

## General-covariant evolution formalism for numerical relativity

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(Received 18 December 2002; published 7 May 2003)

A general-covariant extension of Einstein's field equations is considered with a view to numerical relativity applications. The basic variables are taken to be the metric tensor and an additional four-vector  $Z_\mu$ . Einstein's solutions are recovered when the additional four-vector vanishes, so that the energy and momentum constraints amount to the covariant algebraic condition  $Z_\mu=0$ . The extended field equations can be supplemented by suitable coordinate conditions in order to provide symmetric hyperbolic evolution systems: this is actually the case for either harmonic coordinates or normal coordinates with harmonic slicing.

DOI: 10.1103/PhysRevD.67.104005

PACS number(s): 04.25.Dm

### I. INTRODUCTION

General covariance is a key feature of general relativity. At a first look, Einstein's field equations can be understood as a set of ten second order partial differential equations on the ten unknown metric coefficients  $g_{\mu\nu}$ :

$$R_{\mu\nu} = 8\pi \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right). \quad (1)$$

However, in general relativity, like in many other field theories, physical solutions are given by an equivalence class of field values, related to one another by gauge transformations. General covariance means that the gauge group in Einstein's theory is that of the general (smooth) coordinate transformations

$$y^\mu = f^\mu(x^\nu). \quad (2)$$

Accordingly, the field equations (1) do not (even may not) provide enough information to determine the values of the ten unknown coefficients  $g_{\mu\nu}$ . On the other hand, in numerical applications we must deal with specific metric components. It follows that a numerical evolution system must include a specification of the coordinates as an extra ingredient in order to determine the four kinematical degrees of freedom. Fixing four of the ten metric coefficients, we choose one specific expression for  $g_{\mu\nu}$  out of the equivalence class representing the same physical solution. A general covariant evolution system would be incomplete and conversely, a complete evolution system cannot preserve general covariance.

To be more specific, we will carefully distinguish evolution systems, associated with a particular gauge choice, from evolution formalisms. The latter can be defined as a set of equations that apply to an equivalence class of solutions, prior to a complete gauge specification. Following this way, we will be able to draw a line between general-covariant formalisms, where the equivalence class is defined by the full group of coordinate transformations (2) and partially covariant formalisms, where the equivalence class is defined by any restricted subset of coordinate transformations. This

means also, for instance, that different evolution systems can be obtained from the same evolution formalism, as we will see in the following paragraphs.

#### A. General covariant formalisms

A good example of a general-covariant formalism is provided by recent numerical relativity works [1,2] based upon a well known classical approach. The evolution formalism is given by the original field equations (1), although the Donder [3,4] expression of the Ricci tensor is used to write down the principal part, namely,

$$-\square g_{\mu\nu} + \partial_\mu \Gamma_\nu + \partial_\nu \Gamma_\mu = \dots, \quad (3)$$

where the box symbol stands for the d'Alembert operator on functions and we have noted  $\Gamma^\mu \equiv g^{\rho\sigma} \Gamma^\mu_{\rho\sigma}$  as usual. General covariance is not lost in passing from Eqs. (1)–(3), because we only reordered the partial derivatives in the Ricci tensor.

It is obvious from a comparison between Eq. (3) and the wave equation for  $g_{\mu\nu}$  that we can obtain a symmetric hyperbolic system imposing the well known harmonic coordinate conditions [3,4]:

$$\square x^\mu = -\Gamma^\mu = 0. \quad (4)$$

Although the first proofs of well posedness of the resulting evolution system were well known [5,6], the corresponding proof for the initial-boundary problem, which is highly relevant for numerical relativity applications, has been given recently [1,7,8]. Different evolution systems can be obtained from the general-covariant formalism (3) by modifying the harmonic coordinate conditions (4) [9,10]. One can even add arbitrary "gauge source" terms to the right-hand side of Eq. (4) to obtain a wide class of generalized harmonic evolution systems [11].

A different evolution formalism can be obtained by using the well known 3+1 decomposition [12,13], where one considers the space-time sliced by  $t = \text{const}$  hypersurfaces. The line element can be written as

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij} (dx^i + \beta^i dt)(dx^j + \beta^j dt), \quad (5)$$

where the lapse  $\alpha$  and the shift  $\beta^i$  represent the kinematical degrees of freedom. The field equations (1) can then be translated in terms of the three-dimensional geometry of the slices, namely,

$$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}, \quad (6)$$

$$(\partial_t - \mathcal{L}_\beta)K_{ij} = -\nabla_i\alpha_j + \alpha[(^3)R_{ij} - 2K_{ij}^2 + \text{tr}K K_{ij}], \quad (7)$$

$$(^3)R - \text{tr}(K^2) + (\text{tr}K)^2 = 0, \quad (8)$$

$$\nabla_k(K^k_i - \delta^k_i \text{tr}K) = 0, \quad (9)$$

where we have restricted ourselves to the vacuum case for simplicity.

Let us notice that the coordinate gauge freedom is not limited in any way by translating the four-dimensional (4D) field equations (1) into the 3+1 version (6)–(9). The lapse  $\alpha$  and the shift  $\beta^i$  can take arbitrary values, so that the four gauge degrees of freedom are still at our disposal. Although general covariance is not manifest in the 3+1 equations (6)–(9), their solution space is still invariant under general coordinate transformations (2), because it is equivalent to the corresponding solution space of the 4D equations (1). Conversely, in the 3+1 version (6)–(9), it is manifestly clear that one can evolve the dynamical degrees of freedom  $\gamma_{ij}(t, x^k)$  from any consistent set of initial data  $\{\gamma_{ij}(0, x^k), K_{ij}(0, x^k)\}$  using the evolution equations (6) and (7). The remaining ones (8), (9) can be interpreted as constraints. This diversity in the evolution properties of both sets of equations is not obvious in the 4D version. To summarize: both the 4D equations (1) and the 3+1 equations provide equivalent general-covariant formalisms, although general covariance is apparent only in the 4D version, whereas the evolution properties are manifest only in the 3+1 version (6)–(9).

### B. Extending solution space: taking constraints out

The 3+1 formalism (6)–(9) is especially suited for numerical relativity applications. In this context, one usually takes advantage of the fact that the energy and momentum constraints (8), (9) are first integrals of Eqs. (6), (7). This allows us to enforce the constraints (8), (9) on the initial and boundary data only, or even to use them to monitor the accuracy of the time evolution. But the constraints are not enforced by the time evolution algorithm for interior points, which in its simplest form is based only on Eqs. (6), (7) (free evolution approach [14,15]). This “unconstrained” evolution formalism, although perfectly consistent [16,17], does introduce a strong discrimination between the two sets of equations (6), (7) and (8), (9) that breaks the general covariance of the 3+1 formalism.

To verify this, let us notice first that by replacing the full 3+1 formalism (6)–(9) by the subset (6), (7), we are actually extending the solution space so as to include constraint-violating pairs  $\{\gamma_{ij}, K_{ij}\}$ . Now, as the restricted set of equations (6), (7) corresponds to the space components of Eq. (1), the extended space of solutions will be invariant only under the restricted subset of coordinate transformations (2) that

preserve the time slicing, namely,

$$t' = h(t), \quad (10)$$

$$y^i = f^i(x^j, t).$$

This confirms that general covariance is broken in the unconstrained evolution formalisms.

Let us remark that the extension of the solution space of Eqs. (6)–(9) is a rule, not an exception, among the new formalisms arising after the seminal 1983 work of Choquet-Bruhat and Ruggeri [18], which opened the door to the use of arbitrary shift choices in hyperbolic evolution systems. The bottom line is that the constraints (8), (9) contained in the original 3+1 evolution formalism are at odds with hyperbolicity to the intent that some extension is needed in order to modify the mathematical structure of the formalism without losing the physical solutions. The easiest way of doing this is just taking the constraints (8), (9) out of the system. This is the basic ingredient, although this crucial point can be masked by other manipulations on the evolution equations, such as taking an extra time derivative [18,19], or an extra space derivative [20], or using the constraints to modify the evolution equations (6), (7) [21–26]. The resulting formalisms, when supplemented with suitable coordinate conditions, provide hyperbolic evolution systems that can be used in numerical relativity applications. From our point of view, these formalisms can also be interpreted as providing many nonequivalent ways of extending the solution space of Eqs. (6)–(9) with at least two related common features: constraint equations (8), (9) are left out of the final evolution formalism and general covariance is broken as a result, even before a specific coordinate system is selected.

### C. Extending solution space: extra dynamical fields

A completely different way of extending the solution space is to introduce extra dynamical fields, independent of the metric and its derivatives, into the evolution formalism. This alternative has been independently used by many numerical relativity groups in different ways [27–30]. The key idea in these works was to introduce three supplementary dynamical fields whose evolution equations were obtained by using the momentum constraint (9). As far as these works were focused on numerical relativity applications [31–33], the supplementary quantities were introduced in an *ad hoc* way, breaking even the 3+1 covariance (10) of the formalism. Only very recently [34] the same idea has been implemented in a way which is at least invariant under the restricted subset of coordinate transformations (10): the extra quantities are given by a three-dimensional “zero” vector  $Z_i$  which vanishes for Einstein’s solutions. During numerical evolution, however, nonzero values of  $Z_i$  arise due to truncation errors and the resulting numerical codes actually deal with an extended set of solutions.

Even with this improvement, general covariance is still broken for two different reasons. First of all, although the momentum constraint (9) has been incorporated into the formalism as the right-hand side of the time evolution equation for  $Z_i$ , the energy constraint (8) is still taken out of the time

evolution algorithm. It is obvious that the extension of solution space to “energy constraint violating” modes cannot be invariant under the general coordinate transformations (2). This reason alone could be easily overcome by proceeding along the lines sketched in [36], where every constraint is incorporated into the system by adding an extra “Lagrangian multiplier” quantity: every extra quantity could then be coupled with the previous ones in many different ways so that many more new arbitrary parameters will appear in the resulting evolution formalism. But this would just reinforce the second cause of general-covariance breaking: the lack of covariance of the final set of extra quantities. If one wants to extend solution space by adding extra fields without breaking general covariance, then this set of extra fields should be equivalent to some set of well defined space-time quantities, independent of the time slicing considered.

## II. GENERAL COVARIANT EXTENDED EVOLUTION FORMALISMS

### A. The extended field equations

We propose to extend the field equations (1) in a general covariant way by introducing an extra four-vector  $Z_\mu$ , so that the set of basic fields will consist of the pair  $\{g_{\mu\nu}, Z_\mu\}$ . The original field equations (1) will then be replaced by

$$R_{\mu\nu} + \nabla_\mu Z_\nu + \nabla_\nu Z_\mu = 8\pi \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right). \quad (11)$$

The solutions of the Einstein’s solutions can be easily recognized among the extended set as those satisfying condition [35]

$$Z_\mu = 0, \quad (12)$$

so that the four-vector  $Z_\mu$  will provide a simple way to monitor the quality of numerical simulations or any other kind of approximation scheme. Notice that Eqs. (11) are of mixed order: second order in the metric components  $g_{\mu\nu}$ , but only first order in the extra vector field  $Z_\mu$ . This means, in particular, that terms containing first derivatives of  $Z_\mu$  belong to the principal part and that they are then relevant to the causal structure of the resulting evolution systems, as we will see later.

In order to fully understand the evolution properties of the extended equations, let us translate the manifestly covariant form (11) into the 3+1 language (5). The covariant four-vector  $Z_\mu$  will then be decomposed into its space components  $Z_i$  and the normal component

$$\Theta \equiv n_\mu Z^\mu = \alpha Z^0, \quad (13)$$

where  $n_\mu$  is the unit normal to the  $t = \text{const}$  slices. The 4D equations (11) can then be written in the equivalent form

$$\begin{aligned} (\partial_t - \mathcal{L}_\beta) \gamma_{ij} &= -2\alpha K_{ij}, \quad (14) \\ (\partial_t - \mathcal{L}_\beta) K_{ij} &= -\nabla_i \alpha_j + \alpha [{}^{(3)}R_{ij} + \nabla_i Z_j \\ &\quad + \nabla_j Z_i - 2K_{ij}^2 + (\text{tr}K - 2\Theta)K_{ij}], \quad (15) \end{aligned}$$

$$\begin{aligned} (\partial_t - \mathcal{L}_\beta) \Theta &= \frac{\alpha}{2} [{}^{(3)}R + (\text{tr}K - 2\Theta)\text{tr}K \\ &\quad - \text{tr}(K^2) + 2\nabla_k Z^k - 2(\alpha_k / \alpha)Z^k], \quad (16) \end{aligned}$$

$$(\partial_t - \mathcal{L}_\beta) Z_i = \alpha [\nabla_k (K_i^k - \delta_i^k \text{tr}K) + \partial_i \Theta - (\alpha_i / \alpha)\Theta - 2K_i^k Z_k], \quad (17)$$

where  $\alpha_i$  stands for  $\partial_i \alpha$  and we have restricted ourselves to the vacuum case again.

The evolution properties of the formalism are transparent in the 3+1 version (14)–(17). Only evolution equations appear there, without constraints. One cannot omit any of the equations from (14)–(17) because all them are needed to evolve the extended set of ten dynamical fields  $\{\gamma_{ij}, \Theta, Z_i\}$ . This means that general covariance will not be broken in numerical relativity applications because all the Eqs. (14)–(17) must be used on equal footing in the main algorithm evolving the dynamical fields in time: there is no room left for equation discrimination.

### B. Recovering Einstein’s solutions

The algebraic condition (12) is useful to check *a posteriori* whether a given solution of the extended system (11) is actually a solution of the Einstein’s field equations (1). But it is interesting as well to know *a priori* the necessary and sufficient condition for a given set of initial data to generate a physical solution. In this sense, one can take the divergence of the extended equations (11) to get, allowing for the contracted Bianchi identities,

$$\square Z_\mu + R_{\mu\nu} Z^\nu = 0. \quad (18)$$

This homogeneous second order equation in  $Z_\mu$  ensures that any deviation from the original Einstein equations (1) propagates through light cones and also that a sufficient set of conditions for the initial data to provide physical solutions is given by

$$Z_\mu(0, x^i) = 0, \quad \partial_t Z_\mu(0, x^i) = 0, \quad (19)$$

where the second equation, allowing for Eqs. (16), (17), represents imposing energy and momentum constraints (8), (9) on the initial data.

This means that the algebraic constraint (12) by itself is not a first integral of the extended system (11). Equations (8), (9) appear here as auxiliary conditions so that the full set (8), (9), (12) is preserved by time evolution. From the practical point of view, this means that one can take any set of consistent initial data of Einstein’s equations (1) and use it with a zero initial value of  $Z_\mu$  to get an initial data set for the extended equations (11) that will generate precisely the same solution. The general-covariant gauge-independent equation (18) can be then interpreted as a useful tool to understand the propagation of the initial constraints (19).

### III. COORDINATE CONDITIONS AND SYMMETRIC HYPERBOLIC EVOLUTION SYSTEMS

As stated in the Introduction, the evolution system is not complete until one provides coordinate conditions to fix the four kinematical degrees of freedom. We will consider here two different coordinate conditions leading to a symmetric hyperbolic evolution system. We will start in both cases from one of the two equivalent versions of the extended general-covariant formalism (11).

#### A. A 4D evolution system in harmonic coordinates

Let us use the Donder [3,4] expression of the Ricci tensor to write down the principal part of Eq. (11), namely,

$$-\square g_{\mu\nu} + \partial_\mu(\Gamma_\nu + 2Z_\nu) + \partial_\nu(\Gamma_\mu + 2Z_\mu) = \dots \quad (20)$$

It is obvious from a comparison with the wave equation for  $g_{\mu\nu}$  that we can obtain a symmetric hyperbolic system

$$\square g_{\mu\nu} = \dots, \quad (21)$$

provided that we kill the additional terms in Eq. (20) using the following extension of the well known harmonic coordinate conditions:

$$\square x^\mu = -\Gamma^\mu = 2Z^\mu. \quad (22)$$

The four-vector  $Z_\mu$  can be interpreted in this context as a sort of ‘‘gauge source,’’ along the lines sketched in Ref. [11].

Notice that, in contrast with the classical approach, the compatibility between the reduced system (21) and the coordinate conditions (22) is not an issue in the present context because there are now 14 independent components of the fields  $\{g_{\mu\nu}, Z_\mu\}$  to be determined by the 14 Eqs. (21), (22). A straightforward analysis in the 3+1 framework shows that this is actually the case: the lapse and shift evolution is provided by Eq. (22) and the evolution of the remaining ten degrees of freedom, including the four-vector  $Z_\mu$ , is given by Eq. (21). Notice also that Eq. (22) actually coincides with the classical harmonic coordinate condition (4) for physical solutions, where  $Z_\mu$  vanishes. This is why we talk about ‘‘harmonic coordinates’’ to refer also to condition (22) in the present context.

#### B. A 3+1 evolution system with harmonic slicing

Harmonic coordinates are not flexible enough to be used in most numerical relativity applications. A more suitable choice is the ‘‘harmonic slicing,’’ in which the time coordinate is again assumed to be a harmonic function, but the space coordinates are chosen so that the mixed components  $g_{0i}$  vanish (normal coordinates). We propose here to keep in the extended case the time component of Eq. (22), that is,

$$g_{0i} = 0, \quad \square x^0 = -\Gamma^0 = 2Z^0, \quad (23)$$

which can be translated into the 3+1 language as

$$\beta^i = 0, \quad \partial_t \ln \alpha = -\alpha(\text{tr}K - 2\Theta). \quad (24)$$

The principal part of Eqs. (15)–(17) can be written as

$$\partial_t K_{ij} + \partial_k[\alpha \lambda^k_{ij}] = \dots, \quad (25)$$

$$\partial_t Z_i + \partial_k[\alpha(\delta^k_i(\text{tr}K - \Theta) - K^k_i)] = \dots, \quad (26)$$

$$\partial_t \Theta + \partial_k[\alpha(D^k - E^k - Z^k)] = \dots, \quad (27)$$

where we have noted

$$A_k \equiv \partial_k(\ln \alpha), \quad D_{kij} \equiv \frac{1}{2} \partial_k \gamma_{ij}, \quad (28)$$

$$\lambda^k_{ij} = D^k_{ij} + \frac{\delta^k_i}{2}(A_j + D_j - 2E_j - 2Z_j), \quad (29)$$

$$+ \frac{\delta^k_j}{2}(A_i + D_i - 2E_i - 2Z_i), \quad (30)$$

and  $D_k = \gamma^{ij} D_{kij}$ ,  $E_k = \gamma^{ij} D_{ijk}$ .

One can get a fully first order system in the usual way, by considering both  $A_k$  and  $D_{kij}$  as independent additional quantities. Their evolution equations can be then easily obtained by differentiating Eqs. (14) and (24): namely,

$$\partial_t A_k + \partial_k[\alpha(\text{tr}K - 2\Theta)] = 0, \quad (31)$$

$$\partial_t D_{kij} + \partial_k[\alpha K_{ij}] = 0. \quad (32)$$

The full set of basic independent quantities is then given by  $\{\alpha, \gamma_{ij}, K_{ij}, \Theta, Z_i, A_k, D_{kij}\}$  and the nontrivial principal part of the corresponding evolution systems is given by Eqs. (25)–(27), (31), (32).

The causal structure of the first order system (24)–(32) is simpler than expected: one can easily check from either Eqs. (20), (23) or (24)–(32) that

$$\partial_t(\Gamma_i + 2Z_i) = \partial_t(2E_i - D_i - A_i + 2Z_i) = \dots, \quad (33)$$

[the quantities  $\Gamma_i$  here are just the space components of the quantities  $\Gamma_\mu$  in Eq. (20)] so that one readily identifies three eigenfields propagating along the normal lines. These ‘‘standing modes’’ are the only deviation of our system with respect to the wave equation pattern: all the remaining nontrivial eigenfields propagate along light cones. If we select a specific space direction, along a given unit vector  $u_k$ , then these light cone eigenfields are

$$K_{ij} \pm u_k \lambda^k_{ij}, \quad (\text{tr}K - 2\Theta) \pm u_k A^k. \quad (34)$$

A straightforward calculation shows then that the first order system (24)–(27) is symmetric hyperbolic. The ‘‘symmetrizer’’ can be easily identified starting from quadratic positive-definite ‘‘energy’’ functions which are conserved up to lower order terms. One such ‘‘energy estimate’’ is provided by

$$E = K^{ij}K_{ij} + \lambda^{kij}\lambda_{kij} + (\text{tr}K - 2\Theta)^2 + A^k A_k + (\Gamma^k + 2Z^k)(\Gamma_k + 2Z_k). \quad (35)$$

We are currently working with finite difference algorithms that can take advantage of the symmetric hyperbolicity of the system (24)–(32) to increase the robustness of numerical simulations. Our preliminary results indicate that explicit “energy” expressions of the form (35) can be very useful to devise stable boundary conditions, along the lines sketched in Refs. [8,37].

## ACKNOWLEDGMENTS

The authors are indebted to A. Arbona, J. Massó, and M. Tiglio for useful discussions and comments. This work has been supported by the EU Program “Improving the Human Research Potential and the Socio-Economic Knowledge Base” (Research Training Network Contract HPRN-CT-2000-00137), by the Spanish Ministerio de Ciencia y Tecnología through the research grant No. BFM2001-0988 and by a grant from the Conselleria d’Innovació i Energia of the Govern de les Illes Balears.

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