Neutrino clustering in the galaxy with a global monopole

Tae Hoon Lee* and Bruce H. J. McKellar[†]

School of Physics, University of Melbourne, Victoria 3010, Australia
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In spherically symmetric, static spacetime, we show that only $j=1/2$ fermions can satisfy both Einstein's field equation and Dirac's equation. It is also shown that neutrinos are able to have effective masses and cluster in the galactic halo when they are coupled to a global monopole situated at the galactic core. The astronomical implications of the results are discussed.

I. INTRODUCTION

In the Barriola-Vilenkin global monopole solution [1] to Einstein's field equation, scalar fields with global $O(3)$ broken symmetry are minimally coupled to gravity and the background spacetime has deficits of angle. Nucamendi *et al.* [2] suggested that the global monopole solution could explain the flat rotational velocity curves (FRVC) of stars in galaxies because its energy density is proportional to $1/r^2$ and hence it can be dark matter in the galatic halo. Even though some questions were raised [3] about the global monopole solution for FRVC, various generalized versions of the global monopole were studied as models for dark matter and dark energy [4].

Neutrino clustering was studied by some authors including one of us [5] to explain the continuation of the cosmic ray spectrum beyond the GZK (Greisen-Zatsepen-Kuzmin) cutoff. The origin of the GZK cutoff is degradation of the proton energy by the nuclear resonant scattering process on the cosmic microwave background radiation. If the highest energy protons come from a rather nearby source, proton energy is not lost significantly. It seems likely that there are not enough sources within 50 Mpc to explain the observed GZK-violating events, challenging standard theory. Neutrino-antineutrino(ν-$\bar{\nu}$) annihilation to a Z^0 boson is one possible explanation for the phenomena, because neutrinos can propagate unimpeded from distant ($\geqslant 100$ Mpc) sources of the highest energy to Earth. But for ν-$\bar{\nu}$ annihilation into a Z^0 boson to produce enough super GZK air shower, a neutrino flux at $E_\nu \geqslant 10^{21}$ eV with $m_\nu \sim$ a few eV and a significant clustering of the relic neutrino density in our galactic halo are required [6].

In this article we investigate if neutrinos can coalesce into neutrino clouds in a curved spacetime. Considering fermions coupled to an $O(3)$ triplet of scalar fields in the most general static metric with spherical symmetry, we show that only the total angular momentum $j=1/2$ fermions can satisfy both Einstein's field equation and Dirac's equation. When the global $O(3)$ symmetry of the Lagrangian is spontaneously bro-

ken to $U(1)$ at the ground state of the scalar potential in the shape of a Mexican hat, we can have a global monopole solution similar to that of Barriola and Vilenkin. Being coupled to a global monopole situated at the galactic center, neutrinos are then able to have effective masses and cluster in the galactic halo.

II. GLOBAL $O(3)$ SYMMETRIC MODEL FOR NEUTRINO CLUSTERING

A. General relativistic formulation

The action of the global $O(3)$ symmetric model of scalar fields Φ^m and massive neutrinos $\Psi_n (m,n=1,2,3)$ minimally coupled to gravity is given by

$$S = \int d^4x \sqrt{-g} \mathcal{L}, \tag{1}$$

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi^m \partial_\nu \Phi^m - V(\Phi^2) + \frac{i}{2}(\bar{\Psi}_n \gamma^a \nabla_a \Psi_n$$
$$- \nabla_a \bar{\Psi}_n \gamma^a \Psi_n) - m_n \bar{\Psi}_n \Psi_n + \frac{g_y}{v} \Phi^m \Phi^m \bar{\Psi}_n \Psi_n, \tag{2}$$

where $V(\Phi^2)$ is a scalar potential, $\Phi^2 = \Phi^m \Phi^m$, and the last term is the scalar-neutrino interaction for which we assume the first leading order term preserving $O(3)$ symmetry, scaled by the symmetry breaking scale v of the more fundamental theory. Varying the action with respect to the fields, we obtain following equations for scalar fields Φ^m and neutrinos Ψ_n:

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi^m) - \frac{\partial V}{\partial \Phi^m} + 2\frac{g_y}{v} \bar{\Psi}_n \Psi_n \Phi^m = 0, \tag{3}$$

$$i\gamma^a \nabla_a \Psi_n - m_n \Psi_n + \frac{g_y}{v} \Phi^2 \Psi_n = 0, \tag{4}$$

where the γ^a matrices satisfy the Clifford algebra in a locally flat inertial coordinate

$$\{\gamma^a, \gamma^b\} = -2\eta^{ab}, \tag{5}$$

with

$$\eta^{ab} = \text{Diag}(-1,1,1,1). \tag{6}$$

*Permanent address: Department of Physics, Soongsil University, Seoul 156-743, Korea. Electronic address: thlee@physics.ssu.ac.kr

[†]Electronic address: b.mckellar@physics.unimelb.edu.au

The covariant derivative

$$\nabla_a = e^\mu{}_a(\partial_\mu + \Gamma_\mu) \tag{7}$$

is constructed from the vierbein $e^\mu{}_a$ and spin connection Γ_μ, which we give explicitly in Eqs. (A4)–(A11).

Using the standard definition of the energy-momentum tensor

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = -\frac{e_{a\mu}}{\det\{e\}}\frac{\delta S}{\delta e_a{}^\nu}, \tag{8}$$

we have

$$T_{\mu\nu} = \partial_\mu \Phi^m \partial_\nu \Phi^m - g_{\mu\nu}[\tfrac{1}{2}\partial^\beta \Phi^m \partial_\beta \Phi^m + V(\Phi^2)]$$

$$-\frac{i}{4}[(\bar\Psi_n \gamma_\mu \nabla_\nu \Psi_n - \nabla_\nu \bar\Psi_n \gamma_\mu \Psi_n)$$

$$+(\mu \leftrightarrow \nu)]. \tag{9}$$

The energy-momentum tensor allows us to construct the Einstein equation

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{10}$$

where $G_{\mu\nu}$ is the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R, \tag{11}$$

$R_{\mu\nu}$ is the Ricci tensor and $\kappa = 8\pi G$.

B. Ansatz for the global $O(3)$ model in the spherically symmetric spacetime

When we consider the following potential of scalar fields:

$$V(\Phi^2) = \frac{\lambda}{4}(\Phi^2 - \eta^2)^2, \tag{12}$$

with a constant η, the global $O(3)$ symmetry is spontaneously broken to $U(1)$. The scalar field configuration describing a global monopole is known to be given by the hedgehog ansatz

$$\Phi^m = F(r)\frac{x_m}{r}, \tag{13}$$

with a real function $F(r)$ and $r = (x_m x_m)^{1/2} = (x^2 + y^2 + z^2)^{1/2}$.

Assume the line element of the spherically symmetric, static spacetime as

$$ds^2 = -\delta^2(r)\alpha^2(r)dt^2 + \frac{dr^2}{\alpha^2(r)} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2. \tag{14}$$

In the Cartesian coordinates

$$x^\mu = (t, x^m) = (t, x, y, z) \quad \text{for} \quad m = 1,2,3, \tag{15}$$

the line element can be written as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \tag{16}$$

with the metric tensor

$$g_{\mu\nu} = -\delta^2 \alpha^2 \delta^t{}_\mu \delta^t{}_\nu + \left[\delta_{mn} + \left(\frac{1}{\alpha^2} - 1\right)\frac{x_m x_n}{r^2}\right]\delta^m{}_\mu \delta^n{}_\nu, \tag{17}$$

and its inverse

$$g^{\mu\nu} = -\frac{1}{\delta^2 \alpha^2}\delta^\mu{}_t \delta^\nu{}_t + \left[\delta_{mn} + (\alpha^2 - 1)\frac{x_m x_n}{r^2}\right]\delta^\mu{}_m \delta^\nu{}_n. \tag{18}$$

From the standard definition of Christoffel symbols

$$\Gamma^\mu_{\alpha\beta} = \tfrac{1}{2}g^{\mu\nu}(g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu}), \tag{19}$$

the non-zero components are

$$\Gamma^t_{tm} = \frac{x_m}{2r}\left(\frac{\delta^2{}_{,r}}{\delta^2} + \frac{\alpha^2{}_{,r}}{\alpha^2}\right),$$

$$\Gamma^l_{tt} = \frac{x_l \alpha^2}{2r}(\delta^2 \alpha^2{}_{,r} + \delta^2{}_{,r}\alpha^2),$$

$$\Gamma^l_{mn} = \frac{x_l \alpha^2}{2r}\left[-\frac{\alpha^2{}_{,r}}{\alpha^4}\frac{x_m x_n}{r^2} + (\alpha^{-2} - 1)\right.$$

$$\left.\times\left(\delta_{mn} - \frac{x_m x_n}{r^2}\right)\frac{2}{r}\right], \tag{20}$$

where $\delta^2{}_{,r} \equiv (\partial \delta^2/\partial r)$, ... and the relations

$$\frac{\partial \delta^2}{\partial x_m} = \frac{\partial \delta^2}{\partial r}\frac{\partial r}{\partial x_m} = \frac{\partial \delta^2}{\partial r}\frac{x_m}{r} \equiv \delta^2{}_{,r}x_m/r, \ldots$$

have been used for simplicity.

In the metric given in Eq. (14), the scalar field equation reads

$$\alpha^2 F_{,rr} + \frac{\alpha^2}{2}F_{,r}\left(\frac{\delta^2{}_{,r}}{\delta^2} + 2\frac{\alpha^2{}_{,r}}{\alpha^2} + \frac{4}{r}\right) - \frac{2F}{r^2} - \frac{\partial V}{\partial F}$$

$$+ \frac{2g_y}{v}\bar\Psi_n \Psi_n F = 0, \tag{21}$$

where $F_{,r} \equiv (\partial F/\partial r), F_{,rr} \equiv (\partial^2 F/\partial r^2), \ldots$. Equations of motion for neutrinos and Einstein equation in the spherically symmetric, static spacetime will be given in the next section, and calculations of vierbeins $e^\mu{}_a$ and spin connections Γ_μ in the spacetime will be given in the Appendix.

III. SPHERICAL SYMMETRY AND $j = \frac{1}{2}$ FERMIONS

Using the results of the Appendix, we can show that neutrino field equations

$$i\gamma^a e^\mu{}_a(\partial_\mu + \Gamma_\mu)\Psi_n - m_n\Psi_n + \frac{g_y}{v}\Phi^2\Psi_n = 0, \quad (22)$$

become

$$\gamma^0 \frac{i}{\alpha\delta}\partial_t\Psi_n + \gamma^1 i\alpha\hat{D}_r\Psi_n + \gamma^2 \frac{i}{r}\left(\partial_\theta + \frac{\cos\theta}{2\sin\theta}\right)\Psi_n$$

$$+ \gamma^3\frac{i}{r\sin\theta}\partial_\phi\Psi_n - \left(m_n - \frac{g_y}{v}\Phi^2\right)\Psi_n = 0, \quad (23)$$

in the spherically symmetric and static metric in Eq. (14). Equivalently, we can write the last equation as

$$i\partial_t\Psi_n = \hat{H}\Psi_n, \quad (24)$$

where the Hamiltonian is defined as

$$\hat{H} = \alpha\delta\left[\gamma^0\gamma^1\frac{\alpha}{i}\hat{D}_r + \gamma^1\frac{1}{ir}\hat{k} + \gamma^0\left(m_n - \frac{g_y}{v}\Phi^2\right)\right], \quad (25)$$

with

$$\hat{D}_r = \partial_r + \frac{1}{r} + \frac{1}{4}\left(\frac{\delta^2,_r}{\delta^2} + \frac{\alpha^2,_r}{\alpha^2}\right). \quad (26)$$

Here we have defined the operator $\hat{k}$ as [7,8]

$$\hat{k} \equiv i\gamma^0\gamma^1\left(\gamma^2\frac{1}{i\sqrt{\sin\theta}}\partial_\theta\sqrt{\sin\theta} + \gamma^3\frac{1}{i\sin\theta}\partial_\phi\right), \quad (27)$$

which commutes with the Hamiltonian operator $\hat{H}$

$$[\hat{k}, \hat{H}] = 0. \quad (28)$$

Taking the representation of γ matrices to be direct products of independent Pauli matrices σ_i, ρ_j (i.e. $\vec{\rho} \otimes \vec{\sigma}$ representation) [7,8],

$$\gamma^0 = \rho_2, \quad \gamma^1 = i\rho_1, \quad \gamma^2 = -i\rho_3\sigma_3, \quad \gamma^3 = -i\rho_3\sigma_1, \quad (29)$$

we can represent $\hat{k}$ in 2-dimensional σ space

$$\hat{k} = \sigma_3\frac{1}{i\sqrt{\sin\theta}}\partial_\theta\sqrt{\sin\theta} + \sigma_1\frac{1}{i\sin\theta}\partial_\phi, \quad (30)$$

and we can solve the eigenvalue equation

$$\hat{k}\mathcal{Y}_k^m(\theta,\phi) = k\mathcal{Y}_k^m(\theta,\phi), \quad (31)$$

for the 2-dimensional spinor spherical harmonics [8]

$$\mathcal{Y}_k^m(\theta,\phi) = \frac{e^{im\phi}}{\sqrt{2\pi}}\left[\frac{(j+m)!}{(j-m)!}\right]^{1/2}\frac{\left(\tan\frac{\theta}{2}\right)^{\sigma_2/2}}{\sin^m\theta}\left(\frac{\partial}{\partial\cos\theta}\right)^{j-m}$$

$$\times\frac{\sin^{2j}\theta}{2^j(j-\frac{1}{2})!}\left(\tan\frac{\theta}{2}\right)^{-\sigma_2}u_k, \quad (32)$$

where

$$u_k = \frac{1}{\sqrt{2|k|}}\binom{|k|}{k}, \quad (33)$$

with $k = \pm(j+\frac{1}{2})$ and $j = l \pm \frac{1}{2}$. In the representation (29) for γ matrices, the Hamiltonian is expressed in terms of 2-dimensional ρ matrices

$$\hat{H} = \alpha\delta\left[\rho_3\frac{\alpha}{i}\hat{D}_r + \rho_1\frac{1}{r}\hat{k} + \rho_2\left(m_n - \frac{g_y}{v}\Phi^2\right)\right]. \quad (34)$$

The eigenfunctions of both $\hat{H}$ and $\hat{k}$ can be written as direct products of the ρ-space spinor $\psi_n^{(km)}$ and σ-space spinor $\mathcal{Y}_k^m$,

$$\Psi_n(t,r,\theta,\phi) = e^{-iE_nt}\sum_{-j\leqslant m\leqslant j}\psi_n^{(km)}(r)\mathcal{Y}_k^m(\theta,\phi), \quad (35)$$

and the neutrino equations $i\partial_t\Psi_n = \hat{H}\Psi_n$ then read

$$\frac{E_n}{\alpha\delta}\psi_n^{(km)} = \rho_3\frac{\alpha}{i}\hat{D}_r\psi_n^{(km)} + \left[\rho_1\frac{k}{r} + \rho_2\left(m_n - \frac{g_y}{v}\Phi^2\right)\right]\psi_n^{(km)}. \quad (36)$$

Moreover, if we put

$$\psi_n^{(km)}(r) \equiv \frac{1}{r}(\alpha^2\delta^2)^{-1/4}R_n^{(km)}(r), \quad (37)$$

since

$$\hat{D}_r\psi_n^{(km)} = \frac{1}{r}(\alpha^2\delta^2)^{-1/4}\partial_r R_n^{(km)}, \quad (38)$$

then we get

$$\frac{E_n}{\alpha\delta}R_n^{(km)} = \rho_3\frac{\alpha}{i}\partial_r R_n^{(km)} + \left[\rho_1\frac{k}{r} + \rho_2\left(m_n - \frac{g_y}{v}\Phi^2\right)\right]R_n^{(km)}. \quad (39)$$

Setting

$$R_n^{(km)} = \begin{pmatrix}\varphi_n^{(km)} \\ \chi_n^{(km)}\end{pmatrix}, \quad (40)$$

we obtain the coupled equations

$$\left(\frac{E_n}{\alpha\delta} + i\alpha\partial_r\right)\varphi_n^{(km)} = \left[\frac{k}{r} - i\left(m_n - \frac{g_y}{v}\Phi^2\right)\right]\chi_n^{(km)},$$

$$\left(\frac{E_n}{\alpha\delta} - i\alpha\partial_r\right)\chi_n^{(km)} = \left[\frac{k}{r} + i\left(m_n - \frac{g_y}{v}\Phi^2\right)\right]\varphi_n^{(km)},$$
(41)

asymptotic solutions to which are given in the following section.

Einstein's equation $G_{\mu\nu} = \kappa T_{\mu\nu}$ can be rewritten as

$$R_{\mu\nu} = \kappa(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}T_\alpha{}^\alpha).$$
(42)

Substituting Eq. (9) into the Einstein's equation in Eq. (42) gives

$$R_{\mu\nu} = \kappa\left[\partial_\mu\Phi^m\partial_\nu\Phi^m + g_{\mu\nu}V + \frac{i}{4}g_{\mu\nu}\Sigma - \frac{i}{4}(\bar{\Psi}_n\gamma_\mu\nabla_\nu\Psi_n\right.$$
$$\left. - \nabla_\mu\bar{\Psi}_n\gamma_\nu\Psi_n + \bar{\Psi}_n\gamma_\nu\nabla_\mu\Psi_n - \nabla_\nu\bar{\Psi}_n\gamma_\mu\Psi_n)\right], \quad (43)$$

with

$$\Sigma = e^\alpha{}_c(\bar{\Psi}_n\gamma^c\partial_\alpha\Psi_n - \partial_\alpha\bar{\Psi}_n\gamma^c\Psi_n).$$
(44)

Elements of Ricci tensor calculated from metric coefficients in Eq. (14) and the ansatz for Φ^m in Eq. (13) are substituted into Eq. (43), to give the set of equations.

$$\frac{1}{2\delta^2}(\delta^2\alpha^2)_{,rr} - \frac{(\delta^2\alpha^2)_{,r}}{4\delta^2}\frac{\delta^2_{,r}}{\delta^2} + \frac{1}{r}\frac{(\delta^2\alpha^2)_{,r}}{\delta^2}$$
$$= \kappa\left[-V - \frac{i}{4}\Sigma + \frac{i}{2\delta\alpha}(\bar{\Psi}_n\gamma^0\partial_t\Psi_n - \partial_t\bar{\Psi}_n\gamma^0\Psi_n)\right],$$
(45)

$$-\frac{1}{2\delta^2}(\delta^2\alpha^2)_{,rr} + \frac{(\delta^2\alpha^2)_{,r}}{4\delta^2}\frac{\delta^2_{,r}}{\delta^2} - \frac{1}{r}\alpha^2_{,r}$$
$$= \kappa\left[\alpha^2(F_{,r})^2 + V + \frac{i}{4}\Sigma\right.$$
$$\left. - \frac{i\alpha}{2}(\bar{\Psi}_n\gamma^1\partial_r\Psi_n - \partial_r\bar{\Psi}_n\gamma^1\Psi_n)\right],$$
(46)

$$\frac{1}{r^2}(1-\alpha^2) - \frac{1}{r}\alpha^2_{,r} - \frac{\alpha^2}{2r}\frac{\delta^2_{,r}}{\delta^2}$$
$$= \kappa\left[\frac{F^2}{r^2} + V + \frac{i}{4}\Sigma - \frac{i}{2r}(\bar{\Psi}_n\gamma^2\partial_\theta\Psi_n - \partial_\theta\bar{\Psi}_n\gamma^2\Psi_n)\right],$$
(47)

$$\frac{1}{r^2}(1-\alpha^2) - \frac{1}{r}\alpha^2_{,r} - \frac{\alpha^2}{2r}\frac{\delta^2_{,r}}{\delta^2}$$
$$= \kappa\left[\frac{F^2}{r^2} + V + \frac{i}{4}\Sigma - \frac{i}{2r\sin\theta}(\bar{\Psi}_n\gamma^3\partial_\phi\Psi_n\right.$$
$$\left. - \partial_\phi\bar{\Psi}_n\gamma^3\Psi_n)\right].$$
(48)

The first in above equations is the (t,t) component of Eq. (43), the second the (r,r) component, the third the (θ,θ) component, and the last the (ϕ,ϕ) component.

The (θ,θ) component of the Einstein equation is same as the (ϕ,ϕ) component of the Einstein equation if

$$\bar{\Psi}_n\gamma^2\partial_\theta\Psi_n - \partial_\theta\bar{\Psi}_n\gamma^2\Psi_n$$
$$= \frac{1}{\sin\theta}(\bar{\Psi}_n\gamma^3\partial_\phi\Psi_n - \partial_\phi\bar{\Psi}_n\gamma^3\Psi_n).$$
(49)

Using $\vec{\rho}\otimes\vec{\sigma}$ representation for γ matrices and the mathematical identities,

$$\sigma_3\partial_\theta\mathcal{Y}_k^m = \left(\pm\sigma_3 m\cot\theta \pm \sigma_1\frac{i}{2\sin\theta}\right)\mathcal{Y}_k^m$$
$$\pm\sigma_3 e^{\mp i\phi}\sqrt{(j\pm m+1)(j\mp m)}\mathcal{Y}_k^{m\pm1}, \quad (50)$$

this condition becomes

$$\sum_{m,m'}\psi_n^{\dagger(km')}\rho_1\psi_n^{(km)}\left[\mathcal{Y}_k^{\dagger m'}\left\{\pm\sigma_3(m-m')\cot\theta\right.\right.$$
$$\left.\pm\sigma_1\frac{i}{\sin\theta}\right\}\mathcal{Y}_k^m$$
$$\pm\mathcal{Y}_k^{\dagger m'}\sigma_3 e^{\mp i\phi}\sqrt{(j\pm m+1)(j\mp m)}\mathcal{Y}_k^{m\pm1}$$
$$\left.\mp\mathcal{Y}_k^{\dagger m'\pm1}\sigma_3 e^{\pm i\phi}\sqrt{(j\pm m'+1)(j\mp m')}\mathcal{Y}_k^m\right]$$
$$= \sum_{m,m'}\psi_n^{\dagger(km')}\rho_1\psi_n^{(km)}(m+m')\mathcal{Y}_k^{\dagger m'}\sigma_1\frac{i}{\sin\theta}\mathcal{Y}_k^m.$$
(51)

The condition (51) can be satisfied if

$$m = m',$$

$$m + m' = 2m = \pm1,$$

$$j = \tfrac{1}{2},$$
(52)

since

$$\mathcal{Y}_{k(j=1/2)}^{\pm1/2\pm1} = 0.$$
(53)

Moreover, explicit calculations show us that

$$(\mathcal{Y}_{k(j=1/2)}^{\pm 1/2})^{\dagger}\mathcal{Y}_{k(j=1/2)}^{\pm 1/2}=\frac{1}{4\pi},$$

$$(\mathcal{Y}_{k(j=1/2)}^{\pm 1/2})^{\dagger}\sigma_1\mathcal{Y}_{k(j=1/2)}^{\pm 1/2}=\pm\frac{\sin\theta}{4\pi}\frac{k}{|k|}. \tag{54}$$

These make Eqs. (45)–(48) θ and ϕ independent, which is consistent with the spherically symmetric metric in Eq. (14), when we consider only $(j=\frac{1}{2},m=\frac{1}{2})$ or $(j=\frac{1}{2},m=-\frac{1}{2})$ fermions.

For the case $(j=\frac{1}{2},m=\frac{1}{2})$ or $(j=\frac{1}{2},m=-\frac{1}{2})$ with $k=\pm 1$, the neutrino wave function is

$$\Psi_n(t,r,\theta,\phi)=e^{-iE_nt}\psi_n^{(k,\pm 1/2)}(r)\mathcal{Y}_k^{\pm 1/2}(\theta,\phi). \tag{55}$$

Let $\psi_n(r)\equiv\psi_n^{(k,1/2)}(r)$ or $\psi_n^{(k,-1/2)}(r)$ and

$$\psi_n(r)=\frac{1}{r}(\alpha^2\delta^2)^{-1/4}\binom{\varphi_n}{\chi_n}. \tag{56}$$

In this case the term Σ in Eq. (44) reads

$$\Sigma=-\frac{i}{r^2\alpha^2\delta^2}q_0+\frac{i}{r^2\delta}q_1+\frac{i}{r^2\alpha\delta}q_k, \tag{57}$$

with

$$q_0=\frac{2E_n}{4\pi}r^2\alpha\delta\psi_n^{\dagger}\psi_n=\frac{2E_n}{4\pi}(\varphi_n^*\varphi_n+\chi_n^*\chi_n), \tag{58}$$

$$q_1=\frac{-i}{4\pi}r^2\alpha\delta(\psi_n^{\dagger}\rho_3\partial_r\psi_n-\partial_r\psi_n^{\dagger}\rho_3\psi_n)$$

$$=\frac{-i}{4\pi}(\varphi_n^*\partial_r\varphi_n-\partial_r\varphi_n^*\varphi_n-\chi_n^*\partial_r\chi_n+\partial_r\chi_n^*\chi_n), \tag{59}$$

$$q_k=\frac{1}{2\pi}r\alpha\delta\frac{k}{|k|}\psi_n^{\dagger}\rho_1\psi_n=\frac{1}{2\pi r}\frac{k}{|k|}(\chi_n^*\varphi_n+\varphi_n^*\chi_n), \tag{60}$$

$$q_2=\frac{1}{4\pi}r^2\alpha\delta\psi_n^{\dagger}\rho_2\psi_n=\frac{i}{4\pi}(\chi_n^*\varphi_n-\varphi_n^*\chi_n). \tag{61}$$

Using the last equations Einstein's equations are given by

$$\frac{1}{2\delta^2}(\delta^2\alpha^2)_{,rr}-\frac{(\delta^2\alpha^2)_{,r}}{4\delta^2}\frac{\delta^2_{,r}}{\delta^2}+\frac{1}{r}\frac{(\delta^2\alpha^2)_{,r}}{\delta^2}$$

$$=\kappa\left[-V+\frac{1}{4r^2}\left(\frac{q_0}{\delta^2\alpha^2}+\frac{q_1}{\delta}+\frac{q_k}{\alpha\delta}\right)\right], \tag{62}$$

$$\frac{\alpha^2}{r}\frac{\delta^2_{,r}}{\delta^2}=\kappa\left[\alpha^2(F_{,r})^2+\frac{1}{2r^2}\left(\frac{q_0}{\delta^2\alpha^2}+\frac{q_1}{\delta}\right)\right], \tag{63}$$

$$\frac{1}{r^2}(1-\alpha^2)-\frac{1}{r}\alpha^2_{,r}-\frac{\alpha^2}{2r}\frac{\delta^2_{,r}}{\delta^2}$$

$$=\kappa\left[\frac{F^2}{r^2}+V+\frac{1}{4r^2}\left(\frac{q_0}{\delta^2\alpha^2}-\frac{q_1}{\delta}\right)\right], \tag{64}$$

and the scalar field equation is

$$\alpha^2F_{,rr}+\frac{\alpha^2}{2}F_{,r}\left(\frac{\delta^2_{,r}}{\delta^2}+2\frac{\alpha^2_{,r}}{\alpha^2}+\frac{4}{r}\right)-\frac{2F}{r^2}-\frac{\partial V}{\partial F}$$

$$+\frac{2g_y}{v}\frac{q_2}{r^2\delta\alpha}F=0. \tag{65}$$

IV. ASYMPTOTIC SOLUTIONS FOR LARGE r

As in the case of the global monopole solution [1], one component $\alpha(r)$ of the metric in Eq. (14), far away from the galactic core, can be asymptotically taken as

$$\alpha^2\simeq\alpha_o^2\equiv c-\frac{2M}{r}, \tag{66}$$

with a constant c when

$$F\simeq\eta,\quad V(\eta^2)\simeq 0,\quad \frac{\partial V}{\partial F}(\eta^2)\simeq 0. \tag{67}$$

Analyzing Eqs. (62)–(65) with the assumption that q_i's in Eqs. (58)–(61) are $O(r^0)$, we can get an asymptotic solution for another component of the metric in Eq. (14) as

$$\delta^2\simeq A^2\ln\left(\frac{r}{2M}\right), \tag{68}$$

with a constant A. In this limit with the help of Eq. (56) the neutrino equation (41) becomes

$$\partial_r^2\varphi_n\simeq-q^2(r)\varphi_n, \tag{69}$$

with

$$q(r)\equiv\sqrt{\frac{E_n^2}{\delta^2\alpha_o^4}-\frac{m_{o.n}^2}{\alpha_o^2}}, \tag{70}$$

and $m_{o.n}=m_n-(g_y/v)\eta^2$.

In the region

$$0<\ln\left(\frac{r}{2M}\right)\leq\frac{E_n^2}{A^2\alpha_o^2m_{o.n}^2}, \tag{71}$$

we have an asymptotic solution to Eq. (69)

$$\varphi_n\simeq A_ne^{iq(r)r}+B_ne^{-iq(r)r},$$

$$\chi_n \simeq \frac{i}{m_{o.n}} \left[A_n \left(\frac{E_n}{\alpha_o \delta} - \alpha_o q(r) \right) e^{+iq(r)r} \right.$$

$$\left. + B_n \left(\frac{E_n}{\alpha_o \delta} + \alpha_o q(r) \right) e^{-iq(r)r} \right]. \tag{72}$$

With above asymptotic solutions to Dirac's equation for neutrinos, the q_i in Eqs. (58)–(61) are given by

$$q_0 \simeq \frac{E_n}{\pi} \left[B_n^2 \left\{ 1 + \frac{1}{m_{o.n}^2} \left(\frac{E_n^2}{\alpha_o^2 \delta^2} + \alpha_o^2 q^2(r) \right) \right\} \right.$$

$$\left. + A_n B_n (1 + m_{o.n}^2) \cos(2qr) \right],$$

$$q_1 \simeq \frac{2E_n}{\pi} B_n^2 \frac{q^2}{\delta},$$

$$q_k \simeq \frac{2k}{\pi r |k|} A_n B_n \frac{\alpha_o q}{m_{o.n}} \sin(2qr),$$

$$q_2 \simeq \frac{E_n}{\pi m_{o.n} \alpha_o \delta} [A_n B_n \cos(2qr) + B_n^2],$$

with real constants A_n and B_n such that $|A_n| = |B_n|$. The q_i's are $O(r^0)$ or less and so are consistent with the assumption of Eq. (68). For $r > r_o$ with $\ln(r_o/2M) \equiv E_n^2/A^2 \alpha_o^2 m_{o.n}^2$, $q(r)$ in Eq. (70) is pure imaginary and then wave functions of neutrinos become multiplied by exponentially decaying factors. Therefore we get *neutrino clustering* with the radius r_o. Taking $r_o \simeq 10^{23}$ cm which corresponds to the radius of our galaxy and assuming that there exists a supermassive black hole at the center of the galaxy with a mass $M_{SBH} \simeq 3 \times 10^6 M_\odot$ [12], we can estimate the energy of neutrinos as $E_n \simeq 5 m_{o.n}$, where we have used the relations, $E_n^2/A^2 \alpha_o^2 m_{o.n}^2 \simeq \ln(r_o/2M_{SBH})$ and $A^2 \alpha_o^2 \simeq O(1)$.

V. ASYMPTOTIC SOLUTIONS FOR SMALL r

Next let us study the small r behaviors of components of the metric in Eq. (14), α and δ. For small r near the galactic core we adopt the Thomas-Fermi (TF) approximation as was done in the case of fermion stars [9–11]. In the TF approximation there is at each point in space a Fermi sea of massive neutrinos with the local Fermi momentum $q_F(r)$, which depends only on r because of spherical symmetry. We thus have the energy density and the pressure of neutrinos which depend only on r.

From the local conservation law, $0 = (\bar{\Psi} \gamma^\mu \Psi)_{;\mu} = (1/\sqrt{-g}) \partial_\mu (\sqrt{-g} \bar{\Psi} \gamma^\mu \Psi)$, we have the conserved, total number of neutrinos

$$N = \int d^3 x \sqrt{-g} \bar{\Psi} \gamma^t \Psi = \int d^3 x \sqrt{-g} \Psi^\dagger \Psi e^t_0. \tag{73}$$

The number density of neutrinos in the TF approximation is given by

$$\langle \Psi^\dagger \Psi \rangle_{TF} = \frac{2}{(2\pi)^3} \int d^3 q n_q = \frac{q_F^3(r)}{3\pi^2}, \tag{74}$$

where the Fermi distribution $n_q = \theta(q_F - q)$. From Eq. (74) and the time-dependent Dirac's equation $i\partial_t \Psi_n = E_n \Psi_n$, we can put

$$\left\langle \frac{i}{2\delta\alpha} (\bar{\Psi}_n \gamma^0 \partial_t \Psi_n - \partial_t \bar{\Psi}_n \gamma^0 \Psi_n) \right\rangle_{TF}$$

$$= \frac{2}{(2\pi)^3} \int d^3 q n_q E_n(q) e^t_0 \equiv \rho(r), \tag{75}$$

where ρ is the energy density of neutrinos and $e^t_0 = 1/\delta\alpha$. The stress tensor of neutrinos has the following diagonal elements in the vierbein basis:

$$\left\langle \frac{\alpha}{2i} (\bar{\Psi}_n \gamma^1 \partial_r \Psi_n - \partial_r \bar{\Psi}_n \gamma^1 \Psi_n) \right\rangle_{TF} = p_1,$$

$$\left\langle \frac{1}{2ir} (\bar{\Psi}_n \gamma^2 \partial_\theta \Psi_n - \partial_\theta \bar{\Psi}_n \gamma^2 \Psi_n) \right\rangle_{TF} = p_2, \tag{76}$$

and

$$\left\langle \frac{1}{2ir \sin\theta} (\bar{\Psi}_n \gamma^3 \partial_\phi \Psi_n - \partial_\phi \bar{\Psi}_n \gamma^3 \Psi_n) \right\rangle_{TF} = p_3, \tag{77}$$

respectively.

We assume, in the spherically symmetric spacetime we consider, that $p_1 = p_2 = p_3 \equiv p(r)$. From Dirac's equation (24) and its Hermitian conjugate, we have the relation

$$\frac{i}{\delta\alpha} (\bar{\Psi}_n \gamma^0 \partial_t \Psi_n - \partial_t \bar{\Psi}_n \gamma^0 \Psi_n)$$

$$+ i\alpha (\bar{\Psi}_n \gamma^1 \partial_r \Psi_n - \partial_r \bar{\Psi}_n \gamma^1 \Psi_n)$$

$$+ \frac{i}{r} (\bar{\Psi}_n \gamma^2 \partial_\theta \Psi_n - \partial_\theta \bar{\Psi}_n \gamma^2 \Psi_n)$$

$$+ \frac{i}{r \sin\theta} (\bar{\Psi}_n \gamma^3 \partial_\phi \Psi_n - \partial_\phi \bar{\Psi}_n \gamma^3 \Psi_n)$$

$$= 2m_{eff}(r) \bar{\Psi}_n \Psi_n, \tag{78}$$

with $m_{\text{eff}}(r) = m_n - (g_y/v) F^2(r)$. Since the left hand side of above equation is same as $i\Sigma$ in Eq. (44), we have the following relation in the TF approximation:

$$\frac{i}{2} \langle \Sigma \rangle_{TF} \equiv \rho - 3p = m_{\text{eff}} \frac{v}{2g_y} Q, \tag{79}$$

where we have defined $(v/2g_y)Q(r) \equiv \langle \bar{\Psi}_n \Psi_n \rangle_{TF}$. Thus in the TF approximation the Einstein equations (45)–(48) read

$$\frac{1}{2\delta^2}(\delta^2\alpha^2)_{,rr} - \frac{(\delta^2\alpha^2)_{,r}}{4\delta^2}\frac{\delta^2_{,r}}{\delta^2} + \frac{1}{r}\frac{(\delta^2\alpha^2)_{,r}}{\delta^2}$$

$$= \kappa[-V + \tfrac{1}{2}\rho + \tfrac{3}{2}p], \qquad (80)$$

$$-\frac{1}{2\delta^2}(\delta^2\alpha^2)_{,rr} + \frac{(\delta^2\alpha^2)_{,r}}{4\delta^2}\frac{\delta^2_{,r}}{\delta^2} - \frac{1}{r}\alpha^2_{,r}$$

$$= \kappa[\alpha^2(F_{,r})^2 + V + \tfrac{1}{2}\rho - \tfrac{1}{2}p], \qquad (81)$$

$$\frac{1}{r^2}(1-\alpha^2) - \frac{1}{r}\alpha^2_{,r} - \frac{\alpha^2}{2r}\frac{\delta^2_{,r}}{\delta^2} = \kappa\left[\frac{F^2}{r^2} + V + \frac{1}{2}\rho - \frac{1}{2}p\right], \qquad (82)$$

which are consistent with those in the case of fermion stars [9]. The scalar field equation (21) reads

$$\alpha^2 F_{,rr} + \frac{\alpha^2}{2}F_{,r}\left(\frac{\delta^2_{,r}}{\delta^2} + 2\frac{\alpha^2_{,r}}{\alpha^2} + \frac{4}{r}\right) - \frac{2F}{r^2} - \frac{\partial V}{\partial F} + QF = 0. \qquad (83)$$

If we assume that there exist regular solutions of the metric components, scalar fields, and the energy density and pressure of neutrinos for small r, we can make series expansions of

$$\alpha^2 = \sum_{n\geq 0} a_n r^n, \qquad \delta^2 = \sum_{n\geq 0} b_n r^n$$

$$F = \sum_{n\geq 0} \eta_n r^n, \qquad \rho = \sum_{n\geq 0} \rho_n r^n,$$

$$p = \sum_{n\geq 0} p_n r^n. \qquad (84)$$

Substituting these expansions into Eqs. (80)–(83), we get the following values for the coefficients:

$$a_0 = 1, \quad a_1 = 0, \quad a_2 = -\kappa\left[\frac{1}{3}\rho_0 + \frac{\lambda}{12}\eta^4 + \frac{1}{2}\eta_1^2\right],$$

$$b_1 = 0, \quad b_2 = b_0\kappa[\tfrac{1}{2}\rho_0 + \tfrac{1}{2}p_0 + \tfrac{1}{2}\eta_1^2],$$

$$\eta_0 = 0, \quad \eta_2 = 0, \quad m_n\frac{v}{2g_y}Q_0 = \rho_0 - 3p_0\ldots. \qquad (85)$$

We thus have solutions in the small r region which are

$$F = \eta_1 r + O(r^3),$$

$$\alpha^2 = 1 - \kappa\left[\frac{1}{3}\rho_0 + \frac{\lambda}{12}\eta^4 + \frac{1}{2}\eta_1^2\right]r^2 + O(r^3),$$

$$\delta^2\alpha^2 = 1 + \kappa\left[\frac{1}{6}\rho_0 + \frac{1}{2}p_0 - \frac{\lambda}{12}\eta^4\right]r^2 + O(r^3), \qquad (86)$$

where we have re-parametrized the time as $\sqrt{b_0}\,t \to t$. These solutions are consistent with the results in fermionic stars with a global monopole [11], even if they [11] take $F \simeq 0$ for small r. The value of η_1 is determined by the matching condition that $F(r)$ should be continuous at $r = r_c$, which is the upper bound for small r. This gives us the relation

$$\eta_1 r_c = \eta. \qquad (87)$$

We might put $r_c \simeq 2M_{SBH}$, where M_{SBH} is the mass of the supermassive black hole at the center of our galaxy [12].

VI. NEUTRINO CLUSTERING

In Z-burst models to explain the continuation of the cosmic ray spectrum beyond the GZK cutoff, Weiler [6] and Fargion [13] assumed that the relic neutrinos cluster in galaxies a few times the normal relic density, and found that the required flux of cosmic ray neutrinos is larger than previously suggested. Blanco-Pillado *et al.* [14] suggested that the relic neutrino density in the clustering might be 10^2–10^4 times the mean relic density $n_M \simeq 54\ \mathrm{cm}^{-3}$. Independently in Ref. [5] a neutrino cloud was considered as a sphere with no diffuse boundary such that the relic neutrino density is given by $n_\nu(r) = n_R\theta(R-r) + n_M\theta(r-R)$, where $n_R \simeq 10^{12} - 10^{16}\ \mathrm{cm}^{-3}$ and $R \simeq 10^{14} - 10^{20}\ \mathrm{cm}$. This can moderate the required incident flux of ultrahigh energy neutrinos.

From the results in the previous Secs. IV and V, we can have a cluster of neutrinos with a diffuse boundary such that the density of the relic neutrinos is

$$n_\nu(r) = n_c\theta(r_c - r) + n_o(r)\theta(r - r_c)\theta(r_o - r), \qquad (88)$$

where n_c is the constant neutrino density for small $r(\leq r_c \sim 2M_{SBH})$ as in the Sec. V, and $n_o(r) = n_c r_c^2/r^2$ is the neutrino density for $r_c < r \leq r_o$ as in the Sec. IV. Here $r_o \simeq 10^{23}\ \mathrm{cm}$ (the radius of our galaxy), $r_c \simeq 2M_{SBH} \simeq 10^{12}\ \mathrm{cm}$ (the Schwarzschild radius of a supermassive black hole at the center of the galaxy), and $n_c \geq 10^3(m_{eff}/1\ \mathrm{eV})^3\ \mathrm{cm}^{-3}$ [15].

VII. SUMMARY AND DISCUSSIONS

In the most general static spacetime with spherical symmetry, we explicitly show that only $j = 1/2$ fermions can satisfy Einstein's equation, in the $\vec{\rho} \otimes \vec{\sigma}$ representation of γ matrices [7]. It is also possible to demonstrate this result in other representations of γ matrices. Considering neutrinos coupled to $O(3)$ scalar fields Φ^m via $\Phi^2\bar{\Psi}\Psi$ interaction term, we had a global monopole solution [1] of scalar fields and asymptotic solutions of the metric components to Einstein's equation in Eqs. (66) and (68), for large r. The asymptotic solution of one metric component δ^2 can be given in series of more general functions as $\delta^2(r) \simeq \Sigma_n f_n^2(r)\ln^n(r/2M)$ with $f_n(r \to \infty) = \mathrm{const}$, which do not change the asymptotic behaviors of other solutions in this limit.

We adopted the TF approximation for small r expansions of the energy density, pressure of neutrinos, and so on. Since only s waves are considered in the TF approximation, small r expansions in the Sec. V are consistently related to

asymptotic solutions obtained for large r in the Sec. IV. More rigorous connection between two asymptotic solutions can be made by numerical methods. With the simple assumption, Eq. (88), about the radial dependence of the relic density of neutrinos $n_\nu(r)$ clustered in the galactic halo, further studies of Z-burst models shall be possible. Other realistic calculations for $n_\nu(r)$ are included in a recent work [16] on Z-burst models, even if they [16] used a method based on the collisionless Boltzmann equation to calculate $n_\nu(r)$ and the resultant radial dependence of the neutrino density is a little different from ours. For small r solutions in the Sec. V to have astronomical meaning, the global $O(3)$ symmetry breaking scale η and the scalar self-coupling constant λ in Eq. (12) should be very small, which might be realized in some Majoron models [17]. In these models a scalar field is introduced to provide a Majorana mass for neutrinos. Scalar fields of this type can then provide the neutrino-scalar interactions we have introduced here. The global lepton number of particles in the models is spontaneously broken with a massless Nambu-Goldstone boson, Majoron, and realistically small neutrino masses imply a small value of the lepton number violation scale. In such a case the deficit angle $1 - c \equiv \kappa \eta^2$ in Eq. (66) is negligible [1].

APPENDIX: CALCULATIONS OF VIERBEINS AND SPIN CONNECTIONS

We rewrite the line element in Eq. (14) as

$$ds^2 = \eta_{ab} e^a e^b, \tag{A1}$$

with inverse of η^{ab}, $\eta_{ab} = \text{Diag}(-1,1,1,1)$, and

$$e^0 = \alpha \delta dt, \quad e^1 = \frac{dr}{\alpha}, \quad e^2 = rd\theta, \quad e^3 = r\sin\theta d\phi. \tag{A2}$$

Putting

$$e^a = e^a{}_\mu dx^\mu, \tag{A3}$$

we find the vierbein given by

$$e^a{}_\mu = \begin{pmatrix} \alpha\delta & 0 & 0 & 0 \\ 0 & \dfrac{1}{\alpha}\sin\theta\cos\phi & \dfrac{1}{\alpha}\sin\theta\sin\phi & \dfrac{1}{\alpha}\cos\theta \\ 0 & \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ 0 & -\sin\phi & \cos\phi & 0 \end{pmatrix}, \tag{A4}$$

and their inverse

$$e^\nu{}_b = \begin{pmatrix} \dfrac{1}{\alpha\delta} & 0 & 0 & 0 \\ 0 & \alpha\sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ 0 & \alpha\sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ 0 & \alpha\cos\theta & -\sin\theta & 0 \end{pmatrix} \tag{A5}$$

$$= \begin{pmatrix} \dfrac{1}{\alpha\delta} & 0 & 0 & 0 \\ 0 & \left(\alpha\dfrac{\partial x^m}{\partial r}\right) & \left(\dfrac{1}{r}\dfrac{\partial x^m}{\partial\theta}\right) & \left(\dfrac{1}{r\sin\theta}\dfrac{\partial x^m}{\partial\phi}\right) \end{pmatrix}. \tag{A6}$$

The vierbein satisfy the relations

$$e^a{}_\mu e^\mu{}_b = \delta^a{}_b, \tag{A7}$$

$$e^\nu{}_a e^a{}_\mu = \delta^\nu{}_\mu. \tag{A8}$$

The spin connection which appeared in Eq. (7) is defined as

$$\Gamma_\mu = -\tfrac{1}{4}\gamma^a\gamma^b e_{\nu a} e^\nu{}_{b;\mu}, \tag{A9}$$

with

$$e^\nu{}_{b;\alpha} = e^\nu{}_{b,\alpha} + \Gamma^\nu{}_{\alpha\beta} e^\beta{}_b. \tag{A10}$$

We calculate the spin connections from Eq. (20) and Eq. (A9), and obtain

$$\Gamma_t = \tfrac{1}{4}\gamma^0\gamma^1(2\alpha^2\delta_{,r} + \delta\alpha^2{}_{,r})$$

$$\Gamma_x = \tfrac{1}{2}\gamma^1\gamma^2\frac{\alpha\cos\theta\cos\phi}{r}$$
$$\quad -\frac{1}{2}\gamma^2\gamma^3\frac{\cos\theta\sin\phi}{r\sin\theta} + \frac{1}{2}\gamma^3\gamma^1\frac{\alpha\sin\phi}{r},$$

$$\Gamma_y = \frac{1}{2}\gamma^1\gamma^2\frac{\alpha\cos\theta\sin\phi}{r}$$
$$\quad + \frac{1}{2}\gamma^2\gamma^3\frac{\cos\theta\cos\phi}{r\sin\theta}$$
$$\quad - \frac{1}{2}\gamma^3\gamma^1\frac{\alpha\cos\phi}{r},$$

$$\Gamma_z = -\frac{1}{2}\gamma^1\gamma^2\frac{\alpha\sin\theta}{r}, \tag{A11}$$

which satisfy the Cartan structure equation

$$[\gamma^\nu, \Gamma_\mu] = \gamma^\nu{}_{;\mu}. \tag{A12}$$

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