Brane-induced gravity in more than one extra dimension: Violation of equivalence principle and ghost

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We consider a brane-induced gravity model in more than one extra dimension, regularized by assuming that the bulk gravity is soft in the ultraviolet. We study linear theory about a flat multidimensional space-time and a flat brane. We first find that this model allows for the violation of equivalence between the gravitational and inertial masses of brane matter. We then observe that the model has a scalar ghost field localized near the brane, as well as a quasilocalized massive graviton. The pure tensor structure of four-dimensional gravity on the brane at intermediate distances is due to the cancellation between the extra polarization of the massive graviton and the ghost. This is completely analogous to the situation in the GRS model.

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I. INTRODUCTION AND SUMMARY

In view of the observation [1] that gravity may be localized on a brane embedded in space with one extra dimension of infinite size, it is of interest to study whether there exist mechanisms of (quasi)localization of gravity in spaces with more than one infinite extra dimension. One proposal of this sort has been put forward in Ref. [2]. The basic idea [3] is that radiative effects due to matter residing on the brane may induce new terms in the effective action of multidimensional gravity (cf. Ref. [4]), which concentrate on the brane and dominate the gravitational interactions of brane matter. Thus, the effective action has the form

$$S_{tot}=S_{bulk}^{eff}+S_{brane}. \qquad (1)$$

Here the bulk term involves a $(4+N)$-dimensional metric g_{AB} ($N>1$ is the number of extra dimensions) and at low energy reduces to the $(4+N)$-dimensional Einstein-Hilbert action[1] with the fundamental scale M_*. The brane Einstein-Hilbert term, on the other hand, involves an induced four-dimensional metric $g_{\mu\nu}$ on the brane and has its own mass scale M_{Pl}, which supposedly is determined by dynamics on the brane. It was argued in Ref. [5] that the two scales may be completely different, and, in particular, that the relation

$$M_*\ll M_{Pl} \qquad (2)$$

may hold.

For more than one extra dimension, $N>1$, the model exhibits a potentially interesting UV-IR mixing. Naively, one would expect that at large distances along the brane, the relevant terms in S_{bulk} and S_{brane} are the multidimensional and four-dimensional Einstein-Hilbert terms, respectively, while the brane may be treated as a δ function in transverse directions. This is not the case, however, because of the singularity of the N-dimensional propagator [6–8]. Hence, the behavior of the model at large distances along the brane depends on how the singularity in transverse dimensions is resolved.

One way to resolve this singularity would be to smear the δ function in the brane action. This proposal, however, suffers from the strong coupling problem at unacceptably low energies [6]. Hence, we will not consider this option any longer.

Another proposal [7,8], which is the subject of this paper,[2] is that the bulk gravity is "soft" at distances shorter than M_*^{-1}. Under this assumption, matter on the brane experiences four-dimensional gravity at intermediate distances [7,8]

$$\frac{1}{M_*}\ll r\ll r_c\equiv\frac{M_{Pl}}{M_*^2}, \qquad (3)$$

while the four-dimensional Newton law ceases to hold at both short and ultralarge distances. It is worth noting that this multidimensional brane-induced gravity model, linearized about a flat background, leads to pure tensor [2] four-dimensional gravity on the brane at intermediate distances (3), without an extra scalar inherent in the linearized brane-induced gravity in one extra dimension [3].

These features make brane-induced gravity with $N>1$ potentially interesting, in particular, from the viewpoint of the cosmological constant problem [9]. The violation of the four-dimensional Newton law at ultralarge distances, combined with the absence of an extra scalar interaction on the brane at intermediate scales, is alarming, however, as the same property was present in the model of Ref. [10] which has been found to have a ghost [11,12]. Hence, brane-induced gravity in more than one extra dimension is worth studying in some detail.

In this paper we consider brane-induced gravity, linearized about a flat multidimensional space and a flat brane, mostly at $N>2$; we discuss somewhat the special case N

[1]Leaving aside the issue of the cosmological constant.

[2]Just for brevity, we will call this proposal "brane-induced gravity" in what follows.

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$=2$ toward the end. In Sec. II we neglect complications due to the tensor structure and study a scalar counterpart of the model. We find that once the brane has finite thickness, the equivalence between the gravitational and inertial masses is generally violated for matter on the brane, even in a theory restricted to intermediate scales (3). This is again alarming, since in other models[3] violation of ''charge universality'' (in the gravitational context, the equivalence principle) is a signal of inconsistency [6].

We then proceed in Sec. III to brane-induced gravity itself. We study the linearized field equations, assuming first that the bulk term has the tensor structure of general relativity. We begin with the study of low-mass states which are localized or quasilocalized near the brane. We find in Sec. III A that one such state is a four-dimensional scalar; it is exactly localized on the brane and has negative (tachyonic) mass squared. Another state is a massive four-dimensional graviton.[4] Both masses are of order

$$|m_{tachyon}|\sim|m_{graviton}|\sim r_c^{-1}\equiv\frac{M_*^2}{M_{Pl}}.$$

In Sec. III B we proceed to show that the tachyon is actually a ghost. This can be seen in two ways. One is to study the propagator of the full linearized theory near the tachyon pole and show that the residue has negative sign. Another way is to evaluate the propagator from brane to brane, which describes the gravitational interaction of the matter on the brane. We find that the brane-to-brane propagator is a sum of two terms, one of which has a pole at $p^2=m^2_{graviton}$ with tensor structure appropriate for a massive graviton, while the other is a scalar ghost term (of overall negative sign) with a pole at $p^2=m^2_{tachyon}$. This situation is completely analogous to that in the model of Ref. [10]: at intermediate scales (3), the ghost term cancels out the extra [13] scalar part of the massive graviton propagator, so that the brane-to-brane propagator at intermediate distances has a massless tensor form.

We comment on the case of two extra dimensions, $N=2$, in Sec. III C. There are peculiarities, but the outcome is the same: the model has a tachyonic ghost.

In Sec. IV we generalize by allowing for the most general tensor structure of the linearized bulk equations [in fact, there are only two terms consistent with $(4+N)$-dimensional general covariance]. We again study the case $N>2$ and evaluate the brane-to-brane propagator. We find that it again has a ghost term, although the mass of the ghost is no longer necessarily tachyonic.

Our overall conclusion is that the linearized brane-induced gravity as it stands has a ghost, if the number of extra dimensions is larger than 1. We interpret this property as an indication that this version of induced gravity cannot emerge as a low-energy limit of any consistent microscopic theory.

It is worth noting that a low-energy action of the general form (1) emerges in the string theory context [14]. Furthermore, the hierarchy (2) is also possible in the string theory framework [14]. It would be of interest to understand how string theory resolves the UV-IR ambiguity inherent in the case of more than one extra dimension.

[3]Leaving aside models with extra light four-dimensional degrees of freedom.

[4]The graviton has finite, though very small, width $\Gamma_{graviton}\ll m_{graviton}$, i.e., it is, strictly speaking, quasilocalized.

II. SCALAR MODEL

We begin with a counterpart of the brane-induced gravity with metric perturbations mimicked by a single scalar field Φ. In what follows it will be convenient to consider a thick brane, and take the limit of the delta function brane at the end of the calculations, if desired. It has been argued in Ref. [8] that the loops (and/or nonperturbative effects) involving matter on the brane induce nonlocal terms in the effective action, with the scale of nonlocality set by the brane thickness Δ. At the quadratic level, this effect is modeled by an induced action of the following form [8]:

$$S^{(2)}_{brane}=\frac{M_{Pl}^2}{2}\int d^4x d^Ny d^Ny' f^2(y)\partial_\mu\Phi(x,y)f^2(y')\times\partial^\mu\Phi(x,y'), \qquad (4)$$

where $f(y)$ is a smooth function localized near the brane; it accounts for the thickness of the brane. N is the number of extra dimensions; we concentrate on the case $N>2$. Without loss of generality, f is normalized to unity,

$$\int d^Ny f^2(y)=1, \qquad (5)$$

and is nonzero in a region of size of order Δ. Hereafter $X_A=(x_\mu,y_a)$ are coordinates in $(4+N)$ dimensions, $\mu=0,\dots,3$, $a=4,\dots,N+3$; the signature of the metric is mostly negative.

Let the bulk theory have the effective action $S^{eff}_{bulk}[\Phi]$. There are two more assumptions in the model [8]: (i) The mass scale entering S^{eff}_{bulk} is M_* which is much smaller than M_{Pl}; (ii) the bulk theory is ''soft'' at length scales below $1/M_*$, which we understand as the assumption that the Green's functions of the bulk theory rapidly vanish at high Euclidean momenta.

To consider linearized theory (weak sources), let us neglect the nonlinear terms in the bulk effective action. Then the only relevant term in S^{eff}_{bulk} is quadratic in Φ, and has the form

$$S^{eff,(2)}_{bulk}=-\frac{1}{2}\int d^{N+4}X\Phi(X)\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}\Phi(X)$$

where $\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}$ is the exact inverse propagator of the bulk theory. At low energies, the form factor $\mathcal{F}$ is a constant of order M_*^{2+N} (note that the field Φ is dimensionless). Let us denote the exact propagator of the bulk theory by $D_*(X-X')$, so that

$$D_*(P)=\frac{1}{P^2\mathcal{F}(-P^2)},$$

where $P^2=p^2-p_y^2$, and $p^2=p_\mu p^\mu$. At momenta below M_*, the propagator D_* coincides with the free propagator,

$$D_*(P)=\frac{1}{M_*^{2+N}}\frac{1}{P^2},\quad |P^2|\ll M_*^2, \tag{6}$$

and, by the assumption of softness, $D_*(P)$ rapidly tends to zero at large negative (Euclidean) P^2, with characteristic scale M_*.

A. Scalar propagator

Let the source on the brane be characterized by a spread function $g^2(y)$, and be a δ function in the x coordinates, where, again without loss of generality, g is normalized to unity,

$$\int d^Nyg^2(y)=1. \tag{7}$$

It is convenient to work in the mixed representation, momentum in four dimensions and coordinates in extra dimensions. One has the following equation for the propagator from the brane to everywhere for a given shape of the source:

$$-\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}G_g(p,y')+M_{Pl}^2p^2f^2(y)\int d^Ny'f^2(y')G_g(p,y')=g^2(y), \tag{8}$$

where $\Box^{(4+N)}=-p^2-\partial_y^2$.

Dvali *et al.* [8] proceed under the assumption that $g^2(y)=f^2(y)$ with corrections suppressed by M_*/M_{Pl}. Let us drop this assumption, and see what happens.

Equation (8) has the following solution:

$$G_g(p,y)=D_g(p,y)-\frac{M_{Pl}^2p^2D_{fg}(p)}{1+M_{Pl}^2p^2D_{ff}(p)}\cdot D_f(p,y), \tag{9}$$

where for any function $u(y)$ one defines

$$D_u(p,y)=\int d^Ny'D_*(p,y-y')u^2(y'), \tag{10}$$

and for two functions $u(y),v(y)$ one writes

$$D_{uv}(p)=D_{vu}(p)=\int d^Nyd^Ny'D_*(p,y-y')u^2(y')v^2(y). \tag{11}$$

Let us rewrite the expression (9) in the following suggestive form:

$$G_g(p,y)=\frac{D_g(p,y)}{1+M_{Pl}^2p^2D_{ff}(p)}+\frac{M_{Pl}^2p^2[D_{ff}(p)D_g(p,y)-D_{fg}(p)D_f(p,y)]}{1+M_{Pl}^2p^2D_{ff}(p)}. \tag{12}$$

Now, recall that

$$D_*(p,y-y')=\int d^Np_yD_*(p^2-p_y^2)e^{ip_y\cdot(y-y')}.$$

To evaluate the integral of the form $D_{uv}(p)$, we assume that the brane thickness Δ is much smaller than $1/M_*$, and write for small y and y'

$$D_*(p,y-y')=D_*^{(0)}(p)+D_*^{(2)}(p)\cdot(y-y')^2+\cdots.$$

Clearly,

$$D_*^{(0)}(p)=\int d^Np_yD_*(p^2-p_y^2)$$

and

$$D_*^{(2)}(p)=-\frac{1}{2N}\int d^Np_yp_y^2D_*(p^2-p_y^2). \tag{13}$$

We assume that the latter integrals are convergent at negative (Euclidean) four-momenta, $p^2\leq 0$, because of softness of the propagator D_* at short distances. On dimensional grounds,

$$D_*^{(0)}(|p|\ll M_*)\sim\frac{1}{M_*^4} \tag{14}$$

and

$$D_*^{(2)}(|p|\ll M_*)\sim\frac{1}{M_*^2}. \tag{15}$$

To the first nontrivial order in brane thickness, one has [assuming that u and v are normalized to unity; see Eqs. (5) and (7)]

$$D_{uv}(p)=D_*^{(0)}(p)+D_*^{(2)}(p)\Delta_{uv}^2, \tag{16}$$

where

$$\Delta_{uv}^2=\int d^Nyd^Ny'(y-y')^2u^2(y)v^2(y')$$

explicitly depends on the shapes of the functions $u(y)$ and $v(y)$ and is generically of the order of Δ^2.

At low momenta, $|p|\ll M_*$, one can set $D_{ff}=\mathrm{const}\sim M_*^{-4}$ in the denominators in Eq. (12). Then at intermediate distances (3), the virtuality p^2 is large enough, and one has

$$G_g(p,y)=\frac{D_g(p,y)}{M_{Pl}^2 p^2 D_{ff}(p)}+\frac{D_{ff}(p)D_g(p,y)-D_{fg}(p)D_f(p,y)}{D_{ff}(p)}. \quad (17)$$

This propagator determines the field induced by a weak source of shape $g^2(y)$ in transverse directions, in the theory restricted to intermediate scales (3).

B. Potential on the brane

The interaction between sources with spread functions $g^2(y)$ and $h^2(y)$ is described by an effective four-dimensional propagator, which is the convolution of $G_g(p,y)$ and $h^2(y)$. At intermediate values of the momenta, $M_*^2 \gg p^2 \gg r_c^{-2}$, one has from Eq. (17)

$$G_{eff,4d}(p)=\frac{D_{gh}(p)}{M_{Pl}^2 p^2 D_{ff}(p)}+\frac{D_{gh}(p)D_{ff}(p)-D_{fg}(p)D_{fh}(p)}{D_{ff}(p)}. \quad (18)$$

Keeping terms of order Δ^2, one finds

$$G_{eff,4d}(p)=\frac{D_*^{(0)}(p)+D_*^{(2)}(p)(\Delta_{gh}^2-\Delta_{ff}^2)}{M_{Pl}^2 p^2 D_*^{(0)}(p)}+D_*^{(2)}(p)\times(\Delta_{gh}^2+\Delta_{ff}^2-\Delta_{fg}^2-\Delta_{fh}^2). \quad (19)$$

Consider the first term. Because of the explicit p^2 in the denominator, one can replace $D_*^{(0)}$ and $D_*^{(2)}$ by constants at $p \ll M_*$, i.e., at distances larger than M_*^{-1}. This leads to a four-dimensional Newton potential with nonuniversal gravitational constant:

$$G_{Newton,eff}=\frac{1}{M_{Pl}^2}\left[1+\frac{D_*^{(2)}(0)}{D_*^{(0)}(0)}(\Delta_{gh}^2-\Delta_{ff}^2)\right].$$

According to Eqs. (14) and (15), the nonuniversal correction is of order $\Delta^2 M_*^2$.

This is the main result of this section: the model allows for (weak) violation of the equivalence principle, since the spread functions $g^2(y)$ and $h^2(y)$ may have different shapes, depending on the type of matter residing on the brane.

The second term in Eq. (19) corresponds to a short-ranged force. According to Eq. (13), one has, in coordinate representation,

$$D_*^{(2)}(x)=\frac{1}{2N}\partial_y^2 D_*(x^2-y^2)|_{y=0}.$$

At relatively large distances, $r \gg M_*^{-1}$, the propagator D_* is a free propagator in $(4+N)$ dimensions, up to a factor $1/M_*^{2+N}$. This gives

$$\int dx^0 D_*^{(2)}(x)=\frac{1}{M_*^{2+N}|\mathbf{x}|^{3+N}}, \quad |\mathbf{x}|\gg M_*^{-1},$$

up to a numerical constant of order 1. Hence, the correction to Newton's potential is

$$\Delta V(r)=\frac{1}{M_*^{2+N}r^{3+N}}(\Delta_{gh}^2+\Delta_{ff}^2-\Delta_{fg}^2-\Delta_{fh}^2).$$

This is a short-ranged potential, the "fifth force," which again depends on the composition of matter (the functions g^2 and h^2). It is worth noting that the latter nonuniversality exists also at $N=1$ [7], where the brane-induced gravity does not show any inconsistency.

III. TACHYONIC GHOST

Let us now consider the linearized brane-induced gravity and keep track of its tensor structure. In this section we assume for simplicity that the tensor structure of the linearized equations in the bulk coincides with that in the linearized Einstein theory in $(4+N)$ dimensions. Then the linearized field equation takes the following form:

$$\mathcal{F}(\Box^{(4+N)})G_{AB}(x,y)+M_{Pl}^2 f^2(y)\int dy' f^2(y')G_{AB}^{(4)}(x,y') = T_{AB}(x,y), \quad (20)$$

where $G_{AB}=R_{AB}-(1/2)g_{AB}R$ is the linearized Einstein tensor in $(4+N)$ dimensions, $G_{aB}^{(4)}=0$, and the four-dimensional Einstein tensor $G_{\mu\nu}^{(4)}$ is constructed in terms of four-dimensional components of the metric. The form factor $\mathcal{F}$ has the same properties as above. The function $f^2(y)$ is again the spread function for the induced term.

Let us impose the harmonic gauge

$$\partial_A h_B^A=\frac{1}{2}\partial_B h_A^A, \quad (21)$$

where h_{AB} are perturbations about the Minkowski metric η_{AB}; indices are raised and lowered by the Minkowski metric. Then one has

$$G_{AB}=-\frac{1}{2}\Box^{(4+N)}\left(h_{AB}-\frac{1}{2}\eta_{AB}h_D^D\right), \quad (22)$$

while $G_{\mu\nu}^{(4)}$ remains in its general form

$$G_{\mu\nu}^{(4)}=\frac{1}{2}[\partial_\mu\partial_\lambda h_\nu^\lambda+\partial_\nu\partial_\lambda h_\mu^\lambda-\Box^{(4)}h_{\mu\nu}-\partial_\mu\partial_\nu h_\lambda^\lambda - \eta_{\mu\nu}(\partial_\lambda\partial_\rho h^{\lambda\rho}-\Box^{(4)}h_\lambda^\lambda)]. \quad (23)$$

Hereafter, $\Box^{(4)}=\partial_\mu\partial^\mu$.

A. (Quasi)localized states: Tachyon and massive graviton

Let us consider the sourceless field equation, i.e., Eq. (20) with $T_{AB}=0$, to see whether there exist modes which are (quasi)localized near the brane. The (ab) and $(a\mu)$ components of this equation in the gauge (21) read

$$-\frac{1}{2}\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}\left(h_{ab}-\frac{1}{2}\eta_{ab}h_D^D\right)=0,$$

$$-\frac{1}{2}\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}h_{a\mu}=0.$$

These are the equations of the bulk theory for the corresponding combinations of metrics, and they do not have localized solutions. Hence,

$$h_{ab}=\frac{1}{2}\eta_{ab}h_C^C \tag{24}$$

and

$$h_{a\mu}=0. \tag{25}$$

After taking the trace of Eq. (24), one expresses the (ab) components of the metric in terms of the trace of the four-components,

$$h_{ab}=-\frac{1}{N-2}\eta_{ab}h_\mu^\mu \tag{26}$$

(at this point we specialize to $N>2$). Then the gauge condition (21) with $B=\mu$, together with Eq. (25), give

$$\partial_\mu h_\nu^\mu=-\frac{1}{N-2}\partial_\nu h_\lambda^\lambda.$$

Making use of the above relations, one obtains for the remaining $(\mu\nu)$ components of the field equations

$$-\frac{1}{2}\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}\left(h_{\mu\nu}+\frac{1}{N-2}\eta_{\mu\nu}h_\lambda^\lambda\right)+\frac{M_{Pl}}{2}f^2(y)\cdot\int dy' f^2\left[-\frac{N}{N-2}\partial_\mu\partial_\nu h_\lambda^\lambda-\Box^{(4)}h_{\mu\nu}+\frac{N-1}{N-2}\eta_{\mu\nu}\Box^{(4)}h_\lambda^\lambda\right]=0. \tag{27}$$

The trace of this equation gives

$$-\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}h_\mu^\mu+\tilde{M}_{Pl}^2 f^2(y)\int dy' f^2\cdot\Box^{(4)}h_\mu^\mu=0, \tag{28}$$

where

$$\tilde{M}_{Pl}^2=\frac{2(N-1)}{N+2}M_{Pl}^2. \tag{29}$$

The latter is a scalar equation, and we are interested in its solution localized near the brane. This solution is expressed in terms of the functions $D_f(p,y)$ and $D_{ff}(p)$ introduced in Eqs. (10) and (11). In the mixed representation the solution is

$$h_\mu^\mu(y)=c\cdot D_f(p^2=m_*^2,y), \tag{30}$$

where c is a normalization constant and the mass is determined by the "eigenvalue equation"

$$m_*^2=\frac{1}{\tilde{M}_{Pl}^2 D_{ff}(m_*^2)}. \tag{31}$$

Let us see that the mass squared, m_*^2, is, in fact, negative and real,

$$m_{tachyon}^2\equiv m_*^2<0, \tag{32}$$

$$\mathrm{Im}(m_*^2)=0, \tag{33}$$

so the mode we consider is a tachyon localized near the brane. We first note that

$$|m_*|\sim\frac{M_*^2}{M_{Pl}}\sim r_c^{-1},$$

which is small compared to M_*. Now, one has

$$D_{ff}(p^2)=-\int d^N p_y\frac{|f^2(p_y)|^2}{(-P^2)\mathcal{F}(-P^2)}, \tag{34}$$

where $P^2=p^2-p_y^2$, as before. Since one assumes that the form factor $\mathcal{F}$ rapidly grows at large negative P^2 (the propagator D_* rapidly decays), this integral is convergent in the ultraviolet, and the integrand does not have singularities at $p^2<0$ [a zero of $\mathcal{F}(-P^2)$ at negative P^2 would imply that there is a tachyon in bulk theory]. For $N>2$ the integral is infrared convergent even at $p^2=0$, since for $|P^2|\ll M_*^2$ the form factor $\mathcal{F}$ is constant. For small $p^2<0$ the integral here is a real positive constant, which is of order M_*^{-4} on dimensional grounds, so $D_{ff}(p^2=-|m_*^2|)$ is a negative constant at small $|m_*|$. One concludes that, as long as scales lower than M_* are concerned, there exists a single solution to Eq. (31) which indeed obeys Eqs. (32),(33).

Finally, we have to show that the wave function (30) decays as $|y|\to\infty$. One writes

$$D_f(p,y)=\int d^N p_y\frac{e^{-ip_y\cdot y}f^2(p_y)}{P^2\mathcal{F}(-P^2)}.$$

Large $|y|$ corresponds to small p_y, so at large $|y|$ one has

$$D_f(p^2=m_*^2,y)\propto-\int d^N p_y\frac{e^{-ip_y\cdot y}f^2(p_y)}{p_y^2+|m_*^2|}. \tag{35}$$

Recalling that $f^2(p_y=0)=\int d^N y f^2(y)=1$, one obtains that the wave function (35) has the shape of the N-dimensional Yukawa potential with (small) mass $|m_*|$. Hence the wave function indeed decays as $|y|\to\infty$.

To obtain the complete tensor structure of the tachyon mode, one plugs the solution for the trace, Eq. (30), back into Eq. (27), and obtains, in the mixed representation,

$$-\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}h_{\mu\nu} + M_{Pl}^2 f^2(y) p^2 \int dy' f^2(y') h_{\mu\nu}(p,y') = -c\cdot M_{Pl}^2 D_{ff} p^2 \left[\frac{N}{N-2}\frac{p_\mu p_\nu}{p^2} - \frac{N(N-1)}{(N-2)(N+2)}\eta_{\mu\nu} \right] D_f(p,y), \quad (36)$$

where $p^2 = m_*^2$ for the mode we discuss. This inhomogeneous equation is readily solved. The ab components are found from Eq. (26). In this way one finds the complete expression for the (unnormalized) tachyon mode:

$$h_{\mu\nu}^{(m_*)} = \frac{1}{3}\left(\eta_{\mu\nu} - \frac{N+2}{N-1}\frac{p_\mu p_\nu}{p^2} \right) D_f(y),$$

$$h_{\mu a}^{(m_*)} = 0,$$

$$h_{ab}^{(m_*)} = -\frac{1}{N-1}\eta_{ab} D_f(y), \quad (37)$$

where $p^2 = m_*^2 < 0$ and $D_f(y) = D_f(p^2 = m_*^2; y)$.

For completeness, let us consider (quasi)localized traceless modes, for which $h_\mu^\mu = 0$. For these modes, one obtains from Eq. (27) the following equation:

$$-\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}h_{\mu\nu} + M_{Pl}^2 f^2(y) p^2 \int dy' f^2(y') h_{\mu\nu}(p,y') = 0.$$

The solution to this equation is again of the form

$$h_{\mu\nu}(y) = c_{\mu\nu}\cdot D_f(p^2 = m^2, y),$$

where the $c_{\mu\nu}$ are independent of y, and the mass now obeys

$$m^2 = -\frac{1}{M_{Pl}^2 D_{ff}(m^2)}.$$

We are interested in solutions with $|m| \ll M_*$, which are relevant at low energies. According to Eq. (34), for such a solution the real part of m^2 is positive, and is of order r_c^{-2}. Now, for small positive p^2, the function $D_{ff}(p^2)$ has an even smaller imaginary part, which may be estimated as follows. The integrand in Eq. (34) is a smooth positive function at $p_y^2 \gg p^2$, so this region does not contribute to the imaginary part. The imaginary part comes from the infrared region, and is proportional to

$$\int_0^\epsilon \frac{p_y^{N-1} dp_y}{p_y^2 - p^2 - i0}.$$

The imaginary part of the latter integral is proportional to

$$-ip^{N-2}. \quad (38)$$

So, for $N>2$, the quasilocalized graviton has a small mass $m_{graviton} \equiv m$, where

$$\mathrm{Re}(m) = \frac{1}{M_{Pl}\sqrt{|D_{ff}(0)|}} \sim \frac{M_*^2}{M_{Pl}} \sim r_c^{-1},$$

and an even smaller width,

$$\frac{\Gamma_{graviton}}{m} \sim \frac{m^{N-2}}{M_*^{N-2}}.$$

We conclude that in this model there is a massive four-dimensional graviton with a tiny width. The violation of Newton's law at distances of order r_c is due to the graviton *mass*, not width, in clear contrast to the five-dimensional case [3].

B. Propagators at low energies: The tachyon is a ghost

One way to see that the tachyon is in fact a ghost is to calculate the full propagator $D_{AB,CD}(p;y,y')$ near $p^2 = m_*^2$, i.e., to extract its pole term. This is done in Appendix A. The outcome is

$$D_{AB,CD}^{(pole)}(p;y,y') = -\frac{3}{M_{Pl}^2[D_{ff}(m_*^2)]^2}\cdot\frac{h_{AB}^{(m_*)}(y) h_{CD}^{(m_*)}(y)}{p^2 - m_*^2}, \quad (39)$$

where $h_{AB}^{(m_*)}(y)$ is the (unnormalized) tachyon wave function (37). The overall negative sign here means that the tachyon is indeed a ghost.

The structure of the pole term (39) is precisely what one expects for the contribution of a mode localized near the brane. From Eq. (39) one deduces also that the properly normalized tachyon-ghost mode is

$$h_{AB}^{normalized}(y) = \frac{\sqrt{3}}{M_{Pl}|D_{ff}|} h_{AB}^{(m_*)}.$$

One observes from the latter formula and Eq. (37) that the tachyonic ghost couples to matter on the brane at gravitational strength.

It is perhaps more instructive to study the propagator with both end points on the brane. More precisely, let us consider the source on the brane with the only nonvanishing components $T_{\mu\nu}$, which is distributed in the transverse directions with the same[5] spread function $f^2(y)$ as in Eq. (20),

$$T_{\mu\nu}(x,y) = \theta_{\mu\nu}(x) f^2(y), \quad (40)$$

where $\theta_{\mu\nu}(x)$ is conserved in the four-dimensional sense. The point is to calculate the $(\mu\nu)$ components of the metric

[5] The analysis of the general case of a source with spread function g^2 different from f^2 proceeds along the lines of Sec. II. This analysis is straightforward but not illuminating.

due to this source. This is done in Appendix B, with the result, in mixed representation,

$$h_{\mu\nu}(p,y)=\frac{2}{M_{Pl}^2}\frac{D_f(p,y)}{D_{ff}(p)}\left[\frac{1}{p^2-m^2(p)}\left(\theta_{\mu\nu}-\frac{1}{3}\eta_{\mu\nu}\theta_\lambda^\lambda\right)-\frac{1}{6}\frac{1}{p^2-m_*^2(p)}\eta_{\mu\nu}\theta_\lambda^\lambda\right]+\text{longitudinal part} \tag{41}$$

where

$$m^2(p)=-\frac{1}{M_{Pl}^2 D_{ff}(p)}, \tag{42}$$

$$m_*^2(p)=\frac{1}{\tilde{M}_{Pl}^2 D_{ff}(p)}, \tag{43}$$

and the longitudinal part is proportional to $p_\mu p_\nu$ and vanishes when contracted with the conserved stress-energy [the overall factor 2 in Eq. (41) is due to our definition of M_{Pl}; see Eq. (20)]. Now, the interaction between two sources of the form (40) may be written in terms of the effective four-dimensional propagator $D^{(4)}_{\mu\nu,\lambda\rho}(p)$, so that one has

$$\theta'_{\mu\nu}(p)D^{(4)}_{\mu\nu,\lambda\rho}(p)\,\theta_{\lambda\rho}(p)=\theta'_{\mu\nu}(p)\int d^N y f^2(y)h_{\mu\nu}(y,p),$$

where $h_{\mu\nu}$ is given by Eq. (41). Hence, the effective brane-to-brane propagator is

$$D^{(4)}_{\mu\nu,\lambda\rho}=\frac{2}{M_{Pl}^2}\left[\frac{1}{p^2-m^2(p)}\left(\frac{1}{2}(\eta_{\mu\lambda}\eta_{\nu\rho}+\eta_{\mu\rho}\eta_{\nu\lambda})-\frac{1}{3}\eta_{\mu\nu}\eta_{\lambda\rho}\right)-\frac{1}{6}\frac{1}{p^2-m_*^2(p)}\eta_{\mu\nu}\eta_{\lambda\rho}\right]+\text{longitudinal part}. \tag{44}$$

At low energies the "masses" $m^2(p)$ and $m_*^2(p)$ are constants [up to a tiny p-dependent imaginary part; see Eq. (38)]. Thus, at low energies the propagator (44) corresponds to a massive graviton of mass m (note that the Van Dam–Veltman–Zakharov property indeed holds) and a tachyonic ghost with negative m_*^2. This ghost cancels the contribution of the extra graviton polarization at intermediate momenta $M_*\gg|p|\gg(m,m_*)\sim r_c^{-1}$, so that at these scales the brane-to-brane propagator has the same structure as in general relativity,

$$D^{(4)}_{\mu\nu,\lambda\rho}=\frac{1}{M_{Pl}^2}\frac{1}{p^2}(\eta_{\mu\lambda}\eta_{\nu\rho}+\eta_{\mu\rho}\eta_{\nu\lambda}-\eta_{\mu\nu}\eta_{\lambda\rho})+\text{longitudinal part}.$$

This is precisely the same situation as in the model of Ref. [10]: the reason why the correct tensor structure emerges in the linearized theory at intermediate distances is the existence of a ghost field.

C. $N=2$

The case $N=2$ is somewhat special. Let us first consider the tachyon mode. Equation (24) implies now

$$h_\mu^\mu=0, \tag{45}$$

while h_a^a is arbitrary at this point. The four-dimensional trace of the sourceless equation (20) then gives

$$-\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}h_a^a+\tilde{M}_{Pl}^2 f^2(y)\int dy' f^2\cdot\Box^{(4)}h_a^a=0.$$

This equation has the same structure as Eq. (28), so there again exists a tachyon. At $N=2$, it is the extra-dimensional metric h_{ab} and the traceless part of $h_{\mu\nu}$ that do not vanish in the tachyon mode [in the gauge (21)].

Another point is that the integral (34) is logarithmic at $N=2$, so the estimate for the graviton and tachyon masses is now

$$\mathrm{Re}(m^2)\sim|m_*^2|\sim\frac{M_*^4}{M_{Pl}^2}\log\frac{M_{Pl}}{M_*}.$$

The imaginary part of the graviton mass is suppressed relative to its real part by a logarithm only,

$$\frac{\Gamma_{grav}}{|m|}\sim\frac{1}{\log(M_{Pl}/M_*)}.$$

Yet the graviton width is smaller than its mass.

The tachyon is a ghost at $N=2$ as well. A simple way to see this is to redo the calculation leading to the brane-to-brane propagator. One finds that the expression (44) remains valid at $N=2$, the property (45) being ensured by the appropriate structure of the longitudinal terms. The negative sign of the last term on the right hand side of Eq. (44) tells us that the tachyon is indeed a ghost.

So, in spite of peculiarities, the conclusion for $N=2$ is the same as for $N>2$: the model has a tachyonic ghost.

IV. GENERALIZED MODEL

In this section we drop the assumption that the tensor structure of the linearized bulk equations coincides with that in the linearized Einstein theory and consider the most general tensor structure compatible with the $(4+N)$-dimensional general covariance. The linearized field equation in the bulk theory has the following general form:

$$\mathcal{D}_{ABCD}h^{CD}=0 \tag{46}$$

with some linear operator $\mathcal{D}_{ABCD}$. The symmetry of this operator under $A\leftrightarrow B$, $C\leftrightarrow D$, and $(AB)\leftrightarrow(CD)$ implies the following structure of $\mathcal{D}_{ABCD}$:

$$\mathcal{D}_{ABCD}=a\partial_A\partial_B\partial_C\partial_D+b(\partial_A\partial_B\eta_{CD}+\eta_{AB}\partial_C\partial_D)$$
$$+c(\partial_A\eta_{BC}\partial_D+\partial_B\eta_{AC}\partial_D+\partial_A\eta_{BD}\partial_C+\partial_B\eta_{AD}\partial_C)$$
$$+d\,\eta_{AB}\eta_{CD}+e(\eta_{AC}\eta_{BD}+\eta_{AD}\eta_{BC}),$$

where a,b,c,d,e are as yet arbitrary functions of $\Box^{(N+4)}$. Now, gauge invariance implies

$$\partial_A\mathcal{D}^A_{BCD}=0.$$

This leaves only two possible tensor structures that may appear in $\mathcal{D}_{ABCD}$, namely, the usual Einstein structure and the product of two projectors, $\mathcal{D}_{ABCD}\propto P_{AB}P_{CD}$, where

$$P_{AB}=\Box^{(4+N)}\eta_{AB}-\partial_A\partial_B.$$

In the harmonic gauge (21) one has

$$P_{AB}P_{CD}h^{CD}\equiv\Pi_{AB}=\frac{1}{2}(\Box^{(4+N)}\eta_{AB}-\partial_A\partial_B)\Box^{(4+N)}h^C_C.$$

Consequently, the generalization of Eq. (20) is

$$\mathcal{F}(\Box^{(4+N)})G_{AB}(x,y)+\mathcal{G}(\Box^{(4+N)})\Pi_{AB}(x,y)$$
$$+M^2_{Pl}f^2(y)\int dy'f^2(y')G^{(4)}_{AB}(x,y')=T_{AB}(x,y), \tag{47}$$

where G_{AB} is given by Eq. (22), and a new form factor $\mathcal{G}$ is assumed to have the same ultraviolet properties as the form factor $\mathcal{F}$.

We will not repeat all the steps of the analysis of Sec. III. To see the existence of the localized ghost and find its wave function in this general setup, it suffices to study the structure of the brane-to-brane propagator. We again consider a source of the form (40) and evaluate the $(\mu\nu)$ components of the metric induced by this source. The result is (see Appendix C for calculational details)

$$h_{\mu\nu}(p,y)=\frac{2}{M^2_{Pl}}\left[\frac{D_f(p,y)}{D_{ff}(p)}\frac{1}{p^2-m^2(p)}\left(\theta_{\mu\nu}-\frac{1}{3}\eta_{\mu\nu}\theta^\lambda_\lambda\right)\right.$$
$$\left.-\frac{\tilde{D}_f(p,y)}{\tilde{D}_{ff}}\frac{1}{6}\frac{1}{p^2-m^2_*(p)}\eta_{\mu\nu}\theta^\lambda_\lambda\right]$$
$$+\text{longitudinal part}, \tag{48}$$

where $\tilde{D}_f$ and $\tilde{D}_{ff}$ are defined in a similar way as D_f and D_{ff} [see Eqs. (10), (11)] but with a new function $\tilde{D}_*(p;y,y')$ substituted for $D_*(p;y,y')$. The function $\tilde{D}_*(p;y,y')$ is a solution of the following equation:

$$-\mathcal{O}(\Box^{(4+N)})\cdot\mathcal{F}(\Box^{(4+N)})\cdot\Box^{(4+N)}\tilde{D}_*(p;y,y')=\delta(y-y'),$$

where the operator $\mathcal{O}(\Box^{(4+N)})$ is

$$\mathcal{O}(\Box^{(4+N)})=\frac{N-1+N\mathcal{H}(\Box^{(4+N)})\cdot\Box^{(4+N)}}{N+2-(N+3)\mathcal{H}(\Box^{(4+N)})\cdot\Box^{(4+N)}}$$

with

$$\mathcal{H}(\Box^{(4+N)})=\frac{2\mathcal{G}(\Box^{(4+N)})}{\mathcal{F}(\Box^{(4+N)})}.$$

The "mass" $m^2(p)$ entering Eq. (48) is the same as in Sec. III, while $m^2_*(p)$ is now

$$m^2_*(p)=\frac{1}{2M^2_{Pl}\tilde{D}_{ff}(p^2)}.$$

Thus, the brane-to-brane propagator still has the form (44). The second term in Eq. (44) again has a negative sign, so the model again has a ghost field, but now the mass of the ghost is a solution to the following eigenvalue equation:

$$m^2_*=\frac{1}{2M^2_{Pl}\tilde{D}_{ff}(m^2_*)}. \tag{49}$$

The difference from the case $\mathcal{G}=0$ studied in Sec. III is that the ghost field in principle need not be a tachyon in the general case and that the wave functions of the ghost and graviton have different profiles in the transverse directions. The graviton wave function is again $D_f(p;y)$, while the ghost wave function is $\tilde{D}_f(p;y)$.

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APPENDIX A

Here we calculate the tachyon pole term in the full propagator of the model of Sec. III. We do this by solving Eq. (20) with the conserved right hand side,

$$\partial_A T^A_B=0. \tag{A1}$$

Otherwise T_{AB} is arbitrary. We still use the gauge conditions (21).

We begin with the (ab) components of Eq. (20), which read

$$-\frac{1}{2}\mathcal{F}\Box^{(4+N)}\left(h_{ab}-\frac{1}{2}\eta_{ab}h^D_D\right)=T_{ab}. \tag{A2}$$

We decompose h_{ab} in the following way:

$$h_{ab}=h^T_{ab}+\frac{1}{N}\eta_{ab}h^c_c, \tag{A3}$$

where

$$h^T_{ab}=h_{ab}-\frac{1}{N}\eta_{ab}h^c_c \tag{A4}$$

is the traceless part. The traceless part obeys

$$-\frac{1}{2}\mathcal{F}(\square^{(4+N)})\square^{(4+N)}h^T_{ab}=T_{ab}-\frac{1}{N}\eta_{ab}T^c_c, \tag{A5}$$

while the trace of Eq. (A2) gives

$$h^c_c=-\frac{N}{N-2}h^\mu_\mu+b, \tag{A6}$$

where b obeys

$$-\mathcal{F}(\square^{(4+N)})\square^{(4+N)}b=-\frac{4}{N-2}T^c_c \tag{A7}$$

and hence is equal to

$$b(X)=-\frac{4}{N-2}\int d^{4+N}X'D_*(X-X')T^c_c(X'). \tag{A8}$$

We also have for the overall trace

$$h^C_C=-\frac{2}{N-2}h^\mu_\mu+b. \tag{A9}$$

Let us now consider the $(a\mu)$ components of Eq. (20). They read

$$-\frac{1}{2}\mathcal{F}(\square^{(4+N)})\square^{(4+N)}h_{a\mu}=T_{a\mu}. \tag{A10}$$

Hence,

$$h_{a\mu}(X)=2\int d^{4+N}X'D_*(X-X')T_{a\mu}(X'). \tag{A11}$$

Finally, let us study the $(\mu\nu)$ components of Eq. (20). We need the expression for $\partial_\lambda h^{\lambda\rho}$ that enters $G^{(4)}_{\mu\nu}$. This expression is obtained by making use of the gauge condition

$$\partial_A h^A_\mu\equiv\partial_a h^a_\mu+\partial_\lambda h^\lambda_\mu=\frac{1}{2}\partial_\mu h^C_C. \tag{A12}$$

Now, because of the conservation property (A1), one has

$$\begin{aligned}\partial_a h^a_\mu&=2\int d^{4+N}X'D_*(X-X')\partial_a T^a_\mu(X')\\&=-2\int d^{4+N}X'D_*(X-X')\partial_\lambda T^\lambda_\mu(X').\end{aligned} \tag{A13}$$

Hence, the $(\mu\nu)$ components of Eq. (20) may be written in terms of $h_{\mu\nu}$, the trace h^λ_λ, and the components T^ρ_λ, and T^a_a of the stress-energy tensor. After some algebra one obtains

$$\begin{aligned}&-\mathcal{F}(\square^{(4+N)})\square^{(4+N)}\left(h_{\mu\nu}+\frac{1}{N-2}\eta_{\mu\nu}h^\lambda_\lambda\right)+M^2_{Pl}f^2(y)\cdot\int dy'f^2(y')\left[-\frac{N}{N-2}\partial_\mu\partial_\nu h^\lambda_\lambda-\square^{(4)}h_{\mu\nu}+\frac{N-1}{N-2}\eta_{\mu\nu}\square^{(4)}h^\lambda_\lambda\right]\\&=2T_{\mu\nu}-\frac{2}{N-2}\eta_{\mu\nu}T^a_a-2M^2_{Pl}f^2(y)\int d^Ny'D_f(y')\left[\partial_\lambda\partial_\mu T^\lambda_\nu+\partial_\lambda\partial_\nu T^\lambda_\mu-\frac{2}{N-2}\partial_\mu\partial_\nu T^a_a-\eta_{\mu\nu}\left(\partial_\lambda\partial_\rho T^{\lambda\rho}\right.\right.\\&\left.\left.-\frac{1}{N-2}\square^{(4)}T^a_a\right)\right].\end{aligned} \tag{A14}$$

The trace of this equation gives

$$\begin{aligned}&-\mathcal{F}(\square^{(4+N)})\square^{(4+N)}h^\mu_\mu+\tilde{M}^2_{Pl}f^2(y)\int dy'f^2\cdot\square^{(4)}h^\mu_\mu\\&=\frac{2(N-2)}{N+2}\left[T^\mu_\mu-\frac{4}{N-2}T^a_a+2M^2_{Pl}f^2(y)\int d^Ny'D_f(y')\cdot\left(\partial_\lambda\partial_\rho T^{\lambda\rho}-\frac{1}{N-2}\square^{(4)}T^a_a\right)\right].\end{aligned} \tag{A15}$$

The solution to the latter equation is conveniently written in the mixed representation,

$$\begin{aligned}h^\mu_\mu(p,y)=&\frac{2(N-2)}{N+2}\int d^Ny'D_*(p;y,y')\left(T^\mu_\mu(p,y')-\frac{4}{N-2}T^a_a(p,y')\right)+\frac{4(N-2)}{N+2}\frac{p^2M^2_{Pl}D_f(y)}{1-\tilde{M}^2_{Pl}p^2D_{ff}(p)}\int d^Ny'D_f(p,y')\\&\times\left(\frac{N-1}{N+2}T^\mu_\mu-\frac{p_\mu p_\nu}{p^2}T^{\mu\nu}-\frac{3}{N+2}T^a_a\right),\end{aligned} \tag{A16}$$

where we made use of the relation (29). This expression is still exact. Clearly, it has a pole at $p^2=m_*^2$ [in the second term in the right hand side of Eq. (A16)].

To find the tachyon pole term in $h_{\mu\nu}$, one plugs the solution for h_λ^λ back into Eq. (A14). In terms of Eq. (A14), the tachyon pole in $h_{\mu\nu}$ comes entirely from the pole part in h_λ^λ. One makes use of Eq. (A15) and writes Eq. (A14) in the following form:

$$-\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}h_{\mu\nu} + M_{Pl}^2 f^2(y) p^2 \int dy' f^2(y') h_{\mu\nu}(p,y') = M_{Pl}^2 p^2 f^2(y) \int d^N y' f^2 \left(\frac{N(N-1)}{(N-2)(N+2)} \eta_{\mu\nu} - \frac{N}{N-2} \frac{p_\mu p_\nu}{p^2} \right) h_\lambda^\lambda + \cdots, \tag{A17}$$

where the ellipsis denotes terms that do not contain a pole at $p^2=m_*^2$. This equation is straightforwardly solved, and after some algebra one obtains that the tachyon pole part of $h_{\mu\nu}$ is

$$h_{\mu\nu}^{(pole)}(p,y) = -\frac{1}{3} \frac{1}{p^2-m_*^2} \frac{D_f(p,y)}{M_{Pl}^2 D_{ff}^2} \left(\eta_{\mu\nu} - \frac{N+2}{N-1} \frac{p_\mu p_\nu}{m_*^2} \right) \times \int d^N y' D_f(p,y') \left[\left(\eta_{\lambda\rho} - \frac{N+2}{N-1} \frac{p_\lambda p_\rho}{m_*^2} \right) \times T^{\lambda\rho}(p,y') - \frac{3}{N-1} T_a^a(p,y') \right]. \tag{A18}$$

It remains to find the tachyon pole parts of other metric components. According to Eqs. (A5), (A7), and (A10), the traceless part h_{ab}^T, the term b, and the metric components $h_{a\mu}$ do not have poles at $p^2=m_*^2$. The pole term in h_a^a is determined by the pole term in h_μ^μ through Eq. (A6). Thus, one finds

$$h_{ab}^{(pole)}(p,y) = -\frac{1}{N-2} \eta_{ab} h_\mu^{(pole)\mu}(p,y) = \frac{1}{N-1} \eta_{ab} \frac{1}{p^2-m_*^2} \frac{D_f(p,y)}{M_{Pl}^2 D_{ff}^2} \times \int d^N y' D_f(p,y') \times \left[\left(\eta_{\lambda\rho} - \frac{N+2}{N-1} \frac{p_\lambda p_\rho}{m_*^2} \right) T^{\lambda\rho}(p,y') - \frac{3}{N-1} T_a^a(p,y') \right]. \tag{A19}$$

We see from Eqs. (A18) and (A19) that the pole terms in h_{AB} may indeed be written in the form

$$h_{AB}^{(pole)}(p,y) = \int d^N y' D_{AB,CD}^{(pole)}(p;y,y') T^{CD}(p,y'), \tag{A20}$$

where the pole term in the propagator is given by Eq. (39).

APPENDIX B

Let us calculate the $(\mu\nu)$ components of the metric due to the source of the form (40) with conserved $\theta_{\mu\nu}$. For this particular type of source, the expression (A16) simplifies considerably,

$$h_\mu^\mu(p,y) = -\frac{2(N-2)}{N+2} \frac{D_f(p,y)}{\tilde{M}_{Pl}^2 D_{ff}(p)} \frac{1}{p^2-m_*^2(p)} \theta_\mu^\mu(p), \tag{B1}$$

where $m_*^2(p)$ is given by Eq. (43). We plug this expression into Eq. (A14) and again make use of the properties of the source to obtain

$$-\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}h_{\mu\nu} + M_{Pl}^2 f^2(y) p^2 \int dy' f^2(y') h_{\mu\nu}(p,y') = f^2(y) \left(2\theta_{\mu\nu} - \frac{2}{N+2} \eta_{\mu\nu} \theta_\lambda^\lambda - \frac{N}{N+2} \times \frac{p^2}{p^2-m_*^2(p)} \eta_{\mu\nu} \theta_\lambda^\lambda \right) + \text{longitudinal part}, \tag{B2}$$

where the longitudinal part is proportional to $p_\mu p_\nu$. After some algebra, one finds that the solution has indeed the form (41).

APPENDIX C

Here we sketch the calculations leading to the result (48). The steps are similar to those in Appendixes A and B. First, one considers the (ab) components of Eq. (47) and finds the following generalization of Eq. (A6) [recall that we consider a source of the form (40)]:

$$h_c^c = -\frac{N + \mathcal{H}((N-1)\Box^{(4+N)} + \Box^{(4)})}{N-2+\mathcal{H}((N-1)\Box^{(4+N)} + \Box^{(4)})} h_\mu^\mu. \tag{C1}$$

For the overall trace one has

$$h_C^C = -\frac{2}{N-2+\mathcal{H}((N-1)\Box^{(4+N)} + \Box^{(4)})} h_\mu^\mu. \tag{C2}$$

From the $(a\mu)$ components of Eq. (47) one finds

$$h_{a\mu}=\frac{\mathcal{H}}{N-2+\mathcal{H}((N-1)\Box^{(4+N)}+\Box^{(4)})}\,\partial_a\partial_\mu h^\nu_\nu. \quad \text{(C3)}$$

By making use of the gauge condition (A12) one obtains the following expression for the longitudinal components of the four-dimensional part of the metric:

$$\partial_\nu h^\nu_\mu=-\frac{1-\mathcal{H}\Box^{(N)}}{N-2+\mathcal{H}((N-1)\Box^{(4+N)}+\Box^{(4)})}\,\partial_\mu h^\nu_\nu, \quad \text{(C4)}$$

where $\Box^{(N)}=\delta^{ab}\partial_a\partial_b$. Plugging the expressions (C2),(C4) into the $(\mu\nu)$ components of Eq. (47) one arrives at the following analogue of Eq. (A14):

$$-\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}\left(h_{\mu\nu}+\frac{1+\mathcal{H}\Box^{(N+4)}}{N-2+\mathcal{H}((N-1)\Box^{(4+N)}+\Box^{(4)})}\,\eta_{\mu\nu}h^\lambda_\lambda\right)$$
$$+\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}\frac{\mathcal{H}}{N-2+\mathcal{H}((N-1)\Box^{(4+N)}+\Box^{(4)})}\,\partial_\mu\partial_\nu h^\lambda_\lambda+M^2_{Pl}f^2(y)\cdot\int dy'\,f^2(y')$$
$$\times\left[-\frac{N+\mathcal{H}((N+1)\Box^{(4+N)}-\Box^{(4)})}{N-2+\mathcal{H}((N-1)\Box^{(4+N)}+\Box^{(4)})}\,\partial_\mu\partial_\nu h^\lambda_\lambda-\Box^{(4)}h_{\mu\nu}\right.$$
$$\left.+\frac{N-1+\mathcal{H}N\Box^{(4+N)}}{N-2+\mathcal{H}((N-1)\Box^{(4+N)}+\Box^{(4)})}\,\eta_{\mu\nu}\Box^{(4)}h^\lambda_\lambda\right]=2\,\theta_{\mu\nu}f^2(y). \quad \text{(C5)}$$

The trace of this equation gives

$$-\mathcal{F}(\Box^{(4+N)})\Box^{(4+N)}\frac{N+2+\mathcal{H}(N+3)\Box^{(4+N)}}{N-2+\mathcal{H}((N-1)\Box^{(4+N)}+\Box^{(4)})}h^\mu_\mu+2M^2_{Pl}f^2(y)\int dy'\,f^2\cdot\Box^{(4)}$$
$$\times\frac{N-1+\mathcal{H}N\Box^{(4+N)}}{N-2+\mathcal{H}((N-1)\Box^{(4+N)}+\Box^{(4)})}h^\mu_\mu=2\,\theta^\mu_\mu f^2(y). \quad \text{(C6)}$$

The solution of this equation is

$$h^\mu_\mu=\frac{2\,\theta^\mu_\mu}{1-2p^2D_{ff}(p)}\,\frac{N-2+\mathcal{H}((N-1)\Box^{(4+N)}+\Box^{(4)})}{N+2+\mathcal{H}(N+3)\Box^{(4+N)}}\tilde{D}_f. \quad \text{(C7)}$$

Plugging this result back into Eq. (C5), one obtains after some algebra the desired expression (48).

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