Gluino condensate and long-range fields

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We argue that in the pure $\mathcal{N}=1$ super Yang-Mills theory gauge symmetry is spontaneously broken to the maximal Abelian subgroup. In particular, the colored gluino condensate is nonzero. It invalidates, in a subtle way, the so-called strong-coupling instanton calculation of the (normal) gluino condensate and resolves the long-standing paradox of why its value does not agree with that obtained by other methods.

I. INTRODUCTION

Pure $\mathcal{N}=1$ super Yang-Mills (SYM) theory is known to possess a nonzero gluino condensate $\langle\lambda\lambda\rangle$ whose phase distinguishes between one of the discrete vacua of the theory [1]. The exact value of $\langle\lambda\lambda\rangle$ has been found by several independent methods of controllable deformation to weak coupling. One method [2,3] uses matter supermultiplets whose nonzero Higgs condensate breaks explicitly the gauge group and gives mass to certain fields. One is then able to compute $\langle\lambda\lambda\rangle$ from a single instanton, with extra fermion zero modes contracted via mass terms. The other method [4] compactifies the Euclidean space $R^4 \rightarrow R^3 \times S^1$, so that Bogomol'nyi-Prasad-Sommerfield (BPS) dyons arise as classical saddle points. Again, the gauge $SU(N)$ group is broken, this time spontaneously, by the nonzero expectation value of the Yang-Mills potential A_4 in the compact direction. Dyons have zero fermionic modes saturating $\langle\lambda\lambda\rangle$; it turns out to be independent of the circumference l of the compact dimension. It is then argued that the power of holomorphy allows one to assert the same value in the decompactified limit $l \rightarrow \infty$. We briefly review these methods below. The results of the two seemingly different methods of getting $\langle\lambda\lambda\rangle$ coincide, including the numerical coefficient [4]. The same result which is apparently exact [5], follows independently from a deformation of the $\mathcal{N}=2$ theory—see Ref. [6] for a recent discussion.

In all those approaches, the gauge symmetry is broken by the deformation. Although in all cases the symmetry-breaking parameter tends to zero as one approaches the strong coupling limit, one can ask if the spontaneous breaking of gauge symmetry (i.e., a dynamical Higgs effect) is not a property of the pure SYM theory itself. We present arguments that it is indeed the case. Matter mutliplets which break color explicitly or compactification which breaks it spontaneously, serve as a "seed" to disclose the true nature of the SYM theory in the strong-coupling limit.

Historically, the first calculation of the gluino condensate [7] was directly in the strong-coupling limit of the pure SYM theory. However, a seemingly "clean" calculation of $\langle\lambda\lambda\rangle^N$ in the pure $SU(N)$ SYM theory by saturating it by instanton zero modes yields a value different from the exact result.

This paradox attracted much attention over the years. There have been several attempts in the past to explain the puzzle. It has been suggested [8] that instantons average over the Z_N vacua, or over an additional vacuum with zero gluino condensate [9]. However, Ref. [6] doubts the validity of those arguments.

In this paper, we suggest an alternative strong-coupling calculation of the gluino condensate $\langle\lambda\lambda\rangle^N$. On the one hand it yields the correct result. On the other hand it is very close to the old instanton calculation, and it becomes possible to pinpoint what exactly is wrong there. Namely, the new calculation reduces to the old strong-coupling instanton calculation *provided one neglects long-ranged fields* vanishing as $1/l$ where l is the size of the system. Normally, such fields have no effect on the local properties of the theory, but not in this case: a small perturbation has a dramatic effect because the system is unstable with respect to spontaneous color symmetry breaking.

II. SPONTANEOUS COLOR SYMMETRY BREAKING IN THE COMPACTIFIED SYM THEORY

In this section we briefly review one of the ways to obtain the correct value of the gluino condensate [4].

Let us consider the $SU(2)$ SYM theory compactified to $R^3 \times S^1$ with the "time" dimension x_4 being of circumference l. It should be stressed that it is *not* an introduction of the physical temperature $T=1/l$ as fermions satisfy periodic conditions in the x_4 direction. Therefore, the usual perturbative periodic potential in A_4 does not emerge as in the temperature case: owing to supersymmetry it is zero to all orders of the perturbation theory. We remind the reader that the perturbative potential $V(A_4)$ is zero at $\sqrt{A_4^a A_4^a} = 0, 2\pi/l, 4\pi/l, \ldots$ at which points the Polyakov line (the holonomy) is trivial. If the holonomy is nontrivial (more precisely, if its spatial average is nontrivial) then $V(A_4) > 0$ and the corresponding gauge configuration has an unacceptable volume-divergent positive energy. This is the usual argument against configurations with nontrivial average holonomy in the pure gauge theory [10]. However, in the compact SYM theory the perturbative potential is identically zero for any

A_4 and one is free to consider configurations with any holonomy at spatial infinity.

Choosing the gauge where at spatial infinity $A_4 \rightarrow v \tau^3/2$ one finds that there are two self-dual (L,M) and two anti-self-dual $(\bar{L},\bar{M})$ dyon solutions of the YM equations, with the same asymptotic value of $A_4^3 = v$ at spatial infinity [11,4]. These solutions have all four possible signs of the electric and magnetic charges. The corresponding fields are given explicitly in the Appendix.

The nonperturbative dyon-induced superpotential found in Ref. [4] shows that the minimum (zero) energy is achieved when the weights of the L and M dyons become equal, which happens at

$$\sqrt{A_4^a A_4^a} = v = \frac{\pi}{l}. \tag{1}$$

We notice that this value corresponds to the maximum of the would-be perturbative potential but it is absent. The system settles at the minimum (1) of the nonperturbative potential. It clearly demonstrates that in compactified SYM theory color is spontaneously broken by the Higgs mechanism, with A_4^a playing the role of the Higgs field in the adjoint representation. The symmetry breaking pattern is $SU(2) \rightarrow U(1)$. For higher $SU(N)$ gauge groups the minimum (zero) energy is achieved at [4]

$$A_4 = \text{diag}\left(\frac{N-1}{N}, \frac{N-3}{N}, \ldots, -\frac{N-1}{N}\right)\frac{\pi}{l}. \tag{2}$$

It means that the $SU(N)$ gauge group is spontaneously broken down to the maximal Abelian subgroup $U(1)^{N-1}$, at least at small compactification circumference $l \ll \Lambda^{-1}$ where Λ is the SYM scale parameter.

Equation (2) is not gauge invariant. To put it in a gauge-invariant form one can consider the Polyakov line (the holonomy) along the compactified dimension; its eigenvalues are gauge invariant:

$$P = P \exp\left(i \int_0^l dx^4 \, A_4\right)$$

$$= \text{diag}\left[\exp\left(i\pi \frac{N-1}{N}\right),\right.$$

$$\left.\exp\left(i\pi \frac{N-3}{N}\right) \cdots \exp\left(-i\pi \frac{N-1}{N}\right)\right], \tag{3}$$

$$\text{Tr}\, P = 0. \tag{4}$$

For $SU(2)$ the Polyakov line's eigenvalues are

$$P = \text{diag}(i, -i), \quad \text{Tr}\, P = 0. \tag{5}$$

One dyon can be considered in whatever gauge. However, if we wish to consider the vacuum filled by dyons, we have to take more than one dyon. Two and more dyons can be put together only in the singular "stringy" gauge (see Appendix) where all of them have the same orientation in color space. This orientation is preserved throughout the R^3 volume. The

mere notion of the ensemble of dyons (or monopoles) implies that color symmetry is broken. Of course, once color is aligned, one can always randomize the color orientation by an arbitrary point-dependent gauge transformation, just as the direction of the Higgs field can be randomized but that does not undermine the essence of the Higgs effect. In our case, the eigenvalues of the holonomy [Eqs. (3),(5)] and $\text{Tr}\, P = 0$ are gauge-invariant signatures of the Higgs effect.

Equations (4),(5) do not mean that $\text{Tr}\, P$ is zero identically: it experiences point-to-point fluctuations, of course. For example, if $\text{Tr}\, P$ is measured near the dyon center it will be anything but zero. The statement is that $\text{Tr}\, P \rightarrow 0$ far away from dyon centers. A simple calculation shows that also $\langle \text{Tr}\, P \rangle = 0$ for a Coulomb gas of dyons. As a matter of fact, this is the usual confinement requirement.

Although $A_4 = \pi/l \rightarrow 0$ in the strong-coupling decompactified limit $l \rightarrow \infty$, taken naively, the holonomy (3) remains nontrivial. Unfortunately, it is not a holomorphic quantity so that one cannot prove it rigorously. Nevertheless, we shall argue in the next section that the holonomy does remain nontrivial and that color symmetry remains broken in the decompactified limit. To that end we would need to consider the gluino condensate which is a holomorphic quantity.

Both L and M dyons have two gluino zero modes being the Grassmann partners of the four translational zero modes and thus being related to the dyon field strength:

$$\lambda^{a\alpha}_{\text{zero mode}} = (\sigma_\mu^+)^\alpha_\beta (\sigma_\nu^-)^{\dot\beta}_\gamma \xi^\gamma F^a_{\mu\nu} = (\sigma_i)^\alpha_\gamma \xi^\gamma E_i^a, \tag{6}$$

where $E_i^a = B_i^a$ is the electric field strength of a dyon, see Eqs. (A10),(A13). As shown in Ref. [4] the dyon zero modes saturate the gluino condensate

$$\langle \lambda\lambda \rangle = \langle \epsilon_{\alpha\beta} \lambda^{a\alpha}(x) \lambda^{a\beta}(x) \rangle$$

$$= 2\frac{\Lambda^3}{4\pi v} \int d^3z \, E_i^a(x-z) E_i^a(x-z)$$

$$= \frac{\Lambda^3}{v} \int_0^\infty dr \, r^2 [2F_1^2(r) + F_2^2(r)]$$

$$= \Lambda^3$$

$$\equiv \frac{16\pi^2 M_{\text{PV}}^3}{g^2(M_{\text{PV}})} \exp\left[-\frac{4\pi^2}{g^2(M_{\text{PV}})}\right], \tag{7}$$

where Λ is the renormalization-invariant combination of the Pauli-Villars regularization mass and the bare gauge coupling.[1] The coefficient "2" comes from summing up the (equal) contributions of L and M dyons. The radial functions $F_{1,2}(r)$ are the profile functions of the dyon, see Eqs. (A4),(A5). We remark that it is actually the *anti*-self-dual $\bar{L},\bar{M}$ dyons that lead to the $\langle \lambda\lambda \rangle$ condensate (self-dual L,M dyons lead to $\langle \bar\lambda\bar\lambda \rangle$) but we shall not stress this distinction.

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[1]Λ^3 used here is six times bigger than that used in the QCD convention.

Although technically obtained in the small l limit the result (7) coincides with the exact one in the decompactified strong-coupling limit.

III. INSTANTONS VS DYONS

Let us now recall the strong-coupling instanton calculation of the gluino condensate [7]. Contrary to the dyon, the instanton has four gluino modes for the $SU(2)$ group. Therefore, a single gluino condensate cannot be saturated by an instanton. Instead, one considers a two-point correlation function

$$C(x-y) = \langle \epsilon_{\alpha\beta}\lambda^{a\alpha}\lambda^{a\beta}(x)\epsilon_{\gamma\delta}\lambda^{c\gamma}\lambda^{c\delta}(y)\rangle \qquad (8)$$

which can be saturated by a single instanton. This correlation function does not actually depend on $x-y$ owing to supersymmetry. Therefore, one can evaluate the correlator at $|x-y|\to 0$ using small-size instantons. Since the correlator is $|x-y|$ independent, the same value holds at $|x-y|\to\infty$ where it can be factorized into the product of two gluino condensates $\langle\lambda\lambda\rangle$. This procedure known as "strong-coupling instantons" gives a famous discrepancy factor of $\frac{4}{5}$ as compared to the exact result. We shall show that the evaluation of $C(x-y)$ from an instanton is incorrect both for vanishing and for large $|x-y|$: the seemingly clean calculation has a loophole because of the spontaneous breaking of the gauge group.

We start with a simple algebraic argument showing that instantons do not handle color in a way compatible with supersymmetry. Let us consider the correlation function of two gauge-invariant gluino bilinears like in Eq. (8) but which are not contracted in spinor indices. Since fermion operators anticommute we find that the correlation function must be antisymmetric inside the two pairs of spin indices:

$$\langle\lambda^{a\alpha}\lambda^{a\beta}(x)\lambda^{c\gamma}\lambda^{c\delta}(y)\rangle = \frac{1}{4}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}C(x-y),$$

$$C = \text{const.} \qquad (9)$$

This correlator is actually $|x-y|$ independent since its contraction with $\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}$ is. Therefore one can put $x=y$ in Eq. (9) so that it becomes a one-point average. We next consider a one-point average of gluino fields which are contracted in spin but not in color indices:

$$T^{ab,cd} = \langle\epsilon_{\alpha\beta}\lambda^{a\alpha}\lambda^{b\beta}(x)\epsilon_{\gamma\delta}\lambda^{c\gamma}\lambda^{d\delta}(x)\rangle. \qquad (10)$$

Under gauge transformations this tensor is gauge-rotated with respect to all indices. After averaging over gauge rotations only invariant tensors can result. Fermion statistics requires that $T^{ab,cd}$ is symmetric in (ab) and in (cd). In the $SU(2)$ gauge theory there are only two possible invariant structures made of Kronecker deltas, consistent with symmetry:

$$T^{ab,cd} = A\,\delta^{ab}\delta^{cd} + B(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}). \qquad (11)$$

In higher groups more structures are possible but we do not consider them here. Contracting Eq. (10) once with $\delta^{ab}\delta^{cd}$

and the other time with $\delta^{ac}\delta^{bd}$ we reduce it to Eq. (9) contracted in the first case with $\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}$ and in the second case with $-\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}$ (the minus sign arises from adjusting the order of fermion operators). It gives a system of linear equations on the coefficients A,B:

$$9A + 6B = C,$$

$$3A + 12B = -\frac{1}{2}C \qquad (12)$$

with a unique solution

$$A = \frac{C}{6}, \quad B = -\frac{C}{12}. \qquad (13)$$

Thus, the color structure of the one-point average (10) is unambiguously determined by supersymmetry:

$$\langle\epsilon_{\alpha\beta}\lambda^{a\alpha}\lambda^{b\beta}(x)\epsilon_{\gamma\delta}\lambda^{c\gamma}\lambda^{d\delta}(x)\rangle$$

$$= \frac{C}{6}\left[\delta^{ab}\delta^{cd} - \frac{1}{2}(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})\right]. \qquad (14)$$

The next observation is that the instanton contribution to the left-hand side (LHS) of Eq. (14) fails to reproduce its color structure. There are four gluino zero modes in the instanton background: two are supertranslational and two are superdilatational (or superconformal) [7]. One has to insert those zero modes into Eq. (14), in all possible combinations. A simple exercise in algebra demonstrates that only the color-singlet structure $\delta^{ab}\delta^{cd}$ arises, with the coefficient B being identically zero. This is true not only for exactly coinciding points $x=y$ but also for $x\neq y$. It is true identically, even before one integrates over instanton center and sizes. We have also checked that it does not depend on the gauge in which the instanton field is considered.

To gain further insight, let us introduce a traceless color gluino bilinear operator:

$$\Lambda^{ab} = \epsilon_{\alpha\beta}\left(\lambda^{a\alpha}\lambda^{b\beta} - \frac{\delta^{ab}}{N^2-1}\lambda^{e\alpha}\lambda^{e\beta}\right), \quad \Lambda^{aa} = 0. \quad (15)$$

In the case of the $SU(2)$ group Λ^{ab} belongs to the irreducible dimension-5 "isospin"-2 representation. For higher N the symmetric rank-2 traceless representation is reducible; for example in $SU(3)$ it is a mixture of $\mathbf{8}^d$ (adjoint) and $\mathbf{27}$ representations.

A direct consequence of Eq. (14) is that the one-point average of gluinos in the traceless dimension-5 representation of $SU(2)$ is

$$\langle\Lambda^{ab}(x)\Lambda^{ab}(y)\rangle_{|x-y|\to 0} \to -\frac{5}{6}\langle\lambda\lambda\rangle^2. \qquad (16)$$

In the $SU(N)$ case $-\frac{5}{6}$ is replaced by the general $-(N^2+1)/(N^2-1)/2$; the negative sign is related to that Λ^{ab} is a fermion operator. Meanwhile, the strong-coupling instanton calculation of this quantity (implying $B=0$) yields identical zero. Instantons are "color-blind" and average out any "col-

ored'' operator. Were color symmetry preserved in the pure SYM theory strong-coupling instantons would be all right. This is the first indication that the instantons' failure is related to the actual color symmetry breaking in the theory, but there will be more.[2]

We next consider the correlation function (8) at large separations between x and y. This correlation function has chirality two, meaning that only gauge configurations with unity topological charge can contribute. Instanton is an obvious candidate. From the instanton viewpoint, the correlator is saturated by instantons of size $\rho \sim |x-y|$ [7] but the result turns out to be $\frac{4}{5}$ of the exact one. So far all calculations yielding the correct value were made for a single gluino condensate, whereas the suspicious strong-coupling instanton calculation was for the two-point correlator. Therefore, to pin down the mistake one should perform in parallel a correct calculation but for the two-point correlator of gluino condensates.

There is an alternative strong-coupling calculation of $\langle \lambda\lambda(x)\lambda\lambda(y) \rangle$ stemming from the compactified version of the SYM theory. The unity topological charge can be obtained from any two dyons LL, MM, LM. In the compactified $R^3 \times S^1$ space there are exact classical solutions of all three types. The full eight-parameter static double-monopole MM solution has been known for a while [13]. The LL double-monopole solution can be obtained from the MM one by a gauge transformation. The time-dependent eight-parameter LM solution has been recently constructed explicitly and named "the caloron with non-trivial holonomy" [14,15]. The first two objects have double electric and magnetic charges so that both their electric and magnetic fields decay as $1/r^2$ at large distances. The third object has zero charges so that it is similar to the instanton.

To compute the correlator $\langle \lambda\lambda(x)\lambda\lambda(y) \rangle$ one needs to take one of the three (LL, MM, LM) exact solutions, find their four adjoint fermion zero modes, substitute them into the correlator in question in all possible combinations, and integrate over the solutions' moduli space; then finally sum up the contributions of all three exact solutions.

In practice, the calculation of the correlator depends on the relation between $|x-y|$ and the compactification circumference l. Let us first discuss the "weak-coupling" case of $l \ll |x-y|$. In this case, only part of the moduli space of the exact solutions contribute, corresponding to widely separated "constituent" L, M dyons. Since the field of constituents decreases rapidly beyond their size $\sim l$, the leading contribution comes from one of the dyons staying at the distance $\sim l$ from point x and the other being at the distance $\sim l$ from point y. Their interference can be neglected. Therefore, at $l \ll |x-y|$ the calculation of the correlator just copies (twice) the calculation of the gluino condensate from a single dyon

[4] (recall Sec. II) with an evident result: the correlator is independent of $|x-y|$ and coincides with the square of the (correct) gluino condensate. Notice that all four possible combinations LL, MM, LM, ML contribute exactly $\frac{1}{4} \langle \lambda\lambda \rangle^2$ apiece.

We next turn to the opposite case, $l \gg |x-y| \gg 1/\Lambda$, appropriate to the decompactified strong-coupling limit. One should keep in mind that compactification does not spoil supersymmetry. Owing to supersymmetry $\langle \lambda\lambda(x)\lambda\lambda(y) \rangle$ is independent of $|x-y|$ for any given l. Thus, the correlator must be precisely the same as in the previous case and equal to the square of the gluino condensate. The correct result can be foreseen without calculations.[3]

When $l \to \infty$ while $|x-y|$ is kept fixed, the exact $LL(MM)$ and LM solutions look very different. The LM solution (the caloron) at $l \to \infty$ and fixed size ρ becomes the instanton [14,15]. Its action density is well localized both in x_4 and space. In the leading order in $1/l$ the solution is the usual instanton. The difference with the instanton shows up only in the subleading $1/l$ terms. As to the LL and MM exact solutions, they can be made static by an appropriate gauge choice. At $l \to \infty$ their field is weak everywhere: it is of the order of $1/l$ inside the region of space $\sim l$ and falls as $1/r$ outside that region. The action gets its unity value owing to the integration of a weak field over a large volume.

Naively, one would argue that fields of the order of $1/l \to 0$ are irrelevant for the calculation of the gluino condensate which is a local quantity, and hence one would (i) neglect altogether the LL and MM contributions, and (ii) replace the exact LM field by the instanton. Following this argument, one would conclude that the two-dyon and the instanton calculations are equivalent in the strong-coupling limit. However, we shall see in a moment that this is incorrect.

The LL, LM, MM solutions represent sectors with definite (electric, magnetic) charges $(-2,-2)$, $(0,0)$, and $(2,2)$, respectively. These sectors do not mix up under supersymmetric transformations: it is only their moduli spaces that transform (separately) under supersymmetry. It means that the independence of the correlator $\langle \lambda\lambda(x)\lambda\lambda(y) \rangle$ of $|x-y|$ is satisfied separately for the three sectors. At $l \ll |x-y|$ we know that LL and MM sectors contribute apiece exactly $\frac{1}{4}$ of the gluino condensate squared, whereas the LM sector contributes exactly the other half. Because of supersymmetry, at $l \gg |x-y|$ those configurations contribute precisely the same fractions, despite that the LL, MM fields tend to zero. At the same time the exact LM configuration (i.e., the caloron with nontrivial holonomy) contributes precisely $\frac{1}{2}$ of the gluino condensate squared, whereas the instanton (to which it is reduced if one neglects $1/l$ corrections) is known to contrib-

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[2] It was noticed earlier [12] that multi-instantons do not support the cluster decomposition of gluino correlators. Since Eq. (16) is very general [it can be derived directly from first rearranging gluinos into color-singlet operators and then applying Eq. (9)], instantons' failure to reproduce the equation is another but simpler manifestation of the nonclusterization.

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[3] Out of curiosity, we have computed the one-point average (10) assuming a sum ansatz of dyons at all separations. Surprisingly, it works quite well: the color structure (13) following from supersymmetry is reproduced and the absolute value of the gluino condensate turns out to be only 4% bigger than the exact one. It would be illuminating to compute $\langle \lambda\lambda(x)\lambda\lambda(y) \rangle$ exactly from the $LL(MM)$ and LM solutions.

ute $\frac{4}{5}$. Rather unusual, a vanishing field $A_4 \sim 1/l$ is necessary to maintain the correct result for the local gluino condensate.[4]

The difference between strong-coupling instanton and multidyon calculations becomes even greater for higher $SU(N)$ groups. At large N, the instanton gives only a $O(1/N)$ fraction of the true gluino condensate (see below); where does the rest come from?

In the compactified $SU(N)$ gauge theory, there are $N-1$ "static" dyons $M_1, \ldots, M_{N-1}$ having unit (electric, magnetic) charges with respect to the $N-1$ Abelian subgroups, and one "time-dependent" L dyon [11,4]. When one computes the single gluino condensate in compactified space each of the N configurations contributes equally $1/N\langle\lambda\lambda\rangle$ [4]. Adding up the contributions of $M_1, M_2, \ldots, M_{N-1}$ and L dyons one gets the correct gluino condensate.

To compare it with the strong-coupling instanton calculation, one considers a N-point correlator

$$\langle\lambda\lambda(x_1)\lambda\lambda(x_2)\cdots\lambda\lambda(x_N)\rangle \to \langle\lambda\lambda\rangle^N, \qquad (17)$$

with $x_1 \cdots x_N$ taken far apart. This correlator can be saturated by one instanton but also, in the compactified space, by exact N-dyon solutions. Again, we start with the case $l \ll |x_{mn}|$ where the exact multidyon solutions reduce to widely separated constituents. Each of the N dyon species stay at the distance $\sim l$ from the points $x_1 \cdots x_N$, with all possible N^N permutations. Therefore, the LHS of Eq. (17) is

$$\left(\frac{1}{N}\langle\lambda\lambda\rangle\right)^N N^N = \langle\lambda\lambda\rangle^N \qquad (18)$$

as it should be. By supersymmetry, the LHS of Eq. (17) does not depend on the relation between the compactification circumference l and the separations x_{mn}. Therefore, the same result holds at $l \gg |x_{mn}|$, i.e., in the strong-coupling limit. Meanwhile, only one particular configuration, namely $M_1 M_2, \ldots, M_{N-1}L$, has zero (electric, magnetic) charges with respect to all $U(1)$ subgroups. It is the "caloron with non-trivial holonomy" of Refs. [11,14]. At $l \to \infty$ it becomes the usual instanton of the $SU(N)$ gauge group, *plus* $1/l$ corrections. The instanton contribution to the LHS of Eq. (17) is [5,7,8,17]

$$(\langle\lambda\lambda\rangle_{\text{inst}})^N = \frac{2^N}{(N-1)(3N-1)}\langle\lambda\lambda\rangle^N$$

$$\overset{N\to\infty}{\to} \left(\frac{2e}{N}\langle\lambda\lambda\rangle\right)^N. \qquad (19)$$

Meanwhile, the true caloron contribution to Eq. (17) is

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[4]Experts in the Schwinger model may find the present situation familiar: the chiral condensate in the $2d$ QED is induced by fields that are of the order of $1/l$ where l is the size of the world [16].

$$(\langle\lambda\lambda\rangle_{\text{calor}})^N = \left(\frac{1}{N}\langle\lambda\lambda\rangle\right)^N N!$$

$$\overset{N\to\infty}{\to} \left(\frac{1}{e}\langle\lambda\lambda\rangle\right)^N \qquad (20)$$

where the factor $N!$ comes from the permutations of $M_1, M_2, \ldots, L$. Again, we see that a vanishing distinction between the instanton and the caloron with nontrivial holonomy makes a big difference. Also, contributions from all the rest of multidyon solutions whose field at $l \to \infty$ is $O(1/l)$ (or less) everywhere, are needed to maintain the correct result for the gluino condensate in the strong-coupling limit.

We see, thus, that the nontrivial holonomy is important in determining the gluino condensate in the strong-coupling regime. This is not very usual. The holonomy is a global quantity; the difference between trivial and nontrivial holonomy is the difference between $A_4 = 0$ and $A_4 = \pi/l \to 0$. The fact that this tiny difference plays a crucial role in determining such a local quantity as $\langle\lambda\lambda\rangle$ means that the system is unstable with respect to infinitesimal perturbation breaking color symmetry. In other words, the gauge group is spontaneously broken.

IV. COLOR GLUINO CONDENSATE FROM A DEFORMATION OF $\mathcal{N}=2$ THEORY

In this section we compute directly the value of the color-breaking gluino condensate Λ^{ab}, see Eq. (15). To that end, we consider the compactified version of the $\mathcal{N}=2$ theory. As compared to the pure SYM theory, it has an additional chiral multiplet $(\Psi^{a\alpha}, \Phi^a)$ in the adjoint representation.

The classical potential $g^2 \text{Tr}[\bar\Phi\Phi]^2$ has a flat zero-energy valley which we shall choose in the form

$$\Phi^a = \begin{pmatrix} 0 \\ 0 \\ V \end{pmatrix} \qquad (21)$$

where V is an arbitrary complex number. It breaks the color group $SU(2) \to U(1)$ even without compactification. However, we shall add the mass term for the chiral supermultiplet,

$$m(\epsilon_{\alpha\beta}\Psi^{a\alpha}\Psi^{a\beta} + \Phi^a\Phi^a). \qquad (22)$$

In the decompactified case the mass term drives $V \to 0$ at large m [18,19]. At large m the matter supermultiplet decouples and one is left with the pure SYM theory with a seemingly restored full $SU(2)$ gauge group. Such a conclusion is, however, too hasty. Compactification of the softly broken (by the mass term) $\mathcal{N}=2$ theory is a way to make a gradual transition to the SYM theory, ultimately in the strong coupling regime. We shall see that the $SU(2)$ group is not restored in that limit but remains broken to $U(1)$ by the colored gluino condensate.

Compactifying the x_4 coordinate one finds classical solutions being L,M dyons modified by the presence of the scalar field Φ^a. Assuming the fields are "time"-independent and Φ^a is parallel to A_4^a, the modified M dyon in the regular "hedgehog" gauge is given by [cf. Eqs. (A1),(A2)]

$$\Phi^a = -n_a V \Phi(\sqrt{v^2+V^2}\, r), \quad \Phi(z) = \coth z - \frac{1}{z}, \quad (23)$$

$$A_4^a = -n_a v \Phi(\sqrt{v^2+V^2}\, r)$$

$$\xrightarrow{z\to\infty} -n^a\left(v - \frac{v}{\sqrt{v^2+V^2}}\frac{1}{r}\right), \quad (24)$$

$$A_i^a = \epsilon_{aij}n_j\frac{1-R(\sqrt{v^2+V^2}\, r)}{r}, \quad R(z) = \frac{z}{\sinh z}. \quad (25)$$

Its action is $4\pi l/g^2\sqrt{v^2+V^2}$. The S_- gauge transformation (A7) puts the Φ^a, A_4^a fields along the third color axis, with the asymptotics

$$\Phi^a \simeq \delta^{a3}\left(V - \frac{V}{\sqrt{v^2+V^2}}\frac{1}{r}\right), \quad (26)$$

$$A_4^a \simeq \delta^{a3}\left(v - \frac{v}{\sqrt{v^2+V^2}}\frac{1}{r}\right). \quad (27)$$

The L dyon is obtained by replacing $v \to 2\pi/l - v$ and $V \to -V$ in Eqs. (23)–(25). It is then transformed to the stringy gauge by S_+ [Eq. (A6)] and subsequently gauge-transformed by the time-dependent matrix $U(x^4)$ [Eq. (A11)]. The fields' asymptotics become

$$\Phi^a \simeq \delta^{a3}\left(V - \frac{V}{\sqrt{\left(\frac{2\pi}{l}-v\right)^2+V^2}}\frac{1}{r}\right), \quad (28)$$

$$A_4^a \simeq \delta^{a3}\left(v + \frac{\frac{2\pi}{l}-v}{\sqrt{\left(\frac{2\pi}{l}-v\right)^2+V^2}}\frac{1}{r}\right). \quad (29)$$

The action of the modified L dyon is $4\pi l/g^2\sqrt{(2\pi/l-v)^2+V^2}$.

Both L and M dyons have two λ and two Ψ zero modes. The mass term for Ψ allows one to contract the Ψ zero modes of a dyon. The L,M-induced superpotential is a slight modification of that found in Ref. [4] in the pure SYM case:

$$\mathcal{W}_{\text{dyon}} = (M_{\text{PV}}^{\mathcal{N}=2})^2 m\left[\exp\left(-\frac{4\pi l}{g^2}\sqrt{v^2+V^2}\right)\right.$$
$$\left. + \exp\left(-\frac{4\pi l}{g^2}\sqrt{\left(\frac{2\pi}{l}-v\right)^2+V^2}\right)\right]. \quad (30)$$

Here the Pauli-Villars mass of the full $\mathcal{N}=2$ theory appears in the second power since there are four boson and four fermion zero modes. The factor m arises from the contraction of Ψ zero modes via the mass term. Apparently, the minimum (zero) energy is achieved, as before, at $v = \pi/l$, independently of the vacuum expectation value (VEV) of the matter field Φ^a. From now on, we shall use this value of $\langle A_4 \rangle$.

It is straightforward to calculate the gluino condensate in this setting, basically repeating the steps leading to Eq. (7). The only (technical) difference is that the dyon weight is now proportional to $(M_{\text{PV}}^{\mathcal{N}=2})^2 m$. This quantity is, however, equal to $(M_{\text{PV}}^{\mathcal{N}=1})^3$ at large m (see, e.g., [6]). Therefore, we get the same result as before:

$$\langle \lambda\lambda \rangle = \langle \lambda^1\lambda^1 \rangle + \langle \lambda^2\lambda^2 \rangle + \langle \lambda^3\lambda^3 \rangle = \Lambda^3. \quad (31)$$

The $\mathcal{N}=2$ extension allows us to compute another holomorphic quantity

$$\chi = \left\langle \frac{\Lambda^{ab}\Phi^a\Phi^b}{\Phi^c\Phi^c} \right\rangle \quad (32)$$

where Λ^{ab} is the traceless gluino bilinear in the dimension-5 representation, see Eq. (15). This operator is chiral and transforms under supersymmetry through the parameter ϵ only (not $\bar{\epsilon}$). It gets a contribution from one dyon but cannot acquire corrections either from perturbation theory or from additional dyon pairs. Therefore, we can find the above average at small l and claim that it remains unaltered in the decompactified limit, just as the normal gluino condensate (31) does.

Saturating Λ^{ab} by the two gluino modes of a dyon we obtain [cf. Eq. (7)]:

$$\Lambda^{ab} \to E_i^a E_i^b - \frac{\delta^{ab}}{3}E_i^e E_i^e = (F_2^2(r) - F_1^2(r))$$
$$\times \left(\delta^{a3}\delta^{b3} - \frac{1}{3}\delta^{ab}\right). \quad (33)$$

Equation (33) is written for M dyons; in the case of the time-dependent L dyons Eq. (33) should be gauge rotated by a time-dependent matrix (A11). It is easy to check, however, that this gauge transformation commutes with the color structure in Eq. (33), i.e., leaves it unchanged. Therefore, Eq. (33) is correct both for L and M dyons.

The scalar field Φ^a is directed along the third color axis, according to Eqs. (26),(28). The concrete profile of the Φ^a solution cancels out in the ratio (32). For the same reason the VEV of the Φ field is irrelevant also, although it goes to zero at large m (necessary to pass from the $\mathcal{N}=2$ to the pure SYM

theory). It is only the color direction of Φ^a that matters in Eq. (32), but it is fixed by the dyon solution. Consequently, the average (32) is

$$\chi = \langle \Lambda^{33} \rangle = \frac{2}{3} \langle \lambda^3 \lambda^3 \rangle - \frac{1}{3} \langle \lambda^1 \lambda^1 \rangle - \frac{1}{3} \langle \lambda^2 \lambda^2 \rangle$$

$$= \frac{2}{3} \frac{\Lambda^3}{4\pi v} \int d^3 z [F_2^2(r) - F_1^2(r)] = \frac{2}{3} \Lambda^3 \left(2 - \frac{\pi^2}{6} \right). \quad (34)$$

Combining it with Eq. (31) we find

$$\langle \lambda^1 \lambda^1 \rangle = \langle \lambda^2 \lambda^2 \rangle$$

$$= \Lambda^3 \int dr \ r^2 F_1^2(r)$$

$$= \Lambda^3 \frac{\pi^2 - 6}{18} \simeq 0.214978 \Lambda^3, \quad (35)$$

$$\langle \lambda^3 \lambda^3 \rangle = \Lambda^3 \int dr r^2 F_2^2(r)$$

$$= \Lambda^3 \frac{15 - \pi^2}{9}$$

$$\simeq 0.570044 \Lambda^3. \quad (36)$$

We see that one of the color directions is preferred. In this case it is the 33 direction as we have aligned A_4 at spatial infinity along the third axis. Contrary to $\langle A_4^3 \rangle$ which vanishes in the decompactified limit as $1/l$ the difference in the color components of the gluino condensate remains finite (and computable) in the strong-coupling limit.

Equations (35),(36) demonstrate the dynamical Higgs effect (as there are no elementary Higgs fields in the pure SYM theory). In this case, the colored gluino condensate $\langle \Lambda^{ab} \rangle$ (the composite Higgs field) belonging to the dimension-5 traceless symmetric tensor representation has a nonzero VEV (34) that breaks $SU(2)$ down to the $U(1)$ subgroup in the same sense as the VEV of an elementary Higgs field does.

V. DISCUSSION

The appearance of a nonzero color gluino condensate is, in a sense, trivial. In partially compactified $R^3 \times S^1$ SYM theory the gauge group is apparently spontaneously broken to the maximal Abelian subgroup by dyons, at least when the compact dimension is much less than the Λ scale of the theory. The minimum of the superpotential induced by dyons corresponds to a nonzero A_4 [4] which has to lie in some direction in color space thus breaking the color group. Therefore, in the compactified pure SYM theory a Higgs effect takes place, with A_4 playing the role of the Higgs field in the adjoint representation. Unpleasantly, A_4 is not Lorentz-invariant, but the effect is there. The nonzero value of the colored gluino condensate $\langle \Lambda^{ab} \rangle$ is a Lorentz-invariant manifestation of the same symmetry breaking.

Interestingly, the value of the color gluino condensate we have found does not depend on the compactification circum-

ference l—just as the "normal" gluino condensate is independent of l; they are holomorphic quantities. Therefore, one can claim that color symmetry remains broken in the decompactified strong-coupling limit. An indirect evidence for color symmetry breaking follows from the fact that the correct value of the color-singlet gluino condensate is obtained from field configurations with a nontrivial holonomy. Strong-coupling instantons have a trivial holonomy, and they produce only a color-singlet condensate—in contradiction with Eq. (16) following merely from supersymmetry. But even the color-singlet condensate gets a wrong value from instantons. We have shown in Sec. III that to get the correct value it is insufficient to add other field configurations on top of the instanton: one has to replace the instanton by the caloron with nontrivial holonomy in the first place. A nontrivial holonomy means that there is a privileged direction in color space, which is equivalent to color symmetry breaking.

An additional although so far indirect demonstration of the importance of dyons at strong coupling has come very recently from another end. Using wrapped $D5$ branes [20] or warped deformed conifold [21] the authors obtained the correct all-loop β function of $\mathcal{N}=1$ SYM theory from corresponding supergravity solutions. In addition, both references find the same nonanalytic corrections to the β function, which are naturally associated with the pairs of dyons, not instantons.

Dyons have long-range Coulomb interactions which are Debye-screened in the plasma. It results in magnetic photons getting a mass and in confinement *à la* Polyakov [22]. Polyakov's scenario of confinement in $3d$ is essentially *Abelian*. It implies that the gauge group is spontaneously broken to the maximal Abelian subgroup, that the "charged" gluons get masses via the Higgs mechanism but are confined, and that the Abelian magnetic "photons" get mass from Debye screening. Qualitatively, the same Abelian scenario has been discussed for the $4d$ pure gauge theory by 't Hooft [23]. It can be made quantitative in the weak-coupling regime of the $4d$ $\mathcal{N}=2$ theory softly broken to $\mathcal{N}=1$ [18,24]. The spontaneous breaking of the gauge group to the maximal Abelian subgroup is a welcome feature: the Abelian confinement is well understood, at least on the philosophical level, and has a chance to be ultimately put into a quantitative form following the lines of the references cited above.

The Abelian scenario has clear signatures in the weak-coupling regime but they become not so clear in the strong-coupling limit, especially if the theory confines color and only gauge-invariant correlators are the observables. Strictly speaking, there is no gauge-invariant local order parameter which would distinguish between Abelian confinement and a "true" non-Abelian case. To find out uniequivocal observable signatures of the color-broken scenario and to check whether the pure (not supersymmetric) YM theory has the same features is an intriguing task which we postpone for the future.

APPENDIX: *L,M* MONOPOLES

We give here explicit expressions for the fields of the four dyons in the compactified $R^3 \times S^1$ space, with all four possible signs of the electric and magnetic charges. The two usual Bogomol'nyi-Prasad-Sommerfield (BPS) dyons in the regular ("hedgehog") gauge have the form:

$$A_4^a = \mp n_a v \Phi(vr),$$
(A1)

$$\Phi(z) = \coth z - \frac{1}{z} \overset{z \to \infty}{\longrightarrow} 1 - \frac{1}{z} + O(e^{-z}),$$

$$A_i^a = \epsilon_{aij} n_j \frac{1 - R(vr)}{r},$$
(A2)

$$R(z) = \frac{z}{\sinh z} \overset{z \to \infty}{\longrightarrow} O(ze^{-z}).$$

Here $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$, $n_a = x_a/r$. The upper sign in A_4 corresponds to the self-dual ($E_i^a = F_{i4}^a = B_i^a = \frac{1}{2}\epsilon_{ijk}F_{jk}^a$) and the lower sign to the anti-self-dual ($E_i^a = -B_i^a$) solution. We shall call them M and $\bar{M}$ monopoles, respectively.

The magnetic field strength in the hedgehog gauge is given by two structures:

$$B_i^a = (\delta_{ai} - n_a n_i) F_1(r) + n_a n_i F_2(r), \quad \text{where} \quad \text{(A3)}$$

$$F_1(r) = \frac{1}{r} \frac{d}{dr} R(vr)$$

$$= -v \frac{R(vr) \Phi(vr)}{r}$$

$$= \frac{v^2}{\sinh(vr)} \left(\frac{1}{vr} - \coth(vr) \right)$$

$$= v^2 O(e^{-vr}),$$
(A4)

$$F_2(r) = -\frac{d}{dr} v \Phi(vr)$$

$$= \frac{R^2(vr) - 1}{r^2}$$

$$= \frac{v^2}{\sinh^2(vr)} - \frac{1}{r^2}$$

$$= -\frac{1}{r^2} + v^2 O(e^{-2vr}).$$
(A5)

If there is more than one monopole in the vacuum it is impossible to add them up in the hedgehog gauge: one has to "gauge-comb" them to a gauge where A_4^a has the same asymptotic value at spatial infinity for all monopoles involved, say, along the third color axis. It is achieved with the help of two unitary matrices dependent on the spherical angles θ, ϕ:

$$S_+(\theta, \phi) = e^{-i(\phi/2)\tau^3} e^{i(\theta/2)\tau^2} e^{i(\phi/2)\tau^3}, \quad S_+(n \cdot \tau) S_+^\dagger = \tau^3,$$
(A6)

$$S_-(\theta, \phi) = e^{i(\phi/2)\tau^3} e^{i[(\pi - \theta)/2]\tau^2} e^{i(\phi/2)\tau^3},$$

$$S_- = -i\tau^2 S_+, \quad S_-(n \cdot \tau) S_-^\dagger = -\tau^3.$$
(A7)

We shall gauge-transform the M-monopole field with S_- and the $\bar{M}$-monopole field with S_+. As the result their A_4 components become equal:

$$A_4^{M,\bar{M}} = v \Phi(vr) \frac{\tau^3}{2} = \left[v - \frac{1}{r} + O(e^{-vr}) \right] \frac{\tau^3}{2}.$$
(A8)

On the contrary, the spatial components differ in sign. We write them in spherical components:

$$\pm A_i^{M,\bar{M}} = \begin{cases} A_r = 0, \\[2mm] A_\theta = \frac{R(vr)}{2r}(\tau^1 \sin\phi + \tau^2 \cos\phi), \\[2mm] A_\phi = \frac{R(vr)}{2r}(\tau^1 \cos\phi - \tau^2 \sin\phi) \\[2mm] + \frac{1}{2r}\tan\frac{\theta}{2}\tau^3. \end{cases}$$
(A9)

The azimuthal component of the gauge field has a singularity along the negative z axis, therefore we shall call it the stringy gauge. The field strength, however, has no singularities. The electric field both of M and $\bar{M}$ monopoles in the stringy gauge is

$$E_i^{M,\bar{M}} = \begin{cases} E_r = -\frac{F_2(r)}{2}\tau^3 \overset{r \to \infty}{\longrightarrow} \frac{1}{r^2}\frac{\tau^3}{2}, \\[2mm] E_\theta = \frac{F_1(r)}{2}(-\tau^1 \cos\phi + \tau^2 \sin\phi) \\[2mm] = v^2 O(e^{-vr}), \\[2mm] E_\phi = \frac{F_1(r)}{2}(\tau^1 \sin\phi + \tau^2 \cos\phi) \\[2mm] = v^2 O(e^{-vr}), \end{cases}$$
(A10)

while the magnetic field is $B_i = \pm E_i$. Therefore, the M monopole has (electric, magnetic) charges $(++)$ whereas the $\bar{M}$ one has $(+-)$.

There is a second pair of dyons [11]: a self-dual one with the charges $(--)$ which we shall name L monopole, and an anti-self-dual one with charges $(-+)$ which we shall name

TABLE I. Four dyons of $SU(2)$.

	M	$\bar{M}$	L	$\bar{L}$
electric charge	$+$	$+$	$-$	$-$
magnetic charge	$+$	$-$	$-$	$+$
action, $\dfrac{4\pi l}{g^2}$	v	v	$\dfrac{2\pi}{l}-v$	$\dfrac{2\pi}{l}-v$
top. charge	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$

$\bar{L}$ monopole. They are obtained from Eqs. (A1)–(A5) by replacing $v \to (2\pi/l) - v$. One first transforms them from the hedgehog to the stringy gauge with the help of the unitary matrices S_+ and S_-, respectively. As the result, they get the same asymptotics $A_4(\infty) = [-(2\pi/l) + v + (1/r)]\tau^3/2$. To put the asymptotics in the same form as for $M,\bar{M}$ monopoles [see Eq. (A8)] one makes an additional gauge transformation with the help of the time-dependent matrix

$$U = \exp\left(-i\frac{\pi}{l}x^4\tau^3\right). \tag{A11}$$

This gives the following fields of $L,\bar{L}$ monopoles in the stringy gauge:

$$A_4^{L,\bar{L}} = \left[\left(\frac{2\pi}{l}-v\right)\Phi\left(\left|\frac{2\pi}{l}-v\right|r\right) - \frac{2\pi}{l}\right]\frac{\tau^3}{2} \overset{r\to\infty}{\to}$$

$$= \left(v + \frac{1}{r}\right)\frac{\tau^3}{2}, \tag{A12}$$

$$E_i^{L,\bar{L}} = \begin{cases} E_r = \dfrac{F_2(r)}{2}\tau^3 \overset{r\to\infty}{\to} -\dfrac{1}{r^2}\dfrac{\tau^3}{2}, \\[2mm] E_\theta = -\dfrac{F_1(r)}{2}U(x^4) \\[1mm] \qquad \times(-\tau^1\cos\phi + \tau^2\sin\phi)U^\dagger(x^4), \\[2mm] E_\phi = -\dfrac{F_1(r)}{2}U(x^4) \\[1mm] \qquad \times(\tau^1\sin\phi + \tau^2\cos\phi)U^\dagger(x^4), \end{cases}$$

$$B_i^{L,\bar{L}} = \pm E_i^{L,\bar{L}}. \tag{A13}$$

The "profile" functions $F_{1,2}$ are given by Eqs. (A4),(A5), with the replacement $v \to (2\pi/l) - v$. We notice that "the interior" of the $L,\bar{L}$ dyons, represented by the θ,ϕ field components are time dependent. This is why in the true $3d$ case these objects do not exist.

The properties of the four dyons are summarized in Table I.

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