Large scale structure from the Higgs fields of the supersymmetric standard model

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We propose an alternative implementation of the curvaton mechanism for generating the curvature perturbations which does not rely on a late decaying scalar decoupled from inflation dynamics. In our mechanism the supersymmetric Higgs scalars are coupled to the inflaton in a hybrid inflation model, and this allows the conversion of the isocurvature perturbations of the Higgs fields to the observed curvature perturbations responsible for large scale structure to take place during reheating. We discuss an explicit model which realizes this mechanism in which the μ term in the Higgs superpotential is generated after inflation by the vacuum expectation value of a singlet field. The main prediction of the model is that the spectral index should deviate significantly from unity, $|n-1|\sim 0.1$. We also expect relic isocurvature perturbations in neutralinos and baryons, but no significant departures from Gaussianity and no observable effects of gravity waves in the CMB spectrum.

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I. INTRODUCTION

According to the inflationary paradigm [1], the presently observed large scale universe originated from a very small patch of space which underwent a period of quasiexponentially accelerated expansion known as inflation. In such an inflationary approach, the very largest scales, which are now entering the horizon, would have been in causal contact at very early times. According to inflation this causal contact would have ceased some 50–60 *e*-folds before the end of inflation, corresponding to so-called horizon exit to distinguish it from the horizon entry as observed in the present epoch. Inflation clearly provides an attractive explanation for why the cosmic microwave background (CMB) radiation should have the same temperature from points in the sky which would have been out of causal contact at the time of last scattering (the horizon problem). It also accounts for the observed flatness of the universe $\Omega=1$ (the flatness problem), consistent with the CMB data [2–7].

Another commonly stated success of inflation [1] is the fact that the observed primordial density perturbations, which were first observed by Cosmic Background Explorer (COBE) [8] on cosmological scales just entering the horizon, and which are supposed to be the seeds of large scale structure, could have originated from the quantum fluctuations of the inflaton field, the scalar field which is supposed to be responsible for driving inflation. In this scenario the quantum fluctuations of the inflaton field during the period of inflation become classical perturbations at horizon exit, giving a primordial curvature perturbation which remains constant until the approach of horizon entry [9]. The advantage of this scenario is that the prediction for the nearly scale-invariant spectrum depends only on the form of the inflaton potential, and is independent of what goes on between the end of inflation and horizon entry. The disadvantage is that it provides a strong restriction on models of inflation. The price of such simplicity, with one field being responsible for both inflation and the primordial curvature perturbation often translates into a severe restriction on the parameters of the inflaton potential. This often requires very small values for the couplings and/or the masses which apparently renders many such theories unnatural.

Recently it has been pointed out that in general it is unnecessary for the inflaton field to be responsible for generating the curvature perturbation [10,11]. It is possible that the inflaton only generates a very small curvature perturbation during the period of inflation, which instead may result from the isocurvature perturbations of a curvaton field which subsequently become converted into curvature perturbations in the period after inflation, but before horizon entry [10–13]. Isocurvature perturbations simply mean perturbations which do not perturb the total curvature, usually because the curvaton field contributes a very small energy density ρ_σ during inflation. In the scenarios presented so far [10,11,14–17], the curvaton is assumed to be completely decoupled from inflationary dynamics, and is assumed to be some late-decaying scalar which decays before the time that neutrinos become decoupled.

The reason why the curvaton is assumed to be late-decaying can be understood from the following argument [16]. After reheating the total curvature perturbation can be written as

$$\mathcal{R}=(1-f)\mathcal{R}_r+f\mathcal{R}_\sigma \tag{1}$$

where

$$f=\frac{3\rho_\sigma}{4\rho_r+3\rho_\sigma} \tag{2}$$

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and (on unperturbed hypersurfaces on super-horizon sized scales)

$$\mathcal{R}_i \approx -H\left(\frac{\delta\rho_i}{\dot{\rho}_i}\right) \sim \left(\frac{\delta\rho_i}{\rho_i}\right) \tag{3}$$

where the curvaton density ρ_σ and radiation density ρ_r, arising from the decay of the inflaton, each satisfy their own energy conservation equations and each $\mathcal{R}_i$ remains constant on super-horizon scales. The time evolution of $\mathcal{R}$ on these scales is then given by its time derivative,

$$\dot{\mathcal{R}} \approx -Hf(1-f)\frac{S_{\sigma r}}{3} \tag{4}$$

where $S_{\sigma r}$ is called the entropy perturbation defined by

$$S_{\sigma r} \simeq -3(\mathcal{R}_\sigma - \mathcal{R}_r). \tag{5}$$

The curvaton generates an isocurvature perturbation because initially $\rho_r \gg \rho_\sigma$, and hence $f \ll 1$, so that from Eq. (1) the curvature perturbation is dominated by $\mathcal{R}_r$. However as the universe expands and the scale factor a increases while the Hubble constant H decreases, the curvaton with mass m, whose oscillations have effectively been frozen in by the large Hubble constant, begins to oscillate and act as matter. After this happens ρ_r decreases as a^{-4} while the energy density in the curvaton field ρ_σ has a slower fall-off as a^{-3}. Eventually the curvaton energy density ρ_σ becomes comparable to the radiation density from the inflaton decay ρ_r, and when this happens from Eq. (2) we see that $f \sim 1$ and from Eqs. (4), (5) this leads to the growth of the total curvature perturbation $\mathcal{R}$ from the isocurvature perturbation $\mathcal{R}_\sigma > \mathcal{R}_r$. This mechanism, which allows the curvaton isocurvature perturbations to become converted into the total curvature perturbation, requires the curvaton scalar to be late-decaying.

The main motivation behind the present paper is to propose an alternative curvaton mechanism that removes the necessity for having a late-decaying scalar. To this end we make the following two observations:

(1) The first observation is that hybrid inflation [18,19] involves more than one scalar field and ends by a phase transition in which the fields involved move from the false, vacuum energy dominated potential minimum, towards the global minimum, and start oscillating around it. In the process, the vacuum energy gets redistributed among the fields such that their energy densities are comparable. Thus any isocurvature perturbation in one of the hybrid inflation fields may be converted into curvature perturbations during the onset of reheating. Note that the conversion does not take place until one of the fields decays. Given that the inflaton field is a singlet field, we can choose the curvaton field to be a flat direction during inflation made of a pair of charged fields, where the gauge interactions imply that the curvaton decays first.

(2) The second observation is that the supersymmetric standard model provides a natural candidate for such a pair of charged scalar fields: the two Higgs doublets. In order to allow a flat direction during inflation, we also require that the μ term which usually couples the two Higgs doublets in the minimal supersymmetric standard model must be set to zero. However, providing this obstacle can be overcome, it is natural to explore the attractive possibility that such Higgs fields are the seed of large-scale structure in the Universe. The idea would be that the inflaton gives a very small curvature perturbation, with the Higgs field giving an isocurvature perturbation. The coupling of the Higgs scalars to the inflaton then allows the energy densities of the inflaton and the Higgs field to become of similar magnitude at the onset of reheating, allowing the conversion of isocurvature density perturbations to curvature perturbations.

In the remainder of the paper we shall discuss a supersymmetric hybrid inflation model based on these observations. In this model the superpotential includes, in addition to the two Higgs doublet superfields H_u and H_d of the supersymmetric standard model, also two gauge singlet superfields, one of them ϕ playing the role of the inflaton and the other N coupling to the two Higgs doublets, effectively generating an effective μ term from the coupling NH_uH_d as in the next-to-minimal supersymmetric standard model (NMSSM) [20]. This happens after inflation when it gets a vacuum expectation value (VEV) $N_0 = \langle N \rangle$. The N superfield also couples to the inflaton ϕ as ϕN^2, and therefore acts as a messenger allowing the Higgs fields to couple to the inflaton. In previous discussions of this model [20] we assumed that during inflation the Higgs and N fields are held at the origin, but here we shall show that an alternative inflationary trajectory is possible in which the inflaton ϕ as well as the Higgs doublets are slowly rolling during the inflationary epoch. The isocurvature perturbations of the Higgs fields during inflation are converted into curvature perturbations during the initial stages of the reheating process, when all these fields begin to oscillate with comparable energy densities.

The layout of the rest of the paper is as follows. The model is presented in Sec. II, where the evolution for the background fields is studied. In Sec. III we discuss the nature and evolution of the perturbations during the epoch of inflation, and show that it is natural for the inflaton to give the dominant curvature perturbation, but still too small to account for the COBE value, while the Higgs fields strongly contribute to the isocurvature perturbations. In Sec. IV we describe the evolution of the perturbations during reheating, after the Higgs field has decayed and the Universe is made of a mixture of matter (the oscillating singlet fields) and radiation (Higgs decay products), and show that the isocurvature perturbations are converted into curvature perturbations. In Sec. V we comment on some of the subtleties involved in the transition from inflation to reheating, and the issue of preheating. The predictions of the approach are discussed in Sec. VI, and in Sec. VII we provide a summary.

II. THE MODEL: A NEW INFLATIONARY TRAJECTORY

In this section we revisit the supersymmetric hybrid inflation model based on the superpotential [20]:

$$W = \lambda N H_u H_d - \kappa \phi N^2, \tag{6}$$

where N and ϕ are singlet superfields, and $H_{u,d}$ are the Higgs superfields, and λ,κ are dimensionless couplings. Other cubic terms in the superpotential are forbidding by imposing a global $U(1)_{PQ}$ Peccei-Quinn symmetry. The superpotential in Eq. (6) includes a linear superpotential for the inflaton field, ϕ, typical of hybrid inflation, as well as the singlet N coupling to Higgs doublets as in the NMSSM. In the original version of this model [20] we assumed that during inflation N and H_u, H_d were set to zero. Here we discuss an alternative inflationary trajectory in which these fields may take small values away from the origin, consistent with slow roll inflation.

In order to satisfy the D-flatness condition,[1] we assume the values of the Higgs doublets during inflation to be equal, $H_u = H_d = h$. Inflation takes place below the SUSY breaking scale. Therefore, including the soft SUSY breaking masses, m_ϕ, m_N and m_h, and trilinears A_κ, A_λ, the potential for the real part of the fields is

$$V = V(0) + \frac{\kappa^2}{4}N^4 + \kappa^2(\phi-\phi_c^+)(\phi-\phi_c^-)N^2 + \frac{1}{2}m_\phi^2\phi^2 + \lambda N h^2\left(\frac{A_\lambda}{\sqrt{2}} - \kappa\phi\right) + \frac{\lambda^2}{4}h^4 + \frac{\lambda^2}{2}N^2h^2 + \frac{1}{2}m_h^2 h^2. \tag{7}$$

In our previous work we set the Higgs terms to zero, $h=0$. Allowing the Higgs terms in Eq. (7) we see that the term proportional to $\lambda N h^2$ can induce a non-zero value for h and N during inflation providing that the coefficient of this term is negative $A_\lambda/\sqrt{2} - \kappa\phi < 0$. However, providing that the values of the fields N, H_u and H_d are sufficiently small during inflation, as discussed later, then inflation is controlled by the inflaton ϕ, as in the original model [20], and will end when ϕ reaches one of its critical values $\phi_c^\pm$ given by

$$\kappa\phi_c^\pm = \frac{A_\kappa}{2\sqrt{2}}\left(1 \pm \sqrt{1 - \frac{4m_N^2}{A_\kappa^2}}\right). \tag{8}$$

These critical values correspond to the field dependent mass squared $\kappa^2(\phi-\phi_c^+)(\phi-\phi_c^-)N^2$ changing sign and becoming negative, signaling the end of inflation due to the destabilization of the inflationary trajectory, after which the fields N, ϕ, H_u and H_d then approach their global minimum and acquire their physical vacuum expectation values (VEVs) N_0, ϕ_0, v_u and v_d. For order of magnitude estimations, here on we will take $\phi_c \simeq \phi_c^+ \simeq \phi_c^-$.

The parameters of the potential in Eq. (7) are selected by the following physical requirements. As discussed in [20] the global Peccei-Quinn (PQ) symmetry imposed on the superpotential Eq. (6) solves the strong *CP* problem. When the PQ symmetry is broken by the VEVs of the singlets, it leads to a very light axion in the usual way [22]. The axion scale f_a is set by the VEVs of the singlets, and it is constrained by astrophysical and cosmological observations to be roughly in the window 10^{10} GeV $\leqslant f_a \leqslant 10^{13}$ GeV [23,24]. Given that $f_a \sim \phi_c \sim \phi_0 \sim N_0 \sim 10^{13}$ GeV and the soft breaking term A_k are expected to be of the order of 1 TeV, Eq. (8), this leads to a coupling constant of the order $\kappa \sim 10^{-10}$. The same applies to $\lambda \simeq \kappa$, with $\mu = \lambda N_0 \sim 1$ TeV. The smallness of λ and κ will be explained in a companion paper [25]. Therefore, demanding a zero cosmological constant at the global minimum requires the height of the potential during inflation to be of the order of

$$V(0)^{1/4} \simeq \frac{\sqrt{\kappa}}{2}\phi_c \simeq (10^8 \text{ GeV}), \tag{9}$$

with a Hubble parameter of the order of

$$H = \frac{V(0)^{1/2}}{\sqrt{3}m_P} \simeq 10 \text{ MeV}, \tag{10}$$

where $m_P = M_P/\sqrt{8\pi} = 2.4\times10^{18}$ GeV is the reduced Planck mass.

An important condition for inflation is that the inflaton mass m_ϕ (and also m_h) needs to be small enough in order to ensure the slow roll of the inflaton, as determined by the slow roll parameters defined in the standard way as [1]

$$\epsilon \equiv \frac{1}{2}m_P^2\left(\frac{V'}{V}\right)^2 \ll 1 \tag{11}$$

$$|\eta| \equiv \left|\frac{m_P^2 V''}{V}\right| \ll 1 \tag{12}$$

where V' (V'') are the first (second) derivatives of the potential. In this case we find the slow roll parameters for the ϕ field:

$$\eta_\phi = \frac{m_\phi^2}{3H^2} < 1, \tag{13}$$

$$\epsilon_\phi = \frac{1}{2}\eta_\phi^2\frac{\phi^2}{m_P^2} < 1, \tag{14}$$

where it is understood that these expressions are evaluated at some number of *e*-folds before the end of inflation. Equation (14) is always satisfied once $\eta_\phi < 1$. This would require an inflaton mass of the order of some MeVs at most, say $m_\phi^2 \sim 0.1H^2$. A similar constraint will also apply to the Higgs soft boson masses. This is also compatible with the observational constraint on the spectral index, $n = 0.93 \pm 0.13$ [2], which in terms of the slow-rolling parameters [1], $|n-1| = 2\eta_\phi - 6\epsilon_\phi$, gives $\eta_\phi < 0.03$. The smallness of the inflaton and Higgs soft boson masses which are orders of magnitude smaller than the typical soft breaking values assumed for the trilinear couplings, is explained in a companion paper [25].

We have also included a constant vacuum energy contribution $V(0)$ in the potential in Eq. (7) whose origin we do

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[1] Higgs fields are charged under $SU(2)_L \times U(1)_Y$ interactions, such that there is what is called a D-term contribution in the potential of the form $(g_2^2 + g_1^2)(H_u^2 - H_d^2)^2/8$. A "D-flat" direction is made of a combination of the fields such that the D-term vanishes [21].

not specify. It can be explained once the model is embedded in a supergravity (SUGRA) model [26], where the question of SUSY breaking can be addressed. It should be pointed out however that SUGRA corrections generically give rise to scalar masses contributions of the order of the Hubble parameter during inflation. This is the so-called η problem [27]. Nevertheless, the problem can be avoided [19,26,28,29] by a suitable choice of the SUGRA model, i.e., the Kahler potential and/or the superpotential for the scalar inflaton. Our approach is that it is natural to have scalar masses of the order H^2 during inflation, but rather slightly smaller in order to satisfy Eq. (13). This would only require at most a mild tuning of the parameters in the SUGRA Kähler potential.

We now discuss an inflationary trajectory that will enable the Higgs fields to be non-zero and slowly rolling during the inflationary epoch, and hence to acquire an isocurvature perturbation. The requirement that the h,ϕ fields be slow-rolling during inflation, means that their effective field dependent masses must be smaller than H^2,

$$\frac{\partial^2 V}{\partial h^2} \simeq \lambda^2(3h^2+N^2)+2\lambda N\left(\frac{A_\lambda}{\sqrt{2}}-\kappa\phi\right)+m_h^2<H^2 \quad (15)$$

$$\frac{\partial^2 V}{\partial \phi^2} \simeq \kappa^2 N^2+m_\phi^2<H^2. \quad (16)$$

This requirement implies that both the fields N and h must take small values during inflation, with

$$\frac{N}{\phi_c}<10^{-10}, \quad \frac{h}{\phi_c}<10^{-5}, \quad (17)$$

and also, as already remarked, must have soft masses of order an MeV or less.

The N field-dependent mass is much larger and positive:

$$\frac{\partial^2 V}{\partial N^2} \simeq 2\kappa^2(\phi-\phi_c^+)(\phi-\phi_c^-)\sim O(\mathrm{TeV}^2), \quad (18)$$

where we dropped the small term $3\kappa^2N^2$ using Eq. (17). Thus the N field will oscillate with an amplitude damped by the expansion, following the evolution equation:

$$\ddot{N}+3H\dot{N}+\frac{\partial V}{\partial N}=0, \quad (19)$$

where

$$\frac{\partial V}{\partial N} \approx (2\kappa^2(\phi-\phi_c^+)(\phi-\phi_C^-)+\lambda^2h^2)N+\lambda h^2\left(\frac{A_\lambda}{\sqrt{2}}-\kappa\phi\right)$$

$$\approx \omega_N^2(\phi)N+\lambda h^2\left(\frac{A_\lambda}{\sqrt{2}}-\kappa\phi\right), \quad (20)$$

and we have again dropped the small term κ^2N^3 in the first line using Eq. (17). The frequency of the oscillations is then of the order $\omega_N(\phi)\simeq\kappa\phi_c$.

Apparently, as far as N is concerned, we have violated the slow roll conditions. Since we have seen that N does not disturb the slow roll of h and ϕ we should not be concerned about this. Nevertheless, we shall see that in some sense N can be regarded as slowly rolling, according to the following argument. The typical time scale during inflation is set by the rate of expansion H, but the oscillations of N are much faster than that. On the Hubble time scale as far as the motion of N is concerned all that we can see is the average effect of the oscillations. The motion of N is then given by the "quasi" constant term ("quasi" in the sense that is given by the other fields ϕ and h that are slow-rolling), plus the pure oscillatory term with an amplitude that decays with the expansion. So when compared to the evolution of the other fields, ϕ and h, in a few e-folds the oscillatory term in N will be averaged to zero, and if we require a local minimum $\partial V/\partial N=0$ then we have effectively, from Eq. (20),

$$N(t)\simeq-\frac{\lambda h^2}{\omega_N^2(\phi)}\left(\frac{A_\lambda}{\sqrt{2}}-\kappa\phi\right)\sim\frac{h^2(t)}{\phi_c^2}\phi(t), \quad (21)$$

which relates the inflationary trajectory of the average value of N to the slowly rolling h,ϕ fields. Therefore the oscillations of N are not exactly around zero but its center will move along with the inflaton and Higgs field. Therefore in an effective sense the $N(t)$ field will *also* slowly roll along the valley of minima controlled by the Higgs field and the inflaton. Effectively the *three* fields will follow a slow-roll trajectory,

$$\dot{\phi}(t)\simeq-\eta_\phi H\phi(t), \quad (22)$$

$$\dot{h}(t)\simeq-\eta_h H h(t), \quad (23)$$

$$\dot{N}(t)\simeq-(\eta_\phi+2\eta_h)HN(t), \quad (24)$$

with $N(t)\ll h(t)\ll\phi(t)$, as in Eq. (17) and $\eta_h=m_h^2/(3H^2)$, with $\eta_\phi=m_\phi^2/(3H^2)$ as in Eq. (14).

III. EVOLUTION OF THE FLUCTUATIONS DURING INFLATION

During inflation with several light (slow-rolling) scalar fields ϕ_α, the comoving curvature perturbation [30] can be written as [35]

$$\mathcal{R}=H\sum_\alpha\left(\frac{\dot{\phi}_\alpha}{\sum_\beta\dot{\phi}_\beta^2}\right)Q_\alpha, \quad (25)$$

in terms of the gauge-invariant scalar field amplitude perturbations, the Sasaki-Mukhanov variables Q_i [31],

$$Q_\alpha=\delta\phi_\alpha+\frac{\dot{\phi}_\alpha}{H}\psi. \quad (26)$$

And for the entropy perturbations [36] we have[2]

$$S_{\alpha\beta} \simeq -3H\left(\frac{Q_\alpha}{\dot{\phi}_\alpha} - \frac{Q_\beta}{\dot{\phi}_\beta}\right). \tag{27}$$

These expressions may be compared to the (more intuitive) expressions given earlier in Eqs. (1), (5), where $\mathcal{R}_\alpha = HQ_\alpha/\dot{\phi}$. More general expression can be found in Appendix B, Eqs. (B13) and (B9). The initial curvature perturbation, Eq. (25), will be dominated by the field with the largest velocity, while the field with the smallest velocity dominates the entropy perturbations in Eq. (27) [12,32–35].

At first sight one might think that the N field, with the smallest velocity $\dot{N}$, according to Eq. (24), would give the largest isocurvature perturbation. However, since its evolution is controlled by the Higgs field, the ratio $Q_N/\dot{N}$ turns out to be of the same order of magnitude as $Q_h/\dot{h}$, as discussed in Appendix A. Thus the Higgs fields contribute strongly to the isocurvature perturbations, and in addition dominates the evolution of the N field during inflation according to Eq. (21).

During inflation the curvature perturbation is dominated by the field with the highest velocity, which according to Eq. (22) is the inflaton. For a given quantity A, the spectrum is defined by [1]

$$P_A(k) \equiv \frac{k^3}{2\pi^2}\langle |A|^2 \rangle, \tag{28}$$

where k is the comoving wave number. Using Eq. (25) we find

$$P_{\mathcal{R}}^{1/2} \simeq \frac{3H_*^2}{m_\phi^2 \phi_*}\frac{H_*}{2\pi} \simeq \frac{H_*}{2\pi \eta_\phi \phi_*}, \tag{29}$$

where the subscript "$*$" denotes the time of horizon exit, $H_* a = k$. This is by far smaller than the required COBE value $P_{\mathcal{R}}^{1/2} \simeq 5\times 10^{-5}$ [8], unless we take $\kappa m_\phi \sim 10^{-18}$ GeV, i.e. an inflaton mass of the order of a few eV. Such a tiny value[3] is far smaller than the MeV value we need to satisfy the slow roll conditions, as discussed previously. Therefore it is natural to suppose that the curvature perturbations are initially too small to satisfy the COBE condition, but are generated by the conversion of the isocurvature perturbations from the Higgs field, during reheating, as discussed in the next section.

After horizon exit, the entropy and curvature perturbations evolve independently until the end of inflation. Following Ref. [12], by the time inflation ends the amplitude of the fluctuations can be given in terms of their values at Hubble crossing ($aH_* = k$),

$$\mathcal{R}|_i \sim \frac{H_*}{\dot{\phi}_*}Q_{\phi_*} \simeq -\frac{Q_{\phi_*}}{\eta_\phi \phi_*}, \tag{30}$$

$$S_{\phi h}|_i \sim \frac{8}{3}\frac{H_*}{\dot{h}_*}Q_{h_*} \simeq -\frac{8}{3}\frac{Q_{h_*}}{\eta_h h_*}, \tag{31}$$

with

$$Q_{i*} = \frac{H_*}{\sqrt{2k^3}}. \tag{32}$$

Clearly, given that $h \ll \phi$, we have $\mathcal{R} \ll S_{\phi h}|_i$, so that the isocurvature perturbation from the Higgs field is nice and large, so that when it gets converted into the curvature perturbation during reheating it can account for the COBE observation, as we now discuss.

IV. EVOLUTION OF THE FLUCTUATIONS DURING REHEATING

The slow-rolling regime ends when the inflaton field reaches the critical value ϕ_c. At this point, the effective mass of the N field changes sign, and it starts moving towards the global minimum of the potential at $N_0 \sim O(\phi_c)$. The effective masses of ϕ and h also start increasing, and they also move quickly towards their global minimum values. Around the global minimum, the effective masses $\overline{m}_i^2$ for all the fields are of the same order of magnitude, $\overline{m}_\phi \simeq \overline{m}_N \simeq \overline{m}_h \sim \kappa^2 \phi_c^2 \sim O(1\ \text{TeV})$. Therefore, they all start oscillating with similar frequencies and *amplitudes* of the order of ϕ_c. In this way, the vacuum energy dominating during inflation is equally redistributed among the three fields, $\rho_\phi \sim \rho_N \sim \rho_h$, with the energy density of the oscillating fields behaving as matter.

This will be the situation until the fields decay and transfer their energy density to radiation. Because of the smallness of the singlet couplings, they are very long-lived, with a decay rate of the order $\Gamma_{N,\phi} \simeq \kappa^2 \overline{m}_{N,\phi} \sim O(10^{-17}\ \text{GeV})$, where $\kappa \sim \lambda \sim 10^{-10}$ whose smallness is explained in [25]. Reheating lasts until the singlets completely decay, roughly around the time $H \simeq \Gamma_\phi$. On the other hand, the Higgs field decays much faster through its gauge interactions, with $\Gamma_h \simeq \alpha_W \overline{m}_h \sim O(10\ \text{GeV})$. Because of this, the decay of the Higgs fields can be considered as practically instantaneous when compared to that of the singlets. However, given that $\Gamma_h/\overline{m}_h \sim \alpha_W \sim 0.01$, they will still have time to oscillate several times before decaying. Notice also that $\Gamma_h \gg H$ at the beginning of the oscillating phase. From that point of view, the decay of the Higgs field can also be considered "instantaneous": the Higgs decay before the energy densities in the oscillating fields have been redshifted, so that through the decay a fraction of the vacuum energy dominating during inflation is transferred into radiation. Given that during the

[2]This expression for $S_{\alpha\beta}$ in terms of the scalar field perturbations holds as far as we can neglect the coupling between the scalar, and the ratio $Q_\alpha/\dot{\phi}_\alpha \simeq$const, both of which are a good approximation during inflation.

[3]The tiny mass could be due purely to radiative correction, $\delta m_\phi^2 \sim \kappa^2(\kappa\phi_c)^2$, if the soft-breaking mass is set to zero [26].

decay we can neglect the effect of the expansion, we will also assume that the fluctuations in the Higgs field at the end of inflation are converted into those of the radiation fluid, with $S_{\phi r}=S_{\phi h}|_i$ as the initial condition during reheating.

Therefore, during the reheating era, we are left with a two component fluid, made of a mixture of matter ρ_ϕ (ϕ and N oscillating) and radiation ρ_r (Higgs decay products), but such that initially $\rho_\phi(0)\simeq\rho_r(0)$. The evolution equations for the system can be written as

$$\dot{\rho}_\phi+3H\rho_\phi+\Gamma_\phi\rho_\phi=0, \tag{33}$$

$$\dot{\rho}_r+4H\rho_r-\Gamma_\phi\rho_\phi=0. \tag{34}$$

It will take a while for the initial large fraction of radiation to feel the effect of the inflaton decay products, and at the beginning it is redshifted as usual like $\rho_r\propto a^{-4}$, with $\rho_\phi\propto a^{-3}$ and a being the scale factor. It is only later when the contribution of the decay products become comparable to the already present radiation that the system is fully coupled, and we have $\rho_r\propto a^{-3/2}$. The decay is completed by the time $H\simeq\Gamma_\phi$, and we are left again with radiation which redshifts in the usual way $\rho_r\propto a^{-4}$.

For comoving scales k well outside the Hubble radius, $k\ll aH$, the evolution equations for the fluctuations during reheating can be approximated by[4]

$$\frac{d\mathcal{R}}{d\ln a}\simeq\frac{8\rho_\phi\rho_r}{(3\rho_\phi+4\rho_r)^2}S_{\phi r}, \tag{35}$$

$$\frac{dS_{\phi r}}{d\ln a}\simeq-\frac{12\rho_r}{3\rho_\phi+4\rho_r}S_{\phi r}-\frac{\Gamma_\phi}{H}\frac{\rho_\phi}{\rho_r}\left(\frac{3\rho_\phi+\rho_r}{4\rho_r+3\rho_\phi}\right)S_{\phi r}. \tag{36}$$

Thus, at the beginning of reheating when $\rho_r\simeq\rho_\phi$, the large isocurvature perturbations coming from the Higgs fields in Eq. (31) which dominates the entropy perturbations, acts as a strong source for the curvature perturbations $\mathcal{R}$. Thus the curvature perturbations $\mathcal{R}$ quickly grow to become of the same order of magnitude as the initial entropy perturbation, and afterwards remains practically constant.

The transfer from entropy perturbations to curvature perturbations is illustrated in Fig. 1, based on a numerical integration of the exact evolution equations for the perturbations given in Appendix C, Eqs. (C1)–(C3), together with Eqs. (33), (34), with the boundary conditions in Eqs. (30) and (31). From the numerical integration, we find, asymptotically,

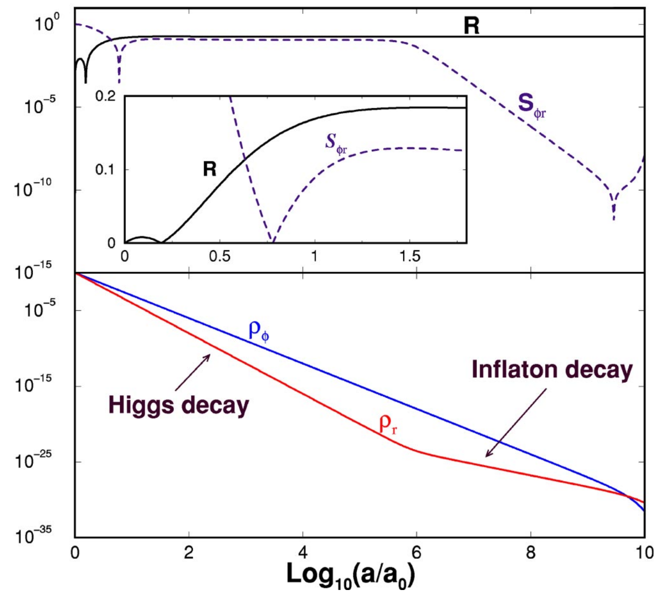

FIG. 1. In the upper panel we show the evolution of the curvature $\mathcal{R}$ (solid line) and entropy $S_{\phi r}$ (dashed line) perturbations during reheating. Both values are normalized to the initial value of $S_{\phi r}$. We have taken $\Gamma_\phi=10^{-17}$ GeV^{-1}, $V(0)^{1/4}=10^8$ GeV, $\phi/h=10^{10}$ and $\eta_\phi=\eta_h$. In the inset in the upper panel we show a blow-up of the initial region on a linear vertical scale. In the lower panel we show the evolution of the energy densities ρ_ϕ and ρ_r, normalized to their initial values.

$$\mathcal{R}\simeq\frac{1}{5}S_{\phi h}|_i. \tag{37}$$

Hence, using Eqs. (31), (A2), we end with the spectrum of adiabatic perturbations

$$P_{\mathcal{R}}^{1/2}\sim\frac{1}{5}P_{S_{\phi r}|_i}^{1/2}\sim\frac{8}{15\eta_h h_*}P_{Q_{h_*}}^{1/2}\sim\frac{8}{15\eta_h}\frac{H_*}{2\pi h_*}, \tag{38}$$

which for $\eta_h\sim0.03$, $H_*\simeq10^{-2}$ GeV and $2\pi h_*\simeq1$ TeV gives the right order of magnitude as measured by COBE.

The entropy perturbation also remains practically constant until the terms due to the decay of the singlets N and ϕ become relevant, and the radiation starts behaving as $\rho_r\propto a^{-3/2}$; the same as the ratio ρ_ϕ/H. From Eq. (36), the evolution for $S_{\phi r}$ is now approximately given by

$$\frac{1}{S_{\phi r}}\frac{dS_{\phi r}}{d\ln a}\simeq-4\frac{\rho_r}{\rho_\phi}-\frac{\Gamma_\phi}{H}\frac{\rho_\phi}{\rho_r}. \tag{39}$$

The main contribution in the above equation comes from the second term, which remains constant in this regime with $\Gamma_\phi\rho_\phi/(H\rho_r)\simeq2.5$, and then $S_{\phi r}\propto a^{-2.5}$. Therefore, when the system of matter and radiation becomes effectively coupled, the entropy perturbation actually decreases due to the relative increase of radiation coming from matter, and this effect is also seen in Fig. 1. Obviously, in this simplest

[4]The exact equations for a two-component fluid obtained from Ref. [36] are given in Appendix B for completeness. They also include the equations for the relative velocity perturbations $V_{\alpha\beta}$. In Appendix C we give the equations and approximate solutions for the scenario we are considering. In this case, in order to understand the qualitative behavior of the numerical solution is enough to consider the evolution equations for $\mathcal{R}$ and $S_{\alpha\beta}$, without including $V_{\alpha\beta}$, Eqs. (C4) and (C5) in Appendix C.

model of reheating once the inflaton completely decays and we are left only with radiation, the relative entropy perturbation vanishes in any case. We have just integrated the equations only until the time $\Gamma_\phi \simeq H$, without continuing the evolution afterwards when all the energy density in matter becomes radiation. However, it is clear that the value of the curvature perturbation, which becomes practically constant near the beginning of reheating, will not subsequently be affected by the details near the end of reheating and subsequent evolution.

V. DISCUSSION

It is worth pointing out a few subtleties regarding details of the transition from inflation to reheating. A detailed description of such transition may be relevant for a more accurate estimation of the final value of $\mathcal{R}$. Here we raise some of the concerns related to this point, and point out to what extent our conclusions can be affected. During inflation the background evolution equations for the fields are approximately given by the slow-roll equations, Eqs. (22)–(24); whereas reheating starts with the fields already oscillating and their energy densities averaged like matter until they decay into radiation. During the transition from one to another, the kinetic energies of the fields start increasing and they cannot be neglected any longer. Indeed it is going to be the N field which first starts moving away from the false vacuum once ϕ reaches the critical value, with its field-dependent squared mass becoming negative, and its VEV and velocity growing accordingly. Its quantum fluctuations Q_N will also feel the same instability, and increase until the effective mass become positive again and the oscillations begin. Given Eq. (25), this may mean that during the transition period it is the N field, with both its kinetic energy and the amplitude of its fluctuations increasing relatively to the others, which will end dominating the curvature perturbation. Still, our conclusion that the Higgs field is responsible for the final value of the adiabatic perturbations holds, although indirectly. It is through the coupling of the Higgs fields to the N field that the quantum fluctuations of the latter are not suppressed during inflation, as it should correspond to a massive field (see Appendix A). We also note that $Q_N/\dot{N} \simeq Q_h/\dot{h}$, so that even if the N kinetic energy dominates before the oscillations begin, we will end in any case with $\mathcal{R} \sim HQ_N/\dot{N} \sim HQ_h/\dot{h}$, and our order of magnitude estimation holds. However, this assumes that the ratio $Q_N/\dot{N}$ remains constant once the field N start moving toward the global minimum. This does not seem unreasonable given that both the background field and its quantum fluctuations will feel the same instability in the effective mass. However, it may happen that as the fields N and h move toward the global minimum with increasing speed $\dot{N}$ and $\dot{h}$, their quantum fluctuations still behave as those of a light field without changing appreciably. This will tend to decrease the final ratios $Q_N/\dot{N}$ and $Q_h/\dot{h}$, and therefore the value of $\mathcal{R}$.

On the other hand, we have started evolving the perturbations during reheating after the Higgs field decay into radiation. As mentioned in the previous section, before the Higgs decay the fields still have time to oscillate several times, such that the energy density is redistributed among the three oscillating scalar fields. During the initial stage of the oscillations, as far as the three species behave *all* on average like matter, the curvature perturbation $\mathcal{R}$ remains practically constant on super-Hubble scales and it is not affected by entropy perturbations [see Eqs. (B11) and (B23) in Appendix B]. In other words, the change in the curvature perturbation is given by the large-scale non-adiabatic (entropic) pressure perturbation, which is negligible as far as the scalar fields behave with similar effective equation of state. However, this argument does not take into account the possible parametric amplification of the scalar quantum fluctuations in a background of oscillating fields, a process that in the context of inflation generically is known as "preheating" [37,38]. Through parametric resonance, induced by the time dependent effective mass term in the evolution equations, field mode amplitudes can grow exponentially with time within certain resonance bands in k space. Thus, preheating becomes a more effective mechanism of transferring energy from the oscillating background fields to quantum fluctuations than the standard perturbative decay of the fields. The question then is whether it also provides a different and efficient source for non-adiabaticity, with super-Hubble ($k \ll aH$) fluctuations exponentially amplified during preheating [39,40]. This would translate into an exponential increase in the curvature perturbation which would be somehow difficult to keep at the level of the COBE observational value. As usual, the answer is model dependent. For example, it has been pointed out that for a massive (mass larger than the Hubble parameter) field during inflation the mechanism is inefficient, because previous to preheating fluctuations are first exponentially damped during inflation [41]. However, this restriction does not apply to light fields during inflation [42], as in the model presented in this paper. Moreover, in hybrid inflation the effect can be stronger due to the presence of negative effective squared masses at the end of inflation [43,44], which are *per se* a source of instabilities in the evolution equations for the fluctuations. To which extend the combined effect on super-Hubble scales is relevant is then a question of *when* the resonance ends [45]. Sooner or later, this occurs due to the backreaction effect of the small-scale (sub-Hubble) modes which also grow resonantly during preheating; in some models it may happen before the curvature perturbation exceeds the COBE value, depending on the value of the inflaton coupling constant. Again, the issue is highly model dependent. Preheating in the present model has been studied previously in Ref. [43], but without neither taking into account the Higgs fluctuations nor metric perturbations. It was shown that the combination of parametric resonance plus tachyonic instability gives rise to the exponential grow of the inflaton and N field fluctuations, and the resonance was stronger on the small-scale perturbations around $k \sim 0.3\overline{m}_\phi$. At the same time and because of that, backreaction effects become important very quickly after only three oscillations of the fields. Given that the tachyonic instability will be still present mainly in the effective N mass, we do not expect this conclusion to change once the Higgs fluctuations are also taken into account. Even they may speed up the end of the resonance, keeping under control the growth of the curvature (and entropy) perturbations. Moreover, an increase due to parametric resonance might compensate for a potential decrease during the end of the slow-roll regime. Nevertheless, the final answer would require the numerical integration of the evolution equations, for both the background fields and quantum fluctuations, from inflation to reheating, which is beyond the scope of the present publication.

VI. SPECTRAL INDEX, GAUSSIANITY AND GRAVITY WAVES

In this section we discuss the main predictions of the mechanism discussed here as compared to both the standard curvaton mechanism, based on the late-decaying scalar, or on standard hybrid inflation.

In the original hybrid inflation model based on the NMSSM [20], where the inflaton was responsible for curvature perturbations, we predicted a flat spectrum of curvature perturbations with a spectral index indistinguishable from $n=1$. In the present NMSSM hybrid inflation model, in which the Higgs field has large isocurvature perturbations which get transferred to curvature perturbations during reheating, we would expect a spectral index controlled by the Higgs fields instead of the inflaton and given by[5]

$$n-1=2\eta_h-6\epsilon_\phi\simeq 2\eta_h\simeq 2m_h^2/(3H^2). \tag{40}$$

The prediction for the spectral index in Eq. (40) involves the soft mass parameter m_h which depends on the details of the supersymmetry breaking mechanism. For example in the extra-dimensional model introduced in the companion paper [25], Higgs soft boson masses are not expected to be much suppressed with respect to the Hubble rate of expansion [25]. Using $n=0.93\pm0.13$ [2] we have an upper bound on the spectral index $n<1.06$ which leads to a constraint on the soft Higgs boson mass of $\eta_h<0.03$. The inflaton mass on the other hand is only constrained by the slow-roll condition $\eta_\phi<1$, since the inflaton does not contribute significantly to curvature perturbations. We would therefore expect typical deviations such as $|n-1|\sim0.1$. If $n=1$ is measured very accurately then this would be evidence for the original inflaton generated curvature perturbations.

In the original single field inflaton model [20] we predicted a Gaussian CMB spectrum. In the present case we would expect small non-Gaussian effects, as discussed in Ref. [16], which contribute quadratically to the Higgs energy density,

$$\frac{\delta\rho_h}{\rho_h}\simeq 2\frac{\delta h}{h}+\frac{(\delta h)^2}{h^2}. \tag{41}$$

Given that the amplitude of the field perturbations at horizon crossing are of the order of the Hubble parameter, $\delta h\sim H_*$ ~10 MeV, but the typical value for the VEV of the Higgs field is $h\sim1$ TeV, the linear term clearly dominates in Eq. (41) given a Gaussian spectrum. The small non-Gaussian effects can be parametrized by [46]

$$|f_{NL}|\simeq\frac{5}{12}\left|\frac{\delta\rho_h/\rho_h}{\mathcal{R}}\right|\sim\frac{75}{8}|\eta_h|, \tag{42}$$

where we have used the approximations in Eqs. (31), (37) and (41). Within the approximations involved in the above equations, and using $\eta_h<0.03$ we then get $|f_{NL}|<0.3$, which is below the expected upper bound by the PLANCK satellite $|f_{NL}|<5$ [46]. Therefore we do not expect significant (observable) non-Gaussian effects in this model.

Concerning gravity waves, as is typical of hybrid inflation models with a low inflation scale, the previous model [20] predicted negligible effects in the CMB spectrum from gravity waves. In the present approach we similarly expect small effects of gravity waves in the CMB spectrum. Gravitational waves are generated with a spectrum [1]

$$P_T^{1/2}\simeq 8\sqrt{2}\,\frac{H_*}{m_P}, \tag{43}$$

and the tensor-scalar ratio, from Eq. (37) and above, is then given by

$$\frac{P_T}{P_{\mathcal{R}}}\simeq(120\pi)^2\epsilon_h\propto\left(\frac{h_*}{m_P}\right)^2, \tag{44}$$

where the value of the Higgs field at the time of horizon exit is typically $h_*\sim1$ TeV, which is negligible compared to the reduced Planck mass m_P. Therefore we do not expect any observable effects of gravity waves in the CMB spectrum.

It has been pointed out that in general for the curvaton models based on a late-decaying scalar the entropy perturbation originated during inflation might survive the reheating era as a relic isocurvature perturbation between radiation and some other present component, say cold dark matter (neutralinos) or neutrinos [15,16]. In the framework described in this paper the Higgs fields can decay into neutralinos and in addition give rise to leptogenesis during the reheating process [47]. Therefore the Higgs isocurvature perturbations might be expected to give rise to relic isocurvature perturbations both in neutralino dark matter and in baryons, as compared to photons. However the Higgs excitations decay near the beginning of the reheating era, and any neutralinos produced at that time are relativistic and share the curvature perturbation with that of the photons. Therefore once the neutralinos decouple from the radiation fluid, their curvature perturbation will remain constant and equal to that of the radiation at the time. On the other hand throughout reheating the radiation will be coupled to the matter energy density through the decay of the inflaton field. For such a coupled system, individual curvature perturbations are not conserved, unlike the late-decaying scalar scenario in [10,11,14–17], and so in our case we expect the isocurvature perturbation of the photons to change in a complicated way during the reheating process. Thus in our case the final neutralino isocur-

[5]This prediction for the spectral index is valid when there is no relic isocurvature perturbation which can significantly affect the CMB power spectrum.

vature perturbation will depend on the details of the reheating process, in particular when the neutralino decouples and becomes non relativistic during the reheating process. It would be interesting to explore this quantitatively in a future work.

VII. SUMMARY

To summarize, we have proposed and discussed a new implementation of the curvaton scenario [10,11,14–17] in which quantum fluctuations of light scalar fields other than the inflaton can be responsible for the generation of the primordial anisotropies in the Universe. Unlike the previous scenarios, we do not assume a late-decaying scalar which is decoupled from the inflaton field. Instead we have proposed a new mechanism based on supersymmetric hybrid inflation in which the isocurvature perturbations of a hybrid field coupled to the inflaton could be transferred to curvature perturbations during the initial stages of reheating. We have further suggested that good candidates for such fields are the Higgs doublets of the supersymmetric standard model, which may have a flat direction during inflation provided that the μ term is generated after inflation. We have discussed in detail a specific model which implements this mechanism, based on a supersymmetric hybrid inflation model which is a variant of the NMSSM, based on the superpotential in Eq. (6). We have shown that for this particular model our mechanism leads to

$$P_{\mathcal{R}}^{1/2} \sim \frac{H_*}{2\pi h_*}, \tag{45}$$

which for a typical value of the Higgs VEV $2\pi h_* \simeq 1$ TeV gives the correct order of magnitude as measured by COBE $P_{\mathcal{R}}^{1/2} \sim 10^{-5}$. This is an important success of both the particular model and of the proposed mechanism in general. The main prediction is that the spectral index is expected to deviate significantly from unity $|n-1| \sim 0.1$. If $n=1$ is measured very accurately then this would be evidence for the original inflaton generated curvature perturbations. No observable signals of non-Gaussianity, or gravity waves are expected in the CMB spectrum. However there may be observable relic isocurvature perturbations between radiation and some other present component, say cold dark matter (neutralinos) or neutrinos which depend on the details of reheating and differ significantly from the expectations of a late-decaying scalar [15,16].

To conclude, we have proposed and discussed the attractive possibility of having the Higgs field of the supersymmetric standard model as being responsible for the large-scale structure of the Universe, providing a further strong link between cosmology and particle physics of the kind recently emphasized in [48]. Eventually, if such a theory as that presented here is realized in nature, it should be possible, from both laboratory and cosmological measurements, to demonstrate the links between particle physics and cosmology inherent in such a model [48].

ACKNOWLEDGMENTS

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APPENDIX A

The equation of motion for the gauge invariant quantum fluctuation Q_α, with comoving wave number k, can be written as [49]

$$\ddot{Q}_\alpha + 3H\dot{Q}_\alpha + \frac{k^2}{a^2}Q_\alpha + \sum_\beta \left[V_{\alpha\beta} - \frac{1}{a^3 m_P^2}\left(\frac{a^3}{H}\dot{\phi}_\alpha\dot{\phi}_\beta\right)^{\cdot}\right] Q_\beta = 0, \tag{A1}$$

where $V_{\alpha_\beta} = \partial^2 V/\partial\phi_\alpha\partial\phi_\beta$. For the inflaton and Higgs fields during inflation, we have $V_{\phi\phi}, V_{hh}, V_{\phi h} \ll H^2$, and therefore the fluctuations Q_ϕ and Q_h will be frozen to a constant value once outside the horizon, $k < aH$, given approximately by the value at horizon crossing,

$$Q_{\alpha *} = \frac{H_*}{\sqrt{2k^3}}, \tag{A2}$$

for $\alpha = \phi, h$. On the other hand, neglecting for simplicity the sub-dominant terms coming from metric contributions, the evolution equation for Q_N reads

$$\ddot{Q}_N + 3H\dot{Q}_N + \left(\frac{k^2}{a^2} + V_{NN}\right) Q_N + V_{N\phi} Q_\phi + V_{Nh} Q_h \simeq 0. \tag{A3}$$

Like in the case of the evolution of the background field N, now the large mass term $V_{NN} \sim O(\kappa^2 \phi_c^2) \gg H^2$ gives rise to oscillations with an amplitude decaying as a^{-1}, but displaced from zero due to the Q_ϕ and Q_h terms. Therefore, after horizon exit, and averaging over the fast oscillations in a Hubble time, the N field fluctuation will also tend to a constant value given by

$$Q_N \simeq \frac{V_{N\phi}}{V_{NN}} Q_\phi + \frac{V_{Nh}}{V_{NN}} Q_h. \tag{A4}$$

Using $V_{NN} \simeq 2\kappa^2 \phi_c^2$, and $V_{N\phi} \simeq 4\kappa^2 \phi_c N - \lambda\kappa h^2$, $V_{Nh} \simeq 2\lambda\kappa\phi_c h$, we have

$$Q_N \simeq \left(2\frac{N}{\phi_c} - \frac{\lambda h^2}{2\kappa\phi_c^2}\right) Q_\phi + \frac{\lambda h}{\kappa\phi_c} Q_h \simeq \frac{\lambda h}{\kappa\phi_c} Q_h. \tag{A5}$$

Therefore, the fluctuation Q_N is suppressed with respect to Q_h by the same factor than the background field N is suppressed with respect to h, Eq. (21), such that

$$\dot{N} \simeq -\frac{\lambda h}{\kappa\phi_c}\dot{h}, \tag{A6}$$

and then

$$\frac{Q_N}{\dot{N}} \simeq -\frac{Q_h}{\dot{h}}. \tag{A7}$$

APPENDIX B

In this appendix we summarize our conventions and notation for the perturbations, and give the evolution equations for curvature and entropy perturbations in a multi-component fluid. These can be found in Refs. [36,12]. Using linear perturbation theory, the equations are given in terms of gauge-invariant quantities $\Delta_c = \delta\rho_c/\rho$, V and η describing the amplitude perturbation of the total density in the comoving frame, velocity and entropy respectively. In addition we have the relative gauge-invariant variables between any two components α and β: entropy $S_{\alpha\beta}$ and relative velocity $V_{\alpha\beta}$.

Gauge-invariant quantities are defined from the original perturbations in the stress-energy tensor and the metric. Linear scalar perturbations of the metric are given by the line element:

$$ds^2 = -(1+2A)dt^2 + 2aB_i dx^i dt + a^2[(1-2\psi)\delta_{ij} + 2E_{ij}]dx^i dx^j, \tag{B1}$$

where ψ is the gauge-dependent metric perturbation. Combining ψ, B and E, one can define the gauge-invariant variable

$$\Phi = -\psi - Ha(B - a\dot{E}), \tag{B2}$$

which is the curvature perturbation in the longitudinal (or zero-shear) gauge $(a\dot{E}_l - B_l = 0)$.

Each fluid component is described by a perfect-fluid stress-energy tensor, with background energy ρ_α and pressure P_α. Although the total stress-energy tensor is conserved, the stress-energy tensor of each component may not be conserved individually. Therefore, the unperturbed continuity equation for a given component α is given in general by

$$\dot{\rho}_\alpha + 3H(\rho_\alpha + P_\alpha) = C_\alpha, \tag{B3}$$

where a dot denotes derivate with respect to time, and C_α are source terms subject to the total energy-momentum conservation constraint $\Sigma_\alpha C_\alpha = 0$. For example, in Eqs. (33) and (34) we have $C_\phi = -C_r = -\Gamma_\phi \rho_\phi$. For later use, we define the equation of state for each component, $w_\alpha = P_\alpha/\rho_\alpha$, and the sound velocity $c^2_{s\alpha} = \dot{P}_\alpha/\dot{\rho}_\alpha$. To simplify notation, we also define $h_\alpha = \rho_\alpha + P_\alpha$.

Perturbations in the stress-energy tensor are given by $\delta\rho_\alpha$, δP_α and the momentum perturbation δq_α (neglecting the anisotropic stress). Gauge-invariant variables $\delta\rho_{c\alpha}$, V_α and η_α are defined as

$$\delta\rho_{c\alpha} = \delta\rho_\alpha + \dot{\rho}_\alpha \frac{\delta q}{\rho + P}, \tag{B4}$$

$$V_\alpha = -\frac{k}{a}\left(\frac{\delta q_\alpha}{\rho_\alpha + P_\alpha} - aB + a^2\dot{E}\right), \tag{B5}$$

$$P_\alpha \eta_\alpha = \delta P_\alpha - c^2_{s\alpha}\delta\rho_\alpha, \tag{B6}$$

such that

$$\delta\rho_c = \sum_\alpha \delta\rho_{c\alpha}, \tag{B7}$$

$$(\rho + P)V = \sum_\alpha (\rho_\alpha + P_\alpha)V_\alpha, \tag{B8}$$

$$S_{\alpha\beta} = \frac{\delta\rho_{c\alpha}}{\rho_\alpha + P_\alpha} - \frac{\delta\rho_{c\beta}}{\rho_\beta + P_\beta}, \tag{B9}$$

$$V_{\alpha\beta} = V_\alpha - V_\beta. \tag{B10}$$

The total entropy perturbation (or non-adiabatic pressure perturbation) $P\eta = \delta P - c_s^2 \delta\rho$ can be written in terms of the individual entropy perturbations η_α for each component and $S_{\alpha\beta}$ as

$$\frac{w}{1+w}\eta = \sum_\alpha \frac{h_\alpha}{h}\frac{w_\alpha \eta_\alpha}{1+w_\alpha} + \frac{1}{2}\sum_\alpha\sum_\beta \frac{h_\alpha h_\beta}{h^2}(c^2_{s\alpha} - c^2_{s\beta})S_{\alpha\beta} + \frac{\Delta_c}{1+w}\sum_\alpha c^2_{s\alpha}\frac{C_\alpha}{3Hh}. \tag{B11}$$

Individual entropy perturbations η_α vanish for adiabatic perturbations, but cannot be neglected for example in the case of a scalar field, in which case they are given by

$$P_\alpha \eta_\alpha = (1 - c^2_{s\alpha})\left(\delta\rho_{c\alpha} - \frac{a}{k}\dot{\rho}_\alpha (V_\alpha - V)\right). \tag{B12}$$

The comoving curvature perturbation is defined by

$$\mathcal{R} = \psi - \frac{H}{\rho + P}\delta q = -\Phi + \frac{aH}{k}V = -\frac{3}{2}\left(\frac{aH}{k}\right)^2 \Delta_c + \frac{aH}{k}V, \tag{B13}$$

where in the last equality we have used the relation between Φ and the comoving total density perturbation through the energy constraint given by

$$\Phi = \frac{3}{2}\left(\frac{aH}{k}\right)^2 \Delta_c. \tag{B14}$$

On the other hand, the curvature perturbation on constant-density hypersurfaces is given by

$$\zeta = -\psi - H\frac{\delta\rho}{\dot{\rho}}, \tag{B15}$$

which is related to the comoving curvature perturbation $\mathcal{R}$ by

$$-\zeta = \mathcal{R} - \frac{2\rho}{9(\rho + P)}\left(\frac{k}{aH}\right)^2 \Phi. \tag{B16}$$

Therefore, for super-Hubble scales, $k \ll aH$, ζ and $\mathcal{R}$ are approximately the same.

We note that we have followed Refs. [12,36] in defining the entropy perturbation $S_{\alpha\beta}$. An alternative definition related to the individual curvature perturbations ζ_α, used in Refs. [10,16], is given by

$$\tilde{S}_{\alpha\beta} = -3H\left(\frac{\delta\rho_\alpha}{\dot{\rho}_\alpha} - \frac{\delta\rho_\beta}{\dot{\rho}_\beta}\right) = 3(\zeta_\alpha - \zeta_\beta). \tag{B17}$$

In terms of $\tilde{S}_{\alpha\beta}$, the total entropy perturbation is then given by

$$\frac{w}{1+w}\eta = \sum_\alpha \frac{h_\alpha}{h}\frac{w_\alpha\eta_\alpha}{1+w_\alpha} + \frac{1}{2}\sum_\alpha\sum_\beta \frac{h_\alpha h_\beta}{h^2}(c_{s\alpha}^2 - c_{s\beta}^2)\tilde{S}_{\alpha\beta}. \tag{B18}$$

Nevertheless, for a system of uncoupled fluids, Eqs. (B9) and (B17) become identical. $S_{\alpha\beta}$ and $\tilde{S}_{\alpha\beta}$ differ by terms proportional to the source terms C_α when there is energy transfer between components. For example, for a 2-component system, their relation can be written as

$$(1-q_\alpha)(1-q_\beta)\tilde{S}_{\alpha\beta} = S_{\alpha\beta} + (q_\alpha - q_\beta)\frac{\delta\rho_c}{\rho+P}, \tag{B19}$$

where $1-q_\alpha = -\dot{\rho}_\alpha/(3Hh_\alpha)$.

The evolution equations for the total density and velocity fluctuations in a flat Universe are given by

$$\Delta_c' = 3w\Delta_c - (1+w)\frac{k}{aH}V, \tag{B20}$$

$$V' = -V - \frac{3}{2}\left(\frac{aH}{k}\right)\Delta_c + \frac{k}{aH}\left(\frac{c_s^2}{1+w}\Delta_c + \frac{w}{1+w}\eta\right), \tag{B21}$$

where prime means derivative with respect to $\ln a$. Thus, taking the derivative in Eq. (B13) and using Eqs. (B20), (B21), the evolution equation for the comoving curvature perturbation reads

$$\mathcal{R}' = (1+3w)\frac{3}{2}\left(\frac{aH}{k}\right)^2\Delta_c - \frac{3}{2}\left(\frac{aH}{k}\right)^2\Delta_c' - \frac{1}{2}(1+3w)\frac{aH}{k}V + \frac{aH}{k}V' \tag{B22}$$

$$= \frac{c_s^2}{1+w}\Delta_c + \frac{w}{1+w}\eta = \frac{2}{3}\left(\frac{k}{aH}\right)^2\frac{c_s^2}{1+w}\Phi + \frac{w}{1+w}\eta. \tag{B23}$$

Finally, for a two-component fluid, such that $C_\alpha = -C_\beta$, the equations for the relative fluctuations $S_{\alpha\beta}$ and $V_{\alpha\beta}$ are given by

$$S_{\alpha\beta}' = -\frac{k}{aH}V_{\alpha\beta} - 3\left(\frac{w_\alpha\eta_\alpha}{1+w_\alpha} - \frac{w_\beta\eta_\beta}{1+w_\beta}\right) - \frac{C_\alpha}{Hh}\left(\frac{h_\beta}{h_\alpha}(1+c_{s\alpha}^2) - \frac{h_\alpha}{h_\beta}(1+c_{s\beta}^2)\right)S_{\alpha\beta} - \frac{C_\alpha}{H}\frac{h}{h_\alpha h_\beta}\frac{w}{1+w}\eta - \frac{C_\alpha}{H}\frac{h}{h_\beta h_\alpha}\left(1 + c_s^2 + \frac{h_\alpha}{h}c_{s\beta}^2 + \frac{h_\beta}{h}c_{s\alpha}^2\right)\frac{\Delta_c}{1+w} + \frac{C_\alpha}{H}\frac{h}{h_\alpha h_\beta}E_{c\alpha}, \tag{B24}$$

$$V_{\alpha\beta}' = \left(3c_{s\alpha}^2\frac{h_\beta}{h} + 3c_{s\beta}^2\frac{h_\alpha}{h} - 1\right)V_{\alpha\beta} + \frac{k}{aH}\left(c_{s\alpha}^2\frac{h_\beta}{h} + c_{s\beta}^2\frac{h_\alpha}{h}\right)S_{\alpha\beta} + \frac{k}{aH}\left(\frac{w_\alpha\eta_\alpha}{1+w_\alpha} - \frac{w_\beta\eta_\beta}{1+w_\beta}\right) + \frac{k}{aH}(c_{s\alpha}^2 - c_{s\beta}^2)\frac{\Delta_c}{1+w} - \frac{C_\alpha}{Hh}\left((1+c_{s\alpha}^2)\frac{h_\beta}{h_\alpha} - (1+c_{s\beta}^2)\frac{h_\alpha}{h_\beta}\right)V_{\alpha\beta}, \tag{B25}$$

where $E_{c\alpha} = E_{c\beta}$ are the gauge invariant perturbations of the energy transfer term in the comoving frame. For the simplest case $C_\alpha = \Gamma_\alpha\rho_\alpha$ then $E_{c\alpha} = \delta\rho_{c\alpha}/\rho_\alpha$.

APPENDIX C

For the scenario considered in this paper, we can take the component "α" to be the oscillating fields behaving like matter ($w_\alpha = c_{s\alpha} = 0$), "$\beta$" then refers to the radiation ($w_\beta = c_{s\beta} = 1/3$) initially coming from the Higgs decay products, and $C_\alpha = C_\phi = -\Gamma_\phi\rho_\phi$. Then, the evolution Eqs. (B23), (B24) and (B25) for the curvature $\mathcal{R}$, $S_{\phi r}$ and $\hat{V}_{\phi r} = (Ha/k)V_{\phi r}$ reduce to

$$\mathcal{R}' = \frac{\Delta_c}{1+w}\left(\frac{\rho_\phi + 4\rho_r}{3\rho_\phi + 4\rho_r}\right) + \frac{8\rho_\phi\rho_r}{(3\rho_\phi + 3\rho_r)^2}S_{\phi r} + \frac{36\rho_\phi\rho_r}{(3\rho_\phi + 3\rho_r)^2}\left(1 + \frac{\Gamma_\phi}{3H}\right)\hat{V}_{\phi r}, \tag{C1}$$

$$S_{\phi r}' = -\left(\frac{k}{aH}\right)^2\hat{V}_{\phi r} - 3\frac{\Delta_c}{1+w} - \frac{12\rho_r}{3\rho_\phi + 4\rho_r}S_{\phi r} - \frac{36\rho_r}{3\rho_\phi + 4\rho_r}\hat{V}_{\phi r} - \frac{\Gamma_\phi}{H}\frac{\rho_\phi}{\rho_r}\left(\frac{3\rho_\phi + \rho_r}{3\rho_\phi + 4\rho_r}\right)S_{\phi r} + \frac{\Gamma_\phi}{H}\frac{9\rho_\phi}{3\rho_\phi + 4\rho_r}\left(1 - 4\frac{\rho_r}{\rho_\phi} + \frac{\Gamma_\phi}{3H}\right)\hat{V}_{\phi r} + \frac{\Gamma_\phi}{H}\frac{\rho_r + 3\rho_\phi}{3\rho_r}\frac{\Delta_c}{1+w}, \tag{C2}$$

$$\hat{V}'_{\phi r} = -\frac{1+3w}{2}\hat{V}_{\phi r} + \frac{8\rho_r}{3\rho_\phi+4\rho_r}\hat{V}_{\phi r} + \frac{2}{3}\frac{\Delta_c}{1+w} + \frac{4\rho_r+\rho_\phi}{3\rho_\phi+4\rho_r}S_{\phi r} + \frac{\Gamma_\phi}{H}\left(\frac{8\rho_r^2-3\rho_\phi^2}{\rho_r(3\rho_\phi+4\rho_r)}\right)\hat{V}_{\phi r}. \tag{C3}$$

For super-Hubble scales such that $k \ll aH$, we can further simplify the system by neglecting the contribution of the terms $\propto (k/aH)^2$, and that of $\Delta_c = 2(k/aH)^2\Phi/3$. In order to understand the qualitative behavior of the solutions to the above system of equations, we can focus on the evolution of the curvature $\mathcal{R}$ and $S_{\phi r}$, without taking into account the terms due to $\hat{V}_{\phi r}$. From the evolution equations we can see that $S_{\phi r}$ and $\hat{V}_{\phi r}$ will behave approximately the same, so both of them will affect the curvature $\mathcal{R}$ in a similar way; then the main effect is obtained by keeping only those term due to $S_{\phi r}$ in the evolution equations. We have checked that the numerical solution of Eqs. (C1)–(C3) is well approximated by that of

$$\mathcal{R}' \simeq \frac{8\rho_\phi\rho_r}{(3\rho_\phi+3\rho_r)^2}S_{\phi r} \tag{C4}$$

$$S'_{\phi r} \simeq -\frac{12\rho_r}{3\rho_\phi+4\rho_r}S_{\phi r} - \frac{\Gamma_\phi}{H}\frac{\rho_\phi}{\rho_r}\left(\frac{3\rho_\phi+\rho_r}{3\rho_\phi+4\rho_r}\right)S_{\phi r}. \tag{C5}$$

In order to solve analytically the equations, we can distinguish two regimes depending on the evolution of the background energy densities ρ_ϕ and ρ_r, as it can be seen in Fig. 1: (a) from the beginning of the reheating period at a_0, with $\rho_r(a_0) \simeq \rho_\phi(a_0)$, up to say a_1, the decay products coming from the singlets have no effect on the radiation, and both components, matter and radiation, behave as if they were decoupled, with $\rho_r \propto a^{-4}$ and $\rho_\phi \propto a^{-3}$; (b) from a_1 to the end of reheating, the singlets decay products start contributing to the background radiation, with $\rho_r \propto a^{-3/2}$ and $\rho_\phi \propto a^{-3}$. Therefore, for $a_0 \leq a \leq a_1$, Eqs. (C4) and (C5) can be solved neglecting the terms proportional to Γ_ϕ/H, and we obtain for $\mathcal{R}$ and $S_{\phi r}$

$$\mathcal{R} \simeq \mathcal{R}(a_0) + \frac{8}{343}S_{\phi r}(a_0)\left(5 - \frac{2a_0^2}{a^2} - \frac{3a_0}{a}\right), \tag{C6}$$

$$S_{\phi r} \simeq \left(\frac{4+3a/a_0}{7a/a_0}\right)^3 S_{\phi r}(a_0), \tag{C7}$$

where $\mathcal{R}(a_0) \ll S_{\phi r}(a_0)$ are the initial values of the perturbations taken as those at the end of inflation. It can be seen that very quickly $\mathcal{R}$ will tend to a constant value with $\mathcal{R} \simeq S_{\phi r}(a_0)/10$. Similarly, the entropy perturbations go to $S_{\phi r} \simeq 0.08 S_{\phi r}(0)$. That is, entropy perturbations act as a source for the curvature perturbations only at the beginning, as far as the ratio ρ_r/ρ_ϕ is not suppressed. Afterwards, $\rho_r \ll \rho_\phi$, and $\mathcal{R}$ remains constant.

Entropy perturbations will not further affect the evolution of the curvature even when the background equations become coupled at a_1, and the effects of Γ_ϕ cannot be neglected. At this point we can still assume that the energy density is dominated by that of the singlets, behaving like matter, with $H \propto a^{-3/2}$. Therefore, from a_1 onwards, the combination $\Gamma_\phi\rho_\phi/(H\rho_r)$ remains constant until the singlets completely decay. Numerically we find $\Gamma_\phi\rho_\phi/(H\rho_r) \simeq 2.5$, independently of the value of Γ_ϕ. Taking this into account, we can now solve Eq. (C5) including the Γ_ϕ term:

$$S_{\phi r}(a > a_1) \simeq 0.07 S_{\phi r}(a_1)\left(\frac{a_1}{a}\right)^{2.5}. \tag{C8}$$

That is, the relative perturbation between matter (singlets) and radiation decreases. With this result, we can see that the source term in the evolution equation for the curvature perturbation,

$$\mathcal{R}' \simeq \frac{8\rho_r}{3\rho_\phi}S_{\phi r} \propto \left(\frac{a}{a_1}\right)^{-2}, \tag{C9}$$

also decreases in time and it is no longer effective, even if by the end of the reheating period $\rho_r \simeq \rho_\phi$ again.

The evolution of the perturbations plotted in Fig. 1 is the result of the numerical integration of the system of Eqs. (C1), (C2), with initial conditions at the end of inflation [12]:

$$\mathcal{R}|_i \simeq \frac{H_*}{\dot{\phi}_*}Q_{\phi_*} \simeq -\frac{Q_{\phi_*}}{\eta_\phi\phi_*}, \tag{C10}$$

$$S_{\phi r}|_i \simeq \frac{8}{3}\frac{H_*}{\dot{h}_*}Q_{h_*} \simeq -\frac{8}{3}\frac{Q_{h_*}}{\eta_h h_*}, \tag{C11}$$

$$\frac{aH}{k}V_{\phi r}|_i \simeq -\frac{1}{3}S_{\phi r}|_i. \tag{C12}$$

Given that for the amplitude of the field perturbations at the time of horizon crossing we have $Q_{\phi *} \simeq Q_{h *} \simeq H_*/\sqrt{2k^3}$, it is more convenient to normalize the above perturbations with respect to the initial value of the entropy perturbation $S_{\phi r}|_i$, such that

$$\left.\frac{\mathcal{R}}{S_{\phi r}}\right|_i \simeq -\frac{3\,\eta_h h_*}{8\,\eta_\phi\phi_*}, \tag{C13}$$

$$\left.\frac{aH}{k}\frac{V_{\phi r}}{S_{\phi r}}\right|_i \simeq -\frac{1}{3}. \tag{C14}$$

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