operators which act on the Hilbert space \mathcal{H} .

The derivative of an operator \mathcal{O} is defined by

$$\partial_\mu \mathcal{O} \equiv [\hat{\partial}_\mu, \mathcal{O}] \quad \text{where} \quad \hat{\partial}_\mu \equiv -i(\theta^{-1})_{\mu\nu} \hat{x}^\nu. \quad (3.4)$$

This derivative satisfies the Leibniz rule and the relations

$$\partial_\mu \hat{x}^\nu = \delta_\mu^\nu \quad \text{and} \quad [\hat{\partial}_\mu, \hat{\partial}_\nu] = i(\theta^{-1})_{\mu\nu}. \quad (3.5)$$

The integral of an operator \mathcal{O} is defined by the trace over the Hilbert space \mathcal{H} as follows:

$$\int d^D x \mathcal{O}(x) \equiv (2\pi)^D \sqrt{\det \theta} \text{Tr}_{\mathcal{H}} \mathcal{O}(x). \quad (3.6)$$

We note that the strength of the gauge field \hat{A}_μ can be written as

$$\hat{F}_{\mu\nu} = [\hat{X}_\mu, \hat{X}_\nu] - i(\theta^{-1})_{\mu\nu}, \quad (3.7)$$

where the anti-Hermitian operator \hat{X}_μ is defined by

$$\hat{X}_\mu \equiv \hat{\partial}_\mu + \hat{A}_\mu. \quad (3.8)$$

In this way the action of the noncommutative Yang-Mills theory is expressed by

$$S = -\frac{1}{4} (2\pi)^D \sqrt{\det \theta} \text{Tr}_{\mathcal{H}} \text{tr}_{U(N)} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu}. \quad (3.9)$$

Here $\text{tr}_{U(N)}$ denotes the trace over the $U(N)$ matrix. In the following sections, we omit the hat on operators for simplicity of the description.

IV. $U(1)$ INSTANTON IN THE CASE OF ANTI-SELF-DUAL NONCOMMUTATIVITY

In this section, we study the noncommutative $U(1)$ instantons on \mathbf{R}^8 in the case that the noncommutativity parameter is anti-self-dual. In a wide subclass of the extended ADHM data, we show that the integer k which appears in the extended ADHM construction should be interpreted as the $D4$ -brane charge rather than the $D0$ -brane charge.

As in the four dimensional case, it is easy to generalize the extended ADHM construction in eight dimensions to the noncommutative space because of its algebraic nature. Since

we define the instantons as ‘‘self-dual’’ configurations, the ‘‘anti-self-dual’’ noncommutativity parameter is expected to be of interest from the viewpoint of the resolution of the instanton moduli space.

Concretely, we introduce the anti-self-dual noncommutativity parameter as follows:

$$\theta^{13} = -\theta^{57} = \theta^{24} = -\theta^{68} = \frac{\zeta}{4} \quad (\zeta > 0). \quad (4.1)$$

This implies the commutation relations of the complex coordinates:

$$\begin{aligned} [z_1, \bar{z}_1] &= -[z_2, \bar{z}_2] = [z_3, \bar{z}_3] = -[z_4, \bar{z}_4] \\ &= -\frac{\zeta}{2}, \quad \text{others are zero.} \end{aligned} \quad (4.2)$$

These relations are the same as those of the harmonic oscillators up to the multiplication of constants. Therefore we define the creation and annihilation operators by

$$a_m^\dagger = \sqrt{\frac{2}{\zeta}} z_m, \quad a_m = \sqrt{\frac{2}{\zeta}} \bar{z}_m \quad \text{for } m=1,3, \quad (4.3)$$

as well as

$$a_m^\dagger = \sqrt{\frac{2}{\zeta}} \bar{z}_m, \quad a_m = \sqrt{\frac{2}{\zeta}} z_m \quad \text{for } m=2,4. \quad (4.4)$$

The number operators can also be defined as

$$n_m = a_m^\dagger a_m = \begin{cases} \frac{2}{\zeta} z_m \bar{z}_m & \text{for } m=1,3, \\ \frac{2}{\zeta} \bar{z}_m z_m & \text{for } m=2,4. \end{cases} \quad (4.5)$$

The Fock space \mathcal{H} on which the creation and annihilation operators (4.3) and (4.4) act is spanned by the direct product of the Fock state: $|n_1 : n_2 : n_3 : n_4\rangle \equiv |n_1\rangle \otimes |n_2\rangle \otimes |n_3\rangle \otimes |n_4\rangle$. The creation and annihilation operators act on each Fock state as follows:

$$\begin{aligned} a_m |n_m\rangle &= \sqrt{n_m} |n_m - 1\rangle, \\ a_m^\dagger |n_m\rangle &= \sqrt{n_m + 1} |n_m + 1\rangle \quad \text{for } m=1, \dots, 4. \end{aligned} \quad (4.6)$$

The noncommutativity of the complex coordinates (4.2) deforms the extended ADHM equations (2.18) as follows:

$$\mu_{\mathbf{R}}^1 = \zeta(\mathbf{1}_{k \times k} + \Xi), \quad \mu_{\mathbf{C}}^1 = \mu_{\mathbf{C}}^2 = \mu_{\mathbf{C}}^{2'} = \mu_{\mathbf{R}}^3 = \mu_{\mathbf{C}}^3 = 0, \quad (4.7)$$

where Ξ is defined by

$$\Xi \equiv \frac{1}{2} (\{B_2^\dagger, B_2\} + \{B_1^\dagger, B_1\} + K^\dagger K + L^\dagger L). \quad (4.8)$$

These equations were originally obtained in Ref. [30].

A. $U(1)$, $k=1$ solution

In this subsection, we start by constructing the $U(1)$, $k=1$ solution explicitly and discuss its D -brane interpretation by calculating the topological charges and the value of the action. It becomes clear that the $U(1)$, $k=1$ solution should be interpreted as the bound state of the $D4$ -brane and the $D8$ -brane with a B field rather than that of the $D0$ -brane and the $D8$ -brane with a B field.

An important fact for the $U(1)$ case is that we are allowed to take $J=K=L=0$ [7]. Then we can obtain the nontrivial solution of the noncommutative version of the extended ADHM equations (4.7) by

$$A_1=A_2=B_1=0, \quad B_2=a, \quad I=\sqrt{\zeta(1+a^2)}, \quad (4.9)$$

where the parameter a is an arbitrary real number. The Dirac-like operator becomes as follows:

$$D_z^\dagger = \begin{pmatrix} \bar{z}_2 + a\bar{z}_4 & -z_1 - az_3 & \sqrt{\zeta(1+a^2)} \\ \bar{z}_1 + a\bar{z}_3 & z_2 + az_4 & 0 \end{pmatrix}. \quad (4.10)$$

The zero mode ψ of D_z^\dagger is a 3×1 matrix which is written as $\psi \equiv (\psi_1 \ \psi_2 \ \xi)^T$. Each component of ψ is explicitly calculated as

$$\begin{aligned} \psi_1 &= -\frac{I}{\delta + a\eta + I^2/2} \frac{1}{\sqrt{1 + I^2(\delta + a\eta + I^2/2)^{-1}}} (z_2 + az_4), \\ \psi_2 &= \frac{I}{\delta + a\eta + I^2/2} \frac{1}{\sqrt{1 + I^2(\delta + a\eta + I^2/2)^{-1}}} (\bar{z}_1 + a\bar{z}_3), \\ \xi &= \frac{1}{\sqrt{1 + I^2(\delta + a\eta)^{-1}}}, \end{aligned} \quad (4.11)$$

where we defined the quantities δ and η by

$$\delta \equiv z_1\bar{z}_1 + \bar{z}_2z_2 + a^2(z_3\bar{z}_3 + \bar{z}_4z_4), \quad (4.12)$$

$$\eta \equiv \bar{z}_4z_2 + \bar{z}_2z_4 + z_1\bar{z}_3 + z_3\bar{z}_1. \quad (4.13)$$

The following formulas are useful for the calculations:

$$\begin{aligned} (\bar{z}_1 + a\bar{z}_3)f(\delta + a\eta) &= f(\delta + a\eta + I^2/2)(\bar{z}_1 + a\bar{z}_3), \\ (\bar{z}_2 + a\bar{z}_4)f(\delta + a\eta) &= f(\delta + a\eta - I^2/2)(\bar{z}_2 + a\bar{z}_4). \end{aligned} \quad (4.14)$$

Substituting the zero mode ψ into the equations (2.10), the field strength form is explicitly obtained as follows:

$$\begin{aligned} F &= F_{z_1\bar{z}_1} [dz_1 \wedge d\bar{z}_1 + dz_2 \wedge d\bar{z}_2 + a^2(dz_3 \wedge d\bar{z}_3 + dz_4 \wedge d\bar{z}_4) \\ &\quad + a(dz_1 \wedge d\bar{z}_3 + dz_2 \wedge d\bar{z}_4 + dz_3 \wedge d\bar{z}_1 + dz_4 \wedge d\bar{z}_2)] \\ &\quad + F_{z_1z_2} [dz_1 \wedge dz_2 + a(dz_1 \wedge dz_4 + dz_3 \wedge dz_2) \\ &\quad + a^2dz_3 \wedge dz_4] + F_{\bar{z}_1\bar{z}_2} [d\bar{z}_1 \wedge d\bar{z}_2 + a(d\bar{z}_1 \wedge d\bar{z}_4 \\ &\quad + d\bar{z}_3 \wedge d\bar{z}_2) + a^2d\bar{z}_3 \wedge d\bar{z}_4], \end{aligned} \quad (4.15)$$

where the nontrivial components are explicitly given by

$$\begin{aligned} F_{z_1\bar{z}_1} &= \frac{I^4}{(\delta + a\eta)(\delta + a\eta + I^2/2)(\delta + a\eta + I^2)} J_3, \\ F_{z_1z_2} &= -\frac{\sqrt{2}I^4}{(\delta + a\eta)(\delta + a\eta + I^2/2)(\delta + a\eta + I^2)} J_+, \\ F_{\bar{z}_1\bar{z}_2} &= \frac{\sqrt{2}I^4}{(\delta + a\eta)(\delta + a\eta + I^2/2)(\delta + a\eta + I^2)} J_-. \end{aligned} \quad (4.16)$$

Here as in the case of the four dimensional noncommutative instantons, we introduced the operators J_+ , J_- , and J_3 [11] by

$$\begin{aligned} J_+ &= \frac{\sqrt{2}}{I^2} (\bar{z}_2 + a\bar{z}_4)(\bar{z}_1 + a\bar{z}_3), \\ J_- &= \frac{\sqrt{2}}{I^2} (z_1 + az_3)(z_2 + az_4), \\ J_3 &= \frac{1}{I^2} [(\bar{z}_2 + a\bar{z}_4)(z_2 + az_4) \\ &\quad - (z_1 + az_3)(\bar{z}_1 + a\bar{z}_3)]. \end{aligned} \quad (4.17)$$

These operators are found to satisfy the Lie algebra of $SU(2)$:

$$[J_+, J_-] = J_3, \quad [J_3, J_\pm] = \pm J_\pm. \quad (4.18)$$

In the rest of this subsection, we discuss the properties of the above solution (4.15). At first, we can explicitly calculate the eight form charge $Q^{(8)}$:

$$Q^{(8)} \equiv \frac{1}{4!(2\pi)^4} \int_{\mathbf{R}^8} F \wedge F \wedge F \wedge F = 0. \quad (4.19)$$

Therefore the solution (4.15) does not have the $D0$ -brane charge.

We can also calculate the value of the action of the solution as

$$\begin{aligned} S &= -\frac{2I^4}{\zeta^2} \int d^8x (F_{z_1z_2} F_{\bar{z}_1\bar{z}_2} + F_{\bar{z}_1\bar{z}_2} F_{z_1z_2} - 2F_{z_1\bar{z}_1} F_{z_1\bar{z}_1}) \\ &= \pi^2 I^8 \left(\frac{\pi\zeta}{2}\right)^2 \text{Tr}_{\mathcal{H}} \frac{1}{(\delta + a\eta)(\delta + a\eta + I^2/2)^2(\delta + \eta + I^2)}. \end{aligned} \quad (4.20)$$

Here we used the integral formula (3.6) and the identity

$$J_+ J_- + J_- J_+ + J_3^2 = \frac{1}{I^4} (\delta + a\eta)(\delta + a\eta + I^2). \quad (4.21)$$

To carry out the trace over the Hilbert space \mathcal{H} , we have to make the quantity $\delta + a\eta$ diagonal. So we make the unitary transformation and define new creation and annihilation operators:

$$\tilde{a}_1 = \frac{1}{\sqrt{1+a^2}}(a_1 + aa_3), \quad \tilde{a}_2 = \frac{1}{\sqrt{1+a^2}}(a_2 + aa_4), \quad (4.22)$$

$$\tilde{a}_3 = \frac{1}{\sqrt{1+1/a^2}}(a_1 - a^{-1}a_3), \quad \tilde{a}_4 = \frac{1}{\sqrt{1+1/a^2}}(a_2 - a^{-1}a_4).$$

These new operators also satisfy the commutation relations of the harmonic oscillators,

$$[\tilde{a}_m, \tilde{a}_n^\dagger] = \delta_{m,n} \quad \text{for } m, n = 1, \dots, 4, \quad \text{others are zero}, \quad (4.23)$$

and the quantity $\delta + a\eta$ is made diagonal in the number basis of new harmonic oscillators (4.22) as

$$\delta + a\eta = \frac{I^2}{2}(\tilde{a}_1^\dagger \tilde{a}_1 + \tilde{a}_2^\dagger \tilde{a}_2) \equiv \frac{I^2}{2}(\tilde{n}_1 + \tilde{n}_2). \quad (4.24)$$

Then we are able to carry out the calculation as follows:

$$\begin{aligned} S &= 16\pi^2 \left(\frac{\pi\xi}{2}\right)^2 \sum_{(\tilde{n}_1, \tilde{n}_2, \tilde{n}_3, \tilde{n}_4) \neq (0,0,0,0)}^{\infty} \\ &\quad \times \frac{1}{(\tilde{n}_1 + \tilde{n}_2)(\tilde{n}_1 + \tilde{n}_2 + 1)^2(\tilde{n}_1 + \tilde{n}_2 + 2)} \\ &= 16\pi^2 \left(\frac{\pi\xi}{2}\right)^2 \sum_{(\tilde{n}_3, \tilde{n}_4)}^{\infty} \sum_{N=1}^{\infty} \frac{1}{N(N+1)(N+2)} \\ &= 4\pi^2 \left(\frac{\pi\xi}{2}\right)^2 \sum_{(\tilde{n}_3, \tilde{n}_4)}^{\infty} = 4\pi^2 V_4, \end{aligned} \quad (4.25)$$

where we used the formula of the summation:

$$\sum_{(\tilde{n}_1, \tilde{n}_2) \neq (0,0)}^{\infty} \langle N | \mathcal{O}(\tilde{n}_1 + \tilde{n}_2) | N \rangle = \sum_{N=1}^{\infty} (N+1) \langle N | \mathcal{O}(N) | N \rangle, \quad (4.26)$$

and the formula of the four dimensional volume V_4 in the operator formalism:

$$V_4 \equiv \int d^4x = \left(\frac{\pi\xi}{2}\right)^2 \sum_{(\tilde{n}_3, \tilde{n}_4)}^{\infty}. \quad (4.27)$$

The appearance of the four dimensional volume V_4 suggests that the solution (4.15) has a four dimensional nature. Therefore it is natural to interpret the solution (4.15) as the non-commutative version of the four dimensional instantons.

This four dimensional nature can be seen more explicitly by transforming the solution. Now let us define new coordinates \tilde{z}_m ($m = 1, \dots, 4$) by

$$\tilde{z}_1 = \frac{1}{\sqrt{1+a^2}}(z_1 + az_3), \quad \tilde{z}_2 = \frac{1}{\sqrt{1+a^2}}(z_2 + az_4), \quad (4.28)$$

$$\tilde{z}_3 = \frac{1}{\sqrt{1+1/a^2}}(z_1 - a^{-1}z_3), \quad \tilde{z}_4 = \frac{1}{\sqrt{1+1/a^2}}(z_2 - a^{-1}z_4).$$

These new coordinates also satisfy the same commutation relations as (4.2):

$$\begin{aligned} [\tilde{z}_1, \tilde{z}_1] &= -[\tilde{z}_2, \tilde{z}_2] = [\tilde{z}_3, \tilde{z}_3] = -[\tilde{z}_4, \tilde{z}_4] \\ &= -\frac{\xi}{2}, \quad \text{others are zero.} \end{aligned} \quad (4.29)$$

Then the solution (4.15) can be rewritten in these new coordinates as

$$\begin{aligned} F &= \frac{\xi^2}{\Delta(\Delta + \xi/2)(\Delta + \xi)} [\tilde{J}_3(d\tilde{z}_1 \wedge d\tilde{z}_1 + d\tilde{z}_2 \wedge d\tilde{z}_2) \\ &\quad - \sqrt{2}\tilde{J}_+ d\tilde{z}_1 \wedge d\tilde{z}_2 + \sqrt{2}\tilde{J}_- d\tilde{z}_1 \wedge d\tilde{z}_2], \end{aligned} \quad (4.30)$$

where we defined the quantity $\Delta \equiv \tilde{z}_1 \tilde{z}_1 + \tilde{z}_2 \tilde{z}_2$ and the following operators:

$$\tilde{J}_+ = \frac{\sqrt{2}}{\xi} \tilde{z}_2 \tilde{z}_1, \quad \tilde{J}_- = \frac{\sqrt{2}}{\xi} \tilde{z}_1 \tilde{z}_2, \quad \tilde{J}_3 = \frac{1}{\xi} (\tilde{z}_2 \tilde{z}_2 - \tilde{z}_1 \tilde{z}_1). \quad (4.31)$$

These operators satisfy the Lie algebra (4.18) of $SU(2)$. The above expression (4.30) is surely the $U(1)$ one instanton solution on the four dimensional space $\tilde{\mathbf{R}}^4$ spanned by the coordinates: \tilde{z}_1 , \tilde{z}_1 , \tilde{z}_2 , and \tilde{z}_2 . So the solution (4.30) has the four form charge over the four dimensional subspace $\tilde{\mathbf{R}}^4$ as

$$\mathcal{Q}^{(4)} \equiv -\frac{1}{2(2\pi)^2} \int_{\tilde{\mathbf{R}}^4} F \wedge F = 4 \sum_{N=1}^{\infty} \frac{1}{N(N+1)(N+2)} = +1. \quad (4.32)$$

From the results (4.19), (4.25) and (4.32), the solution (4.30) should be interpreted as the bound state of the $D4$ -brane and the $D8$ -brane with a B field, and we can naturally interpret the total value of the action (4.25) as the product of the action of the four dimensional instanton over $\tilde{\mathbf{R}}^4$ and the volume of the four dimensional space spanned by the coordinates: \tilde{z}_3 , \tilde{z}_3 , \tilde{z}_4 , and \tilde{z}_4 .

Here we comment on the problem associated with the zero mode, which we encounter when we normalize the ψ . If we rewrite the Dirac-like operator (4.10) by new coordinates (4.28), then the extended ADHM construction reduces to the ADHM construction of the noncommutative instanton in the four dimensional subspace $\tilde{\mathbf{R}}^4$. Therefore this problem of the zero mode is essentially the same as that of the noncommu-

tative instanton in $\widetilde{\mathbf{R}}^4$ [7]. We have used this fact and the procedure of Ref. [11] when we calculate the value of the action (4.25) and the four form charge (4.32).

B. Generalization to $U(1)$, multi- k case

An important point in the previous subsection is that we are able to reduce the extended ADHM construction to the four dimensional ADHM construction by the unitary transformation of the coordinates. In this subsection, we comment on the generalization of this scenario to the more general case.

It is difficult to solve the noncommutative version of the extended ADHM equations (4.7) generally. However there is an interesting subspace in the moduli space of the extended ADHM data. If we take

$$B_1 = w_1 \mathbf{1}_{k \times k}, \quad B_2 = w_2 \mathbf{1}_{k \times k}, \quad K = L = \mathbf{0}_{1 \times k}, \quad (4.33)$$

where w_i ($i=1,2$) are arbitrary complex parameters, then the extended ADHM equations reduce to the equations similar to the four dimensional deformed ADHM equations:

$$\mu_{\mathbf{R}}^1 = \zeta(1 + |w_1|^2 + |w_2|^2) \mathbf{1}_{k \times k}, \quad \mu_{\mathbf{C}}^1 = \mathbf{0}_{k \times k}. \quad (4.34)$$

Here $\mathbf{0}_{1 \times k}$ and $\mathbf{0}_{k \times k}$ denote respectively $1 \times k$ and $k \times k$ matrices whose components are all zero, and $\mathbf{1}_{k \times k}$ denotes a $k \times k$ unit matrix. It is an easy problem to solve these reduced equations (4.34) since the solutions of the deformed ADHM equations in four dimensional case are well known. Some relevant references are [3,6–23].

We can generalize the procedure of the previous subsection to the present case. We are able to extend the definition (4.28) of new coordinates to the case (4.33) straightforwardly. Then the extended Dirac-like operator (2.7) reduces to that of the ADHM construction of the instantons over the four dimensional subspace $\widetilde{\mathbf{R}}^4$. Therefore we can naturally interpret the solution (4.33) as the bound state of the k $D4$ -branes and the $D8$ -brane with a B field, and we can show that the integer k which appears in the extended ADHM construction should be interpreted as the $D4$ -brane charge rather than the $D0$ -brane charge when the noncommutativity parameter is anti-self-dual.

V. RELATIONSHIP WITH SOLUTION GENERATING TECHNIQUE

In this section, we consider the case that the noncommutativity parameter is self-dual. We find the relationship with the solution generating technique and show that the integer k can be interpreted as the charge of the $D0$ -brane bound to the $D8$ -brane with a B field. In four dimensions, this relationship was established by Hamanaka [13]. However in the eight dimensions this relationship has not yet been found. We construct the solution of the extended ADHM equations, which corresponds to the localized instanton solution obtained by using the solution generating technique, and interpret it as the system of k $D0$ -branes and the $D8$ -brane with a B field.

We introduce the self-dual noncommutativity parameter as

$$\theta^{13} = \theta^{57} = \theta^{24} = \theta^{68} = \frac{\zeta}{4} \quad (\zeta > 0). \quad (5.1)$$

This implies the following commutation relations of the complex coordinates:

$$\begin{aligned} [z_1, \bar{z}_1] &= [z_2, \bar{z}_2] = [z_3, \bar{z}_3] = [z_4, \bar{z}_4] \\ &= -\frac{\zeta}{2}, \quad \text{others are zero.} \end{aligned} \quad (5.2)$$

These relations are the same as those of the harmonic oscillators up to the multiplication of constants. Therefore we define the creation and annihilation operators by

$$a_m^\dagger = \sqrt{\frac{2}{\zeta}} z_m, \quad a_m = \sqrt{\frac{2}{\zeta}} \bar{z}_m \quad \text{for } m=1, \dots, 4. \quad (5.3)$$

The number operators can also be defined as

$$n_m = a_m^\dagger a_m = \frac{2}{\zeta} z_m \bar{z}_m \quad \text{for } m=1, \dots, 4. \quad (5.4)$$

As in the previous section, the Fock space \mathcal{H} on which the creation and annihilation operators (5.3) act is spanned by the direct product of the Fock state: $|n_1 : n_2 : n_3 : n_4\rangle \equiv |n_1\rangle \otimes |n_2\rangle \otimes |n_3\rangle \otimes |n_4\rangle$. The creation and annihilation operators act on each Fock state in the same way as Eq. (4.6).

It is easily found that the extended ADHM equations (2.18) in the commutative case are not deformed by the noncommutativity of the coordinates (5.2),

$$\mu_{\mathbf{R}}^1 = \mu_{\mathbf{C}}^1 = \mu_{\mathbf{C}}^2 = \mu_{\mathbf{C}}^{2'} = \mu_{\mathbf{R}}^3 = \mu_{\mathbf{C}}^3 = 0. \quad (5.5)$$

Now let us find the solution of the extended ADHM equations (5.5), which is related to the localized instanton solution obtained by using the solution generating technique. We consider the case of the gauge group $U(1)$ and multi k . It is allowed to take $J=K=L=\mathbf{0}_{1 \times k}$ for the $U(1)$ case. Then the extended ADHM equations (5.5) are simply solved and the solution which might correspond to the localized instanton solution is obtained by

$$A_1 = A_2 = B_1 = \mathbf{0}_{k \times k}, \quad I = \mathbf{0}_{1 \times k}, \quad B_2 = \mathbf{1}_{k \times k}. \quad (5.6)$$

The extended ADHM construction gives rise to the instantons through the zero mode of the Dirac-like operator. So we need to look for the zero mode of the Dirac-like operator:

$$D_z^\dagger = \begin{pmatrix} (\bar{z}_2 + \bar{z}_4) \mathbf{1}_{k \times k} & -(z_1 + z_3) \mathbf{1}_{k \times k} & \mathbf{0}_{k \times 1} \\ (\bar{z}_1 + \bar{z}_3) \mathbf{1}_{k \times k} & (z_2 + z_4) \mathbf{1}_{k \times k} & \mathbf{0}_{k \times 1} \end{pmatrix}. \quad (5.7)$$

Seemingly the Dirac-like operator (5.7) has the only trivial solution. We are however able to construct the nontrivial zero mode of the Dirac-like operator (5.7) by using the par-

tial isometry in the noncommutative setting. Here, in order to write down the zero mode of D_z^\dagger , we prepare an ordering of the states:

$$|n_1:n_2:n_3:n_4\rangle = \prod_{i=1}^4 \frac{1}{\sqrt{n_i!}} (a_i^\dagger)^{n_i} |0:0:0:0\rangle. \quad (5.8)$$

Two sets of four non-negative integers, $\mathbf{m}=(m_1, \dots, m_4)$ and $\mathbf{n}=(n_1, \dots, n_4)$, for which we define $\bar{m}_j = \sum_{i=j}^4 m_i$, and $\bar{n}_j = \sum_{i=j}^4 n_i$ with $j=1, \dots, 4$, are ordered by the following rules: (1) If $\bar{m}_j = \bar{n}_j$ for all $1 \leq j \leq 4$, $\mathbf{m} = \mathbf{n}$; (2) if $\bar{m}_j = \bar{n}_j$ ($j=1, \dots, k-1$) and $\bar{m}_k > \bar{n}_k$ for some k ($1 \leq k \leq 4$), $\mathbf{m} > \mathbf{n}$; (3) if $\bar{m}_j = \bar{n}_j$ ($j=1, \dots, k-1$) and $\bar{m}_k < \bar{n}_k$ for some k ($1 \leq k \leq 4$), $\mathbf{m} < \mathbf{n}$.

We can order all the states by these rules. For example, these rules order the states as

$$|0\rangle\rangle = |0:0:0:0\rangle,$$

$$|1\rangle\rangle = |1:0:0:0\rangle, \quad |2\rangle\rangle = |0:1:0:0\rangle, \quad |3\rangle\rangle = |0:0:1:0\rangle,$$

$$|4\rangle\rangle = |0:0:0:1\rangle, \quad |5\rangle\rangle = |2:0:0:0\rangle, \dots \quad (5.9)$$

The zero mode ψ of D_z^\dagger is a $(2k+1) \times 1$ matrix which is written as $\psi = (\psi_1 \ \psi_2 \ \xi)^T$. Here ψ_1 and ψ_2 are $k \times 1$ matrices respectively, and ξ is a 1×1 matrix. Each component of ψ is explicitly obtained as

$$\psi_1 = \begin{pmatrix} |0\rangle\rangle\langle\langle 0| \\ |0\rangle\rangle\langle\langle 1| \\ \vdots \\ |0\rangle\rangle\langle\langle k-1| \end{pmatrix}, \quad \psi_2 = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \xi = S_k, \quad (5.10)$$

where we defined the shift operator:

$$S_k \equiv \sum_{i=0}^{\infty} |i\rangle\rangle\langle\langle i+k|. \quad (5.11)$$

This shift operator is the typical example of the partial isometry, which is used in the construction by using the solution generating technique, and satisfies

$$S_k S_k^\dagger = 1, \quad S_k^\dagger S_k = 1 - P_k. \quad (5.12)$$

Here we defined the projection operator of rank k by

$$P_k \equiv \sum_{i=0}^{k-1} |i\rangle\rangle\langle\langle i|, \quad (5.13)$$

which satisfies

$$S_k P_k = P_k S_k^\dagger = 0. \quad (5.14)$$

Then we can easily write down the explicit expression of the gauge field (3.8):

$$\begin{aligned} X_\mu &= \psi^\dagger [\hat{\partial}_\mu, \psi] + \hat{\partial}_\mu = \psi^\dagger \hat{\partial}_\mu \psi \\ &= S_k^\dagger \hat{\partial}_\mu S_k, \end{aligned} \quad (5.15)$$

and the strength of gauge field (3.7):

$$F_{\mu\nu} = i(\theta^{-1})_{\mu\nu} (S_k S_k^\dagger - 1) = -i(\theta^{-1})_{\mu\nu} P_k. \quad (5.16)$$

From the noncommutativity parameter (5.1) in this case, the field strength form is written as

$$F = \frac{2}{\zeta} P_k (dz_1 \wedge d\bar{z}_1 + dz_2 \wedge d\bar{z}_2 + dz_3 \wedge d\bar{z}_3 + dz_4 \wedge d\bar{z}_4). \quad (5.17)$$

The solution (5.15) for $k=1$ was originally obtained in Ref. [29] by using the solution generating technique, and it was confirmed that the solution (5.15) preserves 3/16 of supersymmetries by investigating small fluctuations around the solution. The condition for preserving 3/16 of supersymmetries, which was found in Ref. [29], corresponds to choosing the noncommutativity parameter as Eq. (5.1). Then it is confirmed that the solution (5.6) of the extended ADHM equations corresponds to the solution constructed by using the solution generating technique.

In the rest of this section, we study the properties of the above solution (5.15). At first, we are able to calculate the eight form charge $Q^{(8)}$ as

$$Q^{(8)} \equiv \frac{1}{4!(2\pi)^4} \int_{\mathbf{R}^8} F \wedge F \wedge F \wedge F = \text{Tr}_{\mathcal{H}} P_k = k, \quad (5.18)$$

where we used the formula

$$\begin{aligned} dz^1 \wedge d\bar{z}^1 \wedge dz^2 \wedge d\bar{z}^2 \wedge dz^3 \wedge d\bar{z}^3 \wedge dz^4 \wedge d\bar{z}^4 \\ = 16(\text{volume form}). \end{aligned} \quad (5.19)$$

Therefore the integer k which appears in the extended ADHM construction can be regarded as the $D0$ -brane charge. We can also calculate the value of the action for the solution,

$$\begin{aligned} S &= -\frac{1}{2} \int d^8x (F_{13}^2 + F_{57}^2 + F_{24}^2 + F_{68}^2) \\ &= 2\pi^4 \zeta^2 \text{Tr}_{\mathcal{H}} P_k \\ &= 2\pi^4 \zeta^2 k. \end{aligned} \quad (5.20)$$

From the results (5.18) and (5.20), the solution can be interpreted as the system of the k $D0$ -branes and the $D8$ -brane with a B field.

VI. CONCLUSION

In this paper, we have studied the noncommutative version of the extended ADHM construction in the eight dimensional $U(1)$ Yang-Mills theory. We have found that it is an important difference whether the noncommutativity param-

eter is anti-self-dual or self-dual. In the case that the noncommutativity parameter is anti-self-dual, we have shown that the integer k which appears in the extended ADHM construction should be interpreted as the $D4$ -brane charge rather than the $D0$ -brane charge. We have confirmed this fact in a wide subspace of the extended ADHM data by calculating the topological charges.

We have found the relationship with the solution generating technique in the case that the noncommutativity parameter is self-dual. The shift operator of the solution generating technique has naturally appeared in the extended ADHM construction. We have also shown in this case that the integer k can be interpreted as the charge of the $D0$ -brane bound to the $D8$ -brane with a B field.

The natural generalization of our study is to consider the gauge group of the higher rank since our study has been restricted to the case of the $U(1)$ gauge group. It should be checked whether the qualitative nature we have found changes or not. For example, it is of interest whether the

integer k which appears in the extended ADHM construction becomes to be interpreted as the $D0$ -brane charge or not in the case that the noncommutativity parameter is anti-self-dual. Another generalization of our study is to construct the solutions of the BPS equations (2.1) except for the case related to the $Sp(2)$ symmetry in the noncommutative Yang-Mills theory.

The moduli space of the noncommutative instantons in eight dimensions is expected to have much richer structure than that of the noncommutative instantons in four dimensions. Therefore further investigation is necessary to understand the topological structure of gauge fields on the noncommutative \mathbf{R}^8 .

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