Thick domain wall spacetimes with and without reflection symmetry

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We show that different thick domain wall spacetimes, for which the scalar field configuration and the potential are the same, can be found as solutions to the coupled Einstein-scalar field equations, depending on whether or not reflection symmetry on the wall is imposed. Spacetimes with reflection symmetry may be dynamic or static, while the asymmetric ones are static. Asymmetric walls are asymptotically flat on one side and reduce to the Taub spacetime on the other. Examples of asymmetric thick walls in *D*-dimensional spacetimes are given, and previous analysis on the distributional thin-wall limit of the dynamic symmetric thick walls are extended to the asymmetric case. A new family of reflection symmetric, static thick domain wall spacetimes, including previously known Bogomol'nyi-Prasad-Sommerfield walls, is presented.

I. INTRODUCTION

Scalar fields as sources for the Einstein field equations have been the subject of recent interest. Configurations with a plane-parallel symmetry, i.e. scalar field walls, are particularly appealing, since they can be topologically stable. Although in four dimensions they are in conflict with standard cosmology [1], in theories with extra dimensions the possibility of realizing four-dimensional gravity in a 3-brane or thin domain wall [2], already suggested in [3], has attracted wide attention.

Self-gravitating domain walls are solutions to the coupled Einstein-scalar field system, with a potential $V(\phi)$ possessing a (spontaneously broken) discrete symmetry. The coupled system of equations is solved for the field ϕ and the metric tensor components g_{ab}. A topological charge for the wall is obtained by mapping the values of ϕ at spatial infinity to the vacuum manifold in a nontrivial way. To simplify matters, one looks for solutions representing static domain walls; that is, one requires the energy and momentum density to be time independent. Because the field is required to take values at the (degenerate) minima of the potential at spatial infinity on both sides of the wall, the energy and momentum densities are invariant under reflections on the wall's plane.

These features of the domain wall solution, staticity and reflection symmetry, need not be shared by the spacetime metric. As shown by Vilenkin [4], the most general static vacuum solution with plane-parallel and reflection symmetry obtained by Taub [5] cannot be the external spacetime of a true domain wall source (in the absence of a cosmological constant). The first vacuum solution for a spacetime containing an infinitely thin sheet of scalar field compatible with the equation of state of a domain wall was obtained in [4]. The spacetime is not static, but has a de Sitter expansion on the wall's plane. Other vacuum solutions can be found by adding a cosmological constant, or by allowing for curvature on the wall. Once all the possible vacuum solutions are known, they can be appropriately matched across a surface with the Darmois-Israel thin shell formalism, and all the possible solutions representing an infinitely thin wall spacetime have thus been found and classified [6,7]. Infinitely thin walls have distributional energy-momentum tensor fields proportional to a δ distribution supported on the wall's surface. However, as pointed out in [8], a thin wall limit may be a very artificial construction in terms of the coupling constants of the underlying theory, in addition to the fact that distributional curvatures are in general ill defined due to the nonlinearities of general relativity [9].

Dynamic and reflection symmetric spacetimes as solutions to the coupled Einstein-scalar field equations with a symmetry breaking potential have also been found [10,11], representing a thick domain wall with the appropriate boundary conditions for the scalar field at spatial infinity that guarantee topological stability. On the other hand, one can have a static metric as long as the wall is allowed to interpolate between anti–de Sitter (AdS) asymptotic vacua, the so-called Bogomol'nyi-Prasad-Sommerfield (BPS) walls [12,13]. The thin wall limit of these configurations was studied in [14].

Within the thin wall context, [15] envisages the situation in which the reflection symmetry is relaxed and considers static thin domain walls embedded in spacetimes with vanishing cosmological constant. These and the non-reflection symmetric thin domain walls between spacetimes with nega-

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tive and zero cosmological constant are discussed in [16]. Asymmetric thin domain wall spacetimes or asymmetric brane-world scenarios in which the reflection symmetry along the extra dimension was broken by gluing two AdS spacetimes with different cosmological constants have been considered in [17–20], and their possible realization by the introduction of a gauge form field has been proposed in [21]. For thick domain wall spacetimes, a less known result is that the condition of staticity of the metric can be maintained even in the absence of a cosmological constant term, if one is willing to sacrifice reflection symmetry. We only know of an example in the literature so far, for a domain wall in four dimensions, that interpolates between a Minkowski and a Taub spacetime [11]. This solution to the coupled Einstein-scalar field equations has the peculiarity of having the same scalar field potential $V(\phi)$ as the well-known dynamic solution of Ref. [10], which has prompted us to study this issue in more detail, and to consider the extension to higher-dimensional spacetimes.

In what follows we show that two types of thick domain wall spacetimes can be found as solutions of the Einstein-scalar field equations in D dimensions, for which the scalar field ϕ and the potential $V(\phi)$ are the same: (a) reflection symmetric spacetimes, either with a de Sitter expansion on the wall's plane or static ones representing walls embedded in a spacetime with a cosmological constant, and (b) asymmetric static spacetimes, interpolating between Minkowski and Taub spacetimes (rather, their D-dimensional equivalents). They are found with the same boundary conditions on ϕ at infinity, and their energy density is in both cases static and reflection symmetric. Thus, the metric is not uniquely determined but depends on subsidiary conditions imposed on its components.

Both classes of spacetimes cannot be related with a coordinate transformation, but there is a one-to-one correspondence between the dynamic and the asymmetric solutions. We take advantage of this by applying a recently reported method [14] for solving the coupled Einstein-scalar field system to obtain asymmetric solutions, by appropriately scaling the vacuum solutions. Results reported in [14] concerning the thin-wall limit of these solutions are shown to be valid in the asymmetric case. The results are then extended to the general case of a $(D\text{-}2)$-dimensional brane in a D-dimensional spacetime.

In order to obtain further examples, we show how new solutions can be obtained by a different way of scaling vacuum solutions. A particularly interesting class of static solutions representing a parametric family of "double" walls, i.e. walls with energy density concentrated in two parallel sheets, is considered in some detail. These walls reduce to a known BPS thick domain wall [22] for a particular value of the parameter. Other solutions, for theories with less appealing scalar field potentials, are also presented.

II. DYNAMIC VS ASYMMETRIC SOLUTIONS

The most general metric for a 5-dimensional spacetime with a plane-parallel symmetry can be written as

$$g_{ab}=e^{2\mu(\xi)}[-dt_a dt_b+C(\xi,t)^2 dx_a^i dx_b^i]+e^{2\nu(\xi)}d\xi_a d\xi_b \tag{1}$$

where latin indices run over the spatial variables on the brane. The function $\mu(\xi)$ in Eq. (1) is redundant, since only two functions are needed in general. We will keep it, however, and choose it conveniently as a function of $\nu(\xi)$ and $C(\xi,t)$ later. We look for solutions to

$$G_{ab}+g_{ab}\Lambda=T_{ab},$$

$$T_{ab}=\partial_a\phi\partial_b\phi-g_{ab}\left(\frac{1}{2}\partial^c\phi\partial_c\phi+V(\phi)\right), \tag{2}$$

satisfying the requirements

(1) $\phi=\phi(\xi)$.

(2) $V(\phi)$ has a (spontaneously broken) discrete symmetry.

(3) $\phi(\xi)$ takes different values at two different minima of $V(\phi)$ for $|\xi|\to\infty$.

(4) $\phi(\xi)'^2$ is symmetric under reflections in the $\xi=0$ plane (here and in what follows primes denote derivatives with respect to ξ and dots denote derivatives with respect to t).

Following the usual strategy, we will first find $C(\xi,t)$ by imposing the requirements of staticity and reflection symmetry of $\phi(\xi)'^2$ and $V(\phi(\xi))$, given by

$$\phi(\xi)'^2=e^{2\nu}(G_4^4-G_0^0), \tag{3}$$

$$V(\phi(\xi))+\Lambda=-\frac{1}{2}(G_4^4+G_0^0), \tag{4}$$

and then look for solutions $\{\phi(\xi),V(\phi+\Lambda)\}$ in terms of the "warp factor" $\nu(\xi)$. We have from Eq. (2)

$$G_4^0=3e^{-2\mu}\frac{\dot{C}'}{C}=0, \tag{5}$$

therefore $C(\xi,t)$ is the sum of a function for t and a function of ξ. With this, by requiring

$$G_0^0-G_1^1=-2e^{-2\mu}\left(\frac{\dot{C}}{C}\right)^{\cdot}$$
$$+e^{-2\nu}\left[\left(\frac{C'}{C}\right)'+\frac{C'}{C}\left(4\mu'-\nu'+3\frac{C'}{C}\right)\right]=0 \tag{6}$$

for arbitrary $\nu(\xi)$, two types of solutions are possible.

A. Static solutions, with $C=C(\xi)\equiv e^{g(\xi)}$

Since the most general static metric can be written in terms of two functions, we can conveniently set

$$\mu=\frac{1}{4}\nu-\frac{3}{4}g. \tag{7}$$

In this case Eq. (6) is integrated to give $g(\xi) = \beta\xi$ and we have

$$\phi_A'^2 = \frac{3}{4}[\nu'^2 - \beta^2 - \nu''] \tag{8}$$

$$V(\phi)_A = -\frac{3}{8}e^{-2\nu}\nu'' - \Lambda \tag{9}$$

static as required. They will also be reflection symmetric if $\nu(\xi)$ is.

B. Dynamic solutions, with $C = C(t) \equiv e^{h(t)}$

Equation (6) gives $h(t) = \beta t$, and we can now set $\mu = \nu$, obtaining

$$\phi_B'^2 = 3[\nu'^2 - \beta^2 - \nu''] \tag{10}$$

$$V(\phi)_B = -\frac{3}{2}e^{-2\nu}[\nu'' + 3\nu'^2 - 3\beta^2] - \Lambda. \tag{11}$$

While we have ensured that the field's gradient and potential are static and symmetric under reflections in the $\xi = 0$ plane, the spacetimes of solutions **A** and **B** are not. The metric of solutions **A** is manifestly asymmetric, although static

$$(g_A)_{ab} = e^{\nu(\xi)/2 - 3\beta\xi/2}[-dt_a dt_b + e^{2\beta\xi}dx_a^i dx_b^i]$$
$$+ e^{2\nu(\xi)}d\xi_a d\xi_b. \tag{12}$$

Instead, the metric in solutions **B** is dynamic, but symmetric

$$(g_B)_{ab} = e^{2\nu(\xi)}[-dt_a dt_b + e^{2\beta t}dx_a^i dx_b^i] + e^{2\nu(\xi)}d\xi_a d\xi_b. \tag{13}$$

Solutions of type **B** are encountered in the literature, both in 4 and 5 dimensional spacetimes [10,14,22,23], while only one example of those of type **A** has been discussed, in 4 dimensions [11].

The coupled system of equations (2) has now to be solved by proposing a warp factor such that $\{\phi(\xi), V(\phi)\}$ can be integrated. The remarkable point is that the equation for ϕ is the same in both cases. Therefore the warp factors for vacuum solutions in both spacetimes, obtained by integrating the equations $\phi'^2 = 0$ for $\nu(\xi)$, are the same. However, the spacetimes will have different cosmological constants.

Now, in Ref. [14] we presented a method for generating solutions to the system (2) (in 4 dimensions) with a spacetime of type **B** by scaling the vacuum solutions. Specifically, we showed that if $\nu_0(\xi)$ is a vacuum solution with a (non-null) cosmological constant Λ_0, the system can be integrated with the function

$$\nu(\xi) = \delta\nu_0(\xi/\delta) \tag{14}$$

where $0 < \delta < 1$. This holds true for a higher-dimensional wall, and, more importantly, for spacetimes of type **A**. We obtain

$$\phi = \sqrt{\frac{2\Lambda_0}{a}}\sqrt{\delta(1-\delta)}\int_{\xi_0}^{\xi/\delta}e^{\nu_0(\omega)}d\omega \tag{15}$$

$$V(\phi) = \frac{[1 + \delta(a-1)]}{a}\frac{\Lambda_0}{\delta}\exp[2\nu_0(\xi/\delta)(1-\delta)], \quad \Lambda = 0, \tag{16}$$

where $a = 1$ for case **A** and $a = 4$ for case **B**.

So, it is not only possible to generate solutions for asymmetric spacetimes by using this method: the point is that the scalar field and the potential in the asymmetric and the dynamic cases differ by an overall constant only. Therefore, given a theory with a scalar potential, two solutions can be found to the Einstein-scalar field equations with essentially the same scalar field configuration, but representing different spacetimes.

To further illustrate this point, consider the solution found by scaling the vacuum solution $\nu_0(\xi) = -\ln[\cosh(\beta\xi)]$, $\Lambda_0 = 3a^2\beta^2/8$:

$$\nu(\xi) = -\delta\ln[\cosh(\beta\xi/\delta)]. \tag{17}$$

We have

$$\phi(\xi) = \phi_0\tan^{-1}[\sinh(\beta\xi/\delta)], \quad \phi_0 = \sqrt{\frac{2\Lambda_0}{a}}\frac{\sqrt{\delta(1-\delta)}}{\beta}, \tag{18}$$

$$V(\phi) = \frac{[1 + \delta(a-1)]}{\delta}\frac{\Lambda_0}{a}[\cos(\phi/\phi_0)]^{2(1-\delta)}. \tag{19}$$

With the dynamic metric of case **B**, this is just the 5-dimensional analogue of Goetz's solution [10,23]. With the asymmetric metric of case **A**, this is the 5-dimensional analogue of the solution found in [11].

We now wish to make contact with the brane-world scenarios and take the thin wall limit of Eqs. (12),(17) and its curvature tensor fields. In Ref. [14], it was shown that the domain wall spacetime with metric given by Eqs. (13),(17) has a well-defined thin wall limit. The corresponding asymmetric wall shares this property.

It is easy to see that Eqs. (12),(17) are a regular metric in the sense of [9]. We have that both g_{ab} and $(g^{-1})^{ab}$ are locally bounded. Further, with η_{ab} the ordinary Minkowski metric in 5 dimensions, we find that the weak derivative in η_{ab} of g_{ab} exists and is locally square integrable. Hence g_{ab} can be considered as a distributional metric and its curvature tensor fields make sense as tensor distributions. Taking the $\delta \to 0$ limit (in the sense of distributions) we find

$$\lim_{\delta \to 0} g_{ab} = e^{-\beta(|\xi| + 3\xi)/2}[-dt_a dt_b + e^{2\beta\xi}dx_a^i dx_b^i]$$
$$+ e^{-2\beta|\xi|}d\xi_a d\xi_b \tag{20}$$

$$\lim_{\delta \to 0} G_b^a = -\frac{3}{2}\beta\delta(\xi)[\partial_t^a dt_b + \partial_{x^i}^a dx_a^i]. \tag{21}$$

For $\xi > 0$, Eq. (20) is just the Minkowski spacetime, while for $\xi < 0$ it is the 5-dimensional analogue of the Taub solution [5]. By performing two different coordinate transformations, the metrics on both sides of the wall can be cast in a more familiar form (see [11] for this, and for a detailed analysis of geodesics in the 4-dimensional case). Hence, the spacetime with g_{ab} given by Eqs. (12),(17) is an explicit realization of an asymmetric thick domain wall spacetime with a well defined thin domain wall limit.

III. EXTENSION TO D DIMENSIONS

It is straightforward to extend these results for a thick $(D$-2$)$-brane embedded in a D-dimensional spacetime. Writing the metrics as

$$(g_A)_{ab} = e^{2(\nu(\xi) - (D-2)\beta\xi)/(D-1)}[-dt_a dt_b + e^{2\beta\xi} dx_a^i dx_b^i]$$
$$+ e^{2\nu(\xi)} d\xi_a d\xi_b \tag{22}$$

$$(g_B)_{ab} = e^{2\nu(\xi)}[-dt_a dt_b + e^{2\beta t} dx_a^i dx_b^i + d\xi_a d\xi_b] \tag{23}$$

we get

$$\phi(\xi)'^2 = a_D \frac{(D-2)}{(D-1)}[\nu'^2 - \beta^2 - \nu''] \tag{24}$$

$$V(\xi) = -\frac{a_D}{2} \frac{(D-2)}{(D-1)} e^{-2\nu}[\nu'' + (a_D - 1)(\nu'^2 - \beta^2)] - \Lambda, \tag{25}$$

where now $a_D = 1$ for case **A** and $a_D = D - 1$ for case **B**. In particular, with $\nu(\xi)$ given by Eq. (17), solutions (18) and (19) for the field and the potential are found, with $\Lambda_0 = \beta^2 a_D^2 (D-2)/2(D-1)$.

One can now proceed to obtain other solutions by scaling all the vacuum solutions, namely,

$$\nu = \ln[\cosh(\beta\xi)], \quad \Lambda_0 = a_D^2 \frac{\beta^2}{2} \frac{D-2}{D-1} \tag{26}$$

$$= \pm\beta\xi, \quad \Lambda_0 = 0 \tag{27}$$

$$= \ln[\sinh(\beta\xi)], \quad \Lambda_0 = a_D^2 \frac{\beta^2}{2} \frac{D-2}{D-1} \tag{28}$$

when $\beta \neq 0$, and

$$\nu = \ln(\alpha\xi), \quad \Lambda_0 = a_D \frac{\alpha^2}{4} \tag{29}$$

$$= 0, \quad \Lambda_0 = 0 \tag{30}$$

when $\beta = 0$, where α is an integration constant. Notice that with metric **A**, the two solutions (27) correspond to the D-dimensional analogues of Minkowski and Taub spacetimes, respectively. A number of solutions were found in Ref. [14] for the dynamic spacetime of case **B** by scaling these vacuum solutions. For each of these solutions there is a cor-

responding asymmetric one. However, it was shown that among them the only domain wall solution, meaning one that interpolates between two minima of the potential, is (17)–(19) and this is also true for the asymmetric solutions.

It should be stressed that the asymmetric thick branes considered arise as solutions to the Einstein-scalar field equations with a *symmetric potential* possessing a Z_2 symmetry. Furthermore, the spacetime asymmetry cannot be eliminated by a coordinate change. This can be readily seen from the Kretschmann scalar ($K = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$),

$$K_A = \frac{2e^{-4\nu}}{(D-1)^3}[2(D-1)^2 \nu''^2 - 4(D-1)(D-2)\nu''$$
$$\times(\nu'^2 - \beta^2) + (D-2)(2D-3)(\nu'^2 - \beta^2)^2$$
$$+ 2\beta^2(D-2)^2(D-3)(\nu' + \beta)^2] \tag{31}$$

which for the solution (17)–(19) is manifestly asymmetric

$$K_A = \frac{2}{(D-1)^3}\beta^4[\cosh(\beta\xi/\delta)]^{-4(1-\delta)}$$

$$\times\left[-2\frac{1}{\delta^2}(D-1)^2 - 4\frac{1}{\delta}(D-1)(D-2) \right.$$

$$+ (D-2)(2D-3) + 2(D-2)^2$$

$$\left. \times(D-3)\cosh^2(\beta\xi/\delta)e^{-2\beta\xi/\delta} \right] \tag{32}$$

and diverges as $\xi \to \infty$, but goes to zero for $\xi \to +\infty$. The asymmetry is not present in the Ricci scalar, which vanishes for $|\xi| \to \infty$. The corresponding solutions of type **B**, on the other hand, are asymptotically flat. Notice that while the asymmetric solutions are static, they are not in general BPS domain walls.

The fact that the scalar potential for these two different solutions is the same is a consequence of the scaling procedure we have followed. In the next section, we generate other thick domain wall solutions that do not share this property by proposing a different type of scaling.

IV. A SYMMETRIC, STATIC FAMILY OF WALLS

The warp factor for a thick domain-wall solution can be obtained *via* scaling of the vacuum solutions warp factors in more than one way. A very useful one is the following: take two different vacuum solutions with warp factors $\exp(\nu_1)$ and $\exp(\nu_2)$, respectively, and define the warp factor for the thick wall as

$$\nu(\xi) = -\frac{1}{2s}\ln[\exp(-2s\nu_1) + \exp(-2s\nu_2)]. \tag{33}$$

It turns out that the Einstein-scalar field equations can always be integrated with Eq. (33). Naturally, this scaling will provide asymmetric as well as dynamic solutions, and as could be expected, it will work for the spacetimes of arbitrary di-

mensions considered in the previous section. In [23] this type of scaling has been used for a pair of vacuum solutions in a 5-dimensional spacetime of type **B**.

We get solutions with metrics (22) or (23) for $\phi(\xi)$ and $V(\xi)$ as

$$\phi(\xi)=\sqrt{\frac{(D-2)a_D}{D-1}}\frac{\sqrt{2s-1}}{2s}\tan^{-1}[\sinh(s\Delta\nu)] \qquad (34)$$

$$V(\xi)=\frac{a_D}{2}\frac{D-2}{D-1}e^{2\nu}\left\{\frac{\cosh^{-2}(s\Delta\nu)}{4}[(2s-1)(\Delta\nu)'^2\right.$$
$$\left.-a_D(\nu_1'e^{-s\Delta\nu}+\nu_2'e^{s\Delta\nu})^2]+a_D\beta^2\right\} \qquad (35)$$

where $\Delta\nu\equiv\nu_2-\nu_1$. By choosing the vacuum solutions (27), the result (17)–(19) is recuperated. In this case, the parameter s plays the role of the inverse of the wall's thickness, δ^{-1}, but this is not true in general.

The particular case of symmetric, static solutions is found by using vacuum solutions with $\beta=0$, namely taking

$$\nu=-\frac{1}{2s}\ln[1+(\alpha\xi)^{2s}]. \qquad (36)$$

We get, for $D=5$,

$$\phi=\phi_0\tan^{-1}(\alpha^s\xi^s), \quad \phi_0=\frac{\sqrt{3(2s-1)}}{s}, \qquad (37)$$

$$V(\phi)+\Lambda=3\alpha^2\sin(\phi/\phi_0)^{2-2/s}\left[\frac{2s+3}{2}\cos^2(\phi/\phi_0)-2\right], \qquad (38)$$

so that $\Lambda=-6\alpha^2$. In this case, the parameter s cannot be identified with the wall's inverse thickness. Solutions exist only for s a positive integer, and for s even they are not domain walls, since the field takes values at infinity at the same minimum of the potential. For $s=1$, this solution has been presented in [22] in 5 dimensions. A change of coordinates allows one to identify it with the regularized version of the usual Randall-Sundrum brane. For other (odd) values of s, the potential has a local minimum between two global ones. In a region around the origin, the field takes values at this local minimum, and falls to (different) global minima at spatial infinity.

Let us take a closer look at the solutions with s odd. We would like to explore the thin-wall limit of these configurations. Following [14], we introduce a new parameter δ by scaling the solutions (36) so that the metric is now

$$\delta g_{ab}=\left[1+\left(\frac{\alpha\xi}{\delta}\right)^{2s}\right]^{-\delta/s}(-dt_adt_b+dx_a^idx_b^i)bf$$
$$+\left[1+\left(\frac{\alpha\xi}{\delta}\right)^{2s}\right]^{-1/s}d\xi_ad\xi_b. \qquad (39)$$

Notice that the scaling is performed so that this is still a solution to the Einstein-scalar field equations with

$$\phi(\xi)=\phi_0\tan^{-1}\left(\frac{\alpha^s\xi^s}{\delta^s}\right), \quad \phi_0=\frac{\sqrt{3\,\delta(2s-1)}}{s}, \qquad (40)$$

$$V(\phi)+\Lambda=3\alpha^2\sin(\phi/\phi_0)^{2-2/s}$$
$$\times\left[\frac{2s+4\delta-1}{2\delta}\cos^2(\phi/\phi_0)-2\right], \qquad (41)$$

and

$$G_\xi^\xi=6\alpha^2\left[1+\left(\frac{\alpha\xi}{\delta}\right)^{-2s}\right]^{1/s-2} \qquad (42)$$

$$G_t^t=6\alpha^2\left[1+\left(\frac{\alpha\xi}{\delta}\right)^{-2s}\right]^{1/s-2}\left\{1+\frac{1-2s}{2\delta}\left(\frac{\alpha\xi}{\delta}\right)^{-2s}\right\}. \qquad (43)$$

The function $-G_t^t$, i.e. the energy density, has two maxima at

$$\xi_\pm=\pm\delta[(s-1)/(s+2\delta)]^{1/(2s)} \qquad (44)$$

and the wall can in this sense be considered a "double wall" for $s>1$.

It is not difficult to show that the metric (39) is regular in the sense of Ref. [9], thus all the curvature tensor fields make sense as distributions. Taking the distributional limit as $\delta\to0$ of Eqs. (42),(43) we obtain

$$\lim_{\delta\to0}G_\xi^\xi=6\alpha^2,$$

$$\lim_{\delta\to0}G_t^t=6\alpha^2-3\alpha\frac{(2s-1)}{s}\frac{\left[\Gamma\left(1-\frac{1}{2s}\right)\right]^2}{\Gamma\left(2-\frac{1}{s}\right)}\delta(\xi) \qquad (45)$$

corresponding to an infinitely thin domain wall located at $\xi=0$ embedded in an AdS$_5$ spacetime. However, for $\delta\to0$ Eq. (39) is not a regular metric in the differentiable structure arising from the given chart, and we cannot use the approximation theorems of [9] in order to relate the limit of the curvature tensor distributions with the limit of the metric tensor field. Whether or not a metric is regular depends in general on the differentiable structure imposed on the underlying manifold. A different chart may exist for which the resulting differentiable structure gives a regular metric, but this is of no concern to us here.

V. SUMMARY AND DISCUSSION

Thick domain wall solutions are not uniquely determined by the scalar field potential and the boundary conditions on the field at spatial infinity, but depend also on the subsidiary conditions imposed on the spacetime metric. We have shown that a theory with a given scalar field potential admits in general two kinds of solutions, depending on whether or not one demands reflection symmetry on the wall plane. If an appropriate coordinate chart is chosen, the scalar field looks the same in both solutions. However, their spacetimes are

intrinsically different and cannot be related by a global coordinate change. This is readily seen when comparing curvature scalars for both cases. Solutions with reflection symmetry have been shown to have a time-dependent metric, while the asymmetric ones are static. Asymmetric solutions are asymptotically flat on one side of the wall, and become the Taub spacetime on the other side. This result is valid for D-2 walls in D dimensions.

By appropriately choosing the coordinate chart, we have shown that the Einstein equations for both cases can be solved by the same strategy, namely the appropriate scaling of vacuum solutions, allowing us to associate an asymmetric solution to any dynamic one. Using the method of [14] for generating thick wall solutions by scaling thin wall (vacuum) solutions, we have given examples of this, and extend results on the thin-wall limit of dynamic thick wall solutions to the asymmetric case.

A different way of scaling thin wall solutions that also provides thick solutions has been presented, and shown to provide exact solutions to the Einstein-scalar field equations for both cases. As an example, we have found a family of static, symmetric, "double" wall solutions, which contains as a particular case a previously known BPS solution. In the thin-wall limit, the energy density and pressure of these walls correspond to a single infinitely thin sheet.

How four-dimensional gravity arises on non-singular domain walls or thick 3-brane models has been considered in various five-dimensional models with Z_2 symmetry [13,22–24]. It would be interesting to analyze the metric fluctuations in the Z_2-symmetric case of the double domain wall spacetimes with metrics given by Eq. (39). On the other hand, is the spectrum of general linearized tensor fluctuations of the asymmetric walls consistent with four-dimensional gravity on the wall? We leave this interesting question for a future publication.

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