Inflation and Kähler stabilization of the dilaton

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The problems of attempting inflationary model building in a theory containing a dilaton are explained. In particular, I study the shape of the dilaton potential today and during inflation, based on a weakly coupled heterotic string model where corrections to the Kähler potential are assumed to be responsible for dilaton stabilization. Although no specific model building is attempted, if the inflationary energy density is related to the scale of gaugino condensation, then the dilaton may be stabilized close enough to today's value that there is no significant change in the grand unified theory scale coupling. This can occur in a very wide range of models, and helps to provide some justification for the standard predictions of the spectral index. I explain how this result can ultimately be traced to the supersymmetry structure of the theory.

I. INTRODUCTION

The problem of a runaway dilaton in theories arising from the low-energy limit of string theory has been widely discussed. The most frequently quoted mechanism for attempting to stabilize the dilaton is via an effective potential generated when some hidden sector gauge symmetry is driven to strong coupling. The gauginos charged under the strong group condense [1,2], with vacuum expectation values (VEVs) that depend on the string-scale coupling and hence provide an effective non-perturbative superpotential for the dilaton. If more than one group has a gaugino condensate at comparable scales as in the racetrack models [3,4], or if non-perturbative string physics is accounted for as in Kähler stabilization [5–9], then the dilaton may have a non-trivial minimum in its scalar potential, as well as the supersymmetry preserving minimum at zero coupling.

The question of why, during the cosmological evolution of our Universe, we should expect the dilaton to be found in the minimum at non-trivial coupling is also well known [10]. The minimum generated by the above mechanism is typically rather shallow, at least in comparison to the energies of the very early universe. Generically one expects that, starting from an arbitrary value, the dilaton would either not approach or else overshoot such a minimum and roll away to zero coupling. Possible solutions to this tend to invoke friction in the dilaton's equation of motion, generated either from cosmological expansion [11] or higher Kaluza–Klein (KK) modes of the dilaton that are expected to be present at high energies [12]. Here I will concentrate on a different issue: how do we know that, throughout the evolution of the universe, the dilaton's potential had a minimum located in the same place as today? Since each term in the effective supergravity scalar potential couples to the dilaton, at energies in the early universe above the current dilaton mass (typically of order 10–100 TeV), we may expect that the dilaton potential has a different form. For example, during inflation we expect that some term in this scalar potential dominated the energy density of the universe. Since we are not inflating today, there is no reason to expect that this same term is still important and so it does not necessarily play any part in dilaton stabilization today. How can we know whether such a term would have provided a non-trivial dilaton minimum at all, let alone one close to today's value?

This issue is important for various reasons. Of course, various cosmological implications of varying coupling constants have been widely discussed recently in, e.g. [13–16]; however, here we will consider issues relating more to inflation. Firstly, we do not wish to exacerbate the Brustein-Steinhardt problem. For example, if we make the very naïve approximation that the effective dilaton potential instantaneously switched from its inflationary form to today's form at the end of slow-roll inflation,[1] then we do not wish to have to guide it to its new minimum again. Indeed, if this was originally done using higher KK modes as in [12], then a reheat temperature high enough to reexcite these would presumably also be high enough to generate all manner of dangerous relics. Of course, this is just the usual statement of the moduli problem [17] as applied to the dilaton field. However, it is also important for a second, possibly more urgent reason specific to the dilaton (at least in the weakly coupled heterotic theory). Inflationary model building concerns itself in part with generating sufficiently flat potentials for slow roll to occur. A promising way to achieve this without unnatural tuning is to note that, in a globally supersymmetric theory, any potential that is tuned to be flat at tree level will remain so to all orders in perturbation theory so long as supersymmetry (SUSY) remains unbroken. After SUSY breaking, such a potential will become sloped due to quantum corrections. In supergravity (SUGRA) there is also the difficulty that gravitational interactions generate a mass for canonical scalar fields of the same order as their potential: the η problem (see, e.g. [18]). Suggested ways of overcoming the η problem include the use of non-canonical forms for the Kähler potential arising for fields well below the string scale [19,20], or identifying the slow-roll and string moduli fields [21,22], or perhaps by driving inflation via a D term [23,24].

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[1] In this paper we make the assumption that the dilaton itself is not the inflaton.

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Assuming one or other of these is successful, then the slope of the scalar potential will again depend on quantum corrections and it is here that the dilaton plays a crucial role. If these quantum corrections are to be under reasonable calculational control then we had better ensure that during inflation the dilaton is stabilized in the weak-coupling regime of some theory. Even assuming that it is, clearly the spectral index of primordial fluctuations n_k will depend on the location of the dilaton. This conclusion is true irrespective of any renormalization of the coupling constants that must be accounted for in running down from the string scale to the inflationary scale. Therefore, if we wish inflationary model building to be at all predictive, it is essential that we know where the dilaton is stabilized during inflation. Even so, we may still expect that there will be some degeneracy in n_k between different inflationary potentials and different dilaton minima. For example, D-term inflation is often considered to be ruled out, in part because the inflationary scale appears incompatible with results from the Cosmic Background Explorer (COBE) and because the model seems to require matter field values above the Planck scale, rendering calculations unreliable. However, as has been noticed in [25,26] each of these conclusions may be altered if the grand unified theory (GUT) scale gauge coupling is different during inflation. With our present lack of knowledge about the form of the potential during inflation, together with a far from completely satisfactory explanation of how to stabilize the dilaton even today, let alone during less well-understood epochs, it seems almost hopeless to attempt to resolve this situation at present by anything other than very model-dependent statements. Nonetheless, we will see that some reasonably generic, natural mechanisms may provide significant help.

I will assume that the dilaton is stabilized via corrections to the Kähler potential, perhaps arising through non-perturbative string physics. If these are to be relevant in the field-theoretic regime, they should be described in the Kähler potential by some function $g(1/\mathrm{Re}S)$. Much previous work in the subject has concerned itself finding vacua that follow from a specific choice of this function, such as those suggested by Shenker [35]. However, various forms are commonly discussed and there is even some debate as to whether any of them realistically represent the true form of string non-perturbative corrections. Here we will keep this function completely arbitrary, except to assume that its presence in the Kähler potential is capable of providing an acceptable minimum. In Sec. III briefly review the model of Binétruy, Gaillard and Wu [7,8] which treats Kähler stabilization using the linear multiplet formalism. Section III follows the standard path and considers the vacuum today. I will be less interested here in particular examples, but rather will simply list the usual constraints that the corrections to the Kähler potential must satisfy if they are to be capable of providing a phenomenologically reasonable vacuum with, e.g. a vanishing cosmological constant and SUSY breaking at ~ 1 TeV. Having considered the properties of the present vacuum, Sec. IV returns to the full scalar potential and hence examines the vacuum during inflation. Again I stress that we will not be concerned with building specific inflationary models, but rather we will attempt to find a reasonable generic scheme

that ensures the dilaton potential has a minimum during inflation. The key will be to assume that the scalar fields acquire VEVs which depend on the dilaton only via the condensate itself. This is a natural mechanism, being the hidden sector precursor to generation of visible sector SUSY breaking masses. Additionally, in this picture the inflationary energy density is always provided by the matter F terms. This is a standard assumption of much string-inspired inflationary model building, but here it will follow from the above mechanism. Remarkably, the models which have energy density compatible with the COBE bound $V^{1/4} \lesssim 6.7\epsilon^{1.4} \times 10^{16}$ GeV also tend to stabilize the dilaton close to today's value. This is true despite the many orders of magnitude difference between the effective cosmological constants during inflation and today. Similar ideas were pursued in [27] where the authors allowed scalar field VEVs to be generated by vacuum shifting to cancel a D term. However, the results here are more general, and in particular do not depend on any form for the Kähler corrections. Finally, the conclusions will discuss the obtained results and try to understand why they hold. I argue that ultimately, this can be traced to the supersymmetry structure of the higher dimensional theory.

II. THE MODEL

In this paper I assume that, both today and during inflation, the dilaton is stabilized through corrections to the Kähler potential. For convenience, we will work in the linear supermultiplet formalism of [7,8], although since this has been shown to be equivalent to the usual chiral formulation in Ref. [30], we expect the same results to hold there also. In this formalism, for each semi-simple hidden sector gauge group $\mathcal{G}_a$ the gaugino condensate superfields $U_a \sim \mathrm{Tr}(W^\alpha W_\alpha)_a$ are identified with the chiral projections of the vector superfields as

$$U_a = -(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)V_a$$
$$\bar{U}_a = -(\mathcal{D}^\alpha\mathcal{D}_\alpha - 8\bar{R})V_a$$
(2.1)

with the overall vector superfield $V = \Sigma_a V_a$ having the dilaton l as its lowest component: $l = V|_{\theta=\bar\theta=0}$. Compared to the chiral formulation, we have roughly $l \approx 1/(S+\bar{S})$. Considering for simplicity an orbifold compactification, the Kähler potential is

$$K = \ln(V) + g(V) - \sum_I \ln\left(T_I + \bar{T}_I - \sum_A |\Phi_{AI}|^2\right) \quad (2.2)$$

where here I consider only untwisted chiral matter fields Φ_{AI}. The function $g(V)$ is supposed to describe the contribution of string non-perturbative effects to the Kähler potential, and clearly this affects also the dilaton kinetic term. A simple calculation shows this to be

$$\mathcal{L}_{\mathrm{kinetic}} \supset \frac{(lg'+1)}{4l^2}\partial_\mu l\,\partial^\mu l \quad (2.3)$$

where the prime denotes differentiation with respect to l. The string non-perturbative effects also modify the string-scale gauge coupling of the effective field theory:

$$2\pi\alpha = \frac{l}{1+f(l)} \tag{2.4}$$

and in the 4D Einstein frame, f is related to g via

$$V\frac{dg}{dV} = f(V) - V\frac{df}{dV}, \tag{2.5}$$

with $f(0) = g(0) = 0$ as required in the weak-coupling limit.

The gaugino condensates are described by the Veneziano-Yankielowicz superpotential [28,29], appropriately generalized to incorporate SUGRA as well as the presence of several gaugino condensates and/or gauge-invariant matter condensates. There are also superpotentials for the matter fields and terms describing quantum corrections to any possible unconfined gauge groups. It is of course important to preserve the modular invariance of the underlying string theory, and a Green-Schwarz counterterm is introduced to this end (it is assumed that the modular anomaly is completely cancelled by this term). The reader is referred to Refs. [7,8] for details.

Upon solving the equations of motion, the authors find the following expression for the condensates $u_a = U_a|_{\theta=\bar\theta=0}$:

$$
|u_a|^2 = A^2 \exp\left[\ln(l) + g(l) - \frac{(1+f)}{b_a l} \right.
$$
$$
\left. + \frac{b_{E_8} - b_a}{b_a}\sum_I \ln x_I \right] \prod_I |\eta(t_I)|^{4(b_{E_8}-b_a)/b_a}, \tag{2.6}
$$

where $t_I = T_I|_{\theta=\bar\theta=0}$, $x_I \equiv t_I + \bar t_I - \Sigma_A|\phi_{AI}|^2$ and $\eta(t_I)$ is the Dedekind η function, which ensures that Eq. (2.6) is modular invariant. The constants b_a and b_a' are related to the β functions for the condensing gauge group $\mathcal{G}_a$. In particular [8],

$$b_a = \frac{1}{8\pi^2}\left(C_a - \frac13 \sum_A C_a^A \right) \tag{2.7}$$

where C_a and C_a^A are the quadratic Casimirs for the group G_a in the adjoint and matter represenations, respectively. This is $1/8\pi^2$ times the coefficient of the one-loop β function for the coupling, and gives $b_{E_8} = 30/8\pi^2 \approx 0.38$ whereas, for typical choices of hidden sector groups and matter representations [31], $b_a \lesssim 0.1$. The constant of proportionality A^2 in Eq. (2.6) again depends on the group structure, matter representations and Yukawa couplings present in the entire model; we allow this the fairly generous range $10^{-4} - 10^4$ (see [31]) though its value will not be crucial here. Notice that the condensate depends on the dilaton explicitly through the exponentials of both the Kähler potential and the inverse of the string-scale gauge coupling, and implicitly through any dilaton dependence of the matter fields $\phi_{AI} = \Phi_{AI}|_{\theta=\bar\theta=0}$ or t_I moduli. This of course follows from the

interpretation of the condensate scale as the energy at which the logarithmic running of α causes it to become strong. As pointed out in [32] for the whole perturbative model to make sense in the first place, we of course require that the theory started out from reasonably weak coupling, and so we assume without further justification $b_a l \lesssim 1$ at all times.

One practical advantage of Kähler stabilization is that, even in the presence of several condensing gauge groups, the physical properties of the vacuum are dominated by the condensate coming from $\mathcal{G}_+$, the group with the largest beta function coefficient b_+. The condensate scale Λ_c and gravitino mass $\tilde m$ are then given by

$$\Lambda_c = \langle |u_+|^2 \rangle^{1/6} \tag{2.8}$$

$$\tilde m = \frac{b_+}{4}\langle |u_+| \rangle \tag{2.9}$$

in reduced Planck units. So, as usual, the condensate plays the role of an effective non-perturbative superpotential for the dilaton $b_+ u_+/4 \sim e^{K/2}W_{\rm np}$.

The final expression we require is the effective scalar potential for the model. This was presented in [27]. Neglecting the twisted sector matter fields, it is given by the somewhat complicated form

$$
V = \frac{e^K}{1+b_{E_8}l}\sum_I \left[\left| [2\xi(t_I)x_I + 1]B_I - \sum_A \phi_{AI}\frac{\partial W}{\partial\phi_{AI}} \right|^2 \right.
$$
$$
+ x_I\sum_A \left| \frac{\partial W}{\partial\phi_{AI}} + 2\xi(t_I)B_I\bar\phi_{AI} \right|^2 \Bigg]
$$
$$
+ (lg'+1)e^K\left| \frac{b_+ u_+ e^{-K/2}}{4}\left(1 + \frac{1}{b_+ l}\right) - W \right|^2
$$
$$
- 3e^K\left| \frac{b_+ u_+ e^{-K/2}}{4} - W \right|^2 \tag{2.10}
$$

where $\xi(t_I) = [1/\eta(t_I)](d\eta/dt_I)$ and B_I is defined by

$$B_I = \sum_A \phi_{AI}\frac{\partial W}{\partial\phi_{AI}} - W - e^{-K/2}\frac{u_+}{4}(b_{E_8} - b_+). \tag{2.11}$$

Modular invariance of the scalar potential requires that, up to a possible modular invariant function that would contain singularities, the superpotential has the form

$$W = \sum_n c_n \prod_{AI} \phi_{AI}^{p_n^{AI}} \eta(t_I)^{2(p_n^{AI}-1)} \tag{2.12}$$

for some powers p_n^{AI} with Yukawa couplings c_n that we will assume are $\mathcal{O}(1)$. For a cubic superpotential, we take $\Sigma_{AI}p_n^{AI} = 3$ in each term n.

In comparison with the usual chiral formulation, the first line of Eq. (2.10) approximately corresponds to the F terms for the t_I-moduli and matter fields, while the second line contains the dilaton F term together with the usual $-3e^K|W|^2$ piece, bearing in mind the interpretation of the condensate as a non-perturbative contribution to the superpo-

tential. Notice also the factor involving $1/(1+b_{E_8}l)$ which multiplies the moduli and matter F terms; this arises from the Green-Schwarz counterterm preserving modular invariance in the theory.

III. THE DILATON POTENTIAL TODAY

Following the standard assumptions, we assume that at least one matter field in each term in the superpotential and its derivatives has zero VEV today (or at least has a VEV $\ll \Lambda_c$). Minimizing the remaining potential with respect to t_I, the moduli are found [8] to be located at their self-dual points where $[4\xi \, \mathrm{Re}(t_I)+1]=0$. This is in accordance with the general result which states that these self-dual points are always extrema of the scalar potential [33,34]. Such a stabilization of the t_I moduli causes their F terms to vanish and accordingly in these models SUSY is broken by a non-vanishing dilaton F term. Thus the remaining scalar potential is

$$V=\left[(lg'+1)\left(1+\frac{1}{b_+l}\right)^2-3\right]\frac{b_+^2|u_+|^2}{16}. \tag{3.1}$$

In order to obtain a phenomenologically viable model, we would now wish to impose that the dilaton is at the minimum of its potential, in a vacuum with zero cosmological constants and an acceptable values for the gauge coupling and gravitino mass. This corresponds to the conditions

$$(1+b_+l)^2(lg'+1)-3b_+^2l^2|_{l=l_0}=0 \tag{3.2}$$

$$f''+\left|\frac{6b_+^2}{(1+b_+l)^3}\right|_{l=l_0}=0 \tag{3.3}$$

$$f'''-\left|\frac{18b_+^3}{(1+b_+l)^4}\right|_{l=l_0}<0 \tag{3.4}$$

where Eq. (3.2) corresponds to the vanishing of the remaining scalar potential and Eqs. (3.3), (3.4) ensure that the dilaton is at a minimum. Additionally, constraints on the GUT scale coupling and SUSY breaking scales may be interpreted as requiring

$$\left|\frac{l}{2\pi(1+f)}\right|_{l=l_0}=\alpha_0=\frac{1}{25} \tag{3.5}$$

$$\Lambda_c=10^{13} \;\text{GeV}\Rightarrow\widetilde{m}=1 \;\text{TeV}. \tag{3.6}$$

As mentioned in the Introduction, much previous work on these models has now specified particular forms (e.g., those of [35]) for the function $f(l)$ [and hence also $g(l)$] so as to proceed to find viable examples. While it is certainly interesting that this can be done in practice, here I shall pursue a different route and examine the implications of the above constraints more generally.

Firstly, notice that Eq. (3.2) implies

$$lg'+1>\frac{3b_+^2l^2}{(1+b_+l)^2} \tag{3.7}$$

for $l\neq l_0$, 0 as otherwise there would be a vacuum with lower energy.[2] Next consider the gauge coupling. Expanding $f(l)$ around $f(l_0)$ gives

$$\frac{1}{2\pi\alpha}=\frac{1}{2\pi\alpha_0}-\frac{3b_+^2}{(1+b_+l_0)^2}(l-l_0)$$
$$+\frac{1}{l_0}\sum_{n=3}^{\infty}\left|\frac{d^nf}{dl^n}\right|_{l=l_0}\frac{(l-l_0)^n}{n!}, \tag{3.8}$$

where the first and second order terms are evaluated explicitly using Eqs. (3.2) and (3.3). Equation (3.8) determines how the GUT scale coupling changes if the dilaton is located away from its true minimum. What is important to notice is that the first and second order terms always induce a negligible change in α for $b_+|l-l_0|\ll 1$, in other words throughout the entire "weakly-coupled" regime. Any significant change in the GUT scale coupling is therefore traceable to the presence of large higher order derivatives in the expansion (3.8). Indeed, such terms can generically be present, since Eqs. (3.2)–(3.4) do not restrict their magnitude. To paraphrase: if α does not change significantly during inflation, then $|l_i-l_0|$ is sufficiently small that higher order derivatives of f are unimportant. We shall see if this is reasonable in the next section.

IV. THE DILATON POTENTIAL DURING INFLATION

Let us now proceed to examine the dilaton potential during inflation. Since we do not know *a priori* which term is responsible for inflation, at first sight this seems to require the somewhat daunting task of investigating the full scalar potential (2.10). However the COBE bound requires

$$V^{1/4}\leq 6.7\epsilon^{1/4}\times 10^{16} \;\text{GeV}, \tag{4.1}$$

with typical slow-roll parameters ϵ giving $V^{1/4}\approx 10^{14}$ GeV. If we assume this energy density is provided by the VEVs of some scalar field(s) as in hybrid inflation (see, e.g. [21]), then of course we require $\langle\phi_{AI}\rangle\ll 1$ and therefore $W\ll\partial W/\partial\phi_{AI}$ for $\mathcal{O}(1)$ Yukawa couplings. This simplifies the potential to

$$V=\frac{e^K}{1+b_{E_8}l}\sum_I\left[(2\xi x_I+1)e^{-K/2}\frac{u_+}{4}(b_{E_8}-b_+)\right|^2$$
$$+x_I\sum_A\left|\frac{\partial W}{\partial\phi_{AI}}\right|^2\right]+(lg'+1)$$
$$\times e^K\left|\frac{u_+b_+e^{-K/2}}{4}\left(1+\frac{1}{b_+l}\right)\right|^2$$
$$-3e^K\left|\frac{b_+u_+e^{-K/2}}{4}\right|^2. \tag{4.2}$$

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[2]Of course we also have $V=0$ at $l=0$, the SUSY preserving, uncoupled vacuum.

For the theory to be under control, we must also have $|u_+|\ll 1$ in reduced Planck units, but notice that we cannot simply neglect the condensate; until we know where the dilaton is stabilized we do not know the condensate scale in comparison to $\partial W/\partial\phi_{AI}$ (of course, as yet there are no observational constraints on Λ_c during inflation). To proceed further, in this paper I will make the fairly natural assumption that a typical scalar field acquires a VEV connected to the condensate scale Λ_c, at whatever value this turns out to be.[3] A natural expectation would be $\langle\phi_{AI}\rangle\propto\Lambda_c$ for some scalar field(s) ϕ_{AI}. However, there may be some fields which acquire VEVs proportional to some other power of the condensate scale, perhaps due to a global symmetry which restricts the way they appear in the superpotential (see, e.g. [37]). For this reason, let us take a typical scalar VEV to be

$$\langle|\phi_{AI}|^2\rangle\propto|u_+|^{2\lambda}|\eta(t_I)|^{-4} \tag{4.3}$$

where λ is any positive power and the Dedekind η function is present to ensure the correct behavior under modular transformations (the condensate itself being invariant). Of course, this is exactly the mechanism that generically provides SUSY breaking masses in the observable sector today (with $\Lambda_c\to\tilde{m}$ via gravity mediation). Additionally, during inflation this mechanism has the advantage of being able to provide a high inflationary energy scale in a consistent way. The point is that in a string theory, all mass scales under m_s (in particular the inflationary scale $V^{1/4}$) must be generated dynamically. A condensate or gauge-symmetry breaking scalar field VEV will only form if it is energetically permitted. This may be inhibited in the presence of a large inflationary potential from some other source, and so inflation would be in danger of preventing its own SUSY breaking cause [19]. Of course, it is not necessary for all the fields to acquire the same VEVs with the same powers λ; this will be discussed further later. At present, let us assume that all sums are restricted to their dominant term(s). Again, I have ignored the possibility of additional modular invariant functions in the VEVs, and $\mathcal{O}$(few) numerical factors will not affect the argument.

At first sight, it appears that there are now various cases to consider; depending on λ, with $|u_+|^2\ll 1$ we may consider the dominant contribution to V to originate from either the matter, moduli or dilaton F terms during inflation. However, let us consider the stabilization of the t_I moduli. With the scalar VEVs as in Eq. (4.3) and using Eq. (2.6) we have

$$\frac{\partial|u_+|^2}{\partial t_J}=\frac{b_{E_8}-b_+}{b_+}\left[\sum_I\frac{1}{x_I}\frac{\partial x_I}{\partial t_J}+2\xi(t_J)\right]|u_+|^2 \tag{4.4}$$

$$=\frac{b_{E_8}-b_+}{b_+}\left[\frac{2\xi(t_J)(t_J+\bar{t}_J)+1}{x_J}|u_+|^2\right.$$

$$\left.-\lambda\frac{\partial|u_+|^2}{\partial t_J}\sum_{AI}\frac{\langle|\phi_{AI}|^2\rangle}{x_I}\right] \tag{4.5}$$

and so the condensate is extremized at the self-dual points where $4\xi(t_J)\,\mathrm{Re}\,t_J+1=0$. Now, from Eq. (4.2) it is clear that these points also extremize the dilaton F term and the supergravity $-3|W_{\mathrm{np}}|^2$ term, as these only depend on the moduli via the condensate. Consider next the matter F terms. With a cubic superpotential of the form (2.12) and matter VEVs (4.3) we have

$$\left|\frac{\partial W}{\partial\phi_{AI}}\right|^2=|u_+|^{4\lambda}|A_{AI}|^2\prod_{J\neq I}|\eta(t_J)|^{-4} \tag{4.6}$$

where $|A_{AI}|^2=|\Sigma_n c_n p_n^{AI}|^2$ is a constant. Hence for the matter F terms we find

$$e^{-K}(1+b_{E_8}l)\frac{\partial V}{\partial t_J}\supset\sum_{A,I}\left[\frac{\partial K}{\partial t_J}x_I+\frac{\partial x_I}{\partial t_J}+x_I\frac{\partial}{\partial t_J}\right]\left|\frac{\partial W}{\partial\phi_{AI}}\right|^2$$

$$=\sum_{A,I}x_I\left|\frac{\partial W}{\partial\phi_{AI}}\right|^2\left[\sum_K-\frac{1}{x_K}\frac{\partial x_K}{\partial t_J}\right.$$

$$\left.+\frac{1}{x_I}\frac{\partial x_I}{\partial t_J}+2\xi(t_J)(\delta_I^J-1)\right]+\cdots$$

$$=\sum_{A,I\neq J}-\frac{x_I}{x_J}\left|\frac{\partial W}{\partial\phi_{AI}}\right|^2$$

$$\times[2\xi(t_J)(t_J+\bar{t}_J)+1]+\cdots \tag{4.7}$$

where the ellipses represent terms proportional to $\partial|u_+|^2/\partial t_J$. Hence the matter F terms are also minimized at the self-dual points. Considering finally the moduli F terms, from Eqs. (4.2) it is clear that these are *not* minimized at the self-dual points, but rather where $2\xi(t_I)x_I+1=0$. However, at the self-dual points the only remaining pieces of the moduli F terms are

$$\frac{(b_{E_8}-b_+)^2}{4(1+b_{E_8}l)}\sum_I\left|\xi(t_I)u_++\sum_A|\phi_{AI}|^2\right|_2. \tag{4.8}$$

Since $|u_+|^2\ll 1$ for the theory to be under control, it is clear that Eq. (4.8) is negligible compared to either the dilaton or matter F terms in Eq. (4.2) for generic scalar field VEVs of the form (4.3), irrespective of λ. Hence the overall potential is minimized[4] when $4\xi(t_I)\mathrm{Re}(t_I)+1=0$ and we may neglect the moduli F terms henceforth. This approximation will of course eventually cease to be valid as the universe evolves—the remaining small pieces from the moduli F terms will eventually destabilize the potential and drive $\langle|\phi_{AI}|^2\rangle\to 0$ for at least one field in each term in the superpotential and its derivatives, as is the case in the true vacuum today.

Let us now return to our primary goal—the stabilization of the dilaton during inflation. We have seen that, whatever the value of λ the inflationary potential will be dominated by either the matter or dilaton F terms. With $t_I=e^{i\pi/6}$ these are

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[3] In [27,36] the authors followed an alternative route and obtained the inflationary scale by vacuum shifting to cancel a D term.

[4] It may further be shown that the points $t_I=e^{i\pi/6}$ are minima, whereas $t_I=1$ are saddle points of the potential; see [33,34].

$$V = \frac{e^K}{1 + b_{E_8} l} |u_+|^{4\lambda} |\eta(e^{i\pi/6})|^{-8} \sum_{A,I} |A_{AI}|^2 + V_0$$

$$= C \frac{l e^g}{1 + b_{E_8} l} |u_+|^{4\lambda} + V_0 \tag{4.9}$$

for $C = |\eta(e^{i\pi/6})|^{-8} \Sigma_{A,I} |A_{AI}|^2 / 3$ where with $\mathcal{O}(1)$Yukawa couplings we expect $1 \leq C \leq 10$. In Eq. (4.9), V_0 is the form of the scalar potential today as given by Eq. (3.1), although of course as yet we do not know its value during inflation.

As explained in the Introduction, if one is to justify the standard predictions of inflationary models whose slow-roll parameters come from higher-order, coupling-constant dependent terms in the potential (perhaps arising as quantum corrections) then it is necessary that the dilaton be stabilized somewhere near to its value today. Here we do not attempt to solve the Brustein-Steinhardt problem [10] and explain how the dilaton dynamically settled into this minimum (e.g. rather than the one at $l = 0$ which is always present), but we simply try to ensure that if it *can* be solved before inflation, it will not arise again afterwards. In order to achieve a minimum in the dilaton potential close to the one today, it again appears that there are two separate cases to consider. Firstly, λ may be large enough that the matter F terms are typically much smaller than the V_0 contribution (at values of $l \neq l_0$), and therefore V_0 would then drive $l \to l_0$ irrespective of the matter term. Since we must have $|u_+| \ll 1$ for a sub-Planckian condensate, this will occur when $\lambda > 1/2$ as then the matter F terms are typically suppressed compared to the V_0 term. Secondly, it may be that the matter F term itself has a minimum close to $l = l_0$. To examine this case, consider the matter F terms. Using Eqs. (2.5) and (2.6), these are minimized when

$$lg' + 1|_{l=l_i} = \frac{b_{E_8} b_+ l^2}{(2\lambda(1 + b_+ l) + b_+ l)(1 + b_{E_8} l)}\bigg|_{l=l_i}$$

$$\approx \frac{b_{E_8} b_+ l^2}{2\lambda + b_+ l}\bigg|_{l=l_i}, \tag{4.10}$$

where in the second line I make the usual weak-coupling approximations $b_{E_8} l_i$, $b_+ l_i \ll 1$. Comparison with Eq. (3.7) shows that such a value can only exist if

$$\lambda < \frac{b_{E_8}}{6b_+}. \tag{4.11}$$

The point is that the functions $f(l)$ and $g(l)$ are supposed to be determined by non-perturbative aspects of the underlying string model. We have specified phenomenologically desirable properties in Sec. III but once these are fixed, we are not free to retune the functions during inflation.[5] Expanding lg' around $l = l_0$ gives

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[5]In particular this is true if the t_I moduli are fixed at the same place during inflation as today, even if $g(l) \to g(l, t_I)$, though it is not clear that this would allow such a stabilization.

$$\frac{b_{E_8} b_+ l_i^2}{2\lambda + b_+ l_i} = 3b_+^2 l_0^2 + 6b_+^2 l_0 (l_i - l_0)$$

$$+ \sum_{n=2}^{\infty} \frac{d^n(lg')}{dl^n}\bigg|_{l_0} \frac{(l_i - l_0)^n}{n!} \tag{4.12}$$

where I have used Eqs. (3.2), (3.3) and (4.10) to evaluate the zeroth and first order terms. In this equation, the derivatives in the sum are (linear combinations of) those in Eq. (3.8), evaluated at the same place $l = l_0$ and are therefore of the same magnitude. Since the explicit terms on the left-hand side (LHS) and right-hand side (RHS) of Eq. (4.12) are much smaller than those of Eq. (3.8), it is clear that the difference $|l_i - l_0|$ between the dilaton minima during inflation and today will not cause a significant change in α. Additionally, for typical compactifications [31] the RHS of Eq. (4.11) is at least 2/3 so this case smoothly overlaps with the previous one where $l \to l_0$ because of the presence of V_0. We therefore have the remarkable conclusion that *all* VEVs of the form (4.3) will stabilize the dilaton at a value with $\alpha \approx \alpha_0$. As explained in the Introduction, this provides some justification for the usual predictions of inflationary model building.

Let us now investigate the dilaton mass. In comparing this to the Hubble rate, it is of course important to consider the field D with canonical kinetic terms. Equation (2.3) shows that this is given by

$$\frac{1}{4} \partial_\mu D \partial^\mu D = \frac{(lg' + 1)}{4l^2} \partial_\mu l \partial^\mu l \tag{4.13}$$

and hence we now define

$$\eta_D \equiv \frac{1}{V} \frac{d^2 V}{dD^2}\bigg|_{l_i} = \frac{1}{V} \left(\frac{dl}{dD}\right)^2 \frac{d^2 V}{dl^2}\bigg|_{l_i}. \tag{4.14}$$

Again using the approximation $l_i = l_0$ and now considering the case where $l < 1/2$ so that the matter term dominates in its own right, we have

$$\eta_D \approx (lg')' \frac{(2\lambda + b_+ l_i)^2}{b_{E_8} b_+^2 l_i^2} - \frac{4\lambda + 1}{b_+ l_i} \tag{4.15}$$

and so in order to hold the dilaton in place at its minimum during inflation we require

$$(lg')'|_{l_i} \gg \frac{4\lambda + b_+ l_i}{(2\lambda + b_+ l_i)^2} b_{E_8} b_+ l_i \tag{4.16}$$

which is not unreasonable, given that $b_+ l_i \ll 1$. By comparison, at $l = l_0$ we have $(lg')' \approx 6b_+^2 l_0$ but we may expect it to be significantly larger here as higher order curvature terms come into play.

Finally, consider the inflationary energy scale. The energy density during inflation will always be of the order of the matter F terms. This is because either this term dominates in its own right (for $\lambda < 1/2$) or V_0 dominates and attempts to minimize itself at $l = l_0$ where $V_0 = 0$, with this minimization

now leaving a residual piece of the order of the matter F terms. This is in accordance with the standard assumptions of inflationary model building [18,21], but here it emerges as the only residual term if typical scalars acquire VEVs as in Eq. (4.3) and the moduli and dilaton are stabilized. Taking $l_i = l_0$ as a crude approximation, we find

$$V^{1/4} \approx |u_+|^\lambda \left(\frac{le^g}{1 + b_{E_8} l} \right)^{1/4}_{l_0}$$

$$\approx |u_+|^{\lambda + 1/2} A^{-1/2} \exp\left[\frac{1}{8\pi\alpha b_+} \right]_{l_0} \tag{4.17}$$

where in the second line I have used Eq. (2.6) and neglected $\mathcal{O}(1)$ factors from the t_I moduli and Yukawa coupling terms. This may be evaluated using Eqs. (3.5), (3.6) once the full group structure, matter representations and Yukawa couplings of the hidden sector are specified. However, it is clear that a very wide range of inflationary scales are possible using the general scheme (4.3). Purely as an example, a particularly natural choice might be to take $A \approx 1$, $b_+ \approx 0.05$ and $\lambda = 1/3$ so that $|\phi_{AI}| \propto \Lambda_c$. This gives $V \approx 10^{14}$ GeV, which is comfortably compatible with the COBE bound (4.1).

V. CONCLUSIONS

In this paper I have argued that the predictions for the spectral index of fluctuations n_k that follow from inflationary models usually assume that the GUT scale coupling was the same during inflation as today. This could be false if the dilaton was stabilized at a different value during inflation, and we could even lose all calculability if either the theory became strongly coupled or there was no (non-trivial) dilaton minimum. I have not addressed the complicated issue of why we should expect the theory to have a weak-coupling description in the first place [12], but merely assumed that it does. I have then shown that, if dilaton stabilization is due to corrections to the Kähler potential [6,8], then during inflation the dilaton will be stabilized at a value which causes negligible change in α, provided the inflationary energy density arises from VEVs of the form (4.3). I do not claim that this is the only way of achieving such a result. Indeed in [27,36] other mechanisms were proposed. Nor do I claim that it is not possible to achieve such results using racetrack models [4] although perhaps the "competing condensates" makes it less likely. However, the scheme presented here is both natural and generic, relying on the same mechanism as is known to provide VEVS in the visible sector. Additionally, I have not assumed any particular form for the non-perturbative correction $g(l)$, except to suppose that it can provide reasonable phenomenology today. Therefore it is perhaps worthwhile to see if we can understand these results on a slightly deeper level.

In the absence of any corrections to the Kähler potential, we would find α rising as l, but Eq. (3.8) shows that corrections of the form (2.4) lead to a flattening of this dependence, which is tuned to occur around the phenomenologically acceptable value. Of course, as l departs from l_0, higher de-

rivative terms in Eq. (3.8) will eventually cause significant changes in α. The presence of the "flattened slope" in α is precisely a consequence of the vanishing of the cosmological constant in today's vacuum, as expressed in (3.2)–(3.4). Why is this so? The question essentially asks why the (ln of) the coupling should be the Kähler potential for the dilaton. This is of course just a consequence of the supersymmetry structure of the higher dimensional supergravity theory; $D = 10$ SUGRA requires a dilaton to complete its supermultiplet structure, and supersymmetry then dictates how its Kähler potential and coupling to other fields are related, whatever frame is used. Now, the condensate itself depends on the dilaton only through the Kähler potential and α as in Eq. (2.6). Of course, this must be so. Therefore it is clear that there will be a flattening of the condensate's dilaton dependence near today's value l_0 [this is precisely what was tuned in Eqs. (3.2)–(3.4)]. Hence, if we can find a way to make the inflationary energy density depend on the condensate in roughly the same way as the residual form of the scalar potential today, we should expect it to have a flattened dependence near $l = l_0$. Such is the idea behind Eq. (4.3) and it is pleasing that this is a very natural expectation. Can we make the inflationary dilaton minimum l_i close enough to l_0 that α is unaffected? Equation (4.12) shows that the answer is yes, because the same higher order (logarithmic) derivatives that destabilize α also describe the difference between the l_i and l_0. Clearly, this is because the corrections to α and the Kähler potential are related as in Eq. (2.5) which is simply a consequence of supersymmetry. Now, both sides of Eq. (4.12) are $\ll 1$ and so we need to "mix in" far less of the higher order derivative terms in order to satisfy this compared to the amount that would cause an appreciable variation in α.

Although this paper is written with hybrid inflation in mind, such as commonly arises in string inspired constructions, no attempt has been made to specify any inflationary model. In particular, I have not identified a slow-roll field, or the slope of the potential down which it evolves and this deserves some comment. Candidate slow-roll fields could arise from a number of sources. Firstly, it is possible to imagine that whatever the setup the condition (4.3) is not completely stable, so the inflaton is some linear combination of the matter scalar fields. Secondly, we could assume that a VEV of the form (4.3) is not in fact generic, with only certain fields taking this form, others being driven to zero. In this case, one could find a situation where one (combination) of the t_I moduli is left undetermined by the dominant terms in the potential (see, e.g. [21]) thus providing a slow-roll field. These issues have not been dealt with here. However, we have shown that with Eq. (4.3) the dilaton and moduli are minimized so as to provide a constant energy density. Whatever terms destabilize this and lead to the true vacuum must arise from terms in Eq. (2.10) shown to be "negligible" in comparison to those kept in Eq. (4.9). Hence, they are bound to lead to *slow* roll, at least in the vicinity of the potential (4.9). The assumption here is that the VEVs (4.3) can be preserved in that form on a time scale comparable to, or larger than, the slow-roll time scale. The general theme here was not to build new inflationary models, but rather to try to

provide some justification for the predictions of standard ones [18], should they be implemented in a string context. With Kähler stabilization of the dilaton, this appears to be quite plausible. It would be interesting to know whether racetrack models, or more exotic ideas, lead to the same results. In particular, it would be of great interest to know whether this can raise models of gaugino condensation in the strongly coupled heterotic M-theory [38], where the necessity of stabilizing the dilaton and moduli together becomes more apparent. Finally, it is clearly important to try to pursue these ideas into post-inflationary epochs so as to attempt to make contact with apparent experimental results [13–16].

ACKNOWLEDGMENTS

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