SU(3) and nonet breaking effects in $K_L \to \gamma\gamma$ induced by $s \to d+2$ gluons due to an anomaly

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In this paper we study the effects of $s \to d+2$ gluon on $K_L \to \gamma\gamma$ in the standard model. We find that this interaction can induce new sizable SU(3) and U(3) nonet breaking effects in K_L-η, η' transitions and therefore in $K_L \to \gamma\gamma$ due to large matrix elements of $\langle \eta(\eta') | \alpha_s G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a | 0 \rangle$ from the QCD anomaly. These new effects play an important role in explaining the observed value. We also study the effects of this interaction on the contribution to $\Delta m_{K_L - K_S}$.

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It is well known that contributions from an intermediate hadronic state effect play an important role in many low energy processes. Some of the notable examples are $K_L \to \gamma\gamma$ [1,2] and $\Delta m_K = m_{K_L} - m_{K_S}$ [3–7]. For $K_L \to \gamma\gamma$, the direct contribution due to the quark level $s \to d\gamma\gamma$ alone accounts for only a small portion of the amplitude measured experimentally [2,7]. For Δm_K, the direct contribution due to the $\Delta S = 2$ four-quark operator is again only a fraction of the experimental value depending on the value of the bag factor B_K [3,4]. A simple method to estimate the contributions from intermediate hadronic states is the pole dominance approximation in which one assumes that a few low lying resonances saturate the contribution. The commonly identified resonances in the above two cases are π^0, η, and η'. Combined with U(3) flavor symmetry, the $K_L \to \gamma\gamma$ amplitude can be estimated [2,6]. If a U(3) nonet is a good symmetry, the calculations are straightforward. However, not only the nonet but also SU(3) is known to be broken, so there are large uncertainties in these calculations. One should also study if there are some new contributions in the standard model (SM) which have not been examined so far. In this paper we show that indeed there is a new contribution to $K_L \to \gamma\gamma$ and Δm_K. This new contribution comes from the $s \to d+2$ gluon induced $K - \eta(\eta')$ transition, and the intermediate η (η') subsequently decays into $\gamma\gamma$ or changes to another neutral kaon through the usual $\Delta s = 1$ interaction. We find that, because of the large QCD anomaly hadronic matrix element $\langle \eta(\eta') | \alpha_s G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a | 0 \rangle$ $(\tilde{G}^{\mu\nu}_a = \epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta})$, the new contributions are sizable and also induce new sizable SU(3) and U(3) breaking effects.

The decay amplitude A_{dir} of the direct contribution to $K_L \to \gamma\gamma$ from the quark level interaction $s \to d\gamma\gamma$ in the SM has been studied before [1,2,7]. Here we improve the calculations by including QCD corrections which also serve to set up our notation. In the SM, $s \to d\gamma\gamma$ can be generated at one-loop level by exchanging a W boson and quarks with two photons emitted from particles in the loop and particles in the external legs. The QCD corrected effective Hamiltonian for $s \to d\gamma\gamma$ is given by

$$H_{eff}(s \to d\gamma\gamma) = M^{\gamma\gamma}_{IR} + M^{\gamma\gamma}_{R}, \tag{1}$$

where $M^{\gamma\gamma}_{IR}$ is the irreducible contribution with the two photons emitted from particles in the loop. $M^{\gamma\gamma}_{R}$ is the reducible contribution with at least one photon emitted from an external s or d quark.

The irreducible contribution $M^{\gamma\gamma}_{IR}$ is given by [2,7,8]

$$M^{\gamma\gamma}_{IR} = -i \frac{16\sqrt{2}\alpha_{e.m.} G_F}{9\pi} N a_2 \epsilon^{*\mu}(k_2) \frac{1}{2k_1 \cdot k_2} \times \sum_{i=u,c,t} V^*_{id} V_{is} F(x,x_i) \bar{d} \gamma^\rho L R_{\mu\nu\rho} s \epsilon^{*\mu}(k_1). \tag{2}$$

Here $\epsilon^\mu(k)$ is the photon polarization vector with momentum k, $L(R) = [1-(+)\gamma_5]/2$, $N=3$ is the number of colors, $a_2 = c_1 + c_2/N$, $x = 2k_1 \cdot k_2 / m^2_W$, $x_i = m^2_i / m^2_W$, and $R_{\mu\nu\rho} = k_{1\nu}\epsilon_{\mu\rho\sigma\lambda} k^\sigma_1 k^\lambda_2 - k_{2\mu}\epsilon_{\nu\rho\sigma\lambda} k^\sigma_1 k^\lambda_2 + k_1 \cdot k_2 \epsilon_{\mu\nu\rho\sigma}(k_2 - k_1)^\sigma$. The function $F(x,x_i)$ is given by

$$F(x,x_i) = \frac{x_i}{x} \int_0^1 \frac{\ln[1 - y(1-y)x/x_i]}{y} dy. \tag{3}$$

The reducible contribution $M^{\gamma\gamma}_{R}$ is given by [7,8]

$$M^{\gamma\gamma}_{R} = \frac{\sqrt{2}\alpha_{e.m.}}{6\pi} \sum_{i=u,c,t} V^*_{id} V_{is} c^i_{12} \bar{d} \Bigg[\left(\frac{1}{p_d \cdot k_1} - \frac{1}{p_s \cdot k_2} \right) \times \sigma_{\mu\beta}\sigma_{\nu\alpha} k^\beta_1 k^\alpha_2 + 2i \left(\frac{p_{d\mu}}{p_d \cdot k_1} - \frac{p_{s\mu}}{p_s \cdot k_1} \right) \sigma_{\nu\beta} k^\beta_2 + (k_1 \to k_2, k_2 \to k_1; \mu \to \nu, \nu \to \mu) \Bigg] (m_d L + m_s R) \times s \epsilon^{*\mu}(k_1) \epsilon^{*\nu}(k_2). \tag{4}$$

In the above c_i are the Wilson coefficients defined in the following $\Delta S=-1$ effective Hamiltonian [9]:

$$H_{eff}(\Delta S=-1)=\frac{4G_F}{\sqrt{2}}\Bigg[V_{qd}^{*}V_{qs}(c_1O_1+c_2O_2) - \sum_k \sum_{i=u,c,t} V_{id}^{*}V_{is}(c_k^iO_k)\Bigg], \tag{5}$$

where the summation over k is on all possible operators, four-quark operators, and quark-photon and quark-gluon operators, which are defined in Ref. [9]. The operators directly relevant to our calculations to the leading order are

$$O_1=\bar{q}\gamma_\mu Lq\bar{d}\gamma^\mu Ls, \quad O_2=\bar{d}\gamma_\mu Lq\bar{q}\gamma^\mu Ls,$$

$$O_{7\gamma}=\frac{e}{16\pi^2}\bar{d}\sigma_{\mu\nu}F^{\mu\nu}(m_dL+m_sR)s,$$

$$O_{8G}=\frac{g_s}{16\pi^2}\bar{d}\sigma_{\mu\nu}T^aG_a^{\mu\nu}(m_dL+m_sR)s, \tag{6}$$

where $G_a^{\mu\nu}$ and $F^{\mu\nu}$ are the gluon and photon field strengths. Here we have also written down the operator O_{8G} which is needed for the study of $s\to dgg$.

To obtain the amplitude A_{dir} for $K_L\to\gamma\gamma$ from the effective Hamiltonian $H_{eff}(s\to d\gamma\gamma)$, one needs to bind the d and s quarks to form a kaon, which involves long distance nonperturbative QCD effects. This effect cannot be calculated at present and is usually parametrized by a decay constant f_K as $\langle 0|\bar{d}\gamma^\mu\gamma_5 s|\bar{K}^0\rangle=-if_KP_K^\mu$ with f_K determined from data. We have

$$\begin{aligned}A_{dir}(\bar{K}^0\to\gamma\gamma)&=\langle\gamma\gamma|H_{eff}(O_{7\gamma})|\bar{K}^0\rangle\\ &=\frac{2\sqrt{2}\alpha_{e.m.}G_F}{9\pi}f_K[i(Na_2V_{ud}^{*}V_{us}\\ &\quad+3\xi c_{7\gamma}^tV_{td}^{*}V_{ts})F_{\mu\nu}\tilde{F}^{\mu\nu}\\ &\quad+3\xi c_{7\gamma}^tV_{td}^{*}V_{ts}F_{\mu\nu}F^{\mu\nu}],\end{aligned} \tag{7}$$

where $H_{eff}(O_{7\gamma})$ indicates the term proportional to $O_{7\gamma}$ in the effective Hamiltonian of Eq. (5). $\tilde{F}_{\mu\nu}=(1/2)\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$.

In obtaining the above result, we have used the fact that $F(x,x_{c,t})\approx-1/2$ (large $x_{c,t}/x$) and $F(x,x_u)\approx 0$ (small x_u/x). We also neglected small contributions from $c_{7\gamma}^{u,c}$ which are proportional to $x_{u,c}$ [10], but have kept $c_{7\gamma}^t$ which is -0.3 in the SM.

The parameter ξ is an average value of the quantity $\kappa=-(m_K^2/16)(1/p_d\cdot k_1-1/p_s\cdot k_2+1/p_d\cdot k_2-1/p_s\cdot k_2)$. If one assumes that the d and s quarks share the kaon momentum equally, then $\xi=1$ [2]. We have also estimated ξ by calculating the quantity $\langle 0|\kappa\bar{d}(1+\gamma_5)s|\bar{K}^0\rangle$ using the perturbative QCD (PQCD) method and an appropriate distribution amplitude of quarks in the kaon [11]. This approach also obtains a value of order 1 for ξ. One should be aware that the applicability of PQCD may not be good here. However, we find that the contribution related to ξ is not important as long as ξ is of order 1. That is, the precise value of ξ is not important here and we will set ξ to be 1 in our later discussion.

To estimate the irreducible contribution, one needs to know the quantity $a_2=c_1+c_2/N$. Without QCD corrections, $c_1=0$ and $c_2=1$. This gives a $a_2=1/3$. With QCD corrections the value for a_2 will be altered. The leading and next-to-leading order corrections to c_i have been calculated [9]. The values of c_i depend on the renormalization scale μ. Since one does not know precisely where is the matching scale μ, this causes uncertainty in a_2. For example, at the leading order, $a_2=-0.27$ at $\mu\approx 1$ GeV, while at $\mu=1.3$ GeV, $a_2=-0.17$ with $\Lambda_{\overline{MS}}=325$ MeV. At the next-to-leading order the dependences on μ for each of the c_1 and c_2 are reduced, but leave a_2 still sensitive to μ. For example, in the NDR scheme, for $\Lambda_{\overline{MS}}=325$ MeV, a_2 is -0.08 and -0.1 at $\mu=1.0$ GeV and $\mu=1.3$ GeV, respectively. Allowing the QCD parameter $\Lambda_{\overline{MS}}$ to vary within the allowed range 215–435 MeV, a_2 can vary in the range -0.1 to -0.35 depending whether the NDR or HV scheme is used [9]. That is, the value of a_2 is not well determined even from next-to-leading order perturbative calculations. When all effects, perturbative and nonperturbative, are correctly treated, the final physical observables will not depend on the renormalization scale μ. Unfortunately, such a calculation is not possible at present. The parameter a_2 behaves similarly to the one in hadronic B and D decays. In both D and B decays, the parameter a_2 determined from data ($|a_2|\sim 0.2$–0.5) is very different from the factorization value obtained by inserting $c_{1,2}$ at the relevant scale in the expression for a_2 [12]. One would expect a similar thing to happen in kaon decays although the details may be different. To take into account uncertainties in theoretical calculations of a_2, we will treat it as a free parameter and allow it to vary in the range of -0.5 to 0.5. One can also turn the argument around to obtain information about a_2 from $K_L\to\gamma\gamma$ data.

For ξ of order 1, and a_2 in the range of -0.5 to 0.5, we find that the dominant direct $\bar{K}^0\to\gamma\gamma$ amplitude is from the irreducible contribution. We have

$$A_{dir}(K_L\to\gamma\gamma)=i\tilde{A}_{dir}\frac{1}{2}F_{\mu\nu}\tilde{F}^{\mu\nu},$$

$$\tilde{A}_{dir}=\frac{8\alpha_{e.m.}G_F}{9\pi}f_KNa_2\,\mathrm{Re}(V_{ud}^{*}V_{us}). \tag{8}$$

Using $V_{ud}=0.9735$, $V_{us}=0.2196$, and $f_K=1.27f_\pi$ [13], we obtain

$$\tilde{A}_{dir}=2.54\times 10^{-12}a_2 \;\; \mathrm{MeV}^{-1}. \tag{9}$$

For $|a_2|=0.5$, it is only about 35% of the experimental value of 3.5×10^{-12} MeV^{-1} [13]. Without QCD corrections $a_2=1/3$, $\tilde{A}_{dir}$ is about 24% of the total amplitude. There must be some other contributions to this process. These effects may come from contributions with intermediate hadronic states or even contributions from new physics beyond the SM. If one has a good understanding of all SM contributions, one can make a detailed study of new physics beyond the SM. It is probably too early to say that new physics is needed here due to large uncertainties in the possible hadronic intermediate contributions. Therefore we will work within the SM and see how contribution from hadronic intermediate states can affect the results.

Several analyses have been carried out using a pole model with π^0, η, and η' poles to calculate the hadronic intermediate contribution. In this model, the amplitude A_{had} from exchange of intermediate hadronic states is given by [6]

$$\tilde{A}_{had}=\tilde{A}(\pi^0\to\gamma\gamma)\frac{\langle\pi^0|H_W|K_L\rangle}{m_K^2-m_\pi^2}\left[1+\frac{m_K^2-m_\pi^2}{m_K^2-m_\eta^2}\frac{\tilde{A}(\eta\to\gamma\gamma)}{\tilde{A}(\pi^0\to\gamma\gamma)}\left(\frac{1+\delta}{\sqrt{3}}\cos\theta+2\sqrt{\frac{2}{\sqrt{3}}}\rho\sin\theta\right)\right.$$
$$\left.+\frac{m_K^2-m_\pi^2}{m_K^2-m_{\eta'}^2}\frac{\tilde{A}(\eta'\to\gamma\gamma)}{\tilde{A}(\pi^0\to\gamma\gamma)}\left(\frac{1+\delta}{\sqrt{3}}\sin\theta-2\sqrt{\frac{2}{\sqrt{3}}}\rho\cos\theta\right)\right], \tag{10}$$

where θ is the η-η' mixing angle and δ is the SU(3) breaking parameter [6]. The parameter ρ parametrizes the U(3) nonet breaking effect and is defined as

$$\rho=-\sqrt{\frac{3}{8}}\frac{\langle\eta_1|H_W|K^0\rangle}{\langle\pi^0|H_W|K^0\rangle}. \tag{11}$$

In the nonet limit $\rho=1$. A chiral Lagrangian analysis gives $\langle\pi^0|H_W|K_L\rangle=1.4\times10^{-7}m_K^2$ [6]. Using experimental values for $\pi^0,\eta,\eta'\to\gamma\gamma$, $\tilde{A}_{had}$ can be estimated.

The above contributions can be viewed as obtained via $A_{had}=\Sigma_i\langle\gamma\gamma|i\rangle\langle i|\tilde{H}_{eff}(\Delta S=-1)|K_L\rangle$ with $i=\pi^0,\eta,\eta'$ in the pole model approximation. Here $\tilde{H}_{eff}(\Delta S=-1)$ is the full $\Delta S=-1$ effective Lagrangian with the $O_{7\gamma}$ term removed since it has been counted as a contribution to A_{dir}. Therefore A_{dir} and A_{had} are contributions from different sources. In previous calculations the contributions for A_{had} from O_{8G} were not considered [2,6]. We now study in detail the effect of this interaction on $K_L\to\gamma\gamma$.

At the quark-gluon level, O_{8G} induces $s\to d+gg$. To obtain A_{had}, one needs to estimate the contribution from $s\to d+gg$ to $\bar{K}^0$-η,η' through $gg\to\eta,\eta'$. The effective Hamiltonian $M^{gg}_{IR,R}$ for $s\to dgg$, with the color singlet $\bar{d}s$ bispinor product, can be obtained by some simple replacements from $M^{\gamma\gamma}_{IR,R}$. To obtain $M^{gg}_{IR,R}$ one first replaces the photon polarization vectors $\epsilon^\mu(k_1)\epsilon^\nu(k_2)$ by the gluon polarization vector $\epsilon^\mu_a(k_1)\epsilon^\nu_a(k_2)$ with the color index a summed over. Then one replaces $\alpha_{e.m.}$ by $\alpha_s(9/4)/(2N)$ and $\alpha_{e.m.}c^i_{12}$ by $\alpha_s c^i_{8G}/(2N)$ for M^{gg}_{IR} and M^{gg}_R, respectively [7]. The factor $1/(2N)$ comes from picking up the color singlet part.

Similar to the procedure in obtaining the amplitude A_{dir} for $K_L\to\gamma\gamma$, one can obtain the amplitude for $K_L\to gg$. We find that with ξ of order 1, a_2 in the range of -0.5 to 0.5, and $c^t_{8G}\approx-0.15$ as given in the SM, the irreducible contribution, again, dominates the amplitude. We have

$$A(K_L\to gg)=\frac{1}{2N}\frac{2\alpha_sG_F}{\pi}f_KNa_2\,\mathrm{Re}(V^*_{ud}V_{us})i\frac{1}{2}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a. \tag{12}$$

The above interaction can induce large K_L-η,η' transitions and therefore contribution to $K_L\to\gamma\gamma$, because QCD can induce large matrix elements for $\langle\eta(\eta')|\alpha_sG^a_{\mu\nu}\tilde{G}^{\mu\nu}_a|0\rangle$.

The QCD anomaly implies that the divergence of the singlet current $a^1_\mu=\bar{u}\gamma_\mu\gamma_5u+\bar{d}\gamma_\mu\gamma_5d+\bar{s}\gamma_\mu\gamma_5s$ is not zero in the limit of zero quark masses, and is given by

$$\langle\eta(\eta')|\partial^\mu a^1_\mu|0\rangle=\langle\eta(\eta')|2i(m_u\bar{u}\gamma_5u+m_d\bar{d}\gamma_5d+m_s\bar{s}\gamma_5s)|0\rangle-\left\langle\eta(\eta')\left|\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a\right|0\right\rangle, \tag{13}$$

while for the octet current $a^8_\mu=\bar{u}\gamma_\mu\gamma_5u+\bar{d}\gamma_\mu\gamma_5d-2\bar{s}\gamma_\mu\gamma_5s$ one obtains [14]

$$\langle\eta(\eta')|\partial^\mu a^8_\mu|0\rangle=\langle\eta(\eta')|2i(m_u\bar{u}\gamma_5u+m_d\bar{d}\gamma_5d-2m_s\bar{s}\gamma_5s)|0\rangle. \tag{14}$$

Since $m_{u,d}$ are much smaller than m_s, one can neglect terms proportional to $m_{u,d}$. One then obtains

$$\left\langle\eta'(p)\left|\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a\right|0\right\rangle=\sqrt{\frac{3}{2}}(\sqrt{2}f_1\cos\theta+f_8\sin\theta)p^2,$$
$$\left\langle\eta(p)\left|\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a\right|0\right\rangle=\sqrt{\frac{3}{2}}(-\sqrt{2}f_1\sin\theta+f_8\cos\theta)p^2, \tag{15}$$

where $f_{1,8}$ are the singlet and octet pseudoscalar decay constants.

If there is no η-η' mixing and all quark masses are equal, the gg state, being a flavor singlet, can have only transitions to η_1. Because of the η-η' mixing and the different quark masses, both U(3) nonet and SU(3) symmetries are broken. The $K_L \to \eta, \eta'$ transitions induced by $s \to dgg$ will induce nonet and SU(3) breaking in the total amplitude $\tilde{A}^{total}$. Normalizing the signs of each contribution to theoretical calculations, we finally obtain

$$\tilde{A}^{total} = \tilde{A}_{dir} + \tilde{A}(\pi^0 \to \gamma\gamma) \frac{\langle \pi^0 | H_W | K_L \rangle}{m_K^2 - m_\pi^2} \left[1 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_\eta^2} \frac{\tilde{A}(\eta \to \gamma\gamma)}{\tilde{A}(\pi^0 \to \gamma\gamma)} \left(\frac{1 + \delta + \delta^{gg}}{\sqrt{3}} \cos\theta + 2\sqrt{\frac{2}{\sqrt{3}}} (\rho + r^{gg}) \sin\theta \right) + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_{\eta'}^2} \frac{\tilde{A}(\eta' \to \gamma\gamma)}{\tilde{A}(\pi^0 \to \gamma\gamma)} \left(\frac{1 + \delta + \delta^{gg}}{\sqrt{3}} \sin\theta - 2\sqrt{\frac{2}{\sqrt{3}}} (\rho + r^{gg}) \cos\theta \right) \right], \qquad (16)$$

where δ^{gg} and r^{gg} are the SU(3) and nonet breaking induced by the $s \to dgg$ interaction. They are given by

$$\delta^{gg} = -\sqrt{2} f_K f_8 m_K^2 \frac{G_F \,\mathrm{Re}(V_{ud}^* V_{us})}{\langle \pi^0 | H_W | K_L \rangle} a_2,$$

$$r^{gg} = -\frac{f_1}{2 f_8} \delta^{gg}. \qquad (17)$$

We find

$$\delta^{gg} = 0.96 \frac{f_8}{f_K} a_2, \quad r^{gg} = -0.48 \frac{f_1}{f_K} a_2. \qquad (18)$$

We see that the corrections can be sizable and cannot be neglected.

We now provide some details for numerical calculations. There are several parameters involved in $\tilde{A}_{had}$, the mixing angle θ, the decay constants $f_{1,8}$, the SU(3) and U(3) nonet breaking parameters δ and ρ, and the parameter a_2. Chiral perturbation calculations and fitting data not involving $K_L \to \gamma\gamma$ have obtained $\theta \approx -20°$, $\delta \approx 0.17$, $f_8 \approx 1.28 f_\pi$, and $f_1 \approx 1.10 f_\pi$ [15]. We will use these values for these parameters in the calculation of $K_L \to \gamma\gamma$. There is not a reliable estimate for the parameter ρ. Since we are interested to see how the new $s \to dgg$ interaction induces U(3) nonet breaking effect, we will take $\rho = 1$ and attribute nonet breaking solely to r^{gg}. As has been discussed, $s \to dgg$ also induces SU(3) breaking effect. This effect was not included in other fittings. We therefore should include this new SU(3) breaking effect also.

Without the $s \to dgg$ effect, we find that the amplitude $\tilde{A}^{total}$ is equal to $5.5(1 + 0.46 a_2) \times 10^{-12}$ MeV^{-1}, which is considerably larger than the experimental value 3.5×10^{-12} MeV^{-1} [13] for $|a_2| < 0.5$. With the new effect, we find

$$\tilde{A}^{total} = 5.5(1 + 2.14 a_2) \times 10^{-12} \text{ MeV}^{-1}. \qquad (19)$$

To reproduce the central experimental value, a_2 is required to be -0.17, which is a reasonable value to have.

The detailed numerical results depend on several parameters. Even with other parameters fixed, one can also introduce a phase in a_2. To fit the $K_L \to \gamma\gamma$ data, the values for the magnitude and phase of a_2 can vary. We would like to emphasize, however, that the new effect discussed can play an important role in $K_L \to \gamma\gamma$ independent of the details.

The new contributions for $K_L - \eta(\eta')$ transitions also induce a new hadronic intermediate state effect in the K_L and K_S mass difference parameter Re(M_{12}) in the pole dominance approximation. We find [6]

$$2 m_K \,\mathrm{Re}(M_{12}) = \frac{|\langle \pi^0 | H_W | K^0 \rangle|^2}{m_K^2 - m_\pi^2} \left[1 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_\eta^2} \left(\frac{1 + \delta + \delta^{gg}}{\sqrt{3}} \cos\theta + 2\sqrt{\frac{2}{\sqrt{3}}} (\rho + r^{gg}) \sin\theta \right)^2 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_{\eta'}^2} \left(\frac{1 + \delta + \delta^{gg}}{\sqrt{3}} \sin\theta - 2\sqrt{\frac{2}{\sqrt{3}}} (\rho + r^{gg}) \cos\theta \right)^2 \right]. \qquad (20)$$

Without the new effects, the above would lead to $\Delta m_K = -0.5\times 10^{-12}$ MeV, which is a non-negligible portion of the experimental value of 3.5×10^{-12} MeV. With the new effects and $a_2 = -0.17$ as determined from $K_L \to \gamma\gamma$, the contribution to Δm_K is -0.9×10^{-12} MeV, and again it cannot be neglected. The new effect in $K_L \to \pi^0, \eta, \eta'$ transitions can have a sizable contribution to Δm_K.

The $s \to dgg$ process can also induce K_L-glueball mixing, which would also affect $K_L \to \gamma\gamma$ and Δm_{S-L}, as pointed out in Ref. [7], where a light glueball mass 1.4 GeV was used. Recent lattice calculations indicate that the pseudoscalar glueball mass is about 2.3 GeV [16]. With such a large mass the glueball–η (η') mixing contribution should be small and therefore the effects are smaller than the effects discussed earlier.

In conclusion we have evaluated additional contributions to $K_L \to \eta(\eta')$ transitions from $s \to dgg$ in the standard model. These transitions induce new sizable SU(3) and U(3) breaking effects and have significant effects on contributions to $K_L \to \gamma\gamma$ and Δm_K.

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