Perturbative renormalization factors in domain-wall QCD with improved gauge actions

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We evaluate the renormalization factors of the domain-wall fermion system with various improved gauge actions at the one-loop level. The renormalization factors are calculated for the quark wave function, quark mass, bilinear quark operators, and three- and four-quark operators in the modified minimal subtraction ($\overline{\rm MS}$) scheme with dimensional reduction as well as with the naive dimensional regularization. We also present detailed results in mean field improved perturbation theory.

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I. INTRODUCTION

The domain-wall fermion formalism [1–3] offers the possibility of realizing full chiral symmetry at finite lattice spacing when the explicit chiral symmetry breaking term is suppressed exponentially in the fifth dimensional length N_5. This property is understood in terms of the overlap formalism [4] or the Ginsparg-Wilson relation [5,6].

However, the realization of the exact chiral symmetry in the $N_5 \to \infty$ limit is nontrivial. In Ref. [3] it is shown that the explicit breaking term in the axial Ward-Takahashi (WT) identity vanishes exponentially in N_5 only when the eigenvalues of the transfer matrix in the fifth direction are strictly less than unity. The exact chiral symmetry cannot be realized even in the $N_5 \to \infty$ limit if the largest eigenvalue of the transfer matrix becomes unity. Recent studies [7–12] of the chiral properties in quenched domain-wall QCD seem to reveal that this is the case for the strong coupling region around the lattice spacing $a^{-1} \sim 1$ GeV. By investigating the axial Ward-Takahashi identity it is found that a nonzero chiral symmetry breaking term remains even in the $N_5 \to \infty$ limit [11,12], and this residual quark mass extracted by the axial WT identity becomes much larger than the physical u,d quark masses at a reasonable size of $N_5 \sim 20$ for numerical simulation.

The chiral property is improved in the weak coupling region around $a^{-1} \sim 2$ GeV. The value of the residual quark mass in the axial WT identity is much smaller than in the strong coupling region. However, it is still not clear for the standard plaquette gauge action whether chiral symmetry is broken slightly but explicitly [11] or whether the symmetry breaking term vanishes exponentially in N_5 but the decay rate is small [12]. On the other hand, for the renormalization group (RG) improved gauge action [13], it was found in Ref. [11] that the value of the residual mass is much smaller than that for the plaquette action, and furthermore the residual quark mass decays exponentially in N_5 up to $N_5 = 24$, which is consistent with the realization of exact chiral symmetry. We can conclude that chiral symmetry is much better realized with the RG action than with the plaquette gauge action. It is quite reasonable to adopt a combination of the domain-wall fermion and RG improved gauge action for computational simulation.

In this paper we evaluate the renormalization factors that are needed to convert the lattice quantities to the continuum ones for the domain-wall fermion system with various improved gauge actions. We calculate the renormalization factors of the quark wave function, quark mass, bilinear quark operators, the three- and four-quark operators in the modified minimal subtraction ($\overline{\rm MS}$) scheme mainly with dimensional reduction (DRED), and give the relation between DRED and the naive dimensional regularization (NDR). Since the domain-wall height M (the mass in the five-dimensional theory) receives rather large additive quantum corrections, one must employ mean field improved perturbation theory for reliable calculation of the renormalization factors. We will explain this point in detail.

This paper is organized as follows. In Sec. II we present the action and the corresponding Feynman rules. In Sec. III we discuss the general form of the quantum correction and introduce the mean field improvement in order to treat the problem of the additive quantum correction to M. Our main result is given in Sec. IV. The finite part of the renormalization factor is evaluated numerically in Sec. V. We explain how to use the mean field improved results for various renormalization factors in Sec. VI. We close the paper with a brief summary and comments in Sec. VII.

In this paper we take the $SU(N)$ gauge group with the gauge coupling g and the second Casimir $C_F = (N^2-1)/2N$. We set $N=3$ in the numerical calculations for three- and four-quark renormalization factors. The physical quantities are expressed in lattice units and the lattice spacing a is suppressed unless necessary.

II. ACTION AND FEYNMAN RULES

We employ the Shamir domain-wall fermion action [2,3] given by

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$$S_f=\sum_{x,s,y,s'}\overline{\psi}(x,s)D_{\mathrm{DW}\,f}(x,s;y,s')\psi(y,s')+\sum_x m\overline{q}(x)q(x),\tag{1}$$

$$D_{\mathrm{DW}\,f}(x,s;y,s')=D^4(x,y)\delta_{s,s'}+D^5(s,s')\delta_{x,y}+(M-5)\delta_{x,y}\delta_{s,s'},\tag{2}$$

$$D^4(x,y)=\sum_\mu\frac{1}{2}[(1+\gamma_\mu)U_{x,\mu}\delta_{x+\hat{\mu},y}+(1-\gamma_\mu)U^\dagger_{y,\mu}\delta_{x-\hat{\mu},y}],\tag{3}$$

$$D^5(s,s')=\begin{cases}P_R\delta_{2,s'} & (s=1),\\ P_R\delta_{s+1,s'}+P_L\delta_{s-1,s'} & (1<s<N_5),\\ P_L\delta_{N_5-1,s'} & (s=N_5),\end{cases}\tag{4}$$

where x,y are four-dimensional space-time coordinates, and s,s' are fifth-dimensional or "flavor" indices, bounded as $1\leqslant s,s'\leqslant N_5$ with the free boundary condition at both ends. In this paper we will take the $N_5\to\infty$ limit and omit terms suppressed exponentially in N_5. $P_{R/L}$ is the projection matrix $P_{R/L}=(1\pm\gamma_5)/2$, m is the physical quark mass, and the domain-wall height M is a parameter of the theory, which we set as $0<M<2$ in order to realize the massless fermion at the tree level. The quark mass term and quark operators for our calculation are constructed with the four-dimensional quark field defined on the edges of the fifth dimensional space:

$$q(x)=P_R\psi(x,1)+P_L\psi(x,N_5),$$

$$\overline{q}(x)=\overline{\psi}(x,N_5)P_R+\overline{\psi}(x,1)P_L.\tag{5}$$

For the gauge part of the action we employ the following form in four dimensions:

$$S_{\mathrm{gluon}}=\frac{1}{g^2}\left\{c_0\sum_{\mathrm{plaquette}}\mathrm{Tr}U_{\mathrm{pl}}+c_1\sum_{\mathrm{rectangle}}\mathrm{Tr}U_{\mathrm{rtg}}+c_2\sum_{\mathrm{chair}}\mathrm{Tr}U_{\mathrm{chr}}+c_3\sum_{\mathrm{parallelogram}}\mathrm{Tr}U_{\mathrm{plg}}\right\},\tag{6}$$

where the first term represents the standard plaquette action, and the remaining terms are six-link loops formed by a 1×2 rectangle, a bent 1×2 rectangle (chair), and a three-dimensional parallelogram. The coefficients $c_0,\ldots,c_3$ satisfy the normalization condition

$$c_0+8c_1+16c_2+8c_3=1.\tag{7}$$

The RG improved gauge action is defined by setting the parameters to the value suggested by an approximate renormalization group analysis. In the following we will adopt the following choices: $c_1=-0.331,c_2=c_3=0$ (Iwasaki) and $c_1=-0.27,c_2+c_3=-0.04$ (Iwasaki′) [13]; $c_1=-0.252,c_2+c_3=-0.17$ (Wilson) [14]; and $c_1=-1.40686,c_2=c_3=0$ (DBW2) [15] for the RG improved gauge action, as well as $c_1=c_2=c_3=0$ (plaquette) and $c_1=-1/12,c_2=c_3=0$ (Symanzik) [16,17]. With these choices of parameters the RG improved gauge action is expected to realize smooth gauge field fluctuations approximating those in the continuum limit better than with the unimproved plaquette action.

Weak coupling perturbation theory is developed by writing the link variable in terms of the gauge potential

$$U_{x,\mu}=\exp\left[igA_\mu\left(x+\frac{1}{2}\hat{\mu}\right)\right]\tag{8}$$

and expanding in terms of the gauge coupling. We adopt a covariant gauge fixing with a gauge parameter α defined by

$$S_{\mathrm{GF}}=\sum_x\frac{1}{2\alpha}\left[\nabla_\mu A^a_\mu\left(x+\frac{1}{2}\hat{\mu}\right)\right]^2,\tag{9}$$

where $\nabla_\mu f_n\equiv(f_{n+\hat{\mu}}-f_n)$.

The free part of the gluon action in momentum space takes the form

$$S_0=\frac{1}{2}\int_{-\pi}^{\pi}\frac{d^4k}{(2\pi)^4}\sum_{\mu,\nu}A^a_\mu(k)\left[G_{\mu\nu}(k)-\left(1-\frac{1}{\alpha}\right)\hat{k}_\mu\hat{k}_\nu\right]\times A^a_\nu(-k),\tag{10}$$

where

$$G_{\mu\nu}(k)=\hat{k}_\mu\hat{k}_\nu+\sum_\rho(\hat{k}_\rho\delta_{\mu\nu}-\hat{k}_\mu\delta_{\rho\nu})q_{\mu\rho}\hat{k}_\rho\tag{11}$$

with

$$\hat{k}_\mu=2\sin\frac{k_\mu}{2},\tag{12}$$

and $q_{\mu\nu}$ is defined as

$$q_{\mu\nu}=(1-\delta_{\mu\nu})[1-(c_1-c_2-c_3)(\hat{k}^2_\mu+\hat{k}^2_\nu)-(c_2+c_3)\hat{k}^2].\tag{13}$$

The gluon propagator can be written as

$$D_{\mu\nu}(k)=(\hat{k}^2)^{-2}\left[\hat{k}_\mu\hat{k}_\nu+\sum_\sigma(\hat{k}_\sigma\delta_{\mu\nu}-\hat{k}_\nu\delta_{\mu\sigma})\hat{k}_\sigma A_{\sigma\nu}\right]-(1-\alpha)\frac{\hat{k}_\mu\hat{k}_\nu}{(\hat{k}^2)^2}\tag{14}$$

$$=(\hat{k}^2)^{-2}\left[(1-A_{\mu\nu})\hat{k}_\mu\hat{k}_\nu+\delta_{\mu\nu}\sum_\sigma\hat{k}^2_\sigma A_{\nu\sigma}\right]-(1-\alpha)\frac{\hat{k}_\mu\hat{k}_\nu}{(\hat{k}^2)^2},\tag{15}$$

where $A_{\mu\nu}$ is a function of $q_{\mu\nu}$ and $\hat{k}_\mu$; for its form we refer to the original literature [13,16]. In this paper we will adopt the Feynman gauge ($\alpha=1$) without loss of generality, since the renormalization factors for the physical quantities such as quark mass, bilinear quark operators, and three- and four-quark operators do not depend on the choice of the gauge fixing condition.

For the fermion part we need the following three types of propagators in domain-wall QCD. One is the propagator which connects general flavor indices,

$$S(p)_{st}=\sum_{u=1}^{N_5}(-i\gamma_\mu \sin p_\mu+W^-+mM^-)_{su}G_R(u,t)P_R + \sum_{u=1}^{N_5}(-i\gamma_\mu \sin p_\mu+W^++mM^+)_{su}G_L(u,t)P_L, \tag{16}$$

where the mass matrix is

$$W^+=\begin{pmatrix} -W & 1 & & \\ & -W & \ddots & \\ & & \ddots & 1 \\ & & & -W \end{pmatrix},$$

$$W^-_{s,t}=\begin{pmatrix} -W & & & \\ 1 & -W & & \\ & \ddots & \ddots & \\ & & 1 & -W \end{pmatrix}, \tag{17}$$

$$M^+=\begin{pmatrix} & \\ 1 & \end{pmatrix}, \quad M^-=\begin{pmatrix} & 1 \\ & \end{pmatrix}, \tag{18}$$

and $G_{R/L}$ is given by

$$G_R(s,t)=\frac{A}{F}[-(1-m^2)(1-We^{-\alpha})e^{\alpha(-2N_5+s+t)}-(1-m^2)\times(1-We^{\alpha})e^{-\alpha(s+t)} -2W\sinh(\alpha)m(e^{\alpha(-N_5+s-t)}+e^{\alpha(-N_5-s+t)})] +Ae^{-\alpha|s-t|}, \tag{19}$$

$$G_L(s,t)=\frac{A}{F}[-(1-m^2)(1-We^{\alpha})e^{\alpha(-2N_5+s+t-2)} -(1-m^2)(1-We^{-\alpha})e^{\alpha(-s-t+2)} -2W\sinh(\alpha)m(e^{\alpha(-N_5+s-t)}+e^{\alpha(-N_5-s+t)})] +Ae^{-\alpha|s-t|}, \tag{20}$$

$$\cosh(\alpha)=\frac{1+W^2+\sum_\mu \sin^2 p_\mu}{2|W|}, \tag{21}$$

$$A=\frac{1}{2W\sinh(\alpha)}, \tag{22}$$

$$F=1-e^{\alpha}W-m^2(1-We^{-\alpha}), \tag{23}$$

$$W=1-M+\sum_\mu (1-\cos p_\mu). \tag{24}$$

When W becomes negative the fermion propagator is given with the replacement $e^{\pm\alpha}\to -e^{\pm\alpha}$.

The second one is the propagator that connects the physical quark field and the fermion field of general flavor index,

$$\langle q(p)\overline{\psi}(-p,s)\rangle=\frac{1}{F}[i\gamma_\mu \sin mp_\mu-m(1-We^{-\alpha})] \times(e^{-\alpha(N_5-s)}P_R+e^{-\alpha(s-1)}P_L) +\frac{1}{F}\{m[i\gamma_\mu \sin p_\mu-m(1-We^{-\alpha})] -F\}e^{-\alpha}(e^{-\alpha(s-1)}P_R+e^{-\alpha(N_5-s)}P_L), \tag{25}$$

$$\langle \psi(p,s)\overline{q}(-p)\rangle=\frac{1}{F}(e^{-\alpha(N_5-s)}P_L+e^{-\alpha(s-1)}P_R) \times[i\gamma_\mu \sin p_\mu-m(1-We^{-\alpha})] +\frac{1}{F}(e^{-\alpha(s-1)}P_L +e^{-\alpha(N_5-s)}P_R)e^{-\alpha}\{m[i\gamma_\mu \sin p_\mu -m(1-We^{-\alpha})]-F\}, \tag{26}$$

and the third one is the two-point function of the physical quark field,

$$S_q(p)\equiv\langle q(p)\overline{q}(-p)\rangle=\frac{-i\gamma_\mu \sin p_\mu+(1-We^{-\alpha})m}{-(1-e^{\alpha}W)+m^2(1-We^{-\alpha})}. \tag{27}$$

In the continuum limit the physical quark propagator becomes

$$S_q(p)=\frac{(1-w_0^2)}{i\not{p}+(1-w_0^2)m}. \tag{28}$$

Here we notice that the overall factor $1-w_0^2$ with $w_0=1-M$ appears in the quark wave function and mass.

The following two interaction vertices concern the one-loop correction:

$$V^a_{1\mu}(k,p)_{st}=-igT^a[\gamma_\mu \overline{V}_{1\mu}(k,p)+\tilde{V}_{1\mu}(k,p)]\delta_{st}, \tag{29}$$

$$V_{2\mu\nu}^{ab}(k,p)_{st}=\frac{1}{2}g^2\frac{1}{2}\{T^a,T^b\}[\gamma_\mu\tilde{V}_{1\mu}(k,p)+\overline{V}_{1\mu}(k,p)]\delta_{\mu\nu}\delta_{st}, \tag{30}$$

where

$$\overline{V}_{1\mu}(k,p)=\cos\frac{1}{2}(-k_\mu+p_\mu), \tag{31}$$

$$\tilde{V}_{1\mu}(k,p)=i\sin\frac{1}{2}(-k_\mu+p_\mu). \tag{32}$$

In this paper we calculate the one-loop correction to the Green's functions constructed with physical quark fields $q,\overline{q}$. When we carry out the perturbative calculation we first notice that the external line propagator is expanded in terms of the external momentum p_μ and quark mass m as

$$\langle q(p)\overline{\psi}(-p,s)\rangle\to\frac{1-w_0^2}{i\not{p}+(1-w_0^2)m}\left(L(s)-\frac{w_0 i\not{p}}{1-w_0^2}R(s)\right), \tag{33}$$

$$\langle\psi(p,s)\overline{q}(-p)\rangle\to\left(R(s)-L(s)\frac{w_0 i\not{p}}{1-w_0^2}\right)\frac{1-w_0^2}{i\not{p}+(1-w_0^2)m}, \tag{34}$$

where

$$L(s)=(w_0^{(N_5-s)}P_R+w_0^{(s-1)}P_L), \tag{35}$$

$$R(s)=(w_0^{(s-1)}P_R+w_0^{(N_5-s)}P_L). \tag{36}$$

The fifth dimensional indices s,t are summed with this $L(s),R(s)$ and the following form of the propagators concerns in the loop integral:

$$S_{RR}(p)\equiv\sum_{s,t=1}^{\infty}R(s)S(p)_{st}R(t)=-i\gamma_\mu\sin p_\mu\frac{1}{F}\left(-\frac{me^{-\alpha}}{(1-w_0e^{-\alpha})^2}\right)+(w_0-W)\widetilde{G_R}-\frac{1}{F}\frac{m^2e^{-\alpha}}{1-w_0e^{-\alpha}}, \tag{37}$$

$$S_{LL}(p)\equiv\sum_{s,t=1}^{\infty}L(s)S(p)_{st}L(t)=S_{RR}(p), \tag{38}$$

$$S_{RL}(p)\equiv\sum_{s,t=1}^{\infty}R(s)S(p)_{st}L(t)=-i\gamma_\mu\sin p_\mu\tilde{G}_L-\frac{m}{F}\frac{1-We^{-\alpha}}{(1-w_0e^{-\alpha})^2}, \tag{39}$$

$$S_{LR}(p)\equiv\sum_{s,t=1}^{\infty}L(s)S(p)_{st}R(t)=-i\gamma_\mu\sin p_\mu\tilde{G}_R-\frac{m}{F}\frac{e^{-2\alpha}(1-We^{\alpha})}{(1-w_0e^{-\alpha})^2}, \tag{40}$$

$$S_{Lq}(p)\equiv\sum_{s=1}^{\infty}L(s)\langle\psi(p,s)\overline{q}(-p)\rangle=\left(\frac{e^{-\alpha}}{F(1-w_0e^{-\alpha})}\right)\times[im\gamma_\mu\sin p_\mu-(1-We^{\alpha})], \tag{41}$$

$$S_{qR}(p)\equiv\sum_{s=1}^{\infty}\langle q(p)\overline{\psi}(-p,s)\rangle R(s)=S_{Lq}(p), \tag{42}$$

$$S_{qL}(p)\equiv\sum_{s=1}^{\infty}\langle q(p)\overline{\psi}(-p,s)\rangle L(s)=\frac{1}{1-w_0e^{-\alpha}}\frac{1}{F}[i\gamma_\mu\sin p_\mu-m(1-W(p)e^{-\alpha})], \tag{43}$$

$$S_{Rq}(p)\equiv\sum_{s=1}^{\infty}R(s)\langle\psi(p,s)\overline{q}(-p)\rangle=S_{qL}(p), \tag{44}$$

where

$$\widetilde{G_L}=\frac{1}{2W\sinh\alpha}\left[\frac{\sinh\alpha_0-\sinh\alpha}{2w_0\sinh\alpha_0(\cosh\alpha_0-\cosh\alpha)}-(1-m^2)\frac{1}{F}\frac{1-We^{-\alpha}}{(1-w_0e^{-\alpha})^2}\right], \tag{45}$$

$$\widetilde{G_R}=\frac{1}{2W\sinh\alpha}\left[\frac{\sinh\alpha_0-\sinh\alpha}{2w_0\sinh\alpha_0(\cosh\alpha_0-\cosh\alpha)}-(1-m^2)\frac{1}{F}\frac{e^{-2\alpha}(1-We^{\alpha})}{(1-w_0e^{-\alpha})^2}\right]. \tag{46}$$

III. MEAN FIELD IMPROVEMENT

Before evaluating the one-loop correction we discuss the mean field improvement in this section. In the domain-wall formalism the renormalization factor of an n-quark operator O_n has the generic form

$$O_n^{\overline{\rm MS}}(\mu)=ZO_n^{\rm lattice}(1/a), \tag{47}$$

$$Z=(1-w_0^2)^{-n/2}Z_w^{-n/2}Z_{O_n}, \tag{48}$$

where Z_w represents the quantum correction to the normalization factor $1-w_0^2$ of the physical quark fields $q,\overline{q}$, and Z_{O_n} is the vertex correction to O_n. Here we notice that Z_w is written in the form

$$Z_w = 1 + \frac{2w_0}{1-w_0^2}\frac{g^2 C_F}{16\pi^2}\Sigma_w. \tag{49}$$

As is known in perturbative calculations [18] the one-loop correction in Z_w becomes huge for some choice of M because of the tadpole contribution in Σ_w and division with $1-w_0^2$. This reflects the fact that the one-loop correction to the domain-wall height M is additive rather than multiplicative [19]. Rewriting it in the multiplicative form

$$1-\left(w_0 - \frac{g^2 C_F}{16\pi^2}\Sigma_w\right)^2 \rightarrow (1-w_0^2) Z_w \tag{50}$$

can be done only when $g^2 \ll 1$ since the correction Σ_w contains the tadpole contribution and becomes large [18].

To carry out this rewriting reliably, we adopt the mean field improvement as follows:

$$\begin{aligned}
1-&\left(w_0 - \frac{g^2 C_F}{16\pi^2}\Sigma_w\right)^2 \\
&= 1-\left(w_0 + g^2 C_F 2T_{\mathrm{MF}} + \frac{g^2 C_F}{16\pi^2}(-\Sigma_w - 16\pi^2 2T_{\mathrm{MF}})\right)^2 \\
&\rightarrow 1-\left(w_0 + 4(1-u) + \frac{g^2 C_F}{16\pi^2}(-\Sigma_w - 16\pi^2 2T_{\mathrm{MF}})\right)^2 \\
&= 1-\left(w_0^{\mathrm{MF}} + \frac{g^2 C_F}{16\pi^2}(-\Sigma_w - 16\pi^2 2T_{\mathrm{MF}})\right)^2 \\
&= [1-(w_0^{\mathrm{MF}})^2]\left(1 + \frac{2w_0^{\mathrm{MF}}}{1-(w_0^{\mathrm{MF}})^2}\frac{g^2 C_F}{16\pi^2}\right. \\
&\qquad \left.\times(\Sigma_w + 16\pi^2 2T_{\mathrm{MF}})\right) \\
&= [1-(w_0^{\mathrm{MF}})^2] Z_w^{\mathrm{MF}}.
\end{aligned} \tag{51}$$

T_{MF} is the one-loop correction to the mean field factor defined by

$$u = 1 - g^2 C_F \frac{T_{\mathrm{MF}}}{2} + \cdots, \tag{52}$$

where $u = P^{1/4}$ with P being the plaquette. In the second line of Eq. (51) we have replaced the perturbative correction $g^2 C_F T_{\mathrm{MF}}$ to the domain-wall height with the nonperturbative value $2(1-u)$, according to the standard procedure of mean field improved perturbation theory. On the other hand, the perturbative value is still used for Z_w^{MF}, since the mean field improved value $\Sigma_w + 16\pi^2 2T_{\mathrm{MF}}$ becomes small enough.

The procedure for the mean field improvement for the renormalization factor of the general n-quark operator O_n becomes as follows. We factor out the mean field contribution perturbatively from the vertex correction Z_{O_n} and replace it with the nonperturbative one:

$$\begin{aligned}
Z_{O_n} &= \left(1 - g^2 C_F \frac{n}{4} T_{\mathrm{MF}}\right)\left(Z_{O_n} + g^2 C_F \frac{n}{4} T_{\mathrm{MF}}\right) \\
&\rightarrow u^{n/2}\left(Z_{O_n} + g^2 C_F \frac{n}{4} T_{\mathrm{MF}}\right),
\end{aligned} \tag{53}$$

where $u = P^{1/4}$ is evaluated numerically. This leads to the rewriting of the total renormalization factor of O_n,

$$Z \rightarrow Z^{\mathrm{MF}} = [1-(w_0^{\mathrm{MF}})^2]^{-n/2}(Z_w^{\mathrm{MF}})^{-n/2} u^{n/2} Z_{O_n}^{\mathrm{MF}}, \tag{54}$$

where

$$w_0^{\mathrm{MF}} = w_0 + 4(1-u), \tag{55}$$

$$Z_w^{\mathrm{MF}} = Z_w|_{w_0 = w_0^{\mathrm{MF}}} + \frac{4w_0^{\mathrm{MF}}}{1-(w_0^{\mathrm{MF}})^2} g^2 C_F T_{\mathrm{MF}}. \tag{56}$$

Note that the difference between the mean field improved renormalization factor and the unimproved one is of higher order in the perturbative expansion. The renormalization factors of the quark wave function, the quark mass, and the n-quark operator are shifted as

$$Z_2^{\mathrm{MF}} = Z_2|_{w_0 = w_0^{\mathrm{MF}}} + \frac{1}{2} g^2 C_F T_{\mathrm{MF}}, \tag{57}$$

$$Z_m^{\mathrm{MF}} = Z_m|_{w_0 = w_0^{\mathrm{MF}}} - \frac{1}{2} g^2 C_F T_{\mathrm{MF}}, \tag{58}$$

$$Z_{O_n}^{\mathrm{MF}} = Z_{O_n}|_{w_0 = w_0^{\mathrm{MF}}} + \frac{n}{4} g^2 C_F T_{\mathrm{MF}}. \tag{59}$$

Now we have a short comment on the derivation of T_{MF}. The one-loop correction is given by expanding the plaquette value in the gauge coupling and executing the momentum integral:

$$P = 1 - g^2 C_F 2T_{\mathrm{MF}} = 1 - g^2 C_F 2(T + \delta T), \tag{60}$$

where T is the tadpole contribution and δT is the remaining contribution,

$$T = \int_k D_{\mu\mu}(k), \tag{61}$$

$$\begin{aligned}
\delta T = -\int_k \bigg(&\frac{1}{2}[\cos k_\nu D_{\mu\mu}(k) + \cos k_\mu D_{\nu\nu}(k)] \\
&+ 2\sin\frac{k_\mu}{2}\sin\frac{k_\nu}{2} D_{\mu\nu}(k)\bigg),
\end{aligned} \tag{62}$$

with μ, ν unsummed and $\mu \neq \nu$. Here, the momentum integral means

$$\int_k = \int_{-\pi}^{\pi} \frac{d^4 k}{(2\pi)^4}. \tag{63}$$

After numerical integration we get

$$T_{\rm MF}=\begin{cases} 1/8 & (\text{plaquette}),\\ 0.0525664 & (\text{Iwasaki},\quad c_1=-0.331,\quad c_2=c_3=0),\\ 0.0191580 & (\text{DBW2},\quad c_1=-1.40686,\quad c_2=c_3=0),\\ 0.0915657 & (\text{Symanzik},\quad c_1=-1/12,\quad c_2=c_3=0),\\ 0.0552016 & (\text{Iwasaki}',\quad c_1=-0.27,\quad c_2+c_3=-0.04),\\ 0.0482425 & (\text{Wilson},\quad c_1=-0.252,\quad c_2+c_3=-0.17). \end{cases} \tag{64}$$

Here, please notice that the finite part Σ_w and the tadpole factor $T_{\rm MF}$, which is defined through the plaquette P, are gauge independent.

The mean field improved factor can also be defined by perturbative evaluation of the link variable (8). In this case the averaged link variable is written in terms of T and the gauge dependent part $\delta T_{\rm gauge}$,

$$u=\langle U_\mu(n)\rangle=1-C_F g^2\frac{1}{2}[T-(1-\alpha)\delta T_{\rm gauge}]. \tag{65}$$

The gauge dependent term is independent of the choice of gauge action,

$$\delta T_{\rm gauge}=0.0387334. \tag{66}$$

T is given by

$$T=\begin{cases} 0.1549334 & (\text{plaquette}),\\ 0.0947597 & (\text{Iwasaki},\quad c_1=-0.331,\quad c_2=c_3=0),\\ 0.0624262 & (\text{DBW2},\quad c_1=-1.40686,\quad c_2=c_3=0),\\ 0.1282908 & (\text{Symanzik},\quad c_1=-1/12,\quad c_2=c_3=0),\\ 0.0973746 & (\text{Iwasaki}',\quad c_1=-0.27,\quad c_2+c_3=-0.04),\\ 0.0916234 & (\text{Wilson},\quad c_1=-0.252,\quad c_2+c_3=-0.17). \end{cases} \tag{67}$$

As a third choice, one may define the mean field improved factor u through the critical hopping parameter K_c of the Wilson fermion action as

$$u=\frac{1}{8K_c}=1-C_F g^2\frac{1}{2}T_{K_c}, \tag{68}$$

where [20]

$$T_{K_c}=\begin{cases} 0.162858 & (\text{plaquette}),\\ 0.082555 & (\text{Iwasaki},\quad c_1=-0.331,\quad c_2=c_3=0),\\ 0.036483 & (\text{DBW2},\quad c_1=-1.40686,\quad c_2=c_3=0),\\ 0.128057 & (\text{Symanzik},\quad c_1=-1/12,\quad c_2=c_3=0),\\ 0.086167 & (\text{Iwasaki}',\quad c_1=-0.27,\quad c_2+c_3=-0.04),\\ 0.078169 & (\text{Wilson},\quad c_1=-0.252,\quad c_2+c_3=-0.17). \end{cases} \tag{69}$$

IV. ONE-LOOP RENORMALIZATION FACTORS

In this section we derive the renormalization factors at the one-loop level for the quark wave function, the quark mass, the bilinear quark operators, and the three- and four-quark operators in the form of momentum integrals. The loop integral is evaluated numerically in the next section. Matching of the lattice and continuum operators is carried out at the scale μ in the $\overline{\rm MS}$ scheme with dimensional reduction or naive dimensional regularization. The difference between NDR and DRED resides only in the finite parts of the renormalization factors in the continuum. The finite parts on the lattice are derived with mean field improvement. Hereafter we suppress the index ‘‘MF’’ in quantities unless confusion may arise.

A. Quark propagator

The one-loop correction to the physical quark propagator is given by two diagrams. The contribution from the tadpole diagram is given by

$$G_{\rm tad}=\sum_{s=1}^{\infty}\frac{1-w_0^2}{i\not p+(1-w_0^2)m}\left(L(s)-\frac{w_0 i\not p}{1-w_0^2}R(s)\right)\times V^{ab}_{2\mu\nu}(-p,p)\delta^{ab}\int_k D_{\mu\nu}(k)\left(R(s)-L(s)\frac{w_0 i\not p}{1-w_0^2}\right)\times\frac{1-w_0^2}{i\not p+(1-w_0^2)m}. \tag{70}$$

The contribution from the rising sun diagram is written as

$$G_{\rm rs}=\sum_{s,t=1}^{\infty}\frac{1-w_0^2}{i\not p+(1-w_0^2)m}\left(L(s)-\frac{w_0 i\not p}{1-w_0^2}R(s)\right)\int_k V^a_{1\mu}\times(-p,p-k)S(p-k)_{st}V^b_{1\nu}(-(p-k),p)\delta^{ab}D_{\mu\nu}(k)\times\left(R(t)-L(t)\frac{w_0 i\not p}{1-w_0^2}\right)\frac{1-w_0^2}{i\not p+(1-w_0^2)m}. \tag{71}$$

Making use of the summation formulas (37)–(44) and following the calculation in Ref. [18], we obtain the "full" quark propagator at the one-loop level on the lattice and in the continuum with the $\overline{\rm MS}$ scheme as follows:

$$\langle q_{\rm lat}\bar q_{\rm lat}\rangle=\frac{(1-w_0^2)Z_w u^{-1}Z_2^{\rm lat}}{i\not p+(1-w_0^2)Z_w m_{\rm lat}/(uZ_m^{\rm lat})}, \tag{72}$$

$$\langle q_{\overline{\rm MS}}\bar q_{\overline{\rm MS}}\rangle=\frac{Z_2^{\overline{\rm MS}}}{i\not p+m_{\overline{\rm MS}}/Z_m^{\overline{\rm MS}}}, \tag{73}$$

where the mean field improvement is used on the lattice. Comparing these two expressions for the quark propagator, we obtain the following relations:

$$q_{\overline{\rm MS}}=(1-w_0^2)^{-1/2}Z_w^{-1/2}(uZ_2)^{1/2}q_{\rm lat}, \tag{74}$$

$$m_{\overline{\rm MS}}=(1-w_0^2)Z_wZ_mu^{-1}m_{\rm lat}, \tag{75}$$

where

$$Z_2=Z_2^{\overline{\rm MS}}/Z_2^{\rm lat}, \tag{76}$$

$$Z_m=Z_m^{\overline{\rm MS}}/Z_m^{\rm lat}. \tag{77}$$

The explicit forms for the Z factors are given below. The renormalization factor for $1-w_0^2$ is written as

$$Z_w=1+\frac{g^2}{16\pi^2}C_Fz_w^{\rm MF}, \tag{78}$$

$$z_w^{\rm MF}=\frac{2w_0}{1-w_0^2}[\Sigma_w+32\pi^2T_{\rm MF}], \tag{79}$$

$$\Sigma_w=16\pi^2\left\{-2T+4(1-w_0^2)\times\int_k\frac{(T_4-T_3)S_F+T_1(\widetilde{G_L}|_{m=0}+\widetilde{G_R}|_{m=0})}{G_0^2}\right\}. \tag{80}$$

The quark wave function renormalization factor is given by

$$Z_2=1+\frac{g^2}{16\pi^2}C_F[-\log(\mu a)^2+z_2^{\rm MF}], \tag{81}$$

$$z_2^{\rm MF}=\Sigma_1^{\overline{\rm MS}}-\Sigma_1+16\pi^2\frac{T_{\rm MF}}{2}, \tag{82}$$

$$\Sigma_1^{\overline{\rm MS}}=-\frac{1}{2}({\rm DRED}),\frac{1}{2}({\rm NDR}), \tag{83}$$

$$\Sigma_1=16\pi^2\frac{T}{2}+16\pi^2\int_k\left\{\frac{1-w_0^2}{G_0^2}\{\widetilde{G_L}|_{m=0}(-2T_1+2T_5-cT_4)+\widetilde{G_R}|_{m=0}[-2(w_0-W)(T_4-T_3)-cT_3)+f_L^\beta(2T_2^\beta-4T_4s_\beta^2)+f_R^\beta(-2(w_0-W)T_1^\beta-4s_\beta^2]\}+\frac{\theta(\pi^2-k^2)}{(k^2)^2}\right\}-\log\pi^2. \tag{84}$$

The quark mass renormalization factor becomes

$$Z_m=1+\frac{g^2}{16\pi^2}C_F[-3\log(\mu a)^2+z_m^{\rm MF}], \tag{85}$$

$$z_m^{\rm MF}=(\Sigma_2^{\overline{\rm MS}}-\Sigma_2)-(\Sigma_1^{\overline{\rm MS}}-\Sigma_1)-16\pi^2\frac{T_{\rm MF}}{2}, \tag{86}$$

$$\Sigma_2^{\overline{\rm MS}}=-4({\rm DRED}),-2({\rm NDR}), \tag{87}$$

$$\Sigma_2=4\times16\pi^2\int_k\left\{\frac{1}{G_0^2}[T_4F_m^{RL}+2T_1F_m-T_3F_m^{LR}]+\frac{\theta(\pi^2-k^2)}{(k^2)^2}\right\}-4\log\pi^2. \tag{88}$$

Here T is the tadpole factor (61) and $T_{\rm MF}$ is given in Eq. (60). No sum is taken for β. We have used the following shorthand notation: $s_\mu=\sin k_\mu$, $c_\mu=\cos k_\mu$, $\hat s_\mu=\sin(k_\mu/2)$, $\hat c_\mu=\cos(k_\mu/2)$, $c=\Sigma_\mu c_\mu$, $G_0=\hat k^2$, $s^2=\Sigma_\mu s_\mu^2$, $\hat s^2=\Sigma_\mu\hat s_\mu^2$,

$$\widetilde{G_L}|_{m=0}=\frac{1}{2W\,\mathrm{sh}}\left[\frac{\mathrm{sh}_0-\mathrm{sh}}{2w_0\,\mathrm{sh}_0(\mathrm{ch}_0-\mathrm{ch})}-\frac{1-We^{-\alpha}}{F_0(1-w_0e^{-\alpha})^2}\right], \tag{89}$$

$$\widetilde{G_R}|_{m=0}=\frac{1}{2W\,\mathrm{sh}}\left[\frac{\mathrm{sh}_0-\mathrm{sh}}{2w_0\,\mathrm{sh}_0(\mathrm{ch}_0-\mathrm{ch})}-\frac{e^{-2\alpha}}{(1-w_0e^{-\alpha})^2}\right], \tag{90}$$

$$S_F=(w_0-W)\widetilde{G_R}|_{m=0}, \tag{91}$$

$$\begin{aligned} f_L^\beta=&\left(\frac{r}{W}-\frac{\mathrm{ch}}{\mathrm{sh}}g_\beta\right)\widetilde{G_L}|_{m=0}\\ &+\frac{g_\beta}{2W\,\mathrm{sh}}\frac{1}{2w_0\,\mathrm{sh}_0}\frac{1+\mathrm{sh}_0\,\mathrm{sh}-\mathrm{ch}_0\mathrm{ch}}{(\mathrm{ch}_0-\mathrm{ch})^2}\\ &+\frac{1}{W\,\mathrm{sh}}\frac{1}{F^2|_{m=0}}\frac{r\cdot\mathrm{sh}-Wg_\beta(\mathrm{ch}-W)}{(1-w_0e^{-\alpha})^2}\\ &+\frac{g_\beta}{W\,\mathrm{sh}}\frac{1-We^{-\alpha}}{F_0}\frac{w_0e^{-\alpha}}{(1-w_0e^{-\alpha})^3}, \end{aligned} \tag{92}$$

$$g_\beta=\frac{c_\beta+r(\mathrm{ch}-W)}{W\,\mathrm{sh}}, \tag{93}$$

$$\begin{aligned} f_R^\beta=&\left(\frac{r}{W}-\frac{\mathrm{ch}}{\mathrm{sh}}g_\beta\right)\widetilde{G_R}|_{m=0}\\ &+\frac{g_\beta}{2W\,\mathrm{sh}}\frac{1}{2w_0\,\mathrm{sh}_0}\frac{1+\mathrm{sh}_0\,\mathrm{sh}-\mathrm{ch}_0\mathrm{ch}}{(\mathrm{ch}_0-\mathrm{ch})^2}\\ &+\frac{1}{W\,\mathrm{sh}}\frac{g_\beta e^{-2\alpha}}{(1-w_0e^{-\alpha})^3}, \end{aligned} \tag{94}$$

$$F^\beta=(w_0-W)f_R^\beta+r\widetilde{G_R}|_{m=0}, \tag{95}$$

$$F_m=\frac{e^{-\alpha}}{F_0(1-w_0e^{-\alpha})^2}, \tag{96}$$

$$F_m^{RL}=\frac{1-We^{-\alpha}}{F_0}\frac{1}{(1-w_0e^{-\alpha})^2}, \tag{97}$$

$$F_0=1-We^{\alpha}, \tag{98}$$

$$F_m^{LR}=\frac{e^{-2\alpha}}{(1-w_0e^{-\alpha})^2}, \tag{99}$$

$$T_1=\frac{1}{2}\hat{s}^2s^2, \tag{100}$$

$$T_1^\beta==2\hat{s}^2s_\beta^2, \tag{101}$$

$$T_2^\beta=s^2s_\beta^2+4s_\beta^2\hat{c}_\beta^2\sum_\mu\overline{A}_{\beta\mu}\hat{s}_\mu^2-s_\beta^2\sum_\mu\overline{A}_{\beta\mu}s_\mu^2, \tag{102}$$

$$T_3=(\hat{s}^2)^2, \tag{103}$$

$$T_4=\sum_{\mu\nu}\hat{c}_\mu^2\overline{A}_{\mu\nu}\hat{s}_\nu^2, \tag{104}$$

$$T_5=\sum_{\mu\nu}c_\mu\hat{c}_\mu^2\overline{A}_{\mu\nu}\hat{s}_\nu^2, \tag{105}$$

$$\overline{A}_{\mu\nu}=\delta_{\mu\nu}+A_{\mu\nu}. \tag{106}$$

Some fundamental quantities are given by

$$W=1-M+4(1-u)-r\sum_\mu(1-c_\mu),$$

$$w_0=1-M+4(1-u), \tag{107}$$

$$\mathrm{ch}=\cosh(\alpha)=\frac{1+W^2+s^2}{2W},$$

$$\mathrm{ch}_0=\cosh(\alpha_0)=\frac{1+w_0}{2w_0}, \tag{108}$$

$$\mathrm{sh}=\sinh(\alpha),\quad \mathrm{sh}_0=\sinh(\alpha_0). \tag{109}$$

Here we notice that the domain-wall height is shifted by the nonperturbative mean field improvement factor u in this section, which is essential for z_w^{MF} as seen in the previous section.

B. Bilinear operators

We consider the local bilinear operators constructed with physical quark fields,

$$O_\Gamma=\overline{q}\Gamma q, \tag{110}$$

where

$$\Gamma=1(S),\gamma_5(P),\gamma_\mu(V),\gamma_\mu\gamma_5(A),\sigma_{\mu\nu}(T). \tag{111}$$

The one-loop correction to the bilinear quark operators is evaluated in the same way as in Ref. [18]. The operator matching relation is given by

$$O_\Gamma^{\overline{\mathrm{MS}}}(\mu)=(1-w_0^2)^{-1}Z_w^{-1}uZ_\Gamma(\mu a)O_\Gamma^{\mathrm{lat}}(1/a), \tag{112}$$

where the renormalization factor is given by

$$Z_\Gamma=1+\frac{g^2}{16\pi^2}C_F\left[\left(\frac{h_2(\Gamma)}{4}-1\right)\log(\mu a)^2+z_\Gamma^{\mathrm{MF}}\right], \tag{113}$$

$$z_\Gamma^{\mathrm{MF}}=z_\Gamma^{\overline{\mathrm{MS}}}-z_\Gamma^{\mathrm{lat}}+16\pi^2\frac{T_{\mathrm{MF}}}{2}, \tag{114}$$

$$h_2(\Gamma)=4(\mathrm{VA}),16(\mathrm{SP}),0(\mathrm{T}), \tag{115}$$

$$z_\Gamma^{\overline{\mathrm{MS}}}(\mathrm{DRED}) = V_\Gamma^{\overline{\mathrm{MS}}}-1/2=1/2(\mathrm{VA}),7/2(\mathrm{SP}),-1/2(\mathrm{T}), \tag{116}$$

$$z_\Gamma^{\overline{\mathrm{MS}}}(\mathrm{NDR}) = V_\Gamma^{\overline{\mathrm{MS}}}+1/2=0(\mathrm{VA}),5/2(\mathrm{SP}),1/2(\mathrm{T}), \tag{117}$$

$$z_\Gamma^{\mathrm{lat}}=V_\Gamma+\Sigma_1=\frac{h_2(\Gamma)}{4}\log\pi^2+16\pi^2 I_\Gamma+\Sigma_1, \tag{118}$$

$$I_\Gamma=\int_k\left\{\frac{4}{(1-w_0e^{-\alpha})^2G_0^2}\left[e^{-2\alpha}T_3-2\frac{e^{-\alpha}}{F_0}T_1+\frac{X_\Gamma}{F_0^2}\right]-\frac{h_2(\Gamma)}{4}\frac{\theta(\pi^2-k^2)}{(k^2)^2}\right\}, \tag{119}$$

$$X_\Gamma=T_4s^2(\mathrm{SP}),T_2(\mathrm{VA}),\frac{4T_2-T_4s^2}{3}(\mathrm{T}). \tag{120}$$

C. Four-quark operators (DRED)

We consider the following $\Delta S=2$ four-quark operators:

$$O_\pm=\frac{1}{2}[(\overline{q_1}\gamma_\mu^L q_2)(\overline{q_3}\gamma_\mu^L q_4)\pm(\overline{q_1}\gamma_\mu^L q_4)(\overline{q_3}\gamma_\mu^L q_2)], \tag{121}$$

$$O_1=-C_F(\overline{q_1}\gamma_\mu^L q_2)(\overline{q_3}\gamma_\mu^R q_4)+(\overline{q_1}T^a\gamma_\mu^L q_2)\times(\overline{q_3}T^a\gamma_\mu^R q_4), \tag{122}$$

$$O_2=\frac{1}{2N}(\overline{q_1}\gamma_\mu^L q_2)(\overline{q_3}\gamma_\mu^R q_4)+(\overline{q_1}T^a\gamma_\mu^L q_2)\times(\overline{q_3}T^a\gamma_\mu^R q_4), \tag{123}$$

where $\gamma_\mu^{L,R}=\gamma_\mu P_{L,R}$. The operator matching relation between the lattice and the continuum in the $\overline{\mathrm{MS}}$ scheme with DRED is given by

$$O_{4\Gamma}^{\overline{\mathrm{MS}}}(\mu)=(1-w_0^2)^{-2}Z_w^{-2}u^2Z_{4\Gamma}(\mu a)O_{4\Gamma}^{\mathrm{lat}}(1/a), \tag{124}$$

where the renormalization factor is given by

$$Z_{4\Gamma}=1+\frac{g^2}{16\pi^2}[(\delta_\Gamma-2C_F)\log(\mu a)^2+z_{4\Gamma}^{\mathrm{MF}}], \tag{125}$$

$$z_{4\Gamma}^{\mathrm{MF}}=v_\Gamma^{\overline{\mathrm{MS}}}-v_\Gamma+2C_F(\Sigma_1^{\overline{\mathrm{MS}}}-\Sigma_1)+16\pi^2C_FT_{\mathrm{MF}}, \tag{126}$$

$$v_+=\frac{N-1}{N}[(N+2)V_{\mathrm{VA}}-V_{\mathrm{SP}}],$$

$$v_+^{\overline{\mathrm{MS}}}=\frac{(N-2)(N-1)}{N}, \tag{127}$$

$$v_-=\frac{N+1}{N}[(N-2)V_{\mathrm{VA}}+V_{\mathrm{SP}}],$$

$$v_-^{\overline{\mathrm{MS}}}=\frac{(N+2)(N+1)}{N}, \tag{128}$$

$$v_1=NV_{\mathrm{VA}}-\frac{V_{\mathrm{SP}}}{N},\quad v_1^{\overline{\mathrm{MS}}}=\frac{(N-2)(N+2)}{N}, \tag{129}$$

$$v_2=\frac{N^2-1}{N}V_{\mathrm{SP}},\quad v_2^{\overline{\mathrm{MS}}}=\frac{4(N-1)(N+1)}{N}, \tag{130}$$

$$\delta_\Gamma=v_\Gamma^{\overline{\mathrm{MS}}}. \tag{131}$$

Here we notice that the finite part on the lattice can be written in terms of the one-loop correction to bilinear operators. The relation to the NDR is given in Sec. IV F.

D. Three-quark operators (DRED)

We consider the three-quark operators relevant to the proton decay (PD) amplitude,

$$(O_{\mathrm{PD}})_\delta=\varepsilon^{abc}[(\overline{q_1^c})^a\Gamma_X(q_2)^b][\Gamma_Y(q_3)^c]_\delta, \tag{132}$$

where $\overline{q^c}=-q^TC^{-1}$ with $C=\gamma_0\gamma_2$ is a charge conjugated field of q and $\Gamma_X\otimes\Gamma_Y=P_R\otimes P_R,P_R\otimes P_L,P_L\otimes P_R,P_L\otimes P_L$. $\otimes$ acts on the Dirac spinor space representing $[\gamma_X\otimes\gamma_Y]_{\alpha\beta;\gamma\delta}\equiv(\gamma_X)_{\alpha\beta}(\gamma_Y)_{\gamma\delta}$.

The operator matching relation for the three-quark operators is given as follows in the $\overline{\mathrm{MS}}$ scheme with DRED:

$$O_{\mathrm{PD}}^{\overline{\mathrm{MS}}}(\mu)=(1-w_0^2)^{-3/2}Z_w^{-3/2}u^{3/2}Z_{\mathrm{PD}}(\mu a)O_{\mathrm{PD}}^{\mathrm{lat}}(1/a), \tag{133}$$

where the renormalization factors are

$$Z_{\mathrm{PD}}=1+\frac{g^2}{16\pi^2}\left[\left(\delta_{\mathrm{PD}}-\frac{3}{2}C_F\right)\log(\mu a)^2+z_{\mathrm{PD}}^{\mathrm{MF}}\right], \tag{134}$$

$$z_{\mathrm{PD}}^{\mathrm{MF}}=v_{\mathrm{PD}}^{\overline{\mathrm{MS}}}-v_{\mathrm{PD}}+\frac{3}{2}C_F(\Sigma_1^{\overline{\mathrm{MS}}}-\Sigma_1)+16\pi^2C_F\frac{3T_{\mathrm{MF}}}{4}, \tag{135}$$

$$v_{\mathrm{PD}}=\frac{N+1}{2N}[2V_{\mathrm{VA}}+V_{\mathrm{SP}}],\quad v_{\mathrm{PD}}^{\overline{\mathrm{MS}}}=\delta_{\mathrm{PD}}=6\frac{N+1}{2N}. \tag{136}$$

E. Renormalization factor for B_K and B_P

The following quantities are important for K mesons. One is the K meson B parameter B_K, defined by

$$B_K=\frac{\langle K|\overline{s}\gamma_\mu(1-\gamma_5)d\overline{s}\gamma_\mu(1-\gamma_5)d|K\rangle}{\frac{8}{3}\langle K|\overline{s}\gamma_\mu\gamma_5 d|0\rangle\langle 0|\overline{s}\gamma_\mu\gamma_5 d|K\rangle},\qquad(137)$$

which is needed to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix from experiments, and the other is the matrix element divided by the pseudoscalar density,

$$B_P=\frac{\langle K|\overline{s}\gamma_\mu(1-\gamma_5)d\overline{s}\gamma_\mu(1-\gamma_5)d|K\rangle}{\langle K|\overline{s}\gamma_5 d|0\rangle\langle 0|\overline{s}\gamma_5 d|K\rangle},\qquad(138)$$

which can be used to measure the violation of the chiral symmetry, since it should vanish at $m_\pi\to 0$ in the presence of chiral symmetry. The s and d quark fields defining these quantities are the boundary fields given by Eq. (5) and these parameters are written in terms of the four-quark operator O_+ and bilinear quark operators O_A, O_P in the previous subsection. The renormalization factors for B_K and B_P are given by the ratio of those for O_+ and O_A, O_P:

$$Z_{B_K}(\mu a)=\frac{(1-w_0^2)^{-2}Z_w^{-2}Z_+(\mu a)}{(1-w_0)^{-2}Z_w^{-2}Z_A(\mu a)^2}=\frac{Z_+(\mu a)}{Z_A(\mu a)^2}$$
$$=1+\frac{g^2}{16\pi^2}[-4\log(\mu a)+z_{B_K}^{\rm MF}],\qquad(139)$$

$$Z_{B_P}(\mu a)=\frac{(1-w_0^2)^{-2}Z_w^{-2}Z_+(\mu a)}{(1-w_0)^{-2}Z_w^{-2}Z_P(\mu a)^2}=\frac{Z_+(\mu a)}{Z_P(\mu a)^2}$$
$$=1+\frac{g^2}{16\pi^2}[-20\log(\mu a)+z_{B_P}^{\rm MF}],\qquad(140)$$

where

$$z_{B_K}^{\rm MF}=z_+^{\rm MF}-2C_F z_A^{\rm MF},\qquad(141)$$

$$z_{B_P}^{\rm MF}=z_+^{\rm MF}-2C_F z_P^{\rm MF}.\qquad(142)$$

From the explicit notation of Eqs. (114) and (126), the mean field improvement factor $T_{\rm MF}$ is canceled out in $z_{B_K}^{\rm MF}$ and $z_{B_P}^{\rm MF}$. The remaining effect of the mean field improvement is to shift the domain-wall height M.

F. DRED and NDR

In this subsection we list the relation between NDR and DRED in the $\overline{\rm MS}$ scheme. The relations for the quark wave function and the quark mass are given by

$$z_2({\rm NDR})=z_2({\rm DRED})+1,\qquad(143)$$

$$z_m({\rm NDR})=z_m({\rm DRED})+1.\qquad(144)$$

The relations for the bilinear quark operators are

$$z_{\rm VA}({\rm NDR})=z_{\rm VA}({\rm DRED})-1/2,\qquad(145)$$

$$z_{\rm SP}({\rm NDR})=z_{\rm SP}({\rm DRED})-1,\qquad(146)$$

$$z_{\rm T}({\rm NDR})=z_{\rm T}({\rm DRED})+1.\qquad(147)$$

The renormalization factors for the four-quark operators are related by

$$z_+({\rm NDR})=z_+({\rm DRED})-3,\qquad(148)$$

$$z_-({\rm NDR})=z_-({\rm DRED})+2,\qquad(149)$$

$$z_{ij}({\rm NDR})=\begin{pmatrix} z_1({\rm DRED}) & 0\\ 0 & z_2({\rm DRED})\end{pmatrix}+\begin{pmatrix}5/6 & 8\\ 1 & -8/3\end{pmatrix}.\qquad(150)$$

The relation for the three-quark operator is

$$z_{\rm PD}({\rm NDR})=z_{\rm PD}({\rm DRED})+2/3.\qquad(151)$$

Finally, we have the relations for B_K and B_P:

$$z_{B_K}({\rm NDR})=z_{B_K}({\rm DRED})-5/3,\qquad(152)$$

$$z_{B_P}({\rm NDR})=z_{B_P}({\rm DRED})-1/3.\qquad(153)$$

Here we take the color factor $N=3$ for three- and four-quark operators and B_K, B_P.

V. NUMERICAL RESULTS

The finite parts of the renormalization factors in the previous section are numerically calculated. The necessary momentum integration is approximated by a discrete sum of L^4 with $L=64$. For three- and four-quark operators and the parameters B_K, B_P the color factor is set to $N=3$. In Tables I–XIV, the numerical values in DRED schemes without mean field (MF) improvement are given. The numerical error is estimated by varying L from 64 to 60.

The renormalization factors with the MF improvement defined in the previous section are given by the shift in the domain-wall height and subtraction of the tadpole factor $T_{\rm MF}$ as follows:

$$\Sigma_w^{\rm MF}=\Sigma_w|_{w_0=w_0^{\rm MF}}+(16\pi^2)2T_{\rm MF},\qquad(154)$$

$$z_2^{\rm MF}=z_2|_{w_0=w_0^{\rm MF}}+16\pi^2\frac{1}{2}T_{\rm MF},\qquad(155)$$

$$z_m^{\rm MF}=z_m|_{w_0=w_0^{\rm MF}}-16\pi^2\frac{1}{2}T_{\rm MF},\qquad(156)$$

$$z_\Gamma^{\rm MF}=z_\Gamma|_{w_0=w_0^{\rm MF}}+16\pi^2\frac{1}{2}T_{\rm MF},\quad \Gamma={\rm VA, SP, T},\qquad(157)$$

$$z_{4\Gamma}^{\rm MF}=z_{4\Gamma}|_{w_0=w_0^{\rm MF}}+16\pi^2 C_F T_{\rm MF},\quad \Gamma=\pm,1,2,\qquad(158)$$

TABLE I. Results for plaquette action ($c_1=c_2=c_3=0$) without mean field improvement.

M	z_2(DRED)	z_m(DRED)	Σ_w	$z_{V/A}$(DRED)	$z_{S/P}$(DRED)	z_T(DRED)
0.10	−13.6613(11)	13.4795(33)	−51.048195	−17.4958220(51)	−13.4795(33)	−18.8346(11)
0.20	−13.5109(11)	14.1140(34)	−50.744997	−17.347925(44)	−14.1140(35)	−18.4259(11)
0.30	−13.3838(11)	14.6225(33)	−50.488509	−17.223679(14)	−14.6226(32)	−18.0907(11)
0.40	−13.2739(11)	15.0721(33)	−50.266419	−17.116789(15)	−15.0720(34)	−17.7984(11)
0.50	−13.1781(11)	15.4919(33)	−50.072616	−17.024360(22)	−15.4919(34)	−17.5352(11)
0.60	−13.0948(11)	15.8983(33)	−49.903778	−16.944705(23)	−15.8983(34)	−17.2935(11)
0.70	−13.0230(11)	16.3026(33)	−49.758186	−16.876992(13)	−16.3025(33)	−17.0685(11)
0.80	−12.9625(11)	16.7136(33)	−49.635208	−16.8209860(89)	−16.7136(33)	−16.8568(11)
0.90	−12.9133(11)	17.1392(34)	−49.535083	−16.776780(31)	−17.1392(34)	−16.6560(11)
1.00	−12.8760(11)	17.5872(33)	−49.458848	−16.7449780(33)	−17.5872(34)	−16.4642(11)
1.10	−12.8513(11)	18.0673(24)	−49.408369	−16.72701(44)	−18.0682(15)	−16.2799(11)
1.20	−12.8408(11)	18.5869(31)	−49.386472	−16.72314(12)	−18.5871(28)	−16.1018(11)
1.30	−12.8464(11)	19.1603(32)	−49.397179	−16.736511(32)	−19.1604(32)	−15.9285(11)
1.40	−12.8708(11)	19.8031(32)	−49.446110	−16.769859(40)	−19.8032(31)	−15.7587(11)
1.50	−12.9179(11)	20.5366(34)	−49.541138	−16.827118(41)	−20.5366(35)	−15.5906(11)
1.60	−12.9932(11)	21.3919(34)	−49.693506	−16.914213(47)	−21.3918(35)	−15.4217(11)
1.70	−13.1050(11)	22.4162(34)	−49.919841	−17.039647(50)	−22.4162(35)	−15.2475(11)
1.80	−13.2661(11)	23.6896(34)	−50.246184	−17.216692(56)	−23.6895(35)	−15.0591(11)
1.90	−13.4989(11)	25.3751(33)	−50.717592	−17.468473(10)	−25.3751(33)	−14.8329(11)

TABLE II. Results for plaquette action (continued) without mean field improvement.

M	z_+(DRED)	z_-(DRED)	z_1(DRED)	z_2(DRED)	$z_{\rm PD}$(DRED)
0.10	−49.3331(22)	−41.3005(44)	−47.9943(11)	−35.9454(87)	−32.3141(22)
0.20	−48.4171(22)	−41.9492(47)	−47.3391(10)	−37.6374(92)	−32.5399(24)
0.30	−47.6639(22)	−42.4617(43)	−46.7968(11)	−38.9935(86)	−32.7133(21)
0.40	−47.0079(22)	−42.9184(45)	−46.3264(11)	−40.1921(89)	−32.8704(23)
0.50	−46.4200(22)	−43.3550(45)	−45.9091(11)	−41.3117(90)	−33.0271(23)
0.60	−45.8835(22)	−43.7907(45)	−45.5347(11)	−42.3955(90)	−33.1918(23)
0.70	−45.3883(22)	−44.2394(45)	−45.1968(11)	−43.4734(89)	−33.3710(22)
0.80	−44.9276(22)	−44.7128(43)	−44.8918(11)	−44.5696(87)	−33.5704(22)
0.90	−44.4965(22)	−45.2213(46)	−44.6173(10)	−45.7044(91)	−33.7952(23)
1.00	−44.0918(22)	−45.7763(45)	−44.3725(11)	−46.8993(89)	−34.0514(22)
1.10	−43.7112(25)	−46.3936(15)	−44.1583(18)	−48.1819(41)	−34.34815(44)
1.20	−43.3524(23)	−47.0803(36)	−43.9737(13)	−49.5656(75)	−34.6889(17)
1.30	−43.0148(22)	−47.8626(42)	−43.8227(12)	−51.0944(84)	−35.0890(21)
1.40	−42.6974(22)	−48.7641(41)	−43.7085(12)	−52.8086(84)	−35.5620(20)
1.50	−42.3993(22)	−49.8183(47)	−43.6358(10)	−54.7642(92)	−36.1272(24)
1.60	−42.1195(22)	−51.0747(47)	−43.6120(10)	−57.0448(93)	−36.8135(24)
1.70	−41.8547(22)	−52.6077(47)	−43.6469(10)	−59.7764(93)	−37.6636(24)
1.80	−41.5960(22)	−54.5416(48)	−43.7536(10)	−63.1721(94)	−38.7486(24)
1.90	−41.3115(22)	−57.1248(43)	−43.9471(11)	−67.6669(87)	−40.2080(22)

TABLE III. Results for Iwasaki action ($c_1=-0.331$, $c_2+c_3=0$) without mean field improvement.

M	z_2(DRED)	z_m(DRED)	Σ_w	$z_{V/A}$(DRED)	$z_{S/P}$(DRED)	z_T(DRED)
0.10	−5.1519(11)	3.9209(33)	−25.740748	−8.9630147(50)	−3.9210(33)	−10.6437(11)
0.20	−5.0193(11)	4.5020(34)	−25.489672	−8.831595(44)	−4.5020(35)	−10.2748(11)
0.30	−4.9093(11)	4.9536(33)	−25.282889	−8.723012(14)	−4.9537(32)	−9.9795(11)
0.40	−4.8158(11)	5.3423(33)	−25.107854	−8.630858(15)	−5.3423(34)	−9.7270(11)
0.50	−4.7354(11)	5.6970(33)	−24.958176	−8.552108(22)	−5.6970(34)	−9.5038(11)
0.60	−4.6665(11)	6.0333(33)	−24.830209	−8.484921(23)	−6.0333(34)	−9.3021(11)
0.70	−4.6079(11)	6.3618(33)	−24.721839	−8.428287(13)	−6.3617(33)	−9.1171(11)
0.80	−4.5593(11)	6.6907(33)	−24.631970	−8.3817624(90)	−6.6907(33)	−8.9455(11)
0.90	−4.5204(11)	7.0268(34)	−24.560281	−8.345189(31)	−7.0268(34)	−8.7847(11)
1.00	−4.4915(11)	7.3770(33)	−24.507131	−8.3188721(34)	−7.3769(34)	−8.6328(11)
1.10	−4.4731(11)	7.7494(24)	−24.473568	−8.30388(44)	−7.7503(15)	−8.4884(11)
1.20	−4.4664(11)	8.1501(31)	−24.461408	−8.30004(12)	−8.1503(28)	−8.3500(11)
1.30	−4.4727(11)	8.5912(32)	−24.473426	−8.309968(32)	−8.5913(32)	−8.2162(11)
1.40	−4.4943(11)	9.0860(32)	−24.513680	−8.335718(40)	−9.0861(31)	−8.0856(11)
1.50	−4.5343(11)	9.6525(34)	−24.588063	−8.380390(41)	−9.6525(35)	−7.9564(11)
1.60	−4.5974(11)	10.3179(34)	−24.705264	−8.448839(47)	−10.3179(35)	−7.8258(11)
1.70	−4.6905(11)	11.1243(34)	−24.878568	−8.548181(50)	−11.1243(35)	−7.6895(11)
1.80	−4.8249(11)	12.1447(34)	−25.129527	−8.689852(56)	−12.1446(35)	−7.5383(11)
1.90	−5.0209(11)	13.5324(33)	−25.497027	−8.8944944(96)	−13.5324(33)	−7.3485(11)

TABLE IV. Results for Iwasaki action (continued) without mean field improvement.

M	z_+(DRED)	z_-(DRED)	z_1(DRED)	z_2(DRED)	$z_{\rm PD}$(DRED)
0.10	−27.2627(22)	−17.1786(44)	−25.5821(11)	−10.4559(87)	−14.5647(22)
0.20	−26.4373(22)	−17.7781(47)	−24.9941(10)	−12.0053(92)	−14.7768(24)
0.30	−25.7743(22)	−18.2356(43)	−24.5178(11)	−13.2097(86)	−14.9331(21)
0.40	−25.2080(22)	−18.6309(45)	−24.1118(11)	−14.2462(89)	−15.0693(23)
0.50	−24.7090(22)	−18.9988(45)	−23.7573(11)	−15.1919(90)	−15.2008(23)
0.60	−24.2609(22)	−19.3576(45)	−23.4437(11)	−16.0887(90)	−15.3354(23)
0.70	−23.8531(22)	−19.7200(45)	−23.1643(11)	−16.9646(89)	−15.4789(22)
0.80	−23.4788(22)	−20.0966(43)	−22.9151(11)	−17.8418(87)	−15.6361(22)
0.90	−23.1328(22)	−20.4959(46)	−22.6933(10)	−18.7380(91)	−15.8114(23)
1.00	−22.8116(22)	−20.9278(45)	−22.4976(11)	−19.6719(89)	−16.0098(22)
1.10	−22.5127(25)	−21.4056(15)	−22.3282(18)	−20.6676(41)	−16.23873(44)
1.20	−22.2333(23)	−21.9338(36)	−22.1834(13)	−21.7341(75)	−16.5002(17)
1.30	−21.9723(22)	−22.5351(42)	−22.0661(12)	−22.9102(84)	−16.8075(21)
1.40	−21.7283(22)	−23.2291(41)	−21.9785(12)	−24.2296(84)	−17.1717(20)
1.50	−21.4996(22)	−24.0438(47)	−21.9237(10)	−25.7400(92)	−17.6088(24)
1.60	−21.2842(22)	−25.0223(47)	−21.9072(10)	−27.5144(93)	−18.1437(24)
1.70	−21.0777(22)	−26.2300(47)	−21.9364(10)	−29.6648(93)	−18.8138(24)
1.80	−20.8698(22)	−27.7793(48)	−22.0213(10)	−32.3857(94)	−19.6829(24)
1.90	−20.6267(22)	−29.9025(43)	−22.1727(11)	−36.0864(87)	−20.8809(22)

TABLE V. Results for DBW2 action ($c_1=-1.407, c_2+c_3=0$) without mean field improvement.

M	z_2(DRED)	z_m(DRED)	Σ_w	$z_{V/A}$(DRED)	$z_{S/P}$(DRED)	z_T(DRED)
0.10	0.0152(11)	−2.8238(33)	−11.263725	−3.78359680(17)	2.8237(33)	−5.9860(11)
0.20	0.1190(11)	−2.3223(33)	−11.080847	−3.680251(25)	2.3223(34)	−5.6811(11)
0.30	0.2020(11)	−1.9518(33)	−10.936985	−3.597872(14)	1.9518(32)	−5.4478(11)
0.40	0.2704(11)	−1.6464(33)	−10.819677	−3.529968(15)	1.6464(34)	−5.2554(11)
0.50	0.3276(11)	−1.3779(33)	−10.722539	−3.473412(22)	1.3780(34)	−5.0905(11)
0.60	0.3754(11)	−1.1315(33)	−10.641842	−3.426237(23)	1.1315(34)	−4.9455(11)
0.70	0.4151(11)	−0.8974(33)	−10.575304	−3.387277(13)	0.8975(33)	−4.8155(11)
0.80	0.4474(11)	−0.6685(33)	−10.521558	−3.3558918(89)	0.6685(33)	−4.6974(11)
0.90	0.4725(11)	−0.4390(34)	−10.479897	−3.331683(31)	0.4390(34)	−4.5886(11)
1.00	0.4905(11)	−0.2034(33)	−10.450158	−3.3146548(66)	0.2034(33)	−4.4873(11)
1.10	0.5012(11)	0.0449(24)	−10.432689	−3.30550(44)	−0.0458(15)	−4.3921(11)
1.20	0.5041(11)	0.3098(31)	−10.428393	−3.30357(12)	−0.3100(28)	−4.3014(11)
1.30	0.4981(11)	0.6012(32)	−10.438846	−3.310878(32)	−0.6013(32)	−4.2141(11)
1.40	0.4818(11)	0.9290(33)	−10.466525	−3.328712(19)	−0.9291(32)	−4.1286(11)
1.50	0.4531(11)	1.3075(34)	−10.515219	−3.359169(41)	−1.3075(35)	−4.0431(11)
1.60	0.4085(11)	1.7584(34)	−10.590776	−3.405778(31)	−1.7583(34)	−3.9549(11)
1.70	0.3427(11)	2.3164(34)	−10.702549	−3.473869(50)	−2.3164(35)	−3.8597(11)
1.80	0.2469(11)	3.0445(34)	−10.866515	−3.572405(65)	−3.0444(36)	−3.7484(11)
1.90	0.1041(11)	4.0811(33)	−11.113350	−3.7185036(95)	−4.0811(33)	−3.5976(11)

TABLE VI. Results for DBW2 action (continued) without mean field improvement.

M	z_+(DRED)	z_-(DRED)	z_1(DRED)	z_2(DRED)	z_{PD}(DRED)
0.10	−14.4945(22)	−1.2798(44)	−12.2920(11)	7.5299(88)	−3.1623(22)
0.20	−13.8157(22)	−1.8106(46)	−11.8149(11)	6.1928(91)	−3.3588(23)
0.30	−13.2941(22)	−2.1948(43)	−11.4442(11)	5.2047(86)	−3.4960(21)
0.40	−12.8642(22)	−2.5114(45)	−11.1387(11)	4.3904(89)	−3.6090(23)
0.50	−12.4967(22)	−2.7939(45)	−10.8796(11)	3.6746(90)	−3.7126(23)
0.60	−12.1752(22)	−3.0596(45)	−10.6559(11)	3.0174(90)	−3.8140(23)
0.70	−11.8892(22)	−3.3197(45)	−10.4610(11)	2.3933(89)	−3.9180(22)
0.80	−11.6320(22)	−3.5832(43)	−10.2905(11)	1.7827(87)	−4.0289(22)
0.90	−11.3983(22)	−3.8569(46)	−10.1414(10)	1.1706(91)	−4.1496(23)
1.00	−11.1844(23)	−4.1484(45)	−10.0118(11)	0.5424(89)	−4.2839(22)
1.10	−10.9878(25)	−4.4684(15)	−9.9012(18)	−0.1222(41)	−4.43787(44)
1.20	−10.8052(23)	−4.8181(36)	−9.8074(13)	−0.8267(75)	−4.6114(17)
1.30	−10.6354(22)	−5.2162(42)	−9.7322(12)	−1.6034(84)	−4.8154(21)
1.40	−10.4763(22)	−5.6771(43)	−9.6764(11)	−2.4777(86)	−5.0577(21)
1.50	−10.3256(22)	−6.2222(47)	−9.6417(10)	−3.4866(92)	−5.3505(24)
1.60	−10.1804(22)	−6.8855(46)	−9.6312(10)	−4.6888(91)	−5.7132(23)
1.70	−10.0353(22)	−7.7203(47)	−9.6495(10)	−6.1770(93)	−6.1761(24)
1.80	−9.8784(22)	−8.8225(48)	−9.70240(99)	−8.1185(95)	−6.7928(25)
1.90	−9.6743(22)	−10.3995(43)	−9.7951(11)	−10.8830(87)	−7.6788(22)

TABLE VII. Results for Symanzik action ($c_1=-1/12, c_2+c_3=0$) without mean field improvement.

M	z_2(DRED)	z_m(DRED)	Σ_w	$z_{V/A}$(DRED)	$z_{S/P}$(DRED)	z_T(DRED)
0.10	−10.0458(11)	9.5240(33)	−40.074665	−13.8700910(49)	−9.5240(33)	−15.3188(11)
0.20	−9.9009(11)	10.1413(33)	−39.788806	−13.727114(24)	−10.1412(34)	−14.9224(11)
0.30	−9.7791(11)	10.6311(33)	−39.548978	−13.6075270(18)	−10.6311(33)	−14.5997(11)
0.40	−9.6743(11)	11.0606(33)	−39.342775	−13.505042(27)	−11.0606(34)	−14.3199(11)
0.50	−9.5834(11)	11.4588(33)	−39.163976	−13.416707(22)	−11.4588(34)	−14.0693(11)
0.60	−9.5046(11)	11.8418(33)	−39.009135	−13.340761(24)	−11.8417(34)	−13.8404(11)
0.70	−9.4370(11)	12.2205(33)	−38.876383	−13.276319(19)	−12.2204(34)	−13.6283(11)
0.80	−9.3803(11)	12.6037(33)	−38.764914	−13.2230850(32)	−12.6037(33)	−13.4296(11)
0.90	−9.3344(11)	12.9987(34)	−38.674758	−13.181049(45)	−12.9987(35)	−13.2418(11)
1.00	−9.2998(11)	13.4134(33)	−38.606702	−13.150747(12)	−13.4134(33)	−13.0632(11)
1.10	−9.2773(11)	13.8565(24)	−38.562312	−13.13344(45)	−13.8574(15)	−12.8921(11)
1.20	−9.2681(11)	14.3350(31)	−38.544048	−13.12925(12)	−14.3352(28)	−12.7273(11)
1.30	−9.2740(11)	14.8627(32)	−38.555483	−13.141156(36)	−14.8628(31)	−12.5673(11)
1.40	−9.2976(11)	15.4541(34)	−38.601678	−13.171563(56)	−15.4540(35)	−12.4107(11)
1.50	−9.3426(11)	16.1298(34)	−38.689808	−13.224355(40)	−16.1298(35)	−12.2559(11)
1.60	−9.4141(11)	16.9193(32)	−38.830222	−13.304914(22)	−16.9193(32)	−12.1001(11)
1.70	−9.5200(11)	17.8679(34)	−39.038393	−13.421179(55)	−17.8679(35)	−11.9390(11)
1.80	−9.6726(11)	19.0538(34)	−39.338831	−13.586004(38)	−19.0537(34)	−11.7634(11)
1.90	−9.8936(11)	20.6363(34)	−39.774527	−13.821470(50)	−20.6362(35)	−11.5499(11)

TABLE VIII. Results for Symanzik action (continued) without mean field improvement.

M	z_+(DRED)	z_-(DRED)	z_1(DRED)	z_2(DRED)	$z_{\rm PD}$(DRED)
0.10	−39.8843(22)	−31.1922(44)	−38.4356(11)	−25.3974(88)	−24.8428(22)
0.20	−38.9962(22)	−31.8245(45)	−37.8009(11)	−27.0433(90)	−25.0636(23)
0.30	−38.2710(22)	−32.3182(44)	−37.2789(11)	−28.3497(88)	−25.2308(22)
0.40	−37.6431(22)	−32.7541(46)	−36.8283(11)	−29.4948(91)	−25.3804(23)
0.50	−37.0832(22)	−33.1673(45)	−36.4305(11)	−30.5568(90)	−25.5281(23)
0.60	−36.5747(22)	−33.5767(46)	−36.0750(11)	−31.5780(90)	−25.6822(23)
0.70	−36.1074(22)	−33.9957(45)	−35.7555(11)	−32.5878(90)	−25.8487(23)
0.80	−35.6745(22)	−34.4357(44)	−35.4680(11)	−33.6097(87)	−26.0332(22)
0.90	−35.2710(22)	−34.9063(47)	−35.2102(10)	−34.6632(93)	−26.2405(24)
1.00	−34.8936(22)	−35.4188(44)	−34.9811(11)	−35.7690(88)	−26.4766(22)
1.10	−34.5399(25)	−35.9878(14)	−34.7812(19)	−36.9530(40)	−26.74952(39)
1.20	−34.2073(23)	−36.6193(36)	−34.6093(13)	−38.2273(75)	−27.0625(17)
1.30	−33.8953(22)	−37.3386(41)	−34.4692(12)	−39.6342(84)	−27.4301(21)
1.40	−33.6025(22)	−38.1675(48)	−34.3633(10)	−41.2108(94)	−27.8648(24)
1.50	−33.3280(22)	−39.1388(47)	−34.2965(10)	−43.0127(92)	−28.3857(24)
1.60	−33.0701(22)	−40.2990(42)	−34.2750(11)	−45.1183(85)	−29.0194(21)
1.70	−32.8254(22)	−41.7187(48)	−34.3076(10)	−47.6476(94)	−29.8068(24)
1.80	−32.5842(22)	−43.5197(46)	−34.4068(10)	−50.8100(92)	−30.8172(23)
1.90	−32.3141(22)	−45.9435(47)	−34.5857(10)	−55.0298(93)	−32.1861(24)

TABLE IX. Results for Iwasaki′ ($c_1=-0.27, c_2+c_3=-0.04$) without mean field improvement.

M	z_2(DRED)	z_m(DRED)	Σ_w	$z_{V/A}$(DRED)	$z_{S/P}$(DRED)	z_T(DRED)
0.10	−5.5333(11)	4.3931(33)	−26.877090	−9.34607970(10)	−4.3932(33)	−10.9970(11)
0.20	−5.3988(11)	4.9788(34)	−26.621656	−9.212870(42)	−4.9787(35)	−10.6242(11)
0.30	−5.2871(11)	5.4350(33)	−26.410767	−9.1025605(18)	−5.4350(33)	−10.3251(11)
0.40	−5.1918(11)	5.8288(33)	−26.231888	−9.008800(27)	−5.8287(34)	−10.0688(11)
0.50	−5.1098(11)	6.1888(33)	−26.078645	−8.928565(23)	−6.1887(34)	−9.8418(11)
0.60	−5.0394(11)	6.5307(33)	−25.947409	−8.860006(25)	−6.5307(34)	−9.6364(11)
0.70	−4.9794(11)	6.8653(33)	−25.836093	−8.802140(19)	−6.8652(34)	−9.4478(11)
0.80	−4.9295(11)	7.2007(33)	−25.743629	−8.7545520(32)	−7.2007(33)	−9.2725(11)
0.90	−4.8895(11)	7.5439(34)	−25.669734	−8.717095(45)	−7.5439(35)	−9.1082(11)
1.00	−4.8597(11)	7.9020(33)	−25.614815	−8.6901393(17)	−7.9020(33)	−8.9529(11)
1.10	−4.8407(11)	8.2830(24)	−25.579975	−8.67475(45)	−8.2840(15)	−8.8050(11)
1.20	−4.8336(11)	8.6931(31)	−25.567106	−8.67079(12)	−8.6934(28)	−8.6633(11)
1.30	−4.8399(11)	9.1449(32)	−25.579077	−8.680961(36)	−9.1450(31)	−8.5263(11)
1.40	−4.8618(11)	9.6514(34)	−25.620062	−8.707290(34)	−9.6514(34)	−8.3926(11)
1.50	−4.9025(11)	10.2316(34)	−25.696112	−8.753199(41)	−10.2316(35)	−8.2604(11)
1.60	−4.9669(11)	10.9127(33)	−25.816120	−8.8234763(60)	−10.9127(33)	−8.1271(11)
1.70	−5.0619(11)	11.7369(34)	−25.993647	−8.925293(55)	−11.7369(35)	−7.9881(11)
1.80	−5.1991(11)	12.7785(34)	−26.250625	−9.070482(29)	−12.7785(34)	−7.8345(11)
1.90	−5.3990(11)	14.1913(34)	−26.626481	−9.279741(50)	−14.1912(35)	−7.6426(11)

TABLE X. Results for Iwasaki′ (continued) without mean field improvement.

M	z_+(DRED)	z_-(DRED)	z_1(DRED)	z_2(DRED)	z_{PD}(DRED)
0.10	−28.2248(22)	−18.3190(44)	−26.5738(11)	−11.7151(88)	−15.3902(22)
0.20	−27.3904(22)	−18.9221(47)	−25.9790(10)	−13.2766(92)	−15.6030(24)
0.30	−26.7185(22)	−19.3835(44)	−25.4960(11)	−14.4934(88)	−15.7601(22)
0.40	−26.1435(22)	−19.7833(46)	−25.0835(11)	−15.5432(91)	−15.8975(23)
0.50	−25.6361(22)	−20.1564(45)	−24.7228(11)	−16.5032(90)	−16.0306(23)
0.60	−25.1796(22)	−20.5209(46)	−24.4031(11)	−17.4151(90)	−16.1671(23)
0.70	−24.7637(22)	−20.8898(45)	−24.1180(11)	−18.3072(90)	−16.3130(23)
0.80	−24.3814(22)	−21.2737(44)	−23.8634(11)	−19.2019(87)	−16.4732(22)
0.90	−24.0277(22)	−21.6813(47)	−23.6366(10)	−20.1171(93)	−16.6521(24)
1.00	−23.6991(22)	−22.1228(44)	−23.4364(11)	−21.0720(88)	−16.8548(22)
1.10	−23.3932(25)	−22.6116(14)	−23.2629(19)	−22.0906(40)	−17.08897(39)
1.20	−23.1071(23)	−23.1522(36)	−23.1146(13)	−23.1823(75)	−17.3566(17)
1.30	−22.8399(22)	−23.7679(41)	−22.9945(12)	−24.3867(84)	−17.6713(21)
1.40	−22.5901(22)	−24.4782(46)	−22.9047(10)	−25.7370(91)	−18.0440(23)
1.50	−22.3563(22)	−25.3130(47)	−22.8491(10)	−27.2842(92)	−18.4920(24)
1.60	−22.1364(22)	−26.3149(44)	−22.8329(11)	−29.1006(87)	−19.0398(22)
1.70	−21.9264(22)	−27.5496(48)	−22.8636(10)	−31.2983(94)	−19.7250(24)
1.80	−21.7160(22)	−29.1319(46)	−22.9520(10)	−34.0759(91)	−20.6130(23)
1.90	−21.4717(22)	−31.2945(47)	−23.1088(10)	−37.8431(93)	−21.8338(24)

TABLE XI. Results for Wilson action ($c_1=-0.252, c_2+c_3=-0.17$) without mean field improvement.

M	z_2(DRED)	z_m(DRED)	Σ_w	$z_{V/A}$(DRED)	$z_{S/P}$(DRED)	z_T(DRED)
0.10	−4.6207(11)	3.3530(33)	−24.357346	−8.432682000(89)	−3.3530(33)	−10.1259(11)
0.20	−4.4878(11)	3.9318(34)	−24.107889	−8.301041(42)	−3.9318(35)	−9.7575(11)
0.30	−4.3775(11)	4.3809(33)	−23.902639	−8.1922005(19)	−4.3810(33)	−9.4626(11)
0.40	−4.2836(11)	4.7672(33)	−23.729040	−8.099802(27)	−4.7671(34)	−9.2107(11)
0.50	−4.2030(11)	5.1193(33)	−23.580691	−8.020813(22)	−5.1193(34)	−8.9880(11)
0.60	−4.1337(11)	5.4529(33)	−23.453933	−7.953375(25)	−5.4529(34)	−8.7869(11)
0.70	−4.0748(11)	5.7786(33)	−23.346639	−7.896494(19)	−5.7786(34)	−8.6025(11)
0.80	−4.0257(11)	6.1046(33)	−23.257697	−7.8497439(31)	−6.1046(33)	−8.4314(11)
0.90	−3.9865(11)	6.4378(34)	−23.186769	−7.812963(45)	−6.4377(35)	−8.2714(11)
1.00	−3.9572(11)	6.7850(33)	−23.134195	−7.7865037(17)	−6.7850(33)	−8.1204(11)
1.10	−3.9386(11)	7.1543(24)	−23.100997	−7.77141(45)	−7.1552(15)	−7.9768(11)
1.20	−3.9316(11)	7.5517(31)	−23.088966	−7.76753(12)	−7.5519(28)	−7.8394(11)
1.30	−3.9377(11)	7.9895(32)	−23.100847	−7.777532(36)	−7.9896(31)	−7.7068(11)
1.40	−3.9593(11)	8.4808(34)	−23.140662	−7.803408(34)	−8.4807(34)	−7.5776(11)
1.50	−3.9993(11)	9.0441(34)	−23.214259	−7.848538(40)	−9.0440(35)	−7.4500(11)
1.60	−4.0626(11)	9.7062(33)	−23.330281	−7.9176562(61)	−9.7063(33)	−7.3215(11)
1.70	−4.1561(11)	10.5091(34)	−23.501949	−8.017864(55)	−10.5091(35)	−7.1875(11)
1.80	−4.2912(11)	11.5264(34)	−23.750743	−8.160901(29)	−11.5264(34)	−7.0391(11)
1.90	−4.4883(11)	12.9109(34)	−24.115456	−8.367330(50)	−12.9108(35)	−6.8528(11)

TABLE XII. Results for Wilson action (continued) without mean field improvement.

M	z_+(DRED)	z_-(DRED)	z_1(DRED)	z_2(DRED)	$z_{\rm PD}$(DRED)
0.10	−25.8736(22)	−15.7143(44)	−24.1804(11)	−8.9414(88)	−13.4789(22)
0.20	−25.0489(22)	−16.3104(47)	−23.5925(10)	−10.4847(92)	−13.6892(24)
0.30	−24.3867(22)	−16.7642(44)	−23.1163(11)	−11.6826(88)	−13.8436(22)
0.40	−23.8212(22)	−17.1559(46)	−22.7104(11)	−12.7124(91)	−13.9778(23)
0.50	−23.3232(22)	−17.5201(45)	−22.3560(11)	−13.6514(90)	−14.1073(23)
0.60	−22.8760(22)	−17.8750(46)	−22.0425(11)	−14.5410(90)	−14.2398(23)
0.70	−22.4693(22)	−18.2334(45)	−21.7633(11)	−15.4095(90)	−14.3810(23)
0.80	−22.0961(22)	−18.6058(44)	−21.5144(11)	−16.2790(87)	−14.5361(22)
0.90	−21.7514(22)	−19.0009(47)	−21.2930(10)	−17.1673(93)	−14.7091(24)
1.00	−21.4317(22)	−19.4286(44)	−21.0979(11)	−18.0932(88)	−14.9053(22)
1.10	−21.1346(25)	−19.9022(14)	−20.9292(19)	−19.0806(40)	−15.13203(39)
1.20	−20.8572(23)	−20.4259(36)	−20.7853(13)	−20.1384(75)	−15.3913(17)
1.30	−20.5987(22)	−21.0229(41)	−20.6694(12)	−21.3057(84)	−15.6965(21)
1.40	−20.3575(22)	−21.7122(46)	−20.5833(10)	−22.6153(91)	−16.0584(23)
1.50	−20.1324(22)	−22.5234(47)	−20.5309(10)	−24.1174(92)	−16.4941(24)
1.60	−19.9213(22)	−23.4986(44)	−20.5175(11)	−25.8834(87)	−17.0277(22)
1.70	−19.7201(22)	−24.7026(48)	−20.5506(10)	−28.0243(94)	−17.6966(24)
1.80	−19.5188(22)	−26.2497(46)	−20.6406(10)	−30.7369(91)	−18.5654(23)
1.90	−19.2839(22)	−28.3709(47)	−20.7984(10)	−34.4289(93)	−19.7637(24)

TABLE XIII. Values of z_{B_K} and z_{B_P} for plaquette, Iwasaki, and DBW2 gauge actions.

	Plaquette		Iwasaki		DBW2	
M	z_{B_K}(DRED)	z_{B_P}(DRED)	z_{B_K}(DRED)	z_{B_P}(DRED)	z_{B_K}(DRED)	z_{B_P}(DRED)
0.10	−2.6775(21)	−13.388(11)	−3.3614(22)	−16.807(11)	−4.4049(22)	−22.024(11)
0.20	−2.1559(22)	−10.780(11)	−2.8864(23)	−14.432(11)	−4.0017(22)	−20.009(11)
0.30	−1.7341(22)	−8.670(11)	−2.5129(22)	−12.565(11)	−3.6998(22)	−18.499(11)
0.40	−1.3632(23)	−6.816(11)	−2.1924(23)	−10.962(11)	−3.4509(22)	−17.255(11)
0.50	−1.0217(23)	−5.108(11)	−1.9034(22)	−9.517(11)	−3.2342(22)	−16.171(11)
0.60	−0.6976(22)	−3.488(11)	−1.6344(22)	−8.172(11)	−3.0385(22)	−15.193(11)
0.70	−0.3830(23)	−1.915(11)	−1.3777(22)	−6.889(11)	−2.8565(22)	−14.283(11)
0.80	−0.0716(22)	−0.358(11)	−1.1274(22)	−5.637(11)	−2.6829(21)	−13.415(11)
0.90	0.2416(22)	1.208(11)	−0.8789(22)	−4.395(11)	−2.5138(23)	−12.569(11)
1.00	0.5615(22)	2.808(11)	−0.6279(22)	−3.140(11)	−2.3454(23)	−11.727(11)
1.10	0.8941(14)	4.4707(66)	−0.3690(13)	−1.8451(65)	−2.1731(13)	−10.8656(66)
1.20	1.2427(19)	6.2133(98)	−0.0998(19)	−0.4992(98)	−1.9957(20)	−9.9785(98)
1.30	1.6159(22)	8.080(11)	0.1876(21)	0.938(11)	−1.8064(21)	−9.032(11)
1.40	2.0223(21)	10.111(11)	0.5003(21)	2.501(11)	−1.5997(21)	−7.999(11)
1.50	2.4730(22)	12.365(11)	0.8481(22)	4.240(11)	−1.3678(23)	−6.839(11)
1.60	2.9851(23)	14.925(11)	1.2460(23)	6.230(11)	−1.0983(23)	−5.492(11)
1.70	3.5843(23)	17.922(11)	1.7174(23)	8.587(11)	−0.7717(23)	−3.858(12)
1.80	4.3152(23)	21.576(12)	2.3032(23)	11.516(12)	−0.3520(24)	−1.760(12)
1.90	5.2711(22)	26.355(11)	3.0919(22)	15.460(11)	0.2418(21)	1.209(11)

TABLE XIV. Values of z_{B_K} and z_{B_P} for Symanzik, Iwasaki′, and Wilson gauge actions.

	Symanzik		Iwasaki′		Wilson	
M	z_{B_K}(DRED)	z_{B_P}(DRED)	z_{B_K}(DRED)	z_{B_P}(DRED)	z_{B_K}(DRED)	z_{B_P}(DRED)
0.10	−2.8974(22)	−14.487(11)	−3.3019(22)	−16.510(11)	−3.3864(21)	−16.932(11)
0.20	−2.3906(23)	−11.953(11)	−2.8228(23)	−14.114(11)	−2.9128(22)	−14.564(11)
0.30	−1.9843(22)	−9.921(11)	−2.4450(22)	−12.225(11)	−2.5408(22)	−12.704(11)
0.40	−1.6297(23)	−8.148(11)	−2.1201(23)	−10.600(11)	−2.2218(23)	−11.109(11)
0.50	−1.3053(23)	−6.526(11)	−1.8266(23)	−9.133(11)	−1.9344(23)	−9.672(11)
0.60	−0.9993(22)	−4.997(11)	−1.5529(23)	−7.764(11)	−1.6670(23)	−8.335(11)
0.70	−0.7039(22)	−3.520(11)	−1.2913(23)	−6.456(11)	−1.4119(22)	−7.060(11)
0.80	−0.4130(22)	−2.065(11)	−1.0359(22)	−5.180(11)	−1.1634(22)	−5.817(11)
0.90	−0.1216(23)	−0.608(11)	−0.7821(23)	−3.911(11)	−0.9168(23)	−4.584(11)
1.00	0.1751(22)	0.875(11)	−0.5254(22)	−2.627(11)	−0.6677(22)	−3.339(11)
1.10	0.4826(13)	2.4131(65)	−0.2605(13)	−1.3026(64)	−0.4108(13)	−2.0540(65)
1.20	0.8040(19)	4.0200(97)	0.0150(20)	0.0752(98)	−0.1438(20)	−0.7188(98)
1.30	1.1478(21)	5.739(11)	0.3094(21)	1.547(11)	0.1414(21)	0.707(11)
1.40	1.5216(24)	7.608(12)	0.6294(22)	3.147(11)	0.4515(23)	2.258(11)
1.50	1.9369(23)	9.685(11)	0.9856(23)	4.928(11)	0.7970(23)	3.985(11)
1.60	2.4096(22)	12.048(11)	1.3928(22)	6.964(11)	1.1924(22)	5.962(11)
1.70	2.9645(23)	14.822(11)	1.8744(23)	9.372(12)	1.6608(23)	8.304(12)
1.80	3.6452(22)	18.226(11)	2.4720(22)	12.360(11)	2.2436(23)	11.218(11)
1.90	4.5431(23)	22.716(12)	3.2743(23)	16.371(11)	3.0290(23)	15.145(11)

TABLE XV. Fit parameters a_i and b_1 of P(plaquette) as a function of g^2 for the plaquette ($c_1=0$) and RG improved ($c_1=-0.331$) gauge actions. Fit parameters of R(rectangular) for the latter are also included.

Action	$g^2_{\rm max}$		a_1	a_2	a_3	a_4	b_1
Plaquette	1.07143	P	−1.22304(5)	0.26789(2)	−0.00252(5)	0.02313(2)	−0.889706
Iwasaki	2.74348	P	−0.446426(3)	0.0448185(3)	−0.0014248(2)	0.00020329(5)	−0.3062260
		R	−0.571259(2)	0.1004102(6)	−0.0074407(3)	0.00041907(7)	−0.3023588

$$z_{\rm PD}^{\rm MF}=z_{\rm PD}|_{w_0=w_0^{\rm MF}}+16\pi^2C_F\frac{3}{4}T_{\rm MF}, \tag{159}$$

where $w_0=1-M$ and the nonperturbatively shifted $w_0^{\rm MF}$ is defined by

$$w_0^{\rm MF}=1-w_0+4(1-u). \tag{160}$$

VI. TOOLKIT FOR THE MEAN FIELD IMPROVED PERTURBATION THEORY

In this section, we explain step by step how to calculate the renormalization factors in the mean field improved perturbation theory, using our numerical results from the previous section. We assume (quenched) domain-wall QCD with the plaquette or improved gauge actions. Therefore we have two parameters, the bare gauge coupling constant $g^2=6/\beta$ and the domain-wall height M, in addition to the improved coefficient c_1 and c_2+c_3.

(1) First of all, one determines the tadpole factor u. Usually u is determined from the measured plaquette value such that $u=P^{1/4}$. From the existing data for the plaquette values at several β for the plaquette and RG improved (Iwasaki) gauge actions, P may be parametrized as a function of g^2:

$$P=\frac{1+a_1g^2+a_2g^4+a_3g^6+a_4g^8}{1+b_1g^2}. \tag{161}$$

In Table XV the coefficients a_i and b_1 are given [21], where $g^2_{\rm max}$ is the maximum value of the coupling used in the fit. The fits are constrained to satisfy the relation that $c_p=b_1-a_1$, where

$$P=1-c_pg^2+O(g^4) \tag{162}$$

at the leading order of perturbation theory. The value of c_p for various gauge actions is given in Table XVI, together with the values of $c_{R1,R3}$ defined by

$$R1=\frac{1}{3}{\rm Tr}U_{\rm rtg}=1-c_{R1}g^2+O(g^4), \tag{163}$$

$$R2=\frac{1}{3}{\rm Tr}U_{\rm chr}=1-c_{R2}g^2+O(g^4), \tag{164}$$

$$R3=\frac{1}{3}{\rm Tr}U_{\rm plg}=1-c_{R3}g^2+O(g^4), \tag{165}$$

where $R1$, $R2$ and $R3$ are the expectation value of the rectangular, chair, and parallelogram loops, respectively.

At strong coupling, such that $g^2>g^2_{\rm max}$, one had better use the measured value of P, instead of Eq. (161).

(2) One then calculates the mean field improved $\overline{\rm MS}$ coupling $g^2_{\overline{\rm MS}}(\mu)$ at the scale μ from g^2 and P as

$$\frac{1}{g^2_{\overline{\rm MS}}(\mu)}=\frac{P}{g^2}+d_g+c_p+\frac{22}{16\pi^2}\log(\mu a), \tag{166}$$

where d_g, taken from Refs. [22–24], is also given in Table XVI. According to the philosophy of the mean field improvement, an alternative formula seems more natural for the RG improved gauge action [25]:

$$\frac{1}{g^2_{\overline{\rm MS}}(\mu)}=\frac{c_0P+8c_1R1+16c_2R2+8c_3R3}{g^2}+d_g+(c_0\cdot c_p+8c_1\cdot c_{R1}+16c_2\cdot c_{R2}+8c_3\cdot c_{R3})+\frac{22}{16\pi^2}\log(\mu a). \tag{167}$$

(3) One replaces the domain-wall height M with $\tilde{M}=M-4(1-u)$.

TABLE XVI. Values of the perturbative quantities c_P, d_g, c_{Ri}, T, $T-T_{\rm MF}$, and $T_{K_c}-T_{\rm MF}$ for the various gauge actions.

Action	c_1	c_3	c_p	d_g	c_{R1}	c_{R2}	c_{R3}	T	$T-T_{\rm MF}$	$T_{K_c}-T_{\rm MF}$
Plaquette	0	0	1/3	−0.4682	0.5747	0.5228	0.5687	0.1549334	0.0299334	0.037858
Iwasaki	−0.331	0	0.1401	0.1053	0.2689	0.2223	0.2465	0.0947597	0.0421939	0.029989
DBW2	−1.40686	0	0.0511	0.5317	0.1040	0.0818	0.0921	0.0624262	0.0432682	0.017325
Symanzik	−1/12	0	0.2440	−0.2361	0.4417	0.3846	0.4215	0.1282908	0.0367251	0.036492
Iwasaki′	−0.27	−0.04	0.1471		0.2797	0.2342	0.2611	0.0973746	0.042173	0.030965
Wilson	−0.252	−0.17	0.1286	0.1196	0.2439	0.2070	0.2352	0.0916234	0.0433809	0.029927

TABLE XVII. Values of parameters a_i in the interpolation of Σ_w as a function of $w_0 = 1 - \tilde{M}$ for the various gauge actions.

Action	$32\pi^2 T_{\mathrm{MF}}$	a_0	$32\pi^2 T_{\mathrm{MF}} + a_0$	a_1	a_2	a_3	a_4
Plaquette	39.4784	−49.4658	−9.9874	−0.6944	−1.1452	0.6262	−0.7525
Iwasaki	16.6019	−24.5135	−7.9116	−0.4815	−0.8488	0.4240	−0.6429
DBW2	6.0506	−10.4557	−4.4051	−0.2676	−0.5018	0.2253	−0.5031
Symanzik	28.9190	−38.6134	−9.6945	−0.6187	−1.0446	0.5540	−0.7151
Iwasaki′	17.4342	−25.6212	−8.1870	−0.4978	−0.8724	0.4394	−0.6522
Wilson	15.2363	−23.1406	−7.9043	−0.4766	−0.8385	0.4192	−0.6411

TABLE XVIII. Values of parameters a_i in the interpolation of z_2^{MF} as a function of $w_0 = 1 - \tilde{M}$ for the various gauge actions.

Action	$8\pi^2 T_{\mathrm{MF}} + a_0$		a_1	a_2	a_3	a_4	a_5	a_6
	DRED	NDR						
Plaquette	−3.0098	−2.0098	−0.3399	−0.5657	0.3061	−0.3727		
Iwasaki	−0.3442	0.6558	−0.2618	−0.4624	0.2316	−0.3340		
DBW2	2.0037	3.0037	−0.1402	−0.3846	0.0291	−0.0280	0.1037	−0.1933
Symanzik	−2.0735	−1.0735	−0.3142	−0.5337	0.2815	−0.3601		
Iwasaki′	−0.5033	0.4968	−0.2700	−0.4733	0.2393	−0.3381		
Wilson	−0.1504	0.8497	−0.2652	−0.4638	0.2348	−0.3359		

TABLE XIX. Values of parameters a_i in the interpolation of z_m^{MF} as a function of $w_0 = 1 - \tilde{M}$ for the various gauge actions.

Action	$-8\pi^2 T_{\mathrm{MF}} + a_0$		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
	DRED	NDR								
Plaquette	7.7341	8.7341	−4.2837	1.2934	−2.8593	1.1943				
Iwasaki	3.2278	4.2278	−3.7023	1.1366	−0.4148	0.0823	−2.0294	0.7181		
DBW2	−1.7148	−0.7148	−2.3699	0.5504	−1.4802	0.5480	1.2331	−0.4451	−2.0512	0.6116
Symanzik	6.1997	7.1997	−3.9326	1.1268	−2.7623	1.1360				
Iwasaki′	3.5447	4.5447	−3.7840	1.1757	−0.4348	0.0914	−2.0383	0.7237		
Wilson	2.9771	3.9771	−3.6712	1.1320	−0.4158	0.0839	−2.0303	0.7184		

TABLE XX. Values of parameters a_i in the interpolation of z_{Γ}^{MF} as a function of $w_0 = 1 - \tilde{M}$ for the various gauge actions.

Action	Γ	$8\pi^2 T_{\mathrm{MF}} + a_0$		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
		DRED	NDR								
Plaquette	VA	−6.8787	−7.3787	−0.2734	−0.6126	0.3087	−0.3509				
	SP	−7.7341	−8.7341	4.2837	−1.2934	2.8593	−1.1943				
	T	−6.5954	−5.5954	−1.8023	−0.3587	−0.5198	−0.1198				
Iwasaki	VA	−4.1714	−4.6714	−0.2264	−0.4920	0.2233	−0.3074				
	SP	−3.2278	−4.2278	3.7023	−1.1366	0.4148	−0.0823	2.0294	−0.7181		
	T	−4.4831	−3.4831	−1.4031	−0.3538	−0.5281	−0.1163				
DBW2	VA	−1.8047	−2.3047	−0.1441	−0.3251	0.1256	−0.2512				
	SP	1.7148	0.7148	2.3699	−0.5504	1.4802	−0.5480	−1.2331	0.4451	2.0512	−0.6116
	T	−2.9755	−1.9755	−0.9049	−0.2829	−0.5213	−0.1143				
Symanzik	VA	−5.9247	−6.4247	−0.2582	−0.5351	0.2744	−0.3822				
	SP	−6.1997	−7.1997	3.9326	−1.1268	2.7623	−1.1360				
	T	−5.8342	−4.8342	−1.6704	−0.3620	−0.5230	−0.1181				
Iwasaki′	VA	−4.3349	−4.8349	−0.2408	−0.4888	0.2500	−0.3438				
	SP	−3.5447	−4.5447	3.7840	−1.1757	0.4348	−0.0914	2.0383	−0.7237		
	T	−4.5939	−3.5939	−1.4375	−0.3578	−0.5263	−0.1169				
Wilson	VA	−3.9808	−4.4808	−0.2365	−0.4791	0.2454	−0.3416				
	SP	−2.9771	−3.9771	3.6712	−1.1320	0.4158	−0.0839	2.0303	−0.7184		
	T	−4.3105	−3.3105	−1.3949	−0.3590	−0.5231	−0.1186				

TABLE XXI. Values of parameters a_i in the interpolation of z_I^{MF} as a function of $w_0 = 1 - \tilde{M}$ for the various gauge actions.

Action	z_I	$16\pi^2 C_F T_{\mathrm{MF}} + a_0$		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
		DRED	NDR								
Plaquette	z_+	−17.7755	−20.7755	−3.7921	−1.1211	−0.8227	0.4906				
	z_-	−19.5095	−17.5095	5.3566	−2.4018	4.2146	−2.2148				
	z_1	−18.0616	−17.2283	−2.2711	−1.3464	0.0227	−0.7684				
	z_2	−20.6543	−23.3210	11.4490	−3.3084	7.5830	−3.3158				
Iwasaki	z_+	−11.7462	−14.7462	−2.9621	−1.0310	−0.8968	−0.4536				
	z_-	−9.9090	−7.9090	4.0754	−1.6247	3.6731	−1.9229				
	z_1	−11.4375	−10.6042	−1.7924	−1.1399	−0.1300	−0.6906				
	z_2	−8.6739	−11.3406	8.7611	−2.0690	6.7288	−2.8579				
DBW2	z_+	−7.1531	−10.1531	−1.9103	−0.7789	−0.9496	−0.4094				
	z_-	−0.1218	1.8782	2.9921	−1.1429	1.9992	−1.2779	−1.4973	1.1634	2.7409	−1.2539
	z_1	−5.9855	−5.1522	−1.1556	−0.7930	−0.2895	−0.5888				
	z_2	4.5663	1.89963	6.3348	−1.3762	3.8436	−1.8212	−3.0864	1.7158	5.3507	−1.8910
Symanzik	z_+	−15.6167	−18.6167	−3.5212	−1.1030	−0.8459	−0.4747				
	z_-	−16.1930	−14.1930	4.8941	−2.1210	4.0731	−2.1206				
	z_1	−15.7093	−14.8760	−2.1204	−1.2884	−0.0254	−0.7338				
	z_2	−16.5657	−19.2324	10.5000	−2.8547	7.3568	−3.1659				
Iwasaki′	z_+	−12.0779	−15.0779	−3.0365	−1.0404	−0.8846	−0.4650				
	z_-	−10.5214	−8.5214	4.1523	−1.8581	3.7568	−1.7758				
	z_1	−11.8187	−10.9854	−1.8391	−1.1732	−0.1092	−0.6892				
	z_2	−9.4850	−12.1517	8.9385	−2.3597	6.8666	−2.7263				
Wilson	z_+	−11.2758	−14.2758	−2.9482	−1.0361	−0.8815	−0.4672				
	z_-	−9.2925	−7.2925	4.0099	−1.7896	3.7136	−1.7537				
	z_1	−10.9454	−10.1121	−1.7894	−1.1583	−0.1136	−0.6872				
	z_2	−7.9715	−10.6382	8.6425	−2.2482	6.7932	−2.6882				

TABLE XXII. Values of parameters a_i in the interpolation of $z_{\rm PD}^{\rm MF}$ as a function of $w_0=1-\tilde{M}$ for the various gauge actions.

Action	$12\pi^2 C_F T_{\rm MF}+a_0$ DRED	NDR	a_1	a_2	a_3	a_4	a_5	a_6
Plaquette	−14.3397	−13.6730	2.4915	−1.6054	2.3230	−1.3567		
Iwasaki	−7.7345	−7.0678	1.8824	−1.1368	1.9949	−1.1805		
DBW2	−1.2816	−0.6149	1.2163	−0.6239	1.5858	−0.9385		
Symanzik	−12.0114	−11.3447	2.5984	−1.8575	0.5655	−0.1570	1.5948	−0.8303
Iwasaki′	−8.1512	−7.4845	1.9133	−1.2459	2.0505	−1.1324		
Wilson	−7.3006	−6.6339	1.8448	−1.2045	2.0260	−1.1205		

TABLE XXIII. Values of parameters a_i in the interpolation of $z_{BK,P}^{\rm MF}$(DRED) as a function of $w_0=1-\tilde{M}$ for the various gauge actions.

Action		a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
Plaquette	B_K	0.5620	−3.2600	0.5721	−0.6662	0.3607	−1.4980	−0.1876	2.1054	0.3137	−1.7541
	B_P	2.8090	−16.2960	2.8702	−3.3753	1.7750	−7.3424	−0.9065	10.3304	1.5573	−8.6802
Iwasaki	B_K	−0.6274	−2.5482	0.3314	−0.5474	0.3056	−1.4760	−0.2045	2.1211	0.3091	−1.7546
	B_P	−3.1382	−12.7364	1.6692	−2.7886	1.4842	−7.2027	−0.9620	10.3672	1.5174	−8.6625
DBW2	B_K	−2.3449	−1.7033	0.1284	−0.3989	0.2266	−1.4279	−0.2110	2.1224	0.2867	−1.7398
	B_P	−11.7253	−8.5119	0.6501	−2.0474	1.1082	−6.9618	−1.0262	10.3788	1.4224	−8.5939
Symanzik	B_K	0.1755	−3.0186	0.4835	−0.6220	0.3433	−1.5024	−0.1967	2.1251	0.3153	−1.7602
	B_P	0.8768	−15.0883	2.4285	−3.1641	1.6727	−7.3202	−0.9141	10.3648	1.5397	−8.6818
Iwasaki′	B_K	−0.5250	−2.6062	0.3490	−0.5562	0.3107	−1.4839	−0.2043	2.1274	0.3111	−1.7577
	B_P	−2.6257	−13.0268	1.7564	−2.8306	1.5053	−7.2390	−0.9447	10.3860	1.5150	−8.6712
Wilson	B_K	−0.6673	−2.5283	0.3268	−0.5469	0.3049	−1.4727	−0.2047	2.1159	0.3097	−1.7515
	B_P	−3.3370	−12.6378	1.6453	−2.7764	1.4763	−7.2135	−0.9484	10.3706	1.5099	−8.6595

TABLE XXIV. Top: Values of $z_i^{\rm pen}$(DRED) as a function of M, together with interpolated values and relative errors. Bottom: Values of parameters a_i and b_i in the interpolation of $z_i^{\rm pen}$(DRED) as a function of $w_0=1-\tilde{M}$.

M	$z_i^{\rm pen}$(DRED)	Interpolation	Relative error	M	$z_i^{\rm pen}$(DRED)	Interpolation	Relative error
0.10	1.544(1)	1.514	0.019	1.10	1.178(2)	1.191	0.011
0.20	−0.131(1)	−0.105	−0.195	1.20	1.648(1)	1.654	0.004
0.30	−0.448(1)	−0.443	−0.012	1.30	2.246(1)	2.243	0.001
0.40	−0.464(1)	−0.475	−0.024	1.40	3.043(1)	3.030	0.004
0.50	−0.367(1)	−0.384	−0.045	1.50	4.163(1)	4.147	0.004
0.60	−0.213(1)	−0.225	−0.057	1.60	5.865(1)	5.856	0.002
0.70	−0.019(1)	−0.022	−0.163	1.70	8.759(1)	8.768	0.001
0.80	0.211(1)	0.217	0.030	1.80	14.721(1)	14.742	0.001
0.90	0.480(1)	0.493	0.028	1.90	33.357(1)	33.339	0.001
1.00	0.797(1)	0.813	0.020				

Action	a_0	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
All	0.8132	−3.4681	2.3612	−0.8213	2.0186	−0.0127	−0.6956	0.0054	−0.3216

TABLE XXV. Values of parameters a_i in the interpolation of z_{ij}^{MF}(DRED) as a function of $w_0=1-\tilde{M}$ for the various gauge actions.

Action	z_{ii}	$16\pi^2 C_F T_{\mathrm{MF}}+a_0$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
Plaquette	z_{11}	−18.6311	0.7719	−1.8154	1.7127	−1.3019				
	z_{55}	−18.0616	−2.2711	−1.3464	0.0227	−0.7684				
	z_{66}	−20.6543	11.4490	−3.3084	7.5830	−3.3158				
Iwasaki	z_{11}	−10.8167	0.5467	−1.3795	1.4045	−1.1395				
	z_{55}	−11.4375	−1.7924	−1.1399	−0.1300	−0.6906				
	z_{66}	−8.6739	8.7611	−2.0690	6.7288	−2.8579				
DBW2	z_{11}	−3.6286	0.6247	−1.1155	−0.3311	−0.1293	1.3053	−0.5945		
	z_{55}	−5.9855	−1.1556	−0.7930	−0.2895	−0.5888				
	z_{66}	4.5663	6.3348	−1.3762	3.8436	−1.8212	−3.0864	1.7158	5.3507	−1.8910
Symanzik	z_{11}	−15.8928	0.6820	−1.6663	1.6158	−1.2490				
	z_{55}	−15.7093	−2.1204	−1.2884	−0.0254	−0.7338				
	z_{66}	−16.5657	10.5000	−2.8547	7.3568	−3.1659				
Iwasaki′	z_{11}	−11.3034	0.5562	−1.4186	1.4431	−1.1581				
	z_{55}	−11.8187	−1.8391	−1.1732	−0.1092	−0.6892				
	z_{66}	−9.4850	8.9385	−2.3597	6.8666	−2.7263				
Wilson	z_{11}	−10.2878	0.5290	−1.3824	1.4235	−1.1480				
	z_{55}	−10.9454	−1.7894	−1.1583	−0.1136	−0.6872				
	z_{66}	−7.9715	8.6425	−2.2482	6.7932	−2.6882				

Action	z_{ij}	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
Plaquette	z_{12}	0.8427	−4.8888	0.8613	−1.0126	0.5315	−2.2025	−0.2703	3.0984	0.4663	−2.6036
Iwasaki	z_{12}	−0.9415	−3.8209	0.5009	−0.8372	0.4442	−2.1582	−0.2866	3.1060	0.4541	−2.5966
DBW2	z_{12}	−3.5176	−2.5536	0.1952	−0.6143	0.3317	−2.0881	−0.3064	3.1126	0.4259	−2.5776
Symanzik	z_{12}	0.2631	−4.5265	0.7285	−0.9498	0.5018	−2.1934	−0.2742	3.1050	0.4618	−2.6022
Iwasaki′	z_{12}	−0.7877	−3.9080	0.5269	−0.8502	0.4517	−2.1672	−0.2835	3.1087	0.4545	−2.5978
Wilson	z_{12}	−1.0011	−3.7913	0.4935	−0.8332	0.4434	−2.1629	−0.2853	3.1093	0.4533	−2.5969

TABLE XXVI. The renormalization factors for the quark mass and axial current for the DBW2 gauge action at $\beta=1.04, \mu=a^{-1}=2.0$ GeV, $M=1.7$ for various prescriptions to determine $g^2_{\overline{\mathrm{MS}}}$ and the mean link value u.

Coupling	$g^2_{\overline{\mathrm{MS}}}$	$m_{\overline{\mathrm{MS}}}(\mathrm{NDR})/m_{\mathrm{lat}}$		$A_\mu^{\overline{\mathrm{MS}}}(\mathrm{NDR})/A_\mu^{\mathrm{lat}}$	
		$u=P^{1/4}$	$u=(8K_c)^{-1}$	$u=P^{1/4}$	$u=(8K_c)^{-1}$
Eq. (166)	1.411	0.964	1.136	1.014	0.851
Eq. (167)	1.948	0.983	1.125	0.985	0.849
Three-loop ($\Lambda_{\mathrm{QCD}}=240$ MeV)	2.567	1.005	1.113	0.954	0.845

(4) Wave function renormalization factors are given by

$$q_{\overline{\rm MS}}=[(1-w_0^2)Z_w]^{-1/2}(uZ_2)^{1/2}q_{\rm lat}, \tag{168}$$

$$Z_w=1+\frac{g_{\overline{\rm MS}}^2}{16\pi^2}C_F z_w^{\rm MF}, \tag{169}$$

$$z_w^{\rm MF}=\frac{2w_0}{1-w_0^2}[\Sigma_w+32\pi^2 T_{MF}], \tag{170}$$

$$Z_2=1+\frac{g_{\overline{\rm MS}}^2}{16\pi^2}C_F[-2\log(\mu a)+z_2^{\rm MF}], \tag{171}$$

$$z_2^{\rm MF}=\Sigma_1^{\overline{\rm MS}}-\Sigma_2+16\pi^2\frac{T_{\rm MF}}{2}, \tag{172}$$

where $w_0=1-\tilde{M}$. We parametrize $\Sigma_w+32\pi^2 T_{\rm MF}$ and $z_2^{\rm MF}$ as follows:

$$\Sigma_w+32\pi^2 T_{\rm MF}=32\pi^2 T_{\rm MF}+a_0+a_1w_0+a_2w_0^2+a_3w_0^3+a_4w_0^4=32\pi^2 T_{\rm MF}+\sum_{i=0}^{4}a_iw_0^i, \tag{173}$$

$$z_2^{\rm MF}=8\pi^2 T_{\rm MF}+\sum_{i=0}^{4,\ 6}a_iw_0^i. \tag{174}$$

The relative errors of these and the following interpolations are less than a few percent except at points where the value is near zero. We have collected the values of parameters a_i for Σ_w in Table XVII and those for $z_2^{\rm MF}$ in Table XVIII. Here we have two remarks: $z_2^{\rm MF}$ depends on the details of the $\overline{\rm MS}$ scheme, NDR and DRED, and hence both results of a_0 are given; and the interpolation of z_2 by the fourth polynomial of w_0 is not accurate enough for the DBW2 gauge action, so we also employ the sixth polynomial of w_0.

(5) The mass renormalization factor is given by

$$m_{\overline{\rm MS}}=(1-w_0^2)Z_wZ_m\frac{m_{\rm lat}}{u}, \tag{175}$$

$$Z_m=1+\frac{g_{\overline{\rm MS}}^2}{16\pi^2}C_F[-6\log(\mu a)+z_m^{\rm MF}], \tag{176}$$

$$z_m^{\rm MF}=(\Sigma_2^{\overline{\rm MS}}-\Sigma_2)-(\Sigma_1^{\overline{\rm MS}}-\Sigma_1)-16\pi^2\frac{T_{\rm MF}}{2}=-8\pi^2 T_{\rm MF}+\sum_{i=0}^{4,6,8}a_iw_0^i. \tag{177}$$

The values of a_i are given in Table XIX.

(6) The renormalization factors for the bilinear operators are given by

$$O_\Gamma^{\overline{\rm MS}}=\frac{u}{(1-w_0^2)Z_w}Z_\Gamma(\mu a)O_\Gamma^{\rm lat}, \tag{178}$$

$$Z_\Gamma=1+\frac{g_{\overline{\rm MS}}^2}{16\pi^2}C_F[h(\Gamma)\log(\mu a)+z_\Gamma^{\rm MF}], \tag{179}$$

$$z_\Gamma^{\rm MF}=z_\Gamma^{\overline{\rm MS}}-z_\Gamma+16\pi^2\frac{T_{\rm MF}}{2}=8\pi^2 T_{\rm MF}+\sum_{i=0}^{4,6,8}a_iw_0^i, \tag{180}$$

where $h(\Gamma)=0({\rm VA}),6({\rm SP}),-2({\rm T})$. The values of a_i are given in Table XX.

(7) The renormalization factors for the four-quark operators are given by

$$O_I^{\overline{\rm MS}}=\frac{u^2}{[(1-w_0^2)Z_w]^2}Z_{IJ}^{4\Gamma}(\mu a)O_J^{\rm lat}, \tag{181}$$

$$Z_{IJ}^{4\Gamma}=\delta_{IJ}\left\{1+\frac{g_{\overline{\rm MS}}^2}{16\pi^2}[\delta_I\log(\mu a)+z_I^{\rm MF}]\right\}+\frac{g_{\overline{\rm MS}}^2}{16\pi^2}v_{IJ}, \tag{182}$$

$$z_I^{\rm MF}=v_I^{\overline{\rm MS}}-v_I+2C_F(\Sigma_1^{\overline{\rm MS}}-\Sigma_1)+16\pi^2C_FT_{\rm MF}=16\pi^2C_FT_{\rm MF}+\sum_{i=0}^{4,8}a_iw_0^i \tag{183}$$

where $I=+,-,1,2,\ \delta_+=-4,\ \delta_-=8,\ \delta_1=-2,\ \delta_2=16$, and $v_{IJ}=0$ for all I and J except $v_{12}=8$ and $v_{21}=1$ in the NDR scheme. The values of a_i are given in Table XXI.

(8) The renormalization factor for the three-quark operator is given by

$$O_{\rm PD}^{\overline{\rm MS}}=\frac{u^{3/2}}{[(1-w_0^2)Z_w]^{3/2}}Z_{\rm PD}(\mu a)O_{\rm PD}^{\rm lat}, \tag{184}$$

$$Z_{\rm PD}=1+\frac{g_{\overline{\rm MS}}^2}{16\pi^2}[4\log(\mu a)+z_{\rm PD}^{\rm MF}], \tag{185}$$

$$z_{\rm PD}^{\rm MF}=v_{\rm PD}^{\overline{\rm MS}}-v_{\rm PD}+\frac{3}{2}C_F(\Sigma_1^{\overline{\rm MS}}-\Sigma_1)+16\pi^2C_F\frac{3}{4}T_{\rm MF}=12\pi^2C_FT_{\rm MF}+\sum_{i=0}^{4,6}a_iw_0^i. \tag{186}$$

The values of a_i are given in Table XXII.

(9) The renormalization factors for B_K and B_P are given by

$$Z_{B_K}(\mu a)=\frac{Z_+(\mu a)}{Z_A(\mu a)^2}=1+\frac{g_{\overline{\rm MS}}^2}{16\pi^2}[-4\log(\mu a)+z_{B_K}^{\rm MF}], \tag{187}$$

$$z_{B_K}^{\rm MF}=z_{+}^{\rm MF}-2C_F z_A^{\rm MF}=\sum_{i=0}^{9} a_i w_0^i, \qquad (188)$$

$$Z_{B_P}(\mu a)=\frac{Z_+(\mu a)}{Z_P(\mu a)^2}=1+\frac{g_{\overline{\rm MS}}^2}{16\pi^2}[-20\log(\mu a)+z_{B_P}^{\rm MF}], \qquad (189)$$

$$z_{B_P}^{\rm MF}=z_{+}^{\rm MF}-2C_F z_P^{\rm MF}=\sum_{i=0}^{9} a_i w_0^i. \qquad (190)$$

The values of a_i are given in Table XXIII.

(10) The renormalization factors for four-quark operators with $\Delta S=1$ are defined as

$$Q_i^{\overline{\rm MS}}=\frac{1}{[(1-w_0)Z_w]^2}[Z_{ij}^g Q_j^{\rm lat}+Z_i^{\rm pen}Q_{\rm pen}^{\rm lat}], \qquad (191)$$

where $i,j=1,2,\ldots,10$. The renormalization factor for the penguin operators is given by [26]

$$Z_i^{\rm pen}=\frac{g_{\overline{\rm MS}}^2}{48\pi^2}C(Q_i)[-2\log(\mu a)^2+z_i^{\rm pen}], \qquad (192)$$

$$Q_{\rm pen}^{\rm lat}=Q_4+Q_6-\frac{1}{N}(Q_3+Q_5), \qquad (193)$$

$$z_i^{\rm pen}=\frac{\sum_{i=0}^{4} a_i w_0^i}{1+\sum_{i=1}^{4} b_i w_0^i}, \qquad (194)$$

where $C(Q_2)=1$, $C(Q_3)=2$, $C(Q_4)=C(Q_6)=3$, $C(Q_9)=-1$, and $C(Q_i)=0$ for other i. The renormalization factor $z_i^{\rm pen}$ is independent of the gauge action. Furthermore, it does not depend on i for DRED, but

$$z_{4,6,8,10}^{\rm pen}({\rm NDR})=z_i^{\rm pen}({\rm DRED})+1/4, \qquad (195)$$

$$z_{1,2,3,5,7,9}^{\rm pen}({\rm NDR})=z_i^{\rm pen}({\rm DRED})+5/4. \qquad (196)$$

Numerical values for $z_i^{\rm pen}$(DRED) are given in Table XXIV, together with interpolated values and relative errors defined by $(z_i^{\rm pen}-\text{interpolation})/z_i^{\rm pen}$. The values of a_i and b_i for the interpolation are also given in Table XXIV.

For other renormalization factors, we have

$$Z_{ii}^g=u^2\times\begin{cases}1+\frac{1}{16\pi^2}g_{\overline{\rm MS}}^2[2\log(\mu a)+z_{11}^{\rm MF}], & i=1,2,3,4,9,10,\\ 1+\frac{1}{16\pi^2}g_{\overline{\rm MS}}^2[-2\log(\mu a)+z_{55}^{\rm MF}], & i=5,7,\\ 1+\frac{1}{16\pi^2}g_{\overline{\rm MS}}^2[16\log(\mu a)+z_{66}^{\rm MF}], & i=6,8,\end{cases} \qquad (197)$$

$$Z_{ij}^g=\begin{cases}Z_{ji}^g=\frac{1}{16\pi^2}g_{\overline{\rm MS}}^2[-6\log(\mu a)+z_{12}^{\rm MF}], & (i,j)=(1,2),(3,4),(9,10),\\ \frac{1}{16\pi^2}g_{\overline{\rm MS}}^2[6\log(\mu a)+z_{56}^{\rm MF}], & (i,j)=(5,6),(7,8),\\ \frac{1}{16\pi^2}g_{\overline{\rm MS}}^2 z_{65}^{\rm MF}, & (i,j)=(6,5),(8,7),\\ 0, \quad \text{others},\end{cases} \qquad (198)$$

where

$$z_{11}^{\rm MF}=\frac{z_+^{\rm MF}+z_-^{\rm MF}}{2}=16\pi^2 C_F T_{\rm MF}+\sum_{i=0}^{4,6} a_i w_0^i, \qquad (199)$$

$$z_{55}^{\rm MF}=z_1^{\rm MF}-v_{21}=16\pi^2 C_F T_{\rm MF}+\sum_{i=0}^{4} a_i w_0^i, \qquad (200)$$

$$z_{66}^{\rm MF}=z_2^{\rm MF}+v_{21}=16\pi^2 C_F T_{\rm MF}+\sum_{i=0}^{4,8} a_i w_0^i, \qquad (201)$$

$$z_{12}^{\rm MF}=\frac{z_+^{\rm MF}-z_-^{\rm MF}}{2}=\sum_{i=0}^{9} a_i w_0^i, \qquad (202)$$

$$z_{56}^{\mathrm{MF}}=\frac{z_2^{\mathrm{MF}}-z_1^{\mathrm{MF}}+v_{21}-v_{12}}{3}, \quad (203)$$

$$z_{65}^{\mathrm{MF}}=-3v_{21}. \quad (204)$$

The values of a_i are given in Table XXV. Note that

$$z_{56}^{\mathrm{MF}}(\mathrm{DRED})=-z_{12}^{\mathrm{MF}}(\mathrm{DRED}), \quad (205)$$

$$v_{12}(\mathrm{DRED})=v_{21}(\mathrm{DRED})=0, \quad (206)$$

and that

$$z_{11}(\mathrm{NDR})=z_{11}(\mathrm{DRED})-\frac{1}{2}, \quad (207)$$

$$z_{55}(\mathrm{NDR})=z_{55}(\mathrm{DRED})-\frac{1}{6}, \quad (208)$$

$$z_{66}(\mathrm{NDR})=z_{66}(\mathrm{DRED})-\frac{5}{3}, \quad (209)$$

$$z_{12}(\mathrm{NDR})=z_{12}(\mathrm{DRED})-\frac{5}{2}, \quad (210)$$

$$z_{56}(\mathrm{NDR})=z_{56}(\mathrm{DRED})-\frac{7}{2}=-z_{12}(\mathrm{DRED})-\frac{7}{2}, \quad (211)$$

$$z_{65}(\mathrm{NDR})=z_{65}(\mathrm{DRED})-3=-3. \quad (212)$$

If one uses the measured averaged link variable for u instead of $P=u^{1/4}$, one should replace T_{MF} in the above formulas with

$$T_{\mathrm{MF}}\rightarrow T-(1-\alpha)\delta T_{\mathrm{gauge}}, \quad (213)$$

where α is the gauge parameter, $\delta T_{\mathrm{gauge}}=0.0387334$ for all gauge actions, and T is given in Table XVI. Similarly, in the case of using the critical hopping parameter K_c of the Wilson fermion for u, one should make the replacement that $T_{\mathrm{MF}}\rightarrow T_{K_c}$, where $T_{K_c}-T_{\mathrm{MF}}$ is given also in Table XVI.

If one set $u=1$ and $T_{\mathrm{MF}}=0$ in the above steps, one can obtain the renormalization factors in the ordinary perturbation theory without MF improvement.

As an example, we show the renormalization factors for the quark mass (175) and axial current (178) in Table XXVI for the DBW2 gauge action at $\beta=1.04, M=1.7$ [27]. The $g^2_{\overline{\mathrm{MS}}}(\mu=1/a=2.0\ \mathrm{GeV})$'s are obtained using the mean field improved relations (166),(167) and the three-loop running coupling constants of the continuum theory with $\Lambda_{\mathrm{QCD}}=240$ MeV, which are $g^2_{\overline{\mathrm{MS}}}=1.411, 1.948, 2.567$, respectively. [Note that $P=0.7276(1)$ and $R1=0.5227(3)$.] The mean link value $u=\langle U_\mu\rangle$ is determined from either the measured plaquette value $u=P^{1/4}=0.9236$ or the critical K parameter $u=(8K_c)^{-1}=0.85(2)$. The renormalization factors depend mildly on the value of $g^2_{\overline{\mathrm{MS}}}$, while they move more than 10% on changing the method determining the tadpole factor. The values obtained from K_c are closer to those calculated using the nonperturbative techniques[1] for the same parameters in [28].

VII. CONCLUSION

In this paper we evaluated renormalization factors at the one-loop level for the quark wave function, quark mass, bilinear quark operators, and four- and three-quark operators perturbatively. We showed that the mean field improvement, by which the large additive quantum correction to the domain-wall height M is nonperturbatively estimated, makes the perturbative evaluations more reliable. We explained how to obtain the numerical values for the renormalization factors in the MF improved perturbation theory in detail.

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[1] $m_{\overline{\mathrm{MS}}}(\mathrm{NDR})/m_{\mathrm{lat}}=1.40(2)(7), A_\mu^{\overline{\mathrm{MS}}}(\mathrm{NDR})/A_\mu^{\mathrm{lat}}=0.8402(2)$.

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