Exact calculation of ring diagrams and the off-shell effect on the equation of state

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(Received 25 December 2002; revised manuscript received 18 February 2003; published 27 May 2003)

The partition function with ring diagrams at finite temperature is exactly calculated by using contour integrals in the complex energy plane. It contains a pole part with a temperature- and momentum-dependent mass and a phase shift part induced by the off-shell effect in a hot medium. The thermodynamic potentials for the ϕ^4 and ϕ^3 interactions are calculated and compared with the quasiparticle (pole) approximation. It is found that the off-shell effect on the equation of state is remarkable.

DOI: 10.1103/PhysRevD.67.105023 PACS number(s): 11.10.Wx, 24.10.Pa

I. INTRODUCTION

The thermodynamic properties of a system are fully characterized by its thermodynamic potential, with which one can derive all the thermodynamic functions such as pressure, energy density and entropy density from the well-known thermodynamic relations. With the finite temperate field theory [1,2], the partition function can be calculated perturbatively for weakly coupling systems such as the high temperature phase of quantum chromodynamics (QCD). However, the normal perturbation method cannot describe the collective effect, which is now believed to play a very important role [3] in understanding the quark-gluon plasma (QGP) [4] possibly formed in relativistic heavy ion collisions [5]. It is therefore necessary to perform a resummation to include all high-order contributions in the equation of state of the system in a hot and dense medium. In a quasiparticle description, the in-medium effect is reflected in a temperature- and density-dependent mass only. This effective mass is used to explain [6] the difference between the lattice calculation [7] of the equation of state of a QGP and the corresponding Stefan-Boltzmann limit: the particle mass goes up with increasing temperature and then cancels partly the high temperature effect. However, the in-medium effect changes not only the particle mass but also its width. Most discussions concerning the thermal width are focused on its relation to particle decay, while how it contributes to the equation of state is still unclear. To study the off-shell effect on the equation of state is the main goal of this paper.

Ring diagrams are usually considered in the calculation of partition functions [1,2] to avoid infrared divergences and in the quark models [11] to form mesons at the random-phase approximation level. Normally the ring diagrams are calculated only for the static mode [1,2], namely, in the frequency sum only the term with $n=0$ is considered. We will calculate the thermodynamic potential with ring diagrams exactly and investigate the off-shell contribution to the equation of state. We first perform, in a general case, the contour integration instead of the frequency sum in the ring diagrams, and derive the thermodynamic potential which contains the contributions from the quasiparticle with a temperature-dependent mass determined by the pole equation and from the scattering phase shift between the retarded and advanced particle self-energy is due to the off-shell effect. To illustrate the quasiparticle and off-shell contributions to the equation of state related to the study of a QGP, we apply our formulas to the popular models, the ϕ^4 and ϕ^3 theories.

II. FORMULAS

The thermodynamic potential of a system with ring diagrams can be written as

$$\Omega=\Omega_0+\Omega_1+\Omega_{ring}, \tag{1}$$

where

$$\Omega_0=\frac{1}{\beta}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\ln(1-e^{-\beta E_0}) \tag{2}$$

is the free particle contribution with $E_0=\sqrt{m^2+k^2}$ being the particle energy and $\beta=1/T$ being the inverse temperature, and Ω_{ring} is defined in Fig. 1 [1,2]. To make the calculation definite, we consider, in the following, meson ring diagrams only. However, the method can be earily extended to fermions. A shaded circle in Fig. 1 means the particle self-energy Π. To use the standard definition of Ω_{ring}, we have separated Ω_1 with only one self-energy on the ring from Ω_{ring}. After the summation over the rings with a different number of shaded circles, Ω_{ring} can be expressed as [1,2]

$$\Omega_{ring}=\frac{1}{2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\beta}\sum_n[\ln\{1+A(i\omega_n,\mathbf{k})\}-A(i\omega_n,\mathbf{k})], \tag{3}$$

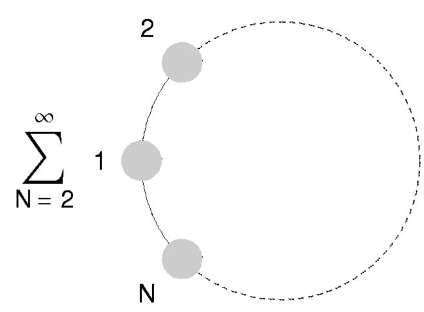

FIG. 1. Ring diagrams with self-energy Π indicated by shaded circles.

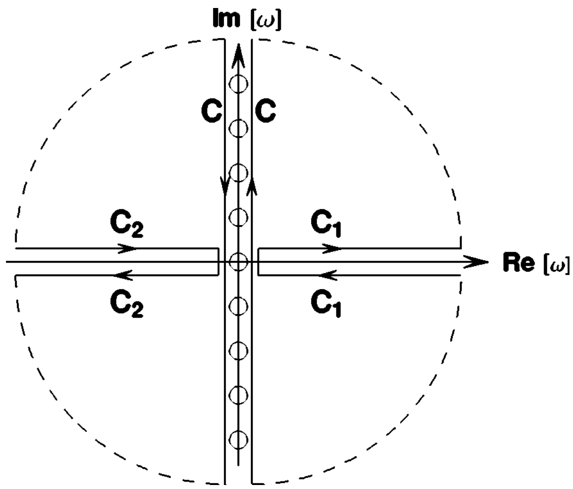

FIG. 2. The integration contours in the complex z plane.

where $\omega_n = 2\pi n/\beta$ with $n = 0, \pm 1, \pm 2, \ldots$ are the Matsubara frequencies of meson field, and function $A(i\omega_n, \mathbf{k})$ is related to the self-energy $\Pi(i\omega_n, \mathbf{k})$:

$$A(i\omega_n,\mathbf{k}) = \frac{\Pi(i\omega_n,\mathbf{k})}{-(i\omega_n)^2 + E_0^2}. \tag{4}$$

By using analytic conjugation and the residue theorem, the frequency summation in Ω_{ring} can be changed into an integration along contours C_1 and C_2 in the complex energy plane, see Fig. 2. Taking into account the general property for the self-energy with complex energy

$$\Pi(z,\mathbf{k}) = \Pi(-z,\mathbf{k}) \tag{5}$$

and the asymptotic behavior

$$\lim_{|z|\to\infty} |z^2 A(z,\mathbf{k})| < \infty,$$

$$\lim_{|z|\to\infty} |z^2 \ln[1 + A(z,\mathbf{k})]| < \infty, \tag{6}$$

the frequency summation is finally written as an integration along the positive real axis [11],

$$\Omega_{ring} = \frac{1}{4\pi i}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\int_0^\infty d\omega\left(1 + \frac{2}{e^{\beta\omega}-1}\right) \times \left(\ln\frac{1 + A_R(\omega,\mathbf{k})}{1 + A_A(\omega,\mathbf{k})} - [A_R(\omega,\mathbf{k}) - A_A(\omega,\mathbf{k})]\right), \tag{7}$$

where the retarded and advanced functions A_R and A_A are defined by

$$A_R(\omega,\mathbf{k}) = -\frac{\Pi_R(\omega,\mathbf{k})}{(\omega + i\eta)^2 - E_0^2},$$

$$A_A(\omega,\mathbf{k}) = -\frac{\Pi_A(\omega,\mathbf{k})}{(\omega - i\eta)^2 - E_0^2}, \tag{8}$$

with the retarded and advanced self-energies $\Pi_R(\omega,\mathbf{k}) = \Pi(\omega + i\eta,\mathbf{k})$ and $\Pi_A(\omega,\mathbf{k}) = \Pi(\omega - i\eta,\mathbf{k})$ and η being an infinitesimal positive constant.

Taking integration by parts for the logarithm term of Eq. (7) and using

$$A_R(\omega,\mathbf{k}) - A_A(\omega,\mathbf{k}) = \frac{i}{2E_0}\left(2\pi\delta(\omega - E_0)\Pi(\omega,\mathbf{k}) - \frac{4E_0}{\omega^2 - E_0^2}\,\mathrm{Im}\,\Pi_R(\omega,\mathbf{k})\right), \tag{9}$$

we have [the other δ function $\delta(\omega + E_0)$ is neglected since the integration over ω is along the positive real axis]

$$\begin{aligned}\Omega_{ring} = &\frac{-1}{4\pi i}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\int_0^\infty d\omega\left(\omega + \frac{2}{\beta}\ln(1 - e^{-\beta\omega})\right) \times \frac{d}{d\omega}\ln\frac{1 + A_R(\omega,\mathbf{k})}{1 + A_A(\omega,\mathbf{k})}\\ &- \frac{1}{4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\left(1 + \frac{2}{e^{\beta E_0}-1}\right)\frac{\Pi(E_0,\mathbf{k})}{E_0}\\ &+ \frac{1}{4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\int_0^\infty \frac{d\omega}{\pi}\left(1 + \frac{2}{e^{\beta\omega}-1}\right)\frac{2\,\mathrm{Im}\,\Pi_R(\omega,\mathbf{k})}{(\omega^2 - E_0^2)}.\end{aligned} \tag{10}$$

From the separation of the logarithm,

$$\ln\frac{1 + A_R(\omega,\mathbf{k})}{1 + A_A(\omega,\mathbf{k})} = \ln\frac{(\omega - i\eta)^2 - E_0^2}{(\omega + i\eta)^2 - E_0^2} + \ln\frac{(\omega + i\eta)^2 - E_0^2 - \Pi_R(\omega,\mathbf{k})}{(\omega - i\eta)^2 - E_0^2 - \Pi_A(\omega,\mathbf{k})}, \tag{11}$$

its derivative can be expressed as a free particle part

$$\frac{d}{d\omega}\ln\frac{(\omega - i\eta)^2 - E_0^2}{(\omega + i\eta)^2 - E_0^2} = 2i\pi\delta(\omega - E_0), \tag{12}$$

and a self-energy-dependent part

$$\begin{aligned}&\frac{d}{d\omega}\ln\frac{(\omega + i\eta)^2 - E_0^2 - \Pi_R(\omega,\mathbf{k})}{(\omega - i\eta)^2 - E_0^2 - \Pi_A(\omega,\mathbf{k})}\\ &= \frac{d}{d\omega}\ln\frac{\omega^2 - E_0^2 - \Pi(\omega,\mathbf{k}) - i(\mathrm{Im}\,\Pi_R(\omega,\mathbf{k}) - 2\omega\eta)}{\omega^2 - E_0^2 - \Pi(\omega,\mathbf{k}) + i(\mathrm{Im}\,\Pi_R(\omega,\mathbf{k}) - 2\omega\eta)}\\ &= -2i\frac{d\tilde{\phi}(\omega,\mathbf{k})}{d\omega}\end{aligned} \tag{13}$$

characterized by the phase shift $\tilde{\phi}$ resulting from the difference between the retarded and advanced self-energies.

The total phase shift can be separated into two parts, a pole part ϕ_0 which leads to the pole equation for the quasiparticle and a scattering phase shift ϕ_s defined in the region $[-\pi/2,\pi/2]$,

$$\tilde{\phi}=\phi_0+\phi_s,$$

$$\phi_0=\pi\theta(\omega^2-E_0^2-\Pi),$$

$$\phi_s(\omega,\mathbf{k})=\arctan\frac{\mathrm{Im}\,\Pi_R(\omega,\mathbf{k})-2\omega\eta}{\omega^2-E_0^2-\Pi(\omega,\mathbf{k})}, \tag{14}$$

where we have made use of the property $\mathrm{Im}\Pi_R(\omega,\mathbf{k})\leqslant 0$, which can be observed from the relation between $\mathrm{Im}\Pi_R(\omega,\mathbf{k})$ and the decay rate [1,2]

$$\omega\frac{d\Gamma}{d^3\mathbf{k}}=-\frac{\mathrm{Im}\,\Pi_R(\omega,\mathbf{k})}{(2\pi)^3(e^{\beta\omega}-1)}. \tag{15}$$

Substituting Eqs. (12) and (13) into Eq.(10) and considering the fact that the derivative of a θ function is a δ function, we obtain

$$\Omega_{ring}=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\left[\frac{1}{\beta}\ln\frac{1-e^{-\beta E}}{1-e^{-\beta E_0}}-\frac{1}{e^{\beta E_0}-1}\frac{\Pi(E_0,\mathbf{k})}{2E_0}\right]$$
$$-\int\frac{d^3\mathbf{k}}{(2\pi)^3}\int_0^\infty\frac{d\omega}{\pi}\frac{1}{e^{\beta\omega}-1}$$
$$\times\left[\phi_s(\omega,\mathbf{k})-\frac{\mathrm{Im}\,\Pi_R(\omega,\mathbf{k})}{\omega^2-E_0^2}\right], \tag{16}$$

where we have neglected the zero-point energy in the vacuum to avoid the ultraviolet divergence. The effective energy $E=\sqrt{m^{*2}+\mathbf{k}^2}$ in Eq. (16) is related to the effective meson mass m^* determined through the pole equation,

$$m^{*2}=m^2+\Pi(E,\mathbf{k}). \tag{17}$$

The first term in the first square bracket of Eq. (16) is the contribution from quasiparticles subtracting the corresponding free particles which will later be canceled in the total thermodynamic potential Ω by Ω_0, and the rest in this bracket is an extra term resulting from the free meson propagator between two self-energies (lines between shaded circles in Fig. 1). The second line of Eq. (16) is due to the imaginary part of the retarded self-energy which is reflected in the scattering phase shift ϕ_s and an extra term resulting also from the free meson propagators in the ring diagrams in Fig. 1. The connection between the phase of the scattering amplitude and the grand canonical potential was first made by Dashen [8] and continued in [9] and [10].

Now we turn to considering the lowest order correction to the thermodynamic potential, namely, Ω_1 in Eq. (1). Since Ω_1 differs from the second term in Eq. (3) in only a sign and a symmetry factor, it can be written as

$$\Omega_1=\gamma_s\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\beta}\sum_n A(i\omega_n,\mathbf{k}), \tag{18}$$

where the γ_s factor is due to the difference between the symmetry factor for self-energy and that for the partition function. The value of γ_s is determined by the interaction, it is $1/4$ for the ϕ^4 theory and $1/6$ for the ϕ^3 theory with the lowest order self-energy.

Putting together Ω_0, Ω_1 and Ω_{ring}, the total thermodynamic potential defined through Eq. (1) is now written as

$$\Omega=\Omega_R+\Omega_I, \tag{19}$$

with the $\mathrm{Re}\,\Pi$- and $\mathrm{Im}\,\Pi$-dependent parts

$$\Omega_R=\frac{1}{\beta}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\ln(1-e^{-\beta E})$$
$$-\left(\frac{1}{2}-\gamma_s\right)\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{e^{\beta E_0}-1}\frac{\Pi(E_0,\mathbf{k})}{E_0},$$

$$\Omega_I=-\int\frac{d^3\mathbf{k}}{(2\pi)^3}\int_0^\infty\frac{d\omega}{\pi}\frac{1}{e^{\beta\omega}-1}\phi_s(\omega,\mathbf{k})$$
$$+\int\frac{d^3\mathbf{k}}{(2\pi)^3}\int_0^\infty\frac{d\omega}{\pi}\frac{1}{e^{\beta\omega}-1}$$
$$\times\left(\frac{1}{2}-\gamma_s\right)\frac{2\mathrm{Im}\Pi_R(\omega,\mathbf{k})}{\omega^2-E_0^2}. \tag{20}$$

It is clear to see that Ω_R contains not only a quasiparticle part with a temperature-dependent mass m^* hidden in the particle energy E, but also an extra term coming from the free meson propagator between two self-energies, and Ω_I arises from the off-shell effect which leads to a phase shift ϕ_s and a similar extra term. For Nambu–Jona-Lasinio (NJL) type [11] interactions, there is no free propagator between two self-energies, thus the extra terms in Ω_R and Ω_I disappear. In this case, the in-medium effect is simple and clear: it results in a quasiparticle with a scattering phase shift. It is also necessary to note that the quasiparticle and phase shift are introduced only from the summation of all ring diagrams. For any ring diagram with a fixed value of self-energy Π, there are no such contributions to Ω.

III. EXAMPLES

With the formulas established in the last section, we can evaluate the equation of state including the resummation effect for any given self-energy. We now consider some examples to illustrate the exact calculation of ring diagrams and compare it with the usually used quasiparticle approximation. The point we will focus on is the contribution from the off-shell effect included in Ω_I.

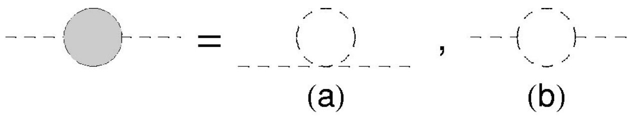

FIG. 3. The leading order self-energies for $\phi^4(a)$ and $\phi^3(b)$.

Let us first consider the $-\lambda\phi^4$ theory. The self-energy to the leading order shown in Fig. 3(a) is evaluated analytically for massless particles $(m=0)$ [1,2]

$$\Pi=\lambda T^2. \tag{21}$$

Since the self-energy is ω and $\mathbf{k}$ independent, the in-medium correction is reflected in the shift of the particle mass only, and there is no off-shell effect. With the formulas we developed in last section, it reads

$$\Omega_I=0,$$

$$\Omega=\Omega_R=\Omega_{quasi}-\frac{1}{4}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\Pi}{k}\frac{1}{e^{k/T}-1},$$

$$\Omega_{quasi}=\int\frac{d^3\mathbf{k}}{(2\pi)^3}T\ln(1-e^{-E/T}),$$

$$E=\sqrt{m^{*2}+k^2},\quad m^{*2}=\Pi. \tag{22}$$

After integrating over the angles and scaling momentum k by temperature T, the quasiparticle and total thermodynamic potentials are both proportional to T^4,

$$\Omega_{quasi}=a(\lambda)T^4,$$

$$\Omega=\left(a(\lambda)-\frac{\lambda}{48}\right)T^4,$$

$$a(\lambda)=\frac{1}{2\pi^2}\int_0^\infty dk\,k^2\ln(1-e^{-\sqrt{\lambda+k^2}}). \tag{23}$$

Note that such temperature scaling arises from the T^2 dependence of the effective mass m^*. For any other temperature dependence, this scaling will be broken.

Figure 4 shows the quasiparticle and full thermodynamic potentials scaled by the corresponding Stefan-Boltzmann limit as functions of the coupling constant λ. We see that both the quasiparticle and the total thermodynamics cannot reach the Stefan-Boltzmann limit, and the deviation becomes more and more significant when the coupling constant increases. This phenomenon is fully due to the quasiparticle mass behavior: The particles become heavy in the hot mean field and this mass transport becomes more and more strong with increasing temperature and coupling constant. If we take the coupling constant λ to be about 0.3, the difference between the lattice calculation of the equation of state of the QGP and the corresponding Stefan-Boltzmann limit (about 15% [7]) can be accounted. We are now certainly not handling QCD, but the spirit here and that used in [6] to fit successfully the lattice calculation are quite the same.

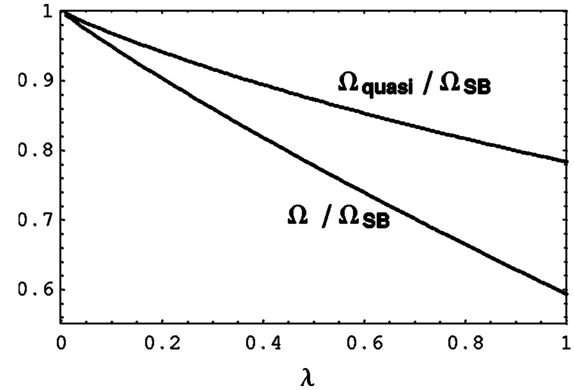

FIG. 4. The quasiparticle and full thermodynamic potentials scaled by the Stefan-Boltzmann limit as functions of the coupling constant in the ϕ^4 theory.

We introduce the so-called static-mode correction [2], which is obtained by taking $n=0$ only in the frequency sum in Eq. (3). The static mode plays an important role in some recent works [3] as it is related to the soft degrees of freedom in an effective theory dealing with high temperature properties of QGP. Figure 5 shows the coupling constant dependence of the static-mode and full calculations of the ring diagrams. We see that only in the small λ region the static-mode calculation is a good approximation, for strong interaction, the contribution from the excited modes cannot be neglected.

In order to see the contribution from the off-shell effect to the equation of state, we further consider the Fock diagram in the $-\lambda\phi^3$ theory. Its self-energy to the leading order, shown in Fig. 3(b), can be explicitly expressed as

$$\Pi(\omega,\mathbf{k})=18\lambda^2\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{4E_1E_2}\Bigg[\{1+f(E_1)+f(E_2)\}\times\left(\frac{1}{\omega-E_1-E_2}-\frac{1}{\omega+E_1+E_2}\right)-\{f(E_1)-f(E_2)\}\times\left(\frac{1}{\omega-E_1+E_2}-\frac{1}{\omega+E_1-E_2}\right)\Bigg], \tag{24}$$

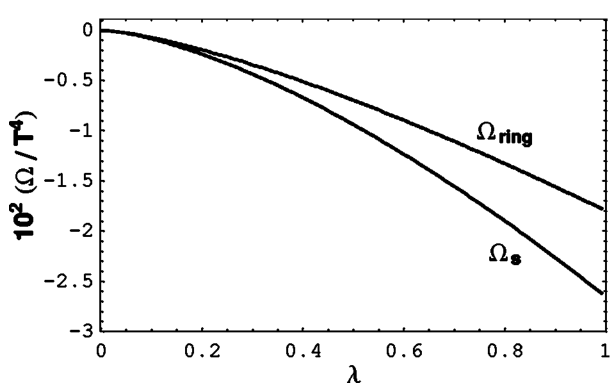

FIG. 5. The static-mode approximation (Ω_s) and the full calculation (Ω_{ring}) of the thermodynamic potential of ring diagrams as functions of the coupling constant in ϕ^4 theory.

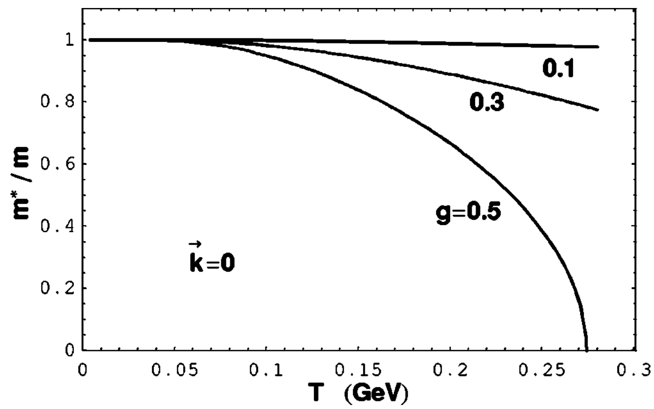

FIG. 6. The effective mass m^* scaled by the free mass m as a function of temperature at $\mathbf{k}=0$ for different coupling constant g in the ϕ^3 theory.

where the factor 18 is the symmetry factor of the ϕ^3 theory, and $f(z)=1/(e^{\beta z}-1)$ is the boson distribution function. The imaginary part of the retarded self-energy,

$$\mathrm{Im}\Pi_R(\omega,\mathbf{k})=-18\pi\lambda^2\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{4E_1E_2} \times[\{1+f(E_1)+f(E_2)\} \times\{\delta(\omega-E_1-E_2)-\delta(\omega+E_1+E_2)\}-\{f(E_1) -f(E_2)\}\{\delta(\omega-E_1+E_2) -\delta(\omega+E_1-E_2)\}], \tag{25}$$

can be simplified as

$$\mathrm{Im}\,\Pi_R(\omega,\mathbf{k})=-\frac{9\lambda^2}{8\pi k}\int_0^\infty dq\,\frac{q}{E_1}[\{\epsilon(\omega-E_1)f(E_1) +f(|\omega-E_1|)\}\theta(1-x_1)\theta(1+x_1) +\{f(E_1)-f(\omega+E_1)\}\theta(1-x_2)\theta(1+x_2)] \tag{26}$$

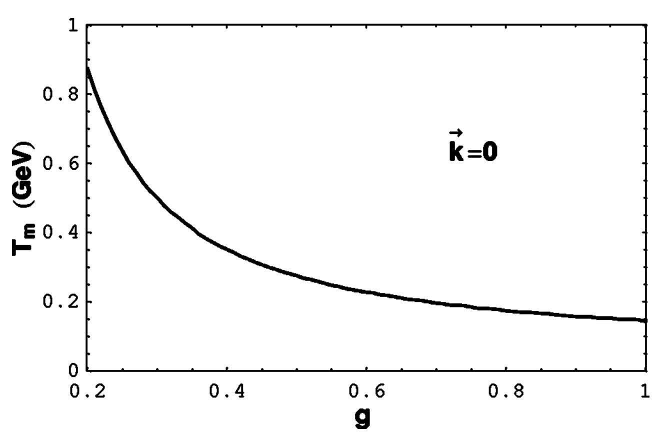

FIG. 7. The maximum temperature T_m for meson field as a function of coupling constant g at $\mathbf{k}=0$ in the ϕ^3 theory.

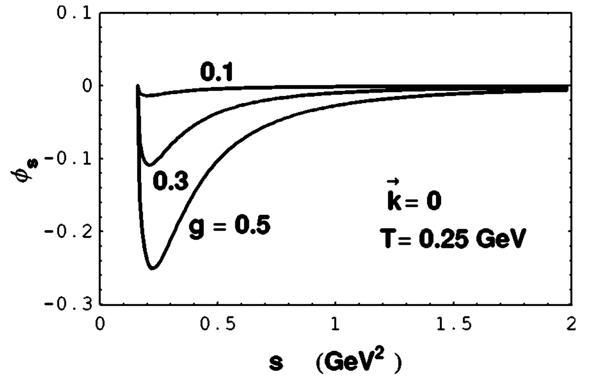

FIG. 8. The scattering phase shift ϕ_s as a function of the Lorentz-invariant variable s at $T=250$ MeV and $\mathbf{k}=0$ for different coupling constants.

with

$$E_1=\sqrt{\mathbf{q}^2+m^2},\quad E_2=\sqrt{(\mathbf{k}-\mathbf{q})^2+m^2},$$
$$x_1=\frac{k^2-\omega^2+2\omega E_1}{2kq},\quad x_2=\frac{k^2-\omega^2-2\omega E_1}{2kq}, \tag{27}$$

and $\epsilon(x)$ being the sign function.

By substituting the self-energy and the imaginary part of the retarded self-energy into Eq. (20), we obtain a rather complicated expression for the thermodynamic potential, which is now impossible to give an analytical result and can only be evaluated numerically. Since the coupling constant λ in the ϕ^3 theory is dimensional, we scale it by a momentum cutoff $\Lambda=1$ GeV to get a dimensionless effective coupling constant $g=\lambda/\Lambda$. For all the calculations below, we choose the meson mass in the vacuum to be $m=200$ MeV. In general, the self-energy Π depends separately on the variables ω^2 and $\mathbf{k}^2$. In free space at $T=0$, it can only be a function of the relativistically invariant combination $s=\omega^2-\mathbf{k}^2$. We find, from numerical calculations, that this is also approximately true for $T\neq 0$, and, since it introduces large computational simplifications, we thus assume this form in the following numerical calculations.

As in the $-\lambda\phi^4$ theory, the summation of ring diagrams in the hot medium leads to the emergence of quasiparticles with a temperature-dependent mass determined by the pole equation (17). Figure 6 shows the temperature dependence of the effective mass m^* for different coupling constants g. Unlike the $-\lambda\phi^4$ theory where the particles obtain mass from the hot medium, the particles in the $-\lambda\phi^3$ theory lose mass in the hot medium. For any given coupling constant, the effective mass drops down with increasing temperature, and at a high-enough temperature the system finally reaches the limit with $m^*=0$. When the system is beyond this maximum temperature T_m determined by $m^*(T_m)=0$, there is no more real mass solution for the pole equation, and it leads to an unphysical jump in thermodynamic potential (19). For the coupling constant $g=0.5$, the effective mass m^* falls quite fast and reaches zero at $T_m=275$ MeV. We show in Fig. 7

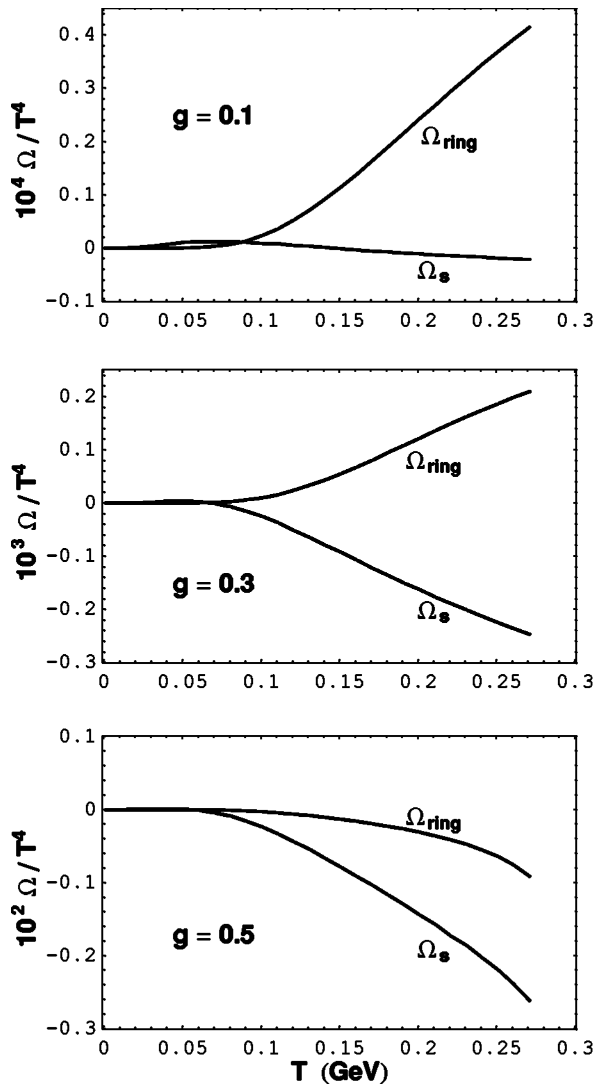

FIG. 9. The static-mode approximation (Ω_s) and the full calculation (Ω_{ring}) of the thermodynamic potential of ring diagrams as functions of the temperature for different coupling constants in the ϕ^3 theory.

the maximum temperature as a function of the coupling constant g. We see that the maximum temperature increases with decreasing g.

The resummation of ring diagrams in the $-\lambda\phi^3$ theory leads to not only an effective mass but also a scattering phase shift due to the off-shell effect. Figure 8 shows the scattering phase shift ϕ_s as a function of the Lorentz-invariant variable s at $T=250$ MeV for different coupling constant. Since $s-m^2-\Pi(s)<0$ corresponding to $m^*/m<1$ for the effective mass, the scattering phase shift is always in the region $0<\phi_s<-\pi/2$ according to the definition of ϕ_s, Eq. (14). It starts at the threshold energy $\sqrt{s}=2m$ for the decay process, then reaches the maximum rapidly, then gets damped slowly, and finally disappears. Because the off-shell effect in the medium arises from the thermal motion and the interaction of particles, the scattering phase shift increases with increasing temperature and coupling constant.

The off-shell effect reflected in the imaginary part of the

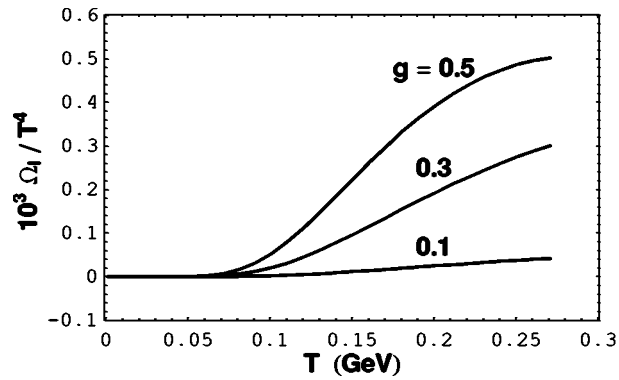

FIG. 10. The thermodynamical potential induced by the off-shell effect in hot medium as a function of the temperature for different coupling constants in the ϕ^3 theory.

retarded self-energy dominates the decay process as shown in Eq. (15). For thermodynamics, the thermal motion of free particles, namely, Ω_0 is obviously the zeroth order contribution, and the effective mass and scattering phase shift result in leading order corrections. Figure 9 shows the difference between the static-mode approximation and the full calculation of the ring diagrams for three values of the coupling constant. While both Ω_s and Ω_{ring} increase with increasing coupling constant (note that the scales for $g=0.1$, 0.3, and 0.5 are different in Fig. 9), the deviation of Ω_{ring} from Ω_s is always significant, and even more important for week couplings. This is quite different from the case in the ϕ^4 theory, see Fig. 5, where the calculation with only static mode is a good approximation in the region of weak coupling. This qualitative difference is mainly from the off-shell effect in the ϕ^3 theory. It is the off-shell effect resulting from the frequency sum over the excited modes that leads to the $\mathrm{Im}\,\Pi$-related part in Ω_{ring}; see the second line of Eq. (16). Figure 10 indicates directly the temperature and coupling constant dependences of Ω_I induced by the off-shell effect. To see how important the off-shell effect on the equation of state is, we plot in Fig. 11 the ratio between the thermodynamical potentials induced by the off-shell effect and by the quasiparticles on the mass shell. Since the off-shell and quasiparticle effects are the same order correction to the thermodynamical potential, we plot the ratio $\Omega_I/(\Omega_R-\Omega_0)$ instead

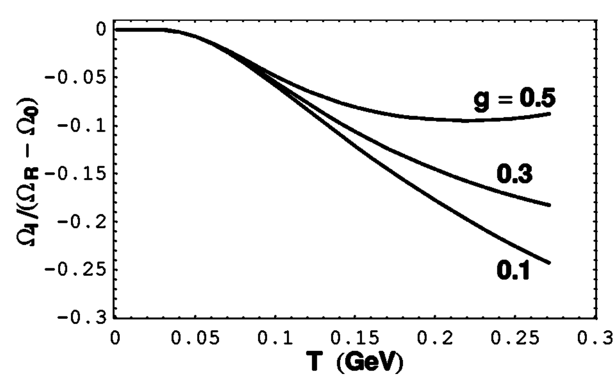

FIG. 11. The ratio between the thermodynamical potentials induced by the off-shell effect and by the quasiparticles in the hot medium in the ϕ^3 theory.

of Ω_I/Ω_R. It is interesting to note that while both Ω_I and $(\Omega_R-\Omega_0)$ get enhanced by strong coupling, the ratio is enlarged in the weak coupling case. The reason is that the effective mass which dominates $(\Omega_R-\Omega_0)$ changes with coupling constant faster than the change in the scattering phase shift which controls Ω_I.

IV. CONCLUSIONS

We have presented an exact calculation of ring diagrams in the frame of the finite temperature field theory in the imaginary time formalism. The resummation of the ring diagrams not only changes the particle mass, but also generates a scattering phase shift, both are collective effects in the hot medium. Only for the system with self-energy in the mean field approximation, the thermodynamics can be described by quasiparticles. In a general case, the corrections from the effective mass and from the scattering phase shift to the equation of state are introduced at the same time. We have applied our formulas to the ϕ^4 and ϕ^3 theories. For the former, with the self-energy to the first order, the static mode can be considered as a good approximation for weak couplings, and the enhanced mass in the hot medium leads to the phenomenon that the system cannot reach the Stefan-Boltzmann limit at high temperatures. For the latter, with the self-energy at Fock level, the suppressed mass in the hot medium results in a maximum temperature where the particles become massless and the system starts to collapse. The off-shell effect in the ϕ^3 theory generates a scattering phase shift simultaneously when the quasiparticles are formed. The total thermodynamic potential contains a quasiparticle part and an off-shell part. The off-shell part increases with increasing temperature and coupling constant, while its contribution to the total thermodynamics is enhanced in the case of weak couplings. For the coupling constant $g=0.1$, the ratio of the off-shell part to the quasiparticle part reaches 25% at high temperatures. The formulas developed here will be extended to discuss more realistic systems.

ACKNOWLEDGMENTS

This work was supported in part by the grants NSFC19925519, 10135030, and 10105005, and the national research program G2000077407.

[1] J.I. Kapusta, *Finite-Temperature Field Theory* (Cambridge University Press, Cambridge, England, 1989).
[2] M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, 2000).
[3] J.P. Blaizot, E. Iancu, and A. Rebhan, Phys. Rev. Lett. **83**, 2906 (1999); J.P. Blaizot and E. Iancu, Phys. Rep. **359**, 355 (2002).
[4] For instance, see *Quark-Gluon Plasma*, edited by R.C. Hwa (World Scientific, Singapore, 1990).
[5] For instance, see Proceedings of the QM'01 Conference, Stony Brook, 2001 [Nucl. Phys. A698 (2002)].
[6] A. Peshier, B. Kaempfer, O.P. Pavlenko, and G. Soff, Phys. Rev. D **54**, 2399 (1996); P. Levai and U. Heinz, Phys. Rev. C **57**, 1879 (1998).
[7] For instance, see, F. Karsch, Nucl. Phys. **A698**, 199c (2002).
[8] R. Dashen, S. Ma, and H.J. Bernstein, Phys. Rev. **187**, 345 (1969).
[9] R.E. Norton, Ann. Phys. (N.Y.) **170**, 18 (1986).
[10] S. Jeon and P.J. Ellis, Phys. Rev. D **58**, 045013 (1998).
[11] J. Hüfner, S.P. Klevansky, P. Zhuang, and H. Voss, Ann. Phys. (N.Y.) **234**, 225 (1994); P. Zhuang, J. Hüfner, and S.P. Klevansky, Nucl. Phys. **A576**, 525 (1994).