Naturally light invisible axion in models with large local discrete symmetries

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We show that by introducing appropriate local $Z_N(N\geqslant 13)$ symmetries in electroweak models it is possible to implement an automatic Peccei-Quinn symmetry, at the same time keeping the axion protected against gravitational effects. Although we consider here only an extension of the standard model and a particular 3-3-1 model, the strategy can be used in any kind of electroweak model. An interesting feature of this 3-3-1 model is that if we add (i) right-handed neutrinos, (ii) the conservation of the total lepton number, and (iii) a Z_2 symmetry, the Z_{13} and the chiral Peccei-Quinn $U(1)_{\rm PQ}$ symmetries are both accidental symmetries in the sense that they are not imposed on the Lagrangian but are just a consequence of the particle content of the model, its gauge invariance, renormalizability, and Lorentz invariance. In addition, this model has no domain wall problem.

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I. INTRODUCTION

It is well known that an elegant way to solve the strong *CP* problem is by introducing a chiral $U(1)_{\rm PQ}$ [1] symmetry which also implies the existence of a pseudo Goldstone boson—the axion [2]. This particle becomes an interesting candidate for dark matter if its mass is of the order of 10^{-5} eV [3–5]. It was also recently argued [6] that an axion-photon oscillation can explain the observed dimming supernovas [7] if the axion has a rather small mass: $\sim 10^{-16}$ eV. Which ever of these possibilities (if any) is realized in nature, the existence of a light invisible axion can be prevented by gravity, since it induces renormalizable and nonrenormalizable effective interactions which explicitly break any global symmetry, in particular the $U(1)_{\rm PQ}$ symmetry, and the axion can gain a mass that is greater than the mass coming from instanton effects [8]. This can be avoided if the dimension of the effective operators is $d \gtrsim 12$. Hence, unless d is high enough, invisible axion models do not solve the strong *CP* problem in a natural way. Here we will show how in two electroweak models the axion is protected against gravity effects; however, the strategy can be used in any electroweak model.

It was pointed out several years ago by Krauss and Wilczek that a local gauge symmetry, say $\mathcal{U}(1)$, can masquerade as discrete symmetries $Z_N \subset \mathcal{U}(1)$ to an observer equipped with only low-energy probes [9]. This means that these symmetries evade the no-hair theorem [10]; i.e., unlike continuous symmetries they must be respected even for gravitational interactions. The only implication of the original gauge symmetry for the low-energy effective theory is the absence of interaction terms forbidden by the Z_N symmetry. For instance, if there were more charged scalar fields in the theory, the discrete symmetry would forbid many couplings that were otherwise possible.

Here we will not worry about the origin of this Z_N [or $\mathcal{U}(1)$] symmetry [11–15]. For instance, it might be that the Z_N symmetries come from a fifth dimension, as was shown for the case of $U(1)_Y$ in Ref. [12]. We will simply assume that at very high energies we have a model of the form $\mathcal{U}(1)\otimes G_{EW}$ where $\mathcal{U}(1)$ is a local symmetry (maybe a subgroup of a larger symmetry) which is broken to Z_N at a high energy scale and G_{EW} is an electroweak model, i.e., $\mathcal{U}(1)\otimes G_{EW}\to Z_N\otimes G_{EW}$. We will use the existence of these local Z_N symmetries in order to protect the axion against gravitational effects. We get this by enlarging, if necessary, the representation content of the model, so that we can impose symmetries with $N\geqslant 13$. In addition, the $U(1)_{\rm PQ}$ symmetry (and under some conditions also the Z_N symmetry) is an automatic symmetry, in the sense that it is not imposed on the Lagrangian but is just a consequence of the particle content of the model, its gauge invariance, renormalizability, and Lorentz invariance.

In these circumstances we show how a naturally light invisible axion (it is almost a singlet ϕ under the gauge symmetry), which is also protected against the effects of quantum gravity, can be obtained in the context of electroweak models as follows. Effective operators like $\phi^{N-1}/M_{\rm Pl}^{(N-1)-4}$ are automatically suppressed by the (local) Z_N symmetry. At the same time this symmetry makes the $U(1)_{\rm PQ}$ symmetry an automatic symmetry of the classical Lagrangian or, as in the 3-3-1 model considered here, both Z_N and $U(1)_{\rm PQ}$ are already automatic symmetries of the model under the conditions discussed in Sec. III. For instance, a Z_{13} symmetry implies that the first nonforbidden operator is of dimension 13 and it implies a contribution to the axion mass square of

$(v_\phi)^{11}/M_{\rm Pl}^9 \approx 10^{-21}$ eV^2 or $10^{-11} m_a^2$, if $m_a \sim \Lambda_{\rm QCD}^2/v_\phi \approx 10^{-5}$ eV is the instanton induced mass (we have used $M_{\rm Pl}=10^{19}$ GeV and $v_\phi = 10^{12}$ GeV). The naturalness of the Peccei-Quinn (PQ) solution to the θ-strong problem is not spoiled since in this case we have $\theta_{\rm eff} \propto v_\phi^N/M_{\rm Pl}^{N-4}\Lambda_{\rm QCD}^4$ [16], and it means that $\theta_{\rm eff}\propto 10^{-11}$ for $N=13$. Recently we applied this strategy in the context of an extension of the electroweak standard model [17]; now we will apply this procedure to a model with 3-3-1 symmetry.

Hence, we see that it is necessary to search for models that have a representation content large enough to allow the implementation of a discrete symmetry Z_N with $N\geqslant 13$. In the context of a $SU(2)_L\otimes U(1)_Y$ model we have enlarged the representation content by adding several Higgs multiplets, right-handed neutrinos, and the scalar singlet necessary to make the axion invisible. Hence, it is possible to accommodate a Z_{13} symmetry while keeping a general mixing among fermions of the same charge [17]. Larger Z_N symmetries are possible if we add more scalar doublets in such a way as to generate appropriate texture of the fermionic mass matrices. On the other hand, we will show in this work that in a 3-3-1 model the minimal representation content plus right-handed neutrinos admit enough large discrete symmetries. In these models the addition of a singlet (or a decuplet [18]) is the reason for maintaining the axion invisible; however, in the 3-3-1 model the axion picture is a mixture of the Dine *et al*. invisible axion [20] and the Kim heavy quark axion [21]. Nevertheless, unlike the model of Ref. [21], here the exotic quarks are already present in the minimal version of the model.

This paper is organized as follows. In Sec. II we review briefly the new invisible axion in the context of an extension of the standard model [17]. In Sec. III we consider one 3-3-1 model. Our conclusions and some phenomenological consequences are in the last section.

II. AN EXTENSION OF THE STANDARD MODEL

Let us consider the invisible axion in an extension of the $SU(2)_L\otimes U(1)_Y$ model. The representation content is the following: $Q_L=(ud)_L^T\sim(\mathbf{2},1/3)$, $L_L=(\nu l)_L^T\sim(\mathbf{1},-1)$ denote any quark and lepton doublet; $u_R\sim(\mathbf{1},\mathbf{4/3})$, $d_R\sim(\mathbf{1},-2/3)$, $l_R\sim(\mathbf{1},-2)$, $\nu_R\sim(\mathbf{1},0)$ are the right-handed components; and we will assume that each charge sector gains mass from a different scalar doublet [19], i.e., Φ_u, Φ_d, Φ_l, and Φ_ν generate Dirac masses for u-like and d-like quarks, charged leptons, and neutrinos, respectively [all of them of the form $(\mathbf{2},+1)=(\varphi^+,\ \varphi^0)^T$]. We also add a neutral complex singlet $\phi\sim(\mathbf{1},0)$ as in Refs. [20,21], a singly charged singlet $h^+\sim(\mathbf{1},+2)$ as in Zee's model [22], and finally, a triplet $\vec{T}\sim(\mathbf{3},+2)$ as in Ref. [23]. The introduction of right-handed neutrinos seems a natural option in any electroweak model if neutrinos are massive particles, as strongly suggested by solar [24], reactor [25], and atmospheric [26] neutrino data.

Next, we will impose the following (local in the sense discussed above) Z_{13} symmetry among those fields:

$$Q_L\to\omega_5 Q_L,\ \ u_R\to\omega_3 u_R,\ \ d_R\to\omega_5^{-1}d_R, \tag{1}$$

$$L\to\omega_6 L,\ \ \nu_R\to\omega_0\nu_R,\ \ l_R\to\omega_4 l_R,$$

$$\Phi_u\to\omega_2^{-1}\Phi_u,\ \ \Phi_d\to\omega_3^{-1}\Phi_d,\ \ \Phi_l\to\omega_2\Phi_l,$$

$$\Phi_\nu\to\omega_6^{-1}\Phi_\nu,\ \ \phi\to\omega_1^{-1}\phi,\ \ \vec{T}\to w_4^{-1}\vec{T},$$

$$h^+\to\omega_1 h^+$$

with $\omega_k=e^{2\pi i k/13}$, $k=0,1,\ldots,6$. With this representation content and the Z_{13} symmetry defined in Eq. (1) the allowed Yukawa interactions and the scalar potential are automatically invariant under a $U(1)_{\rm PQ}$ chiral symmetry. The PQ charges are quantized after imposing an extra Z_3 symmetry with parameters denoted by $\tilde{\omega}_0$, $\tilde{\omega}_1$, and $\tilde{\omega}_1^{-1}$. Under Z_3 the fields transform as follows:

$$\Phi_u,\Phi_l,T,\nu_R\to\tilde{\omega}_1(\Phi_u,\Phi_l,T,\nu_R),$$

$$\Phi_\nu,\phi,u_R,l_R\to\tilde{\omega}_1^{-1}(\Phi_\nu,\phi,u_R,l_R), \tag{2}$$

while all the other fields remain invariant, i.e., transform with $\tilde{\omega}_0$. As we said before it is possible to implement larger Z_N symmetries if more scalar doublets are added in such a way as to generate appropriate texture of the fermionic mass matrices.

The PQ assignment is the following:

$$
\begin{aligned}
&u_L'=e^{i(2/5)\alpha X_d}u_L,\quad d_L'=e^{-i\alpha X_d}d_L,\\
&l_L'=e^{-i(4/5)\alpha X_d}l_L,\quad \nu_L'=e^{i(3/5)\alpha X_d}\nu_L,\\
&\phi_u^{0\,\prime}=e^{-i(4/5)\alpha X_d}\phi_u^0,\quad \phi_d^{0\,\prime}=e^{-2i\alpha X_d}\phi_d^0,\\
&\phi_l^{0\,\prime}=e^{-i(8/5)\alpha X_d}\phi_l^0,\quad \phi_\nu^{0\,\prime}=e^{-i(6/5)\alpha X_d}\phi_\nu^0,\\
&\phi_u^{+\,\prime}=e^{i(3/5)\alpha X_d}\phi_u^+,\quad \phi_d^{+\,\prime}=e^{-i(3/5)\alpha X_d}\phi_d^+,\\
&\phi_l^{+\,\prime}=e^{-i(8/5)\alpha X_d}\phi_l^+,\quad \phi_\nu^{+\,\prime}=e^{i(1/5)\alpha X_d}\phi_\nu^+,\\
&T^{0\,\prime}=e^{-i(8/5)\alpha X_d}T^0,\quad T^{+\,\prime}=e^{i(1/5)\alpha X_d}T^+,\\
&T^{++\,\prime}=e^{i(6/5)\alpha X_d}T^{++},\quad h^{+\,\prime}=e^{i(1/5)\alpha X_d}h^+,\\
&\phi'=e^{-i(6/5)\alpha X_d}\phi.
\end{aligned} \tag{3}
$$

The axion is invisible since it is almost singlet as in Refs. [20,21]; the scalar triplet is only a small correction for it.

Some axion models [27] lead to the formation of domain walls in the evolution of the universe, which could be inconsistent with the standard cosmology [28]. The domain wall number is defined as [29]

$$N_{DW}=\left|\sum_{f=L}\mathrm{Tr}\,X_f T_a^2(f)-\sum_{f=R}\mathrm{Tr}\,X_f T_a^2(f)\right|, \tag{4}$$

where X_f denotes the PQ charge of the quark f and there is no summation on a; we have $\mathrm{Tr}\,T_aT_b=(1/2)\delta_{ab}$ for $\mathbf{3}$ and $\mathbf{3}^*$. Using the PQ assignment in Eq. (3) we obtain N_{DW}

$=|(9/5)X_d|$. If $X_d=\pm 5$ we have $N_{\rm DW}=9$. (We choose $X_d=-5$ in order to have $\bar{\theta}\to\bar{\theta}+2\pi k$, $k=0,\ldots,8$.) The $N=9$ vacua can be characterized by

$$\langle u_L^\dagger u_R\rangle=\mu_u^3 e^{-i2\pi(2k/9)},\ \ \langle d_L^\dagger d_R\rangle=\mu_d^3 e^{i2\pi(5k/9)},$$
$$\langle\phi_u^0\rangle=v_u e^{i(\beta_u-2\pi(2k/9))},\ \ \langle\phi_d^0\rangle=v_d e^{i(\beta_d-2\pi(5k/9))},$$
$$\langle\phi_l^0\rangle=v_l e^{i(\beta_l-2\pi(4k/9))},\ \ \langle\phi_\nu^0\rangle=v_\nu e^{i(\beta_\nu-2\pi(3k/9))},$$
$$\langle T^0\rangle=v_T e^{i(\beta_T-2\pi(4k/9))},\ \ \langle\phi\rangle=v_\phi e^{i(\beta_\phi-2\pi(3k/9))},\tag{5}$$

where $k=0,\ldots,8$.

In this extension of the standard electroweak model only the $U(1)_{\rm PQ}$ is an automatic symmetry and its charges are quantized only if we add an extra Z_3 symmetry. As we showed before, there are genuine discrete symmetries that are not broken by the instanton effects, so this model suffers from the domain wall problem, and it must be solved by any of the methods proposed in the literature [29,30]. Moreover, in this model the Z_{13} symmetry is not automatic. For more details, see Ref. [17].

III. THE AXION IN A 3-3-1 MODEL

The so called 3-3-1 models are interesting candidates for physics at the TeV scale [31–34]. In fact, some years ago Pal [18] pointed out that the strong *CP* question is solved elegantly in those models. The point was that the Yukawa couplings of these models automatically contain a Peccei-Quinn symmetry [1] if a simple discrete symmetry was also imposed in order to avoid a trilinear term in the scalar potential. Here we will consider one particular 3-3-1 model in which only three scalar triplets are needed [32], but we introduce also a scalar singlet, $\phi\sim(\mathbf{1},\mathbf{1},0)$. In this 3-3-1 model if we add (i) right-handed neutrinos, (ii) the conservation of the total lepton number L, and (iii) a Z_2 symmetry defined below, we have that both Z_{13} and $U(1)_{\rm PQ}$ are accidental symmetries of the classical Lagrangian (in the sense discussed in Sec. I).

Before considering the implementation of a naturally light and invisible axion in the context of the 3-3-1 model of Ref. [32], let us briefly review the model.

In the quark sector we have

$$Q_{mL}=(d_m,u_m,j_m)_L^T\sim(\mathbf{3},\mathbf{3}^*,-1/3),\ \ m=1,2,$$
$$Q_{3L}=(u_3,d_3,J)_L^T\sim(\mathbf{3},\mathbf{3},2/3),\tag{6}$$

and the respective right-handed components in the singlets

$$u_{\alpha R}\sim(\mathbf{3},\mathbf{1},2/3),\quad d_{\alpha R}\sim(\mathbf{3},\mathbf{1},-1/3),\ \ \alpha=1,2,3,$$
$$J_R\sim(\mathbf{3},\mathbf{1},5/3),\ \ j_{mR}\sim(\mathbf{3},\mathbf{1},-4/3).\tag{7}$$

In the scalar sector, this model has only three triplets,

$$\chi=(\chi^-,\chi^{--},\chi^0)^T,\rho=(\rho^+,\rho^0,\rho^{++})^T,$$
$$\eta=(\eta^0,\eta_1^-,\eta_2^+)^T,\tag{8}$$

transforming as $(\mathbf{1},\mathbf{3},-1),(\mathbf{1},\mathbf{3},1)$, and $(\mathbf{1},\mathbf{3},0)$, respectively. Finally, in this model leptons transform as triplets $(\mathbf{3}_a,0)$ where $a=e,\mu,\tau$:

$$\Psi_{aL}=(\nu_a,\ l_a,\ E_a)_L^T,\tag{9}$$

and the corresponding right-handed singlets

$$\nu_{aR}\sim(\mathbf{1},\mathbf{1},0),\ \ l_{aR}\sim(\mathbf{1},\mathbf{1},-1),\ \ E_{aR}\sim(\mathbf{1},\mathbf{1},+1),\tag{10}$$

and we have added right-handed neutrinos which are not present in the minimal version of the model. Hence, we see that the model has 15 multiplets, including right-handed neutrinos and the singlet ϕ [in fact, as usual we have to add a scalar singlet $\phi\sim(\mathbf{1},\mathbf{1},0)$ in order to make the axion invisible [20,21]] and it will admit, under the three conditions introduced above, automatic Z_{13} and $U(1)_{\rm PQ}$ symmetries as we will show in the following.

With the quark and scalar multiplets above we have the Yukawa interactions

$$-\mathcal{L}_Y^q=\overline{Q}_{iL}(F_{i\alpha}u_{\alpha R}\rho^*+\tilde{F}_{i\alpha}d_{\alpha R}\eta^*)+\overline{Q}_{3L}(G_{1\alpha}u_{\alpha R}\eta+\tilde{G}_{1\alpha}d_{\alpha R}\rho)+\lambda_1\overline{Q}_{3L}J'_{1R}\chi+\lambda_{im}\overline{Q}_{iL}j_{mR}\chi^*+\text{H.c.},\tag{11}$$

where repeated indices mean summation. In the lepton sector we have

$$-\mathcal{L}_Y^l=G_{ab}^\nu\overline{(\Psi)}_{aL}\nu_{bR}\eta+G_{ab}^l\overline{(\Psi)}_{aL}l_{bR}\rho+G_{ab}^E\overline{(\Psi)}_{aL}E_{bR}\chi+\text{H.c.}\tag{12}$$

In both Yukawa interactions above a general mixing in each charge sector is allowed. If we want to implement a given texture for the quark and lepton mass matrices we have to introduce more scalar triplets and a larger Z_N symmetry will be possible in the model. Interesting possibilities are the cases where N is a prime number (see below).

The most general L-, Z_2-, and gauge-invariant scalar potential is

$$V_{331}=\sum_{x=\eta,\rho,\chi,\phi}\mu_x^2T_x^\dagger T_x+\lambda_1(\eta^\dagger\eta)^2+\lambda_2(\rho^\dagger\rho)^2+\lambda_3(\chi^\dagger\chi)^2+(\eta^\dagger\eta)[\lambda_4(\rho^\dagger\rho)+\lambda_5(\chi^\dagger\chi)]+\lambda_6(\rho^\dagger\rho)(\chi^\dagger\chi)+\lambda_7(\rho^\dagger\eta)\times(\eta^\dagger\rho)+\lambda_8(\chi^\dagger\eta)(\eta^\dagger\chi)+\lambda_9(\rho^\dagger\chi)(\chi^\dagger\rho)+\lambda_\phi(\phi^*\phi)^2+\phi^*\phi\sum_{k=\eta,\rho,\chi}\lambda_{\phi k}T_k^\dagger T_k+(\lambda_{10}\phi\epsilon^{ijk}\eta_i\rho_j\chi_k+\text{H.c.}),\tag{13}$$

where the μ's are parameters with dimension of mass, the λ's are dimensionless parameters, and we have denoted $T_x = \eta,\rho,\chi,\phi$ and $T_k = \eta,\rho,\chi$. Now we can explain the motivation for the three conditions assumed at the beginning of the section. (i) With the present experimental data [24–26] in any electroweak model right-handed neutrinos are no longer avoided under the assumption that neutrinos are massless. (ii) In Eq. (11) a Majorana mass term is still possible among right-handed neutrinos, say $M_R\overline{(\nu^c)_R}\nu_R$; on the other hand, in the scalar potential it is possible to have the quartic term $\chi^\dagger\eta\rho^\dagger\eta$, which also violates the total lepton number. Both terms are avoided by imposing the conservation of the total lepton number L. (iii) The trilinear term in the scalar potential $\eta\rho\chi$ is avoided if we impose a Z_2 symmetry under which J_R, j_{mR}, χ, ϕ are odd and all the other fields are even. In these conditions, the Yukawa interactions in Eqs. (11), (12) and the scalar potential in Eq. (13) are automatically invariant under the (local) Z_{13} symmetry:

$$
\begin{aligned}
&Q_{iL}\to\omega_5^{-1}Q_{iL}, \quad Q_{3L}\to\omega_5 Q_{3L},\\
&u_{\alpha R}\to\omega_1 u_{\alpha R}, \quad d_{\alpha R}\to\omega_1^{-1}d_{\alpha R},\\
&J_R\to\omega_3 J_R, \quad j_{mR}\to\omega_3^{-1}j_{mR},\\
&\Psi_L\to w_0\Psi_L, \quad l_R\to\omega_6^{-1}l_R,\\
&\nu_R\to\omega_4^{-1}\nu_R, \quad E_R\to\omega_2^{-1}E_R,\\
&\eta\to\omega_4\eta, \quad \rho\to\omega_6\rho,\\
&\chi\to\omega_2\chi, \quad \phi\to\omega_1\phi,
\end{aligned}
\tag{14}
$$

where $\omega_k = e^{2\pi i k/13}$, $k=0,\ldots,6$. Notice that if N is a prime number the singlet ϕ can transform under this symmetry with any assignment (but the trivial one); otherwise we have to be careful with the way we choose the singlet ϕ to transform under the Z_N symmetry. This symmetry implies that the lowest-order effective operator that contributes to the axion mass is $\phi^{13}/M_{\rm Pl}^9$, which gives a mass of the order of $(v_\phi)^{11}/M_{\rm Pl}^9$ and also keeps the $\bar{\theta}$ parameter small, as discussed previously.

It happens that, like the Z_{13} symmetry, the $U(1)_{\rm PQ}$ is also automatic, i.e., a consequence of the gauge symmetry and renormalizability of the model, in the interactions in Eqs. (11), (12), and (13). Let us see the PQ charge assignment for the fermions in the model:

$$
\begin{aligned}
&u'_L = e^{-i\alpha X_u}u_L, \;\; d'_L = e^{-i\alpha X_d}d_L, \quad l'_L = e^{-i\alpha X_l}l_L,\\
&\nu'_L = e^{-i\alpha X_\nu}\nu_L, \;\; j'_L = e^{-i\alpha X_j}j_L, \;\; J'_L = e^{-i\alpha X_J}J_L,\\
&E'_L = e^{-i\alpha X_E}E_L,
\end{aligned}
\tag{15}
$$

and in the scalar sector we have the following PQ charges:

$$
\begin{aligned}
&\eta'^0 = e^{-2i\alpha X_u}\eta^0 = e^{+2i\alpha X_d}\eta^0,\\
&\eta_1'^- = e^{-i\alpha(X_u+X_d)}\eta_1^- = e^{+i(X_u+X_d)}\eta_1^-,\\
&\eta_2^{+\prime} = e^{-i\alpha(X_J+X_u)}\eta_2^+ = e^{+i\alpha(X_j+X_d)}\eta_2^+,\\
&\rho'^0 = e^{+2i\alpha X_u}\rho^0 = e^{-2i\alpha X_d}\rho^0,\\
&\rho'^+ = e^{-i\alpha(X_u+X_d)}\rho^+ = e^{+i\alpha(X_u+X_d)}\rho^+,\\
&\rho'^{++} = e^{-i\alpha(X_J+X_d)}\rho^{++} = e^{+i\alpha(X_j+X_u)}\rho^{++},\\
&\chi'^- = e^{-i\alpha(X_u+X_J)}\chi^- = e^{+i\alpha(X_d+X_j)}\chi^-,\\
&\chi'^{--} = e^{-i\alpha(X_d+X_J)}\chi^{--} = e^{+i\alpha(X_u+X_j)}\chi^{--},\\
&\chi'^0 = e^{-2i\alpha X_J}\chi^0 = e^{+2i\alpha X_j}\chi^0,\\
&\phi' = e^{-2iX_j}\phi.
\end{aligned}
\tag{16}
$$

From Eqs. (15) and (16) we obtain the following relations:

$$
X_d = -X_u = X_l = -X_\nu, \quad X_j = -X_J = -X_E. \tag{17}
$$

In the present model, although we have two independent PQ charges (say, X_d and X_j), the known quark contributions to $\bar{\theta}$ which are proportional to X_d cancel out exactly. Only X_j is important for solving the CP problem:

$$
\bar{\theta}\to\bar{\theta} - 2\alpha\sum_{\text{all quarks}} X_f = \bar{\theta} - 2\alpha X_j. \tag{18}
$$

Hence we can assume that $X_d=0$ and the only relevant PQ transformations are

$$
\begin{aligned}
&j'_L = e^{-i\alpha X_j}j_L, \;\; J'_L = e^{i\alpha X_j}J_L,\\
&E'_L = e^{-i\alpha X_j}E_L, \;\; \eta_2^{+\prime} = e^{i\alpha X_j}\eta_2^+,\\
&\rho'^{++} = e^{i\alpha X_j}\rho^{++}, \;\; \chi'^- = e^{i\alpha X_j}\chi^-\\
&\chi'^{--} = e^{i\alpha X_j}\chi^{--}, \;\; \chi'^0 = e^{2i\alpha X_j}\chi^0,\\
&\phi' = e^{-2i\alpha X_j}\phi.
\end{aligned}
\tag{19}
$$

Notice that, as in Ref. [21], we have an invisible axion (it is almost a singlet; see below), and the PQ charge that solves the strong CP problem is the charge of the exotic quarks. However, unlike in Ref. [21] the heavy quark is already present in the minimal version of this 3-3-1 model. The condition $X_u = -X_d$ is not allowed in the context of the standard $SU(2)_L\otimes U(1)_Y$ model since in this case it is not possible to shift the $\bar{\theta}$.

We have seen above that the known quark contributions to $\bar{\theta}$ cancel out exactly. This also happens in the domain wall number $N_{\rm DW}$ defined in Eq. (4). Using the PQ charges in Eqs. (15) and (16) we obtain $N_{DW}=X_j$ and since we have always chosen $X_j=1$ we see that, as in Kim's axion model, there is no domain wall problem in this 3-3-1 axion model. We stress that the contributions of the known quarks cancel out exactly even if we assume that $X_d\neq 0$. Moreover, we will see in Sec. IV that the coupling of the axion with the photon also does not depend at all on the PQ charge of the usual quarks and leptons since there is an exact cancellation among them. However, if $X_d \neq 0$ there are still couplings with the usual fermions. Notwithstanding, since X_d does not play any role in the solution of the strong *CP* problem, we can assume at the very start that $X_d = 0$, i.e., that the nontrivial PQ transformations are those in Eq. (19). This means that at the tree level there is no coupling of the axion with the known quarks and charged leptons.

We can verify that in fact the axion is almost singlet. After redefining the neutral fields as usual, $T_k^0 = (v_k + \mathrm{Re}\, T_k^0 + i\, \mathrm{Im}\, T_k^0)/\sqrt{2}$, $\phi = (v_\phi + \mathrm{Re}\, \phi + i\, \mathrm{Im}\, \phi)/\sqrt{2}$, with $k = \eta, \rho, \chi$, we obtain the constraint equations $t_x = 0$ (where $x = \eta, \rho, \chi, \phi$)

$$t_\eta = \mathrm{Re}\left[\mu_\eta^2 v_\eta + \lambda_1 |v_\eta|^2 v_\eta + \frac{1}{2}(\lambda_4 |v_\rho|^2 + \lambda_5 |v_\chi|^2) v_\eta + \frac{\lambda_{10}}{2} v_\rho v_\chi v_\phi + \lambda_{\phi\eta} |v_\phi|^2 v_\eta \right],$$

$$t_\rho = \mathrm{Re}\left[\mu_\rho^2 v_\rho + \lambda_2 |v_\rho|^2 v_\rho + \frac{1}{2}(\lambda_4 |v_\eta|^2 + \lambda_6 |v_\chi|^2) v_\rho + \frac{\lambda_{10}}{2} v_\eta v_\chi v_\phi + \lambda_{\phi\rho} |v_\phi|^2 v_\rho \right],$$

$$t_\chi = \mathrm{Re}\left[\mu_\chi^2 v_\chi + \lambda_3 |v_\chi|^2 v_\chi + \frac{1}{2}(\lambda_5 |v_\eta|^2 + \lambda_6 |v_\rho|^2) v_\chi + \frac{\lambda_{10}}{2} v_\eta v_\rho v_\phi + \lambda_{\phi\chi} |v_\phi|^2 v_\chi \right],$$

$$t_\phi = \mathrm{Re}\left[\mu_\phi^2 v_\phi + \lambda_\phi |v_\phi|^2 v_\phi + \frac{\lambda_{10}}{2} v_\eta v_\rho v_\chi + \frac{1}{2}(\lambda_{\phi\eta} |v_\eta|^2 + \lambda_{\phi\rho} |v_\rho|^2 + \lambda_{\phi\chi} |v_\chi|^2) v_\phi \right],$$

$$\mathrm{Im}(v_\phi v_\eta v_\rho v_\chi) = 0. \tag{20}$$

Notice that the solution with $v_\eta, v_\rho, v_\chi, v_\phi \neq 0$ is allowed. For instance, just for an illustration, assuming for simplicity that all vacuum expectation values (VEV) and parameters (but μ_ϕ and μ_χ) are real, $\lambda_1 v_\eta^2, \lambda_2 v_\rho^2, |\lambda_{\phi k} v_k v_\phi| \ll |\lambda_{10} v_k v_{k'}|$, we obtain

$$v_\phi^2 \approx -\frac{\mu_\phi^2}{\lambda_\phi}, \quad v_\chi^2 \approx -\frac{\mu_\chi^2}{\lambda_3},$$

$$v_\eta \approx -\lambda_{10} \frac{v_\rho v_\chi v_\phi}{\mu_\eta^2}, \quad v_\rho \approx -\lambda_{10} \frac{v_\eta v_\chi v_\phi}{\mu_\rho^2}, \tag{21}$$

and the self-consistent condition $\lambda_{10}^2 v_\chi^2 v_\phi^2 \approx \mu_\eta^2 \mu_\rho^2$. With all VEVs real the pseudoscalar mass eigenstates, in the basis $\mathrm{Im}(\eta^0, \rho^0, \chi^0, \phi)$, are given by

$$G_1^0 = \frac{1}{(v_\eta^2 + v_\rho^2)^{1/2}} (-v_\eta, v_\rho, 0, 0),$$

$$G_2^0 = \frac{1}{\sqrt{N}} \left(\frac{s_1}{s_2} v_\rho^2 v_\eta, \frac{s_1}{s_2} v_\eta^2 v_\rho, \frac{s_2}{s_1} v_\phi^2 v_\chi, \frac{s_2}{s_1} v_\chi^2 v_\phi \right),$$

$$a = \frac{1}{(v_\chi^2 + v_\phi^2)^{1/2}} (0, 0, -v_\chi, v_\phi),$$

$$A^0 = \frac{1}{\sqrt{N}} (v_\rho v_\chi v_\phi, v_\eta v_\chi v_\phi, v_\eta v_\rho v_\phi, v_\eta v_\rho v_\chi), \tag{22}$$

where $s_1 = (v_\chi^2 + v_\phi^2)^{1/2}$, $s_2 = (v_\eta^2 + v_\rho^2)^{1/2}$, and $N = v_\eta^2 [v_\rho^2 (v_\chi^2 + v_\phi^2) + v_\chi^2 v_\phi^2] + v_\rho^2 v_\chi^2 v_\phi^2$; $G_{1,2}^0$ are genuine Goldstone bosons (absorbed when the gauge bosons Z, Z' become massive), and a is the pseudo Goldstone axion. If $|v_\phi| \gg |v_k|$ we see that $a \simeq \mathrm{Im}\, \phi$ and we have an invisible axion [20]. The usual restriction coming from red giants implies $|v_\phi| > 10^9$ GeV [35]; A^0 is a heavy pseudoscalar with $m_A^2 = -(\lambda_{10}/8C)\{v_\eta^2 [v_\rho^2 (v_\chi^2 + v_\phi^2) + v_\chi^2 v_\phi^2] + v_\rho^2 v_\chi^2 v_\phi^2\}$ with $\lambda_{10} < 0$ and we have defined $C = v_\phi v_\eta v_\rho v_\chi$.

IV. CONCLUSION

We have built invisible axion models in which the axion is naturally light (protected against quantum gravity effects) because of a Z_{13} discrete symmetry. In the context of a $SU(2) \otimes U(1)$ model this symmetry and a Z_3 have to be imposed and new fields have to be added. On the other hand, in a 3-3-1 model with right-handed neutrinos added, the Z_{13} is automatic if we impose the conservation of the total lepton number and a Z_2 symmetry. Moreover, in both models $U(1)_{\mathrm{PQ}}$ is an accidental symmetry. This means that at low energy the gauge symmetries are $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes Z_{13} \otimes Z_3$ or $SU(3)_C \otimes SU(3)_L \otimes U(1)_Y \otimes Z_{13} \otimes Z_2$. Notice, however, that in the context of the standard model, even by imposing L conservation and the Z_3 symmetry, the Z_{13} symmetry is not automatic, but in the 3-3-1 model L conservation and Z_2 make Z_{13} an automatic symmetry.

Hence, we have implemented an invisible and naturally light axion in a multi-Higgs-boson extension of the standard model and also in a 3-3-1 model. Unlike the axion of the first model, in the 3-3-1 model considered here the minimal representation content (plus right-handed neutrinos) is already enough to implement a local Z_{13} symmetry. As we said before, the interesting discrete symmetries are those in which N is a prime number. In this case Z_N has no subgroup except itself and the identity [36]. Hence, the next interesting symmetry should be Z_{17}, which will allow an even lighter axion, i.e., $m_{a(\mathrm{gravity})}^2 = (v_\phi)^{15}/M_{\mathrm{Pl}}^{13} \sim 10^{-67}$ eV^2 or $10^{-57} m_a^2$. Similarly, in this case the contribution to the θ parameter is rather small, $\overline{\theta}_{(\mathrm{gravity})} \sim 10^{-43}$.

We now consider some phenomenological consequences of the axion in the 3-3-1 model. (The case of the model of

Sec. II has been considered in Ref. [17].) In general, the axion-photon coupling is given by

$$c_{a\gamma\gamma}=\tilde{c}_{a\gamma\gamma}-1.95, \tag{23}$$

where the first term is defined as

$$\tilde{c}_{a\gamma\gamma}=\frac{1}{N_{DW}}\sum_{\text{all fermions}} X_f Q_f^2, \tag{24}$$

with $N_{DW}=1$ in this model since it has no domain wall problem as in Kim's model [21]; X_f and Q_f are the PQ and electromagnetic charge, respectively, of the fermion *f*. The term -1.95 comes from the light quark PQ anomalies and it exists only if these quarks carry PQ charges. In Eq. (24) the contribution proportional to X_d cancels out exactly even if we assume that $X_d\neq 0$. So we have in general that $c_{a\gamma\gamma}=-(2/3)X_j-1.95$, or $c_{a\gamma\gamma}=2.62\ (-1.28)$ for $X_j=+1(-1)$. In our case in particular, since $X_d=0$, we have just the first contribution in Eq. (23), i.e., $c_{a\gamma\gamma}=\tilde{c}_{a\gamma\gamma}=\pm(2/3)$. We stress that the contributions of the PQ charges of the known quarks cancel out exactly in $\bar{\theta}$ in Eq. (18), in the domain wall number given in Eq. (4), and also in Eq. (24). Thus, if we assign PQ charges to those quarks it implies only a coupling with the axion at the tree level. On the other hand, if we assume that only the exotic fermions of the model carry a PQ charge, the coupling with the known quarks and leptons arises at higher order in perturbation theory.

We have imposed at the very start the conservation of the total lepton number and a Z_2 symmetry. Another possibility is to impose the discrete symmetry Z_{13}. In this case only the $U(1)_{\rm PQ}$ symmetry is automatic and the quartic *L*-violating term $\chi^\dagger\eta\rho^\dagger\eta$ is allowed. This term implies a new relation among the PQ charges of the known particles and the exotic ones: $X_j=3X_d$. Taking also into account Eqs. (17), it can be shown that there is a surviving symmetry $Z_3\subset U(1)_{\rm PQ}$, which implies a domain wall problem [37].

The 3-3-1 model in which the leptons transform as $\Psi_{aL}=(\nu_a,\ l_a,\ l_a^c)_L^T$ needs also the introduction of a scalar sextet $S\sim(\mathbf{6},0)$ [or singlet charged leptons $E\sim(\mathbf{1},1)$] [38,39]. In this case the Yukawa interaction in the quark sector is given by Eq. (11) and in the leptonic sector we have

$$-\mathcal{L}_Y^l=G^\nu_{ab}\overline{\Psi}_{aL}\nu_{bR}\eta+G_{ab}\overline{(\Psi)^c}_{aL}\psi_{bL}S+G'_{ab}\epsilon_{ijk}\overline{(\Psi)^c}_{iaL}\psi_{jbL}\eta_k+\text{H.c.}, \tag{25}$$

where G^ν_{ab} is an arbitrary 3×3 matrix while G_{ab} (G') is a symmetric (antisymmetric) matrix and we have omitted some $SU(3)$ indices. Notice that this model has only 13 multiplets (including right-handed neutrinos and the singlet ϕ) so we cannot have a symmetry as large as Z_{13}. However, by adding more scalar multiplets as in Ref. [40] it may be possible to implement, automatically, a large enough Z_N symmetry. The supersymmetric version of the model can also be considered since in this model, without considering right-handed neutrinos, there are 23 chiral superfields [in the same case, the minimal supersymmetric standard model (MSSM) has 14 chiral superfields] [41].

In the models considered in this work the axion couples to neutrinos too. This coupling may have astrophysical and/or cosmological consequences; we can also implement hard [42], soft [43], or spontaneous [44] *CP* violation.

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