Decoherence and records for the case of a scattering environment

P. J. Dodd and J. J. Halliwell

Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom
(Received 20 January 2003; published 23 May 2003)

Using nonrelativistic many body quantum field theory, a master equation is derived for the reduced density matrix of a dilute gas of massive particles undergoing scattering interactions with an environment of light particles. The dynamical variable that naturally decoheres (the pointer basis) is essentially the local number density of the dilute gas. Earlier master equations for this sort of system (such as that derived by Joos and Zeh) are recovered on restricting to the one-particle sector for the distinguished system. The derivation shows explicitly that the scattering environment stores information about the system by "measuring" the number density. This therefore provides an important example of the general connection between decoherence and records indicated by the decoherent histories approach to quantum theory. It also brings the master equation for this system into a form emphasizing the role of local densities, which is relevant to current work in deriving hydrodynamic equations from quantum theory.

I. INTRODUCTION

The notion of decoherence plays a central role in studies of emergent classicality and the foundations of quantum theory generally [1]. While it is usually regarded as signifying the loss of quantum coherence for a system of interest, it may also usefully be regarded as an indication of the degree to which information about the system is stored somewhere in the system or in its immediate environment [2,3]. It is in this way that decoherence is related to "generalized measurements." An important application of these ideas is in quantum cosmology [4]. In applying quantum theory to the very early universe, there are no actual measuring devices to measure what was happening. The process of decoherence, however, guarantees that measurements we make in the present are correlated with alternatives in the past.

These ideas are perhaps most transparent when formulated in terms of the decoherent histories approach to quantum theory [2,4–8]. Other approaches to decoherence, such as Zurek's "einselection" approach [1,9,10], related density matrix approaches [11] or quantum state diffusion [12,13], may be equally useful for analyzing these issues, but will not be explored here. It is the aim of this paper, continuing in the spirit of Ref. [3], to investigate the connection between decoherence and information storage. To fix ideas, we briefly review the decoherent histories approach (although the general results of this paper are by no means specific to that approach).

In the decoherent histories approach [2,4–8], probabilities are assigned to histories via the formula

$$p(\alpha_1, \alpha_2, \dots) = \text{Tr}(C_{\underline{\alpha}} \rho C_{\underline{\alpha}}^\dagger) \qquad (1)$$

where $C_{\underline{\alpha}}$ denotes a time-ordered string of projectors at times $t_1, \dots, t_n$,

$$C_{\underline{\alpha}} = P_{\alpha_n} e^{-(i/\hbar)H(t_n - t_{n-1})} P_{\alpha_{n-1}} \cdots e^{-(i/\hbar)H(t_2 - t_1)} P_{\alpha_1} \qquad (2)$$

and $\underline{\alpha}$ denotes the string $\alpha_1, \alpha_2, \dots, \alpha_n$. We are interested in sets of histories which satisfy the condition of decoherence, which is that the decoherence functional

$$D(\underline{\alpha}, \underline{\alpha}') = \text{Tr}(C_{\underline{\alpha}} \rho C_{\underline{\alpha}'}^\dagger) \qquad (3)$$

is zero when $\underline{\alpha} \neq \underline{\alpha}'$. Decoherence implies the weaker condition that $\text{Re}D(\underline{\alpha}, \underline{\alpha}') = 0$ for $\underline{\alpha} \neq \underline{\alpha}'$, and this is equivalent to the requirement that the above probabilities satisfy the probability sum rules.

The stronger condition of decoherence is the more interesting one since it is related to the existence of records—information storage about the histories somewhere in the system. More precisely, if the initial state is pure, decoherence means that there exist a set of alternatives at the final time t_n which are perfectly correlated with the alternatives $\alpha_1, \dots, \alpha_n$ at times $t_1, \dots, t_n$ [5,14]. This follows because, with a pure initial state $|\Psi\rangle$, the decoherence condition implies that the states $C_{\underline{\alpha}}|\Psi\rangle$ are an orthogonal set. It is therefore possible to introduce a projection operator R_β (which is generally not unique) such that

$$R_{\underline{\beta}} C_{\underline{\alpha}} |\Psi\rangle = \delta_{\underline{\alpha}\underline{\beta}} C_{\underline{\alpha}} |\Psi\rangle. \qquad (4)$$

It follows that the extended histories characterized by the chain $R_{\underline{\beta}} C_{\underline{\alpha}} |\Psi\rangle$ are decoherent, and one can assign a probability to the histories $\underline{\alpha}$ and the records $\underline{\beta}$, given by

$$p(\alpha_1, \alpha_2, \dots, \alpha_n; \beta_1, \beta_2, \dots, \beta_n)$$
$$= \text{Tr}(R_{\beta_1 \beta_2, \dots, \beta_n} C_{\underline{\alpha}} \rho C_{\underline{\alpha}}^\dagger). \qquad (5)$$

This probability is then zero unless $\alpha_k = \beta_k$ for all k, in which case it is equal to the original probability $p(\alpha_1, \dots, \alpha_n)$. Hence, either the α's or the β's can be completely summed out of Eq. (5) without changing the probability, so the probability for the histories can be entirely replaced by the probability for the records at a fixed moment of time at the end of the history:

$$p(\underline{\alpha}) = \text{Tr}[R_{\underline{\alpha}} \rho(t_n)] = \text{Tr}(C_{\underline{\alpha}} \rho C_{\underline{\alpha}}^\dagger). \qquad (6)$$

Conversely, the existence of records $\beta_1, \ldots, \beta_n$ at some final time perfectly correlated with earlier alternatives $\alpha_1, \ldots, \alpha_n$ at $t_1, \ldots, t_n$ implies decoherence of the histories.

These issues are most usefully investigated in the context of particular models, and it then becomes possible to ask some more precise questions: Which dynamical variables in the environment store the information about the decoherent histories? Or what is essentially the same thing, how are the "pointer basis" variables stored in the environment? How is the amount of decoherence related to the amount of information stored?

Reference [3] investigated these questions in the context of the quantum Brownian model (QBM), which consists of a particle of large mass M moving in a potential $V(x)$ and linearly coupled to a bath of harmonic oscillators. The total system is therefore described by the action

$$S_T[x(t), q_n(t)] = \int dt \left[\frac{1}{2} M \dot{x}^2 - V(x) \right] + \sum_n \int dt \left[\frac{1}{2} m_n \dot{q}_n^2 \right.$$
$$\left. - \frac{1}{2} m_n \omega_n^2 q_n^2 - c_n q_n x \right]. \tag{7}$$

In the traditional discussion of decoherence in this model, it is shown that for a continuum of oscillators in a thermal state, the influence functional or density matrix become approximately diagonalized in position. This may be seen, for example, through the master equation for the reduced density matrix $\rho(x,y)$ of the distinguished system [15], which in the high temperature limit is

$$\frac{\partial \rho}{\partial t} = \frac{i\hbar}{2m} \left(\frac{\partial^2 \rho}{\partial x^2} - \frac{\partial^2 \rho}{\partial y^2} \right) - \frac{i}{\hbar} [V(x) - V(y)] \rho - \gamma(x-y)$$
$$\times \left(\frac{\partial \rho}{\partial x} - \frac{\partial \rho}{\partial y} \right) - \frac{2m\gamma kT}{\hbar^2} (x-y)^2 \rho. \tag{8}$$

In Ref. [3], the issue of how the information about position is stored in the environment was addressed. The system is linear in the oscillators, so the classical and quantum dynamics coincide for the environment. Classically, the equations of motion of the environment of oscillators are

$$m_n \ddot{q}_n + m_n \omega_n^2 q_n = -c_n x(t). \tag{9}$$

The solution to this equation, with fixed $p_n(0)$, $q_n(0)$ is

$$q_n(\tau) = q_n(0) \cos \omega_n \tau + \frac{p_n(0)}{m_n \omega_n} \sin \omega_n \tau$$
$$- \frac{c_n}{m_n \omega_n} \int_0^\tau dt \ x(t) \ \sin \omega_n(\tau - t) \tag{10}$$

$$p_n(\tau) = p_n(0) \cos \omega_n \tau - m_n \omega_n q_n(0) \sin \omega_n \tau$$
$$- c_n \int_0^\tau dt \ x(t) \ \cos \omega_n(\tau - t) \tag{11}$$

where $p_n = m\dot{q}_n$. From this solution, one can see that at the final time τ, the positions and momenta of the environment of oscillators depend on the particle's trajectory $x(t)$ via the temporally nonlocal quantities

$$X_n^s = \int_0^\tau dt \ x(t) \ \sin \omega_n(\tau - t) \tag{12}$$

$$X_n^c = \int_0^\tau dt \ x(t) \ \cos \omega_n(\tau - t). \tag{13}$$

These are essentially the Fourier modes of the particle's trajectory $x(t)$. Hence, each oscillator stores a single Fourier mode of the trajectory, and therefore by using a large number of oscillators, information about many Fourier modes is stored from which the approximate trajectory may be recovered. Furthermore, since it is the Fourier modes that are naturally registered in the environment, rather than positions at each moment of time, decoherence is in fact most clearly seen in terms of the variables (12), (13), rather than position, as shown in Ref. [3]. The variables are nonlocal in time so it can only be seen at the level of an influence functional expressed in path integral language, rather than a master equation. Hence, in this model, it was possible to see exactly how the environment stored the information about the system's trajectory in configuration space. Furthermore, a detailed quantitative estimate of the information content was also carried out in Ref. [3].

Although a very illustrative model, the quantum Brownian motion model is not the most relevant model for decoherence in physically interesting situations. Far more physically significant is the case in which the environment is a set of light particles which interact with the distinguished particle by a scattering process. The resulting master equation, first derived by Joos and Zeh, is very similar in form to the QBM case, Eq. (8) [11,16,17]. But the dynamics of the environment, and therefore the means of information storage, are rather different.

The aim of this paper is to investigate the connection between decoherence and records in the case of decoherence by a scattering environment. In some ways it is simpler, since, in the usual assumption of widely separated time scales for system and environment dynamics, each environmental particle scatters briefly off the distinguished particles, and moves freely thereafter, carrying some information about the distinguished particles. This process can therefore be described by a Markovian master equation, and the process of information storage and decoherence may be described in a moment by moment manner (unlike the quantum Brownian motion case, where the environment oscillators store information about the entire history). This means in fact that we do not need to make use of the full machinery of the decoherence functional—it is sufficient in fact to look at the evolution of the reduced density operator.

In the quantum Brownian motion model case, the system variables that the environment measures were actually identified quite simply, from the classical equations of motion. In the scattering case, the system variables measured most di-

rectly by the environment are also determined quite easily, by examining simple scattering processes. In particular, suppose we consider the scattering of some light particles off a dilute gas of a set of more massive particles with coordinates $\mathbf{q}_j$. Then, it follows quite straightforwardly from simple scattering theory (and we will in fact demonstrate this) that the scattering amplitude is proportional to the Fourier transform of the number density of the massive particles,

$$N_{\mathbf{k}} = \sum_j e^{i\mathbf{k}\cdot\mathbf{q}_j}. \tag{14}$$

This means that, loosely speaking, for a known interaction potential, measurements of the initial and final momenta of the scattering environment determine the number density of the distinguished system.

Of course, the number density is closely related to position, which is normally held to be the preferred basis in these calculations. But, following the lead of the oscillator model, we expect decoherence to look simplest in terms of the dynamical variables which are most simply and directly stored in the environment. Our aim is therefore to give a derivation of the master equation which emphasizes the central role played by the number density. We have found that the derivation is in fact most transparent in terms of nonrelativistic many body quantum field theory, where the number density appears very naturally. We will give an alternative and more general derivation of the master equation, using many body theory, which brings out the role of local number density more clearly, hence showing the connection with records.

It is pertinent at this stage to mention the Lindblad form of the master equation [18], which is the most general possible form a master equation can take under the assumption that the evolution is Markovian (a condition well satisfied in a wide variety of interesting models). The Lindblad master equation is

$$\frac{d\rho}{dt} = -i[H,\rho] - \frac{1}{2}\sum_{j=1}^{n} (\{L_j^\dagger L_j,\rho\} - 2L_j\rho L_j^\dagger). \tag{15}$$

Here, H is the Hamiltonian of the distinguished subsystem (sometimes modified by terms depending on the L_j) and the n operators L_j model the effects of the environment. The master equation of quantum Brownian motion, for example, is of this form with

$$L = \left(\frac{4m\gamma kT}{\hbar^2}\right)^{1/2} x + i\left(\frac{\gamma}{2mkT}\right)^{1/2} p \tag{16}$$

as described in Refs. [19,13]. [Actually, the master equation (8) is not strictly of the Lindblad form, and as a consequence can suffer from a violation of positivity [20]. However, the difference between Eq. (8) and the Lindblad form with L given by Eq. (16) is of the order of $1/T$ which does not matter for high temperatures.]

The Lindblad operators L_j determine the basis in which the density operator tends to become approximately diagonal, or what is essentially the same, the sets of variables describing an approximately decoherent set of histories. This

may be seen from the formal solution to the Lindblad equation [21]. Consider the case of a single Lindblad operator $L = L_R + iL_I$, where L_R, L_I are Hermitian. Divide the finite time interval $[0,t]$ into K subintervals, so that $t = K\delta t$, and let $\delta t \to 0$, $K \to \infty$, holding t constant. Then, the formal solution to the Lindblad equation is obtained by taking the limit $\delta t \to 0$, $K \to \infty$ (with t fixed) of the expression

$$\rho(t) = \left(\frac{\delta t}{\pi}\right)^K \int d^2 l_1 \cdots d^2 l_K \prod_{m=1}^{K} \exp\left(\frac{\delta t}{2}(\ell_m^* L - \ell_m L^\dagger)\right)$$

$$\times \exp\left(-\frac{\delta t}{2}|L - \ell_m|^2\right)\exp\left(-\frac{i}{\hbar}H'\delta t\right)\rho(0)$$

$$\times \prod_{m=1}^{K} \exp\left(\frac{i}{\hbar}H'\delta t\right)\exp\left(-\frac{\delta t}{2}|L - \ell_m|^2\right)$$

$$\times \exp\left(-\frac{\delta t}{2}(\ell_m^* L - \ell_m L^\dagger)\right). \tag{17}$$

Here, $H' = H + (i\hbar/4)[L,L^\dagger]$ and the ℓ_m are complex numbers at the discrete moments of time labeled by m. We use the notation

$$|L - \ell_m|^2 \equiv (L_R - \operatorname{Re}\ell_m)^2 + (L_I - \operatorname{Im}\ell_m)^2. \tag{18}$$

The ordering of the operators at each moment of time is irrelevant in the limit $\delta t \to 0$ (although the operators at different times are time ordered, according to increasing m). That this is the solution is readily verified by explicit computation. The solution has the form of a "measurement process" of the L's, continuous in time, with "feedback" via the terms $(\ell_m^* L - \ell_m L^\dagger)$ [22]. That is, one can see that the effect of the environment is to produce a tendency towards diagonality in L.

We shall show that a many-body theory derivation of the master equation in the case of a scattering environment leads to a master equation of the Lindblad form (under the assumption that the environment dynamics are much faster than the system dynamics), and that the Lindblad operators are essentially the local number density. The previous forms of the master equation are recovered in the one-particle sector for the system of massive particles.

This work grew out of a more ambitious program, in the context of the decoherent histories approach to quantum theory, which aims to give a very general account of emergent classicality. In particular, it is asserted that at sufficiently coarse-grained scales, the local densities (number, momentum, energy) define a set of habitually decohering variables, even in the absence of an environment [2,23]. This is because they are locally conserved, and therefore slowly varying when coarse grained over sufficiently large volumes, and thus are expected to be approximately decoherent (because exactly conserved quantities are exactly decoherent in the histories approach). This is therefore a different mechanism for decoherence than the usual mechanism of decoherence through an environment. Hence, in order to close the gap between the familiar system-environment models and the less familiar hydrodynamic models without an obvious

environment, it is useful to rewrite the system-environment models in terms of local densities as we do here.

In Sec. II, we briefly review many body field theory, and carry out the simple scattering calculation leading to the result that the scattering particles effectively measure the local number density, Eq. (14).

In Sec. III, we use many body field theory to derive the master equation for the system, using a slow motion limit for the gas of massive particles. As anticipated it has the Lindblad form with the Lindblad operators proportional to the Fourier-transformed number density $N_{\mathbf{k}}$.

In Sec. IV, we show that our master equation reduces to an earlier result of Gallis and Fleming [17] in the one-particle sector for the gas of massive particles. (This is essentially the same as the master equation of Joos and Zeh [11] but the comparison with Gallis and Fleming is more direct.)

The master equation of Secs. III and IV does not have any dissipation and is analogous to the Lindblad equation of quantum Brownian motion with L proportional to x. In Sec. V, we go beyond the slow-motion limit to derive a master equation with dissipative terms.

We summarize and conclude in Sec. VI.

II. MANY BODY FIELD THEORY

The dynamics of a many body system is very conveniently handled using many body quantum field theory. We now set up the formalism of many body field theory [24,25] which we will use to derive the master equation. We consider a set of nonrelativistic system particles described by a field $\psi(\mathbf{x})$ interacting through a potential $\phi(\mathbf{x})$ with an environment described by a field $\chi(\mathbf{x})$. The total system is described by the Hamiltonian

$$H = \int d^3x \left(\frac{1}{2M} \boldsymbol{\nabla}\psi^\dagger(\mathbf{x})\cdot\boldsymbol{\nabla}\psi(\mathbf{x}) + \frac{1}{2m}\boldsymbol{\nabla}\chi^\dagger(\mathbf{x})\cdot\boldsymbol{\nabla}\chi(\mathbf{x}) \right)$$
$$+ \int d^3x\, d^3x'\, \psi^\dagger(\mathbf{x})\psi(\mathbf{x}')\phi(\mathbf{x}-\mathbf{x}')\chi^\dagger(\mathbf{x}')\chi(\mathbf{x}) \quad (19)$$

(for simplicity, we set $\hbar = 1$ hereafter). In this language, the number densities $N(\mathbf{x})$ and $n(\mathbf{x})$ of the system and environment fields are

$$N(\mathbf{x}) = \psi^\dagger(\mathbf{x})\psi(\mathbf{x}) \quad (20)$$

$$n(\mathbf{x}) = \chi^\dagger(\mathbf{x})\chi(\mathbf{x}). \quad (21)$$

[This is the field-theoretic version of Eq. (14).]

The above relations are also more conveniently written in terms of $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$, the annihilation operators for the system and environment, respectively, and the Hamiltonian then is

$$H = \sum_{\mathbf{q}} (E_{\mathbf{q}} a_{\mathbf{q}}^\dagger a_{\mathbf{q}} + \omega_{\mathbf{q}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}}) + \frac{1}{V} \sum_{\mathbf{k}_1' + \mathbf{k}_2' = \mathbf{k}_1 + \mathbf{k}_2}$$
$$\times \nu(\mathbf{k}_2' - \mathbf{k}_2) a_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_1'} b_{\mathbf{k}_2'} \quad (22)$$

where $E_{\mathbf{q}} = \mathbf{q}^2/2M$, $\omega_{\mathbf{q}} = \mathbf{q}^2/2m$, V is the spatial volume of the system (which we assume is in a box) and

$$\nu(\mathbf{k}) = \int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\phi(\mathbf{x}). \quad (23)$$

The Fourier transformed number densities are

$$N_{\mathbf{k}} = \sum_{\mathbf{q}} a_{\mathbf{q}}^\dagger a_{\mathbf{q}+\mathbf{k}} \quad (24)$$

$$n_{\mathbf{k}} = \sum_{\mathbf{q}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}+\mathbf{k}} \quad (25)$$

and one may see that the Hamiltonian has the more concise form

$$H = \sum_{\mathbf{q}} (E_{\mathbf{q}} a_{\mathbf{q}}^\dagger a_{\mathbf{q}} + \omega_{\mathbf{q}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}}) + \frac{1}{V} \sum_{\mathbf{k}} \nu(\mathbf{k}) N_{\mathbf{k}} n_{-\mathbf{k}} \quad (26)$$

$$= H_0 + H_{int}. \quad (27)$$

From these relations we see that the environment couples to the number density of the system. It is this feature of many body field theory that makes it the appropriate medium for the derivation of the master equation emphasizing the role of number density.

The S matrix is

$$S = T \exp\left(-i \int_{-\infty}^{\infty} dt\ H_{int}(t) \right) \quad (28)$$

where

$$H_{int}(t) = \frac{1}{V} \sum_{\mathbf{k}} \nu(\mathbf{k}) N_{\mathbf{k}}(t) n_{-\mathbf{k}}(t) \quad (29)$$

and here

$$N_{\mathbf{k}}(t) = \sum_{\mathbf{q}} a_{\mathbf{q}}^\dagger a_{\mathbf{q}+\mathbf{k}}\ e^{i(E_{\mathbf{q}} - E_{\mathbf{q}+\mathbf{k}})t} \quad (30)$$

$$n_{\mathbf{k}}(t) = \sum_{\mathbf{q}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}+\mathbf{k}}\ e^{i(\omega_{\mathbf{q}} - \omega_{\mathbf{q}+\mathbf{k}})t}. \quad (31)$$

We may now use this formalism to look at a simple scattering situation to determine how the environment stores information about the system. In the quantum Brownian motion case, the nature of information storage was determined in essence by solving the classical equations of motion. A similar strategy works here too. Let us suppose that the distinguished system is classical, and consider what happens when the environment scatters off it. Suppose the environment starts in an initial momentum state $|\mathbf{k}_0\rangle$ and scatters into a final state $|\mathbf{k}_f\rangle$. The scattering amplitude for this process, to first order, is

$$\langle \mathbf{k}_f | S | \mathbf{k}_0 \rangle = \frac{i}{V} \int_{-\infty}^{\infty} dt \sum_{\mathbf{k}} \nu(\mathbf{k}) N_{\mathbf{k}}(t) \langle \mathbf{k}_f | n_{-\mathbf{k}}(t) | \mathbf{k}_0 \rangle$$

$$= \frac{i}{V} \nu(\mathbf{k}) \int dt N_{\mathbf{k}}(t) e^{i(\omega_{\mathbf{k}_f} - \omega_{\mathbf{k}_0})t} \qquad (32)$$

where $\mathbf{k} = \mathbf{k}_f - \mathbf{k}_0$. This simple result shows that a single scattering event by the environment stores information about the Fourier transform (in space and time) of the number density. It is in this sense that the number density has a preferred status—this is the variable that is measured most directly by the environment and is the exact analogy of the relations Eqs. (12), (13) in the quantum Brownian case.

The measured variables above are, of course, nonlocal in time, involving a temporal Fourier transform of the number density, so they cannot, in fact, be compatible with a Markovian master equation. Under a reasonable slow motion assumption, the system time scale is much slower than the environment time scale, and we may ignore the time dependence in $N_{\mathbf{k}}(t)$, yielding

$$\langle \mathbf{k}_f | S | \mathbf{k}_0 \rangle = \frac{i}{V} \nu(\mathbf{k}) \ N_{\mathbf{k}} \delta(\omega_{\mathbf{k}_f} - \omega_{\mathbf{k}_0}). \qquad (33)$$

This corresponds more directly to a Markovian master equation, as we shall see.

It remains to briefly sketch the connection between these results and the discussion in Sec. I of records in the decoherent histories approach. We imagine that the environment consists of a very large number of particles which scatter off the system particles. Each scattering event consists of an incoming environment particle with momentum $\mathbf{k}_0$ scattered into a final state of momentum $\mathbf{k}_f$, as outlined above. After the scattering event, which is essentially instantaneous (on the time scale of system dynamics), we may imagine that the scattered particle propagates freely and may be measured at any time in the future. Therefore, the records in the decoherent histories approach consist of projections at the end of this histories onto the momenta of all the scattered environment particles, from which the number densities $N_{\mathbf{k}}$ of the system at a series of times may be retrodicted.

III. DERIVATION OF THE MASTER EQUATION

Following the method first used by Joos and Zeh [11], we may derive the master equation for the reduced density operator ρ of the system by considering the scattering of the environment off the system, to second order in interactions. We assume that the system and environment are initally uncorrelated, so the total density operator is

$$\rho_T = \rho_0 \otimes \rho_{\mathcal{E}}. \qquad (34)$$

We also assume that each scattering event takes place on a time scale which is extremely short compared to the time scale of system dynamics. This means that in an interval of time Δt which is long for the environment but short for the system, we may write

$$\rho_T(t + \Delta t) = S \rho_T(t) S^{\dagger} \qquad (35)$$

(where we are using the interaction picture). Expanding Eq. (35) to second order, the S-matrix may be written as

$$S = 1 + iU_1 - U_2 \qquad (36)$$

where

$$U_1 = -\int_{-\infty}^{\infty} dt \ H_{int}(t) \qquad (37)$$

and

$$U_2 = \frac{1}{2} \int dt_1 \int dt_2 \ \mathrm{T}(H_{int}(t_1) H_{int}(t_2)). \qquad (38)$$

The requirement of unitarity, $S^{-1} = S^{\dagger}$, implies that $U_1 = U_1^{\dagger}$ and

$$U_2 + U_2^{\dagger} = U_1^2. \qquad (39)$$

We will therefore write

$$U_2 = \frac{1}{2} U_1^2 + iB \qquad (40)$$

where $B = B^{\dagger}$, so we now have

$$S = 1 + i(U_1 - B) - \frac{1}{2} U_1^2. \qquad (41)$$

Inserting this in Eq. (35), we obtain

$$\frac{d\rho_T}{dt} \Delta t = i[U_1 - B, \rho_T] + U_1 \rho_T U_1 - \frac{1}{2} U_1^2 \rho_T - \frac{1}{2} \rho_T U_1^2. \qquad (42)$$

We now trace Eq. (42) over the environment to obtain the master equation for the system density operator ρ. As is usual in this sort of model, we assume that the environment is so large that its state is essentially unaffected by the interaction with the system. Since the total density operator starts out in the factored state (34), this then means that, to a good approximation, ρ_T persists in the approximately factored form $\rho \otimes \rho_{\mathcal{E}}$, and we may insert this in the right-hand side of Eq. (42) [26]. We thus obtain the preliminary form for the master equation

$$\frac{d\rho}{dt} \Delta t = i[\mathrm{Tr}_{\mathcal{E}}(U_1 \rho_{\mathcal{E}}) - \mathrm{Tr}_{\mathcal{E}}(B \rho_{\mathcal{E}}), \rho]$$

$$+ \mathrm{Tr}_{\mathcal{E}} \left(U_1 \rho_T U_1 - \frac{1}{2} U_1^2 \rho_T - \frac{1}{2} \rho_T U_1^2 \right). \qquad (43)$$

We now work out these terms in more detail. We first consider the simple but useful slow motion approximation, in which we ignore the time dependence of $N_{\mathbf{k}}(t)$. This implies that

$$U_1 \approx -\frac{1}{V} \sum_{\mathbf{k}} \nu(\mathbf{k}) N_{\mathbf{k}} \sum_{\mathbf{q}} b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}-\mathbf{k}} \ 2\pi \delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q}-\mathbf{k}}). \qquad (44)$$

The important terms for decoherence are the final three terms on the right-hand side of Eq. (44). When traced, these give

$$\mathrm{Tr}_{\mathcal{E}}\left(U_1\rho_T U_1 - \frac{1}{2}U_1^2\rho_T - \frac{1}{2}\rho_T U_1^2 \right)$$

$$= \frac{1}{V^2}\sum_{\mathbf{kk'}} c(\mathbf{k},\mathbf{k'})\left(N_{\mathbf{k'}}\rho N_{\mathbf{k}} - \frac{1}{2}N_{\mathbf{k}}N_{\mathbf{k'}}\rho - \frac{1}{2}\rho N_{\mathbf{k}}N_{\mathbf{k'}} \right)$$

$$(45)$$

where

$$c(\mathbf{k},\mathbf{k'}) = 4\nu(\mathbf{k})\nu(\mathbf{k'})\sum_{\mathbf{qq'}} \delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q-k}})\delta(\omega_{\mathbf{q'}} - \omega_{\mathbf{q'-k'}})$$

$$\times \langle b_{\mathbf{q}}^\dagger b_{\mathbf{q-k}} b_{\mathbf{q'}}^\dagger b_{\mathbf{q'-k'}} \rangle_{\mathcal{E}}. \qquad (46)$$

We will take the environment to be a thermal state, which is diagonal in the momentum states. It follows that

$$\langle b_{\mathbf{q}}^\dagger b_{\mathbf{q-k}} b_{\mathbf{q'}}^\dagger b_{\mathbf{q'-k'}} \rangle_{\mathcal{E}} \propto \delta_{\mathbf{q},\mathbf{q'-k'}}\delta_{\mathbf{q'},\mathbf{q-k}}. \qquad (47)$$

This implies $\mathbf{k} = -\mathbf{k'}$, and also that the two delta-functions are the same in Eq. (46). We then interpret the square of the delta-function in the usual way

$$[\delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q-k}})]^2 = \delta(0)\delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q-k}}) = \frac{\Delta t}{2\pi}\delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q-k}}).$$

$$(48)$$

We now have

$$c(\mathbf{k},\mathbf{k'}) = \delta_{\mathbf{k},-\mathbf{k'}}c(\mathbf{k})\frac{\Delta t}{2\pi} \qquad (49)$$

where

$$c(\mathbf{k}) = \frac{2}{\pi}|\nu(\mathbf{k})|^2 \sum_{\mathbf{q}} \delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q-k}})\langle b_{\mathbf{q}}^\dagger b_{\mathbf{q-k}} b_{\mathbf{q-k}}^\dagger b_{\mathbf{q}} \rangle_{\mathcal{E}}$$

$$= \frac{2}{\pi}|\nu(\mathbf{k})|^2 \sum_{\mathbf{q}} \delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q-k}})\langle b_{\mathbf{q}}^\dagger b_{\mathbf{q}} \rangle_{\mathcal{E}}$$

$$\times (\langle b_{\mathbf{q-k}}^\dagger b_{\mathbf{q-k}} \rangle_{\mathcal{E}} + 1). \qquad (50)$$

The terms involving environment averages have the usual thermal form (for a bosonic environment)

$$\langle b_{\mathbf{q}}^\dagger b_{\mathbf{q}} \rangle_{\mathcal{E}} = \frac{1}{e^{\beta(\omega_{\mathbf{q}} - \mu)} - 1} \qquad (51)$$

where $\beta = 1/kT$ with T temperature and μ is the chemical potential.

The form of Eq. (49) means that the important terms in the master equation are of the Lindblad form:

$$\mathrm{Tr}_{\mathcal{E}}\left(U_1\rho_T U_1 - \frac{1}{2}U_1^2\rho_T - \frac{1}{2}\rho_T U_1^2 \right)$$

$$= \frac{\Delta t}{V^2}\sum_{\mathbf{k}} c(\mathbf{k})\left(N_{\mathbf{k}}\rho N_{\mathbf{k}}^\dagger - \frac{1}{2}N_{\mathbf{k}}^\dagger N_{\mathbf{k}}\rho - \frac{1}{2}\rho N_{\mathbf{k}}^\dagger N_{\mathbf{k}} \right)$$

$$(52)$$

where we have used the fact that $N_{\mathbf{k}}^\dagger = N_{-\mathbf{k}}$. The remaining two terms in Eq. (43) clearly just modify the unitary dynamics of the system. First we have

$$\mathrm{Tr}_{\mathcal{E}}(U_1\rho_{\mathcal{E}}) = \frac{1}{V}\sum_{\mathbf{k}} \nu(\mathbf{k})N_{\mathbf{k}}\sum_{\mathbf{q}} \langle b_{\mathbf{q}}^\dagger b_{\mathbf{q-k}} \rangle_{\mathcal{E}} 2\pi$$

$$\times \delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q-k}}). \qquad (53)$$

Clearly from the term $\langle b_{\mathbf{q}}^\dagger b_{\mathbf{q-k}} \rangle_{\mathcal{E}}$ this expression will be zero unless $\mathbf{k} = 0$, and therefore it is proportional to N, the total particle number operator (although the overall coefficient will need to be regularized). This therefore contributes a term to the master equation of the form $[N,\rho]$. We assume that there is a fixed number of system particles so it is reasonable to take this term to be zero.

The other remaining term in Eq. (43) involves the time ordering terms in U_2 and is a bit more complicated to evaluate. Fortunately, the detailed form of this expression is not needed here, and it can in fact be easily shown that this term has the form

$$\mathrm{Tr}_{\mathcal{E}}(B\rho_{\mathcal{E}}) = \Delta t\sum_{\mathbf{k}} d(\mathbf{k})N_{\mathbf{k}}N_{\mathbf{k}}^\dagger \qquad (54)$$

for some coefficient $d(\mathbf{k})$ which we will not need. Inserting all these results in Eq. (43), the factors of Δt all drop out, and we obtain, in the Schrödinger picture,

$$\frac{d\rho}{dt} = -i\left[H_0 - \sum_{\mathbf{k}} d(\mathbf{k})N_{\mathbf{k}}N_{\mathbf{k}}^\dagger, \rho \right] + \sum_{\mathbf{k}} c(\mathbf{k})\left(N_{\mathbf{k}}\rho N_{\mathbf{k}}^\dagger \right.$$

$$\left. - \frac{1}{2}N_{\mathbf{k}}^\dagger N_{\mathbf{k}}\rho - \frac{1}{2}\rho N_{\mathbf{k}}^\dagger N_{\mathbf{k}} \right). \qquad (55)$$

As desired, this is the Lindblad form with the Lindblad operators given by

$$L_{\mathbf{k}} = c^{1/2}(\mathbf{k})N_{\mathbf{k}}. \qquad (56)$$

We have therefore produced a derivation of the master equation for a scattering environment which shows very clearly the connection between the preferred basis (diagonalization in the Lindblad operators) and the information storage about the system, as indicated by the simple scattering calculation, Eq. (33).

It is interesting to note that the decoherence effect is second order in interactions, but we were able to anticipate it from the simple first order calculation, Eq. (33). The reason for this is the relationship Eq. (40), which shows that the important part of the second order terms is the square of the first order terms, and this is a consequence of unitarity.

IV. COMPARISON WITH PREVIOUS WORKS

It is useful to check that the master equation we have derived reproduces known results when we restrict to the one-particle sector for the system. We will compare the results of this to the derivation of Gallis and Fleming [17] (which is essentially the same as Joos and Zeh [11], and Diosi [16]).

In the one-particle sector we may work with a density matrix $\rho(\mathbf{k},\mathbf{k}') = \langle \mathbf{k}|\rho|\mathbf{k}'\rangle$, or equivalently $\rho(\mathbf{x},\mathbf{y})$ in the position representation. We use the relations

$$[N_{\mathbf{q}},a_{\mathbf{k}}^{\dagger}] = a_{\mathbf{k}-\mathbf{q}}^{\dagger} \tag{57}$$

$$[N_{\mathbf{q}},a_{\mathbf{k}}] = -a_{\mathbf{k}+\mathbf{q}}. \tag{58}$$

These relations imply that

$$N_{\mathbf{q}}\rho(\mathbf{k},\mathbf{k}')N_{-\mathbf{q}} = \rho(\mathbf{k}-\mathbf{q},\mathbf{k}'-\mathbf{q}) \tag{59}$$

$$N_{-\mathbf{q}}N_{\mathbf{q}}\rho(\mathbf{k},\mathbf{k}') = \rho(\mathbf{k},\mathbf{k}') \tag{60}$$

$$\rho(\mathbf{k},\mathbf{k}')N_{-\mathbf{q}}N_{\mathbf{q}} = \rho(\mathbf{k},\mathbf{k}'). \tag{61}$$

In the position representation, this means

$$N_{\mathbf{k}}\rho(\mathbf{x},\mathbf{y})N_{-\mathbf{k}} = e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\rho(\mathbf{x},\mathbf{y}). \tag{62}$$

The master equation for the one-particle density operator $\rho(\mathbf{x},\mathbf{y})$ is then

$$\frac{\partial\rho(\mathbf{x},\mathbf{y})}{\partial t} = -i\langle\mathbf{x}|[H_0,\rho]|\mathbf{y}\rangle - F(\mathbf{x}-\mathbf{y})\rho(\mathbf{x},\mathbf{y}) \tag{63}$$

where

$$F(\mathbf{x}-\mathbf{y}) = \frac{1}{(2\pi)^5}\int d^3q\, d^3k\, |\nu(\mathbf{k})|^2 n_{\mathbf{q}}(n_{\mathbf{q}-\mathbf{k}}+1)\delta(\omega_{\mathbf{q}}$$
$$- \omega_{\mathbf{q}-\mathbf{k}})(1-e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}). \tag{64}$$

Note that the term involving the coefficient $d(\mathbf{k})$ in Eq. (55) drops out because $[N_{\mathbf{k}}N_{\mathbf{k}}^{\dagger},\rho]=0$ in the one-particle sector.

To compare this with the Gallis-Fleming result [17], we first introduce the quantity

$$f(\mathbf{k},\mathbf{k}') = \frac{m}{2\pi}\nu(\mathbf{k}-\mathbf{k}') \tag{65}$$

(which appears in the usual Born approximation to the first order scattering). Then, letting $\mathbf{k}\rightarrow-\mathbf{k}+\mathbf{q}$ in Eq. (64), we get

$$F(\mathbf{r}) = \frac{1}{(2\pi)^3 m^2}\int d^3q\, d^3k\, |f(\mathbf{q},\mathbf{k})|^2 n_{\mathbf{q}}(n_{\mathbf{k}}+1)\delta(\omega_{\mathbf{q}}-\omega_{\mathbf{k}})$$
$$\times(1-e^{i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}). \tag{66}$$

The delta-function implies that $\mathbf{q}^2=\mathbf{k}^2$, and we find that

$$F(\mathbf{r}) = \frac{1}{(2\pi)^3 m}\int dq\, q^3 n_q(n_q+1)\int d\Omega\, d\Omega'\, |f(\mathbf{q},\mathbf{k})|^2$$
$$\times(1-e^{i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}). \tag{67}$$

This in fact agrees with Gallis and Fleming if we identify q/m as their $v(q)$, the speed of the incoming particles, and $(1/2\pi^2)q^2 n_q(n_q+1)$ as the density of particles with speed q. In the one particle sector there is therefore agreement with earlier work. (At least up to an overall numerical factor which we could not rectify. However, we have also spotted some small and probably insignificant numerical errors in Ref. [17]).

Mention should also be made of the master equations derived by Unruh and Zurek, which used a field as an environment for a particle [27], and Anastopoulos and Zoupas [28], which used a photon field and as an environment for a spinor field. Also of relevance is the general account of the derivation of master equations given by Omnès [29]. These works are rather different to the present paper.

V. BEYOND THE SLOW MOTION APPROXIMATION

The derivation above assumed, in essence, that the system dynamics are infinitely slow. Not surprisingly, the resulting master equation does not involve dissipation, since, in the approximation used, the system is essentially at rest for the time scale of a single scattering event. It is analogous to the master equation of quantum Brownian motion with the Lindblad operator L proportional to x, Eq. (16). To get a more realistic equation with dissipation, we therefore need to go beyond the slow motion approximation.

Because the local number density is a locally conserved quantity, it obeys a continuity equation of the form

$$\dot{N}_{\mathbf{k}} = -i\mathbf{k}\cdot\mathbf{P}_{\mathbf{k}} \tag{68}$$

where $\mathbf{P}_{\mathbf{k}}$ is the local momentum density

$$\mathbf{P}_{\mathbf{k}} = \sum_{\mathbf{q}}\left(\mathbf{q}+\frac{1}{2}\mathbf{k}\right)a_{\mathbf{q}}^{\dagger}a_{\mathbf{q}+\mathbf{k}}. \tag{69}$$

It is reasonable to expect that the master equation will involve this operator when we go beyond the infinitely slow limit. We now briefly repeat the derivation of the master equation, this time allowing a slow time dependence in $N_{\mathbf{k}}(t)$.

We have

$$U_1 = \frac{1}{V}\sum_{\mathbf{k}}\nu(\mathbf{k})\sum_{\mathbf{q}}b_{\mathbf{q}}^{\dagger}b_{\mathbf{q}-\mathbf{k}}\int dt\, N_{\mathbf{k}}(t)e^{i(\omega_{\mathbf{q}}-\omega_{\mathbf{q}-\mathbf{k}})t}. \tag{70}$$

To take into account the time dependence of $N_{\mathbf{k}}(t)$, we write

$$N_{\mathbf{k}}(t) = N_{\mathbf{k}} + t\dot{N}_{\mathbf{k}} + \cdots \qquad (71)$$

where $\dot{N}_{\mathbf{k}}$ is given in terms of the momentum density, Eq. (69). Inserting this in Eq. (70), the factor of t may be rewritten in terms of a delta-function derivative, yielding

$$U_1 = \frac{1}{V} \sum_{\mathbf{k}} \nu(\mathbf{k}) \sum_{\mathbf{q}} b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}-\mathbf{k}} [N_{\mathbf{k}} \delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q}-\mathbf{k}})$$
$$- i\dot{N}_{\mathbf{k}} \delta'(\omega_{\mathbf{q}} - \omega_{\mathbf{q}-\mathbf{k}}) + \cdots]. \qquad (72)$$

We now use this expression for U_1 in the derivation of the master equation. So, for example, we get, in place of Eq. (45),

$$\mathrm{Tr}_{\mathcal{E}}(U_1^2 \rho_{\mathcal{E}}) = \frac{\Delta t}{V^2} \sum_{\mathbf{k}} c(\mathbf{k}) N_{\mathbf{k}}^{\dagger} N_{\mathbf{k}} \rho$$
$$+ \frac{i}{V^2} \sum_{\mathbf{k}} b(\mathbf{k})(N_{\mathbf{k}} \dot{N}_{\mathbf{k}}^{\dagger} - N_{\mathbf{k}}^{\dagger} \dot{N}_{\mathbf{k}}) \rho + \cdots \qquad (73)$$

where $c(\mathbf{k})$ is given by Eq. (50) and

$$b(\mathbf{k}) = |\nu(\mathbf{k})|^2 \sum_{\mathbf{q}} \delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q}-\mathbf{k}}) \delta'(\omega_{\mathbf{q}} - \omega_{\mathbf{q}-\mathbf{k}})$$
$$\times \langle b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}} \rangle_{\mathcal{E}} (\langle b_{\mathbf{q}-\mathbf{k}}^{\dagger} b_{\mathbf{q}-\mathbf{k}} \rangle_{\mathcal{E}} + 1). \qquad (74)$$

This coefficient may in fact be shown to be simply related to $c(\mathbf{k})$. The delta-function derivative may be dealt with by noting the formal relation

$$\delta(x)\delta'(x) = \frac{1}{2} \frac{\partial}{\partial x} [\delta(x)]^2. \qquad (75)$$

Now note that

$$\omega_{\mathbf{q}} - \omega_{\mathbf{q}-\mathbf{k}} = \frac{1}{2m}(2\mathbf{k} \cdot \mathbf{q} - \mathbf{k}^2). \qquad (76)$$

It follows that the delta-function derivatives may be expressed in terms of derivatives with respect to q_i as

$$\delta'(\omega_{\mathbf{q}} - \omega_{\mathbf{q}-\mathbf{k}}) = \frac{2m}{\mathbf{k}^2} k_i \frac{\partial}{\partial q_i} \delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q}-\mathbf{k}}). \qquad (77)$$

Inserting these relations in Eq. (74) and integrating by parts yields

$$b(\mathbf{k}) = -\frac{1}{2} |\nu(\mathbf{k})|^2 \sum_{\mathbf{q}} [\delta(\omega_{\mathbf{q}} - \omega_{\mathbf{q}-\mathbf{k}})]^2 \frac{2m}{\mathbf{k}^2}$$
$$\times k_i \frac{\partial}{\partial q_i} [\langle b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}} \rangle_{\mathcal{E}} (\langle b_{\mathbf{q}-\mathbf{k}}^{\dagger} b_{\mathbf{q}-\mathbf{k}} \rangle_{\mathcal{E}} + 1)] \qquad (78)$$

where we will interpret the square of the delta-function as in Eq. (48).

Now for simplicity work in the high temperature limit, so

$$\langle b_{\mathbf{q}-\mathbf{k}}^{\dagger} b_{\mathbf{q}-\mathbf{k}} \rangle_{\mathcal{E}} \ll 1 \qquad (79)$$

and

$$\langle b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}} \rangle_{\mathcal{E}} \approx e^{\mu\beta} e^{-\beta\omega_{\mathbf{q}}}. \qquad (80)$$

It follows that

$$k_i \frac{\partial}{\partial q_i} \langle b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}} \rangle_{\mathcal{E}} = -\beta k_i \frac{\partial\omega_{\mathbf{q}}}{\partial q_i} \langle b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}} \rangle_{\mathcal{E}} = -\beta \frac{\mathbf{k} \cdot \mathbf{q}}{m} \langle b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}} \rangle_{\mathcal{E}} \qquad (81)$$

and we arrive at the very simple result

$$b(\mathbf{k}) = \frac{\beta}{2} \Delta t c(\mathbf{k}). \qquad (82)$$

It is not difficult to see that we then arrive at a master equation which is once again of the Lindblad form, but this time with Lindblad operators of the form

$$L_{\mathbf{k}} = c^{1/2}(\mathbf{k}) \left(N_{\mathbf{k}} - i\frac{\beta}{2}\dot{N}_{\mathbf{k}} \right) = c^{1/2}(\mathbf{k}) \left(N_{\mathbf{k}} - \frac{\beta}{2}\mathbf{k} \cdot \mathbf{P}_{\mathbf{k}} \right) \qquad (83)$$

(up to terms of the order of β^2, which can be dropped in the approximation we are using). This is clearly closely analogous to the QBM result, Eq. (16). (A closely analogous formula appears in Diosi's paper [16].)

VI. SUMMARY AND DISCUSSION

We have given a derivation of the master equation describing a many-body system interacting with a reasonably general class of environments. The form of the master equation emphasizes the central role of the local number density, which is the system variable measured most directly by the environment in a scattering situation.

We did not in fact give a specific form for the interaction between the system and environment since it was not necessary to illustrate the general points we are making. Some specific forms for this interaction are discussed elsewhere [11,17].

The derivation reduces to familiar results of Gallis and Fleming [17], Diosi [16], and Joos and Zeh [11], when we restrict to the one-particle sector of the many-body field theory. The many-body derivation confers some advantages of the usual derivations (which consider scattering theory in quantum mechanics) in that they avoid essentially classical assumptions about fluxes of scattering particles. Our derivation also has the possibility of being extended to a low temperature regime (and to Bose-Einstein condensation, for example) and to fermionic environments, although we do not discuss this here.

Note added. After completion of this work we became aware of two closely related works [30,31] which also consider decoherence by a scattering environment, similar to Joos and Zeh, and to Gallis and Fleming. These works are different from ours in that they do not consider many body

field theory, as we do. However, the authors of Ref. [31] also found a small discrepancy over a numerical factor in the results of Gallis and Fleming (see the discussion at the end of Sec. IV above). We agree with Ref. [31].

[1] See for example, W. Zurek, Phys. Today **44**(10), 36 (1991).

[2] M. Gell-Mann and J.B. Hartle, Phys. Rev. D **47**, 3345 (1993).

[3] J.J. Halliwell, Phys. Rev. D **60**, 105031 (1999).

[4] J.B. Hartle, in *Gravitation and Quantifications*, Proceedings of the 1992 Les Houches Summer School, edited by B. Julia and J. Zinn-Justin (Elsevier Science, Amsterdam, 1995).

[5] M. Gell-Mann and J.B. Hartle, in *Complexity, Entropy and the Physics of Information*, edited by W. Zurek, SFI Studies in the Sciences of Complexity Vol. VIII (Addison-Wesley, Reading, MA, 1990); in *Proceedings of the Third International Symposium on the Foundations of Quantum Mechanics in the Light of New Technology*, edited by S. Kobayashi, H. Ezawa, Y. Murayama, and S. Nomura (Physical Society of Japan, Tokyo, 1990).

[6] R.B. Griffiths, J. Stat. Phys. **36**, 219 (1984); Phys. Rev. Lett. **70**, 2201 (1993).

[7] R. Omnès, J. Stat. Phys. **53**, 893 (1988); **53**, 933 (1988); **53**, 957 (1988); **57**, 357 (1989); Ann. Phys. (N.Y.) **201**, 354 (1990); Rev. Mod. Phys. **64**, 339 (1992).

[8] J.J. Halliwell, Ann. N.Y. Acad. Sci. **775**, 726 (1994).

[9] W.H. Zurek, Philos. Trans. R. Soc. London, **A356**, 1793 (1998).

[10] W.H. Zurek, Phys. Rev. D **24**, 1516 (1981); **26**, 1862 (1982).

[11] E. Joos and H.D. Zeh, Z. Phys. B **59**, 223 (1985).

[12] N. Gisin and I.C. Percival, J. Phys. A **25**, 5677 (1992); **26**, 2233 (1993); **26**, 2245 (1993); Phys. Lett. A **167**, 315 (1992).

[13] J. Halliwell and A. Zoupas, Phys. Rev. D **52**, 7294 (1995); **55**, 4697 (1997).

[14] M. Gell-Mann and J.B. Hartle, in *Proceedings of the 4th Drexel Symposium on Quantum Nonintegrability*, edited by D.H. Feng and B.L. Hu (International Press, Singapore, 1997); also available as gr-qc/9509054.

[15] See for example, A. Caldeira and A. Leggett, Physica A **121**, 587 (1983). The literature on quantum Brownian motion mod-els is considerable. A selection is G.S. Agarwal, Phys. Rev. A **3**, 828 (1971); **4**, 739 (1971); H. Dekker, *ibid.* **16**, 2116 (1977); Phys. Rep. **80**, 1 (1991); G.W. Ford, M. Kac, and P. Mazur, J. Math. Phys. **6**, 504 (1965); H. Grabert, P. Schramm, and G.-L. Ingold, Phys. Rep. **168**, 115 (1988); V. Hakim and V. Ambegaokar, Phys. Rev. A **32**, 423 (1985); J. Schwinger, J. Math. Phys. **2**, 407 (1961); I.R. Senitzky, Phys. Rev. **119**, 670 (1960). Some derivations of the master equation of quantum Brownian motion are, B.L. Hu, J. Paz, and Y. Zhang, Phys. Rev. D **45**, 2843 (1992); **47**, 1576 (1993); J.J. Halliwell and T. Yu, *ibid.* **53**, 2012 (1996).

[16] L. Diosi, Europhys. Lett. **30**, 63 (1995).

[17] M.R. Gallis and G.N. Fleming, Phys. Rev. A **42**, 38 (1990).

[18] G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).

[19] L. Diósi, Europhys. Lett. **22**, 1 (1993).

[20] V. Ambegaokar, Ber. Bunsenges Phys. Chem. **95**, 400 (1991). (This author in turn cites a private communication from P. Pechukas.)

[21] L. Diósi, N. Gisin, J. Halliwell, and I.C. Percival, Phys. Rev. Lett. **74**, 203 (1995).

[22] C.M. Caves and G.J. Milburn, Phys. Rev. A **36**, 5543 (1987).

[23] J.J. Halliwell, Phys. Rev. D **58**, 105015 (1998); Phys. Rev. Lett. **83**, 2481 (1999).

[24] D.N. Zubarev, *Nonequilibrium Statistical Thermodynamics* (Consultants Bureau, New York, 1974).

[25] A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[26] C.W. Gardiner, *Quantum Noise* (Springer-Verlag, Berlin, 1991).

[27] W.G. Unruh and W.H. Zurek, Phys. Rev. D **40**, 1071 (1989).

[28] C. Anastopoulos and A. Zoupas, Phys. Rev. D **58**, 105006 (1998).

[29] R. Omnès, Phys. Rev. A **56**, 3383 (1997).

[30] B. Vacchini, quant-ph/0301039.

[31] K. Hornberger and J.E. Sipe, quant-ph/0303094.