Final-state phases in $B\rightarrow$ baryon-antibaryon decays

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The recent observation of the decay $\overline{B}^0 \rightarrow \Lambda_c^+ \overline{p}$ suggests that related decays may soon be visible at e^+e^- colliders. It is shown how these decays can shed light on strong final-state phases and amplitudes involving the spectator quark, both of which are normally expected to be small in B decays.

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I. INTRODUCTION

Phases in B decays arising from final-state interactions are an important gateway to the observation of direct CP violation. The pattern of decays to $D\pi$, $D^*\pi$, $D\rho$, and related states has been elaborated recently by the CLEO [1,2], BaBar [3], and Belle [4–6] Collaborations. Some amplitudes for decays involving the weak subprocess $b\rightarrow c\overline{u}d$ obey isospin triangle relations. In certain cases these triangles have nonzero area, indicating non-zero final-state phases between different contributing amplitudes [7]. Some decays governed by the Cabibbo-suppressed subprocess $b\rightarrow c\overline{u}s$ also involve amplitude triangles with apparently non-zero area, though not yet at a statistically significant level [7,8]. One would expect this behavior if flavor SU(3) is a good symmetry for B decays.

The decays of B mesons to charmed baryon–charmless antibaryon pairs also obey simple isospin relations and flavor-SU(3) regularities [9,10]. Models for these decays [11–15] have been published that allow estimates of their rates. The recent observation of the decay $\overline{B}^0\rightarrow\Lambda_c^+\overline{p}$ by the Belle Collaboration [16] with a branching ratio $\mathcal{B}(\overline{B}^0\rightarrow\Lambda_c^+\overline{p})=(2.19^{+0.56}_{-0.49}\pm0.32\pm0.57)\times10^{-5}$ indicates that such processes are within experimental reach at existing e^+e^- colliders. (Many early models [11–14] overestimated branching ratios to baryon-antibaryon final states but contain useful theoretical techniques.) The present paper indicates how these data may be useful in elaborating final-state phases among different amplitudes contributing to the decays. It also indicates how one can test for suppression of decay amplitudes involving the spectator quark.

We shall discuss the decomposition of $\overline{B}^0\rightarrow\Lambda_c^+\overline{p}$ and related decays into invariant amplitudes of flavor SU(3) in Sec. II. The triangles formed by these amplitudes, and their significance for final-state interactions, are discussed in Sec. III. We conclude with some experimental prospects in Sec. IV. Conventions for the quark composition of baryons are given in the Appendix.

II. INVARIANT AMPLITUDES OF FLAVOR SU(3)

The weak Hamiltonian giving rise to the subprocess $b\rightarrow c\overline{u}d$ transforms as the $I=1$, $I_3=-1$ member of an octet of flavor SU(3). The $\overline{B}$ mesons $b\overline{q}(\overline{q}=-\overline{u},\overline{d},\overline{s})$ form a 3^*. [Recall that $(-\overline{u},\ \overline{d})$ is an isodoublet.] Thus the SU(3) representations of the initial state are those in the product

$$3^*\times8=3^*+6+15^*. \tag{1}$$

The $\Lambda_c^+=c[ud]$ belongs to a flavor-SU(3) antitriplet (3^*) along with the $\Xi_c^+=c[su]$ and the $\Xi_c^0=c[sd]$. The brackets indicate antisymmetry with respect to flavor. For decays to a final state of a 3^* charmed baryon and an octet antibaryon, all three representations in Eq. (1) occur. Hence there must be three independent invariant amplitudes of flavor SU(3) characterizing such decays. Similarly, in the Cabibbo-suppressed decays governed by $b\rightarrow c\overline{u}s$, the weak Hamiltonian transforms as the strange charged isodoublet member of a flavor octet, so the invariant amplitudes are the same.

Charmed baryons belonging to a flavor-SU(3) sextet (6) also have been seen, consisting of an isotriplet $\Sigma_c^{++}=cuu$, $\Sigma_c^+=c(ud)$, $\Sigma_c^0=cdd$, an isodoublet $\Xi'^+_c=c(us)$, $\Xi'^0_c=c(ds)$, and an isosinglet $\Omega_c^0=css$. The parentheses indicate symmetry with respect to flavor. Similarly, one can consider not only octet but also (anti)decuplet antibaryons. In Table I we summarize the SU(3) representations that contribute to each class of decays.

An economical tensor notation was utilized by Savage and Wise to describe these processes [10]. We illustrate with the 3^*+8 final state. We use subscripts to denote the components of a 3^* representation of $SU(3)_f$ and use superscripts to denote the components of a 3 representation. The $\overline{B}$ mesons, in a 3^* representation as mentioned, can then be written as $(-B^-,\ \overline{B}^0,\ \overline{B}_s^0)\equiv B_i$. The charmed baryons in a 3^* representation can be expressed as $(-\Xi_c^0,\ \Xi_c^+,\ \Lambda_c^+)\equiv(\Xi_c)_i$. The octet of charmless baryons, on the other hand, can be represented by a two-index tensor:

$$N^i_j\equiv\begin{pmatrix} -\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & \Sigma^+ & p \\ -\Sigma^- & \Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & n \\ -\Xi^- & \Xi^0 & -2\Lambda/\sqrt{6} \end{pmatrix}. \tag{2}$$

TABLE I. Invariant amplitudes in the direct channel contributing to $\overline{B}\rightarrow$ (charmed baryon)+(antibaryon) decays via the subprocess $b\rightarrow c\overline{u}d$ or $b\rightarrow c\overline{u}s$.

Charmed baryon / Antibaryon	3^*	6
8	3^*+6+15^*	3^*+6+15^*
10^*	6	3^*+15^*

TABLE II. Invariant amplitudes in the crossed channel contributing to $\bar{B}\to$ (charmed baryon)+(antibaryon) decays via the subprocess $b\to c\bar{u}d$ or $b\to c\bar{u}s$.

Charmed baryon / Antibaryon	3*	6
8	$1+8_D+8_F$	8_D+8_F+10
10*	8	8+10

The weak Hamiltonian responsible for the Cabibbo-favored quark subprocess $b\to cd\bar{u}$ and the Cabibbo-suppressed $b\to cs\bar{u}$, belonging to an $SU(3)_f$ octet as mentioned above, can similarly be written as

$$H^i_j\sim(d\bar{u})V_{ud}+(s\bar{u})V_{us}=\begin{pmatrix}0&0&0\\V_{ud}&0&0\\V_{us}&0&0\end{pmatrix},\qquad(3)$$

where V_{ud} and V_{us} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements: $V_{us}/V_{ud}\simeq\lambda\simeq0.2256$. The effective Hamiltonian for the decays $B\to\Xi_c\bar{N}$ can be written in terms of invariant amplitudes α, β and γ [10]:

$$H_{\rm eff}=\alpha\Xi_c^iN_i^jH_j^kB_k+\beta\Xi_c^iH_i^jN_j^kB_k+\gamma\Xi_c^iB_iH_k^jN_j^k,\qquad(4)$$

where we sum over repeated indices. Expanding the sum would give us the amplitudes for the relevant processes. [Remember to multiply each amplitude by $(-1)^{n_{\bar{u}}}$, where $n_{\bar{u}}$ is the number of $\bar{u}$ quarks in the antibaryon.]

Two equivalent notations are helpful to visualize possible relations among invariant amplitudes. The second is particularly relevant when certain dynamical assumptions are made.

(1) The process $3^*\times8\to3^*\times8$ in the crossed channel reads

$$3^*\times3\to1+8_D+8_F\to8\times8,\qquad(5)$$

where D and F denote the two ways of coupling an octet to a pair of octets. The singlet S and octet amplitudes D and F (suitably normalized) are related to α, β, and γ by

$$\alpha=D+F,\quad\beta=D-F,\quad\gamma=S-\frac{2}{3}D.\qquad(6)$$

The S, D, F notation is that (aside from normalization) used by Li and Wu [9]. In Table II we summarize the SU(3) representations that contribute to each class of decay, including also sextet charmed baryons and antidecuplet antibaryons. We see, of course, that the number of invariant amplitudes is the same as in the direct channel.

(2) A topological expansion of amplitudes [17,18] yields three invariant amplitudes, of which two are associated with the subprocess $b\to cd\bar{u}$ or $b\to cs\bar{u}$, with an additional light quark-antiquark pair produced from the vacuum [Fig. 1(a)], and one is associated with the exchange process $b\bar{d}\to c\bar{u}$ or $b\bar{s}\to c\bar{u}$, in which two such pairs are produced from the vacuum [Fig. 1(b)]. We call the first two amplitudes a_1 and a_2 and the third amplitude a_E (to denote exchange). Explicit definitions of these amplitudes are given below. Consider the amplitudes for $\bar{B}$ to decay to 6 quarks ($cq_{w'}\,q_v$ and $\bar{q}_s\,\bar{q}_w\,\bar{q}_v$) via color-suppressed processes as shown in Fig. 1(a). With the c quark staying at the top, there are 2 permutations for $\{q_{w'}\ q_v\}$ and 6 permutations for $\{\bar{q}_s\ \bar{q}_w\ \bar{q}_v\}$. Thus there are 12 possible color-suppressed diagrams contributing to a specific amplitude. The amplitudes of the 12 diagrams are denoted by A^{lm}_{ijk}, where lm is a permutation of $\{w'\ v\}$ and ijk is a permutation of $\{s\ \ w\ v\}$. The color-suppressed amplitude for $\bar{B}$ to decay to a charmed baryon and an antibaryon is then a weighted sum of the 12 amplitudes, with the weights being the products of the coefficient of cq_lq_m in the quark composition of the charmed baryon and that of $\bar{q}_i\bar{q}_i\bar{q}_k$ in the quark composition of the antibaryon. It turns out that each color-suppressed amplitude is a linear combination of a_1 and a_2, with

$$a_1=\frac{1}{2}(A^{[w'v]}_{[sw]v}+A^{[w'v]}_{[sv]w}),\qquad(7)$$

$$a_2=\frac{1}{2}(A^{[w'v]}_{[ws]v}+A^{[w'v]}_{[wv]s}).\qquad(8)$$

Here $A^{[lm]}_{[ij]k}\equiv(A^{lm}_{ijk}-A^{lm}_{jik})-(A^{ml}_{ijk}-A^{ml}_{jik})$ and 1/2 is merely a normalization factor. Similarly, E^{lm}_{ijk} is used to denote the amplitude for $\bar{B}$ to decay to 6 quarks via an exchange process as shown in Fig. 1(b). Here lm is a permutation of $\{v_1\ v_2\}$ and ijk is a permutation of $\{w\ \ v_1\ v_2\}$. Since the two quark-

TABLE III. $SU(3)_f$ predictions of the amplitudes for $\bar{B}\to$ an SU(3) 3* charmed baryon and an octet antibaryon. CF= Cabibbo favored; CS=Cabibbo suppressed.

CF decay	Amplitude	CS decay	Amplitude
$\bar{B}^0\to\Lambda_c^+\bar{p}$	a_1+a_E	$\bar{B}^0_s\to\Xi_c^+\bar{\Sigma}^-$	$\lambda(a_1+a_E)$
$\bar{B}^0_s\to\Xi_c^0\bar{\Xi}^0$	$-a_2$	$\bar{B}^0\to\Xi_c^0\bar{n}$	$-\lambda a_2$
$\bar{B}^0_s\to\Lambda_c^+\bar{\Sigma}^-$	$-a_1$	$\bar{B}^0\to\Xi_c^+\bar{p}$	$-\lambda a_1$
$\bar{B}^0\to\Xi_c^0\bar{\Sigma}^0$	$-(a_1+a_2+a_E)/\sqrt{2}$	$\bar{B}^0_s\to\Xi_c^0\bar{\Sigma}^0$	$-\lambda(a_1+a_E)/\sqrt{2}$
$\bar{B}^0\to\Xi_c^0\bar{\Lambda}$	$(a_1-a_2+a_E)/\sqrt{6}$	$\bar{B}^0_s\to\Xi_c^0\bar{\Lambda}$	$\lambda(a_1+2a_2+a_E)/\sqrt{6}$
$\bar{B}^0\to\Xi_c^+\bar{\Sigma}^-$	a_E	$\bar{B}^0_s\to\Lambda_c^+\bar{p}$	λa_E
$B^-\to\Xi_c^0\bar{\Sigma}^-$	$-(a_1+a_2)$	$B^-\to\Xi_c^0\bar{p}$	$-\lambda(a_1+a_2)$

TABLE IV. Invariant amplitudes in a topological expansion for $\overline{B}\to$ (charmed baryon)+(antibaryon) decays via the subprocess $b\to c\overline{u}d$ or $b\to c\overline{u}s$.

Charmed baryon / Antibaryon	3^*	6
8	$a_1,\ a_2,\ a_E$	$b_1,\ b_2,\ b_E$
10^*	c	$d,\ d_E$

antiquark pairs ($q_{v_1}\overline{q}_{v_1}$ and $q_{v_2}\overline{q}_{v_2}$) are both produced from the vacuum, a_E should not depend on the ordering of v_1 and v_2. One finds that all exchange amplitudes for $\overline{B}$ to decay to a charmed baryon and an antibaryon are multiples of

$$a_E=\frac{1}{2}(E^{[v_1v_2]}_{[v_1v_2]w}+E^{[v_1v_2]}_{[wv_2]v_1}-E^{[v_1v_2]}_{[wv_1]v_2}). \tag{9}$$

The topological decompositions of the amplitudes are presented in Table III. They are in agreement with those obtained from Eq. (4) if we set

$$a_1=-\gamma,\qquad a_2=-\beta,\qquad a_E=\alpha+\gamma. \tag{10}$$

In particular, if processes involving the spectator quark are suppressed, as has been argued for heavy-quark decays (see, e.g., the discussion in [18]), one expects $|a_E|\ll|a_1|,|a_2|$, and hence an approximate symmetry

$$\alpha=-\gamma. \tag{11}$$

We shall explore the consequences of this relation in the next section.

More generally, the topological amplitudes contributing to each type of process are summarized in Table IV. For decays to $6+8$, both $q_{w'}q_v$ and $q_{v_1}q_{v_2}$ are symmetrized and therefore

$$b_1=\frac{1}{2}(A^{(w'v)}_{[sw]v}+A^{(w'v)}_{[sv]w}), \tag{12}$$

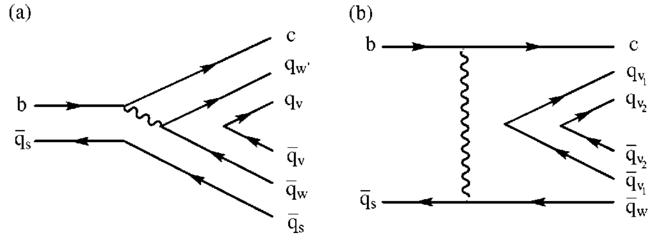

FIG. 1. Diagrams for $\overline{B}\to$ a charmed baryon and an antibaryon. (a) Color-suppressed diagram. $q_{w'}=d$ for Cabibbo-favored decays and $q_{w'}=s$ for Cabibbo-suppressed decays; $\overline{q}_w=\overline{u}$. (b) Exchange diagram. $\overline{q}_s=\overline{d}$ for Cabibbo-favored decays and $\overline{q}_s=\overline{s}$ for Cabibbo-suppressed decays; $\overline{q}_w=\overline{u}$.

$$b_2=\frac{1}{2}(A^{(w'v)}_{[ws]v}+A^{(w'v)}_{[wv]s}), \tag{13}$$

$$b_E=\frac{1}{2}(E^{(v_1v_2)}_{[wv_1]v_2}+E^{(v_1v_2)}_{[wv_2]v_1}), \tag{14}$$

where $A^{(lm)}_{[ij]k}\equiv(A^{lm}_{ijk}-A^{lm}_{jik})+(A^{ml}_{ijk}-A^{ml}_{jik})$ and $E^{(lm)}_{[ij]k}$ is defined in a similar way. Note that, if q_{v_1} and q_{v_2} are identical, only one term in Eq. (14) contributes. For decays to 3^*+10^*, there is no exchange diagram since q_{v_1} and q_{v_2} are antisymmetrized in a 3^* charmed baryon but $\overline{q}_{v_1}$ and $\overline{q}_{v_2}$ are symmetrized in a 10^* antibaryon; and

$$c=A^{[w'v]}_{(swv)}/\sqrt{2}\equiv\sum_{\sigma}\ (A^{w'v}_{\sigma\{swv\}}-A^{vw'}_{\sigma\{swv\}})/\sqrt{2}, \tag{15}$$

where the sum runs over all permutations σ of $\{s\ w\ v\}$. For decays to $6+10^*$,

$$d=A^{(w'v)}_{(swv)}\equiv\sum_{\sigma}\ (A^{w'v}_{\sigma\{swv\}}+A^{vw'}_{\sigma\{swv\}}), \tag{16}$$

and $d_E=E^{(v_1v_2)}_{(wv_1v_2)}$ is defined in a similar fashion. In Tables V–VII we summarize the corresponding amplitudes for de-

TABLE V. $SU(3)_f$ predictions of the amplitudes for $\overline{B}\to$(6 charmed baryon + octet antibaryon). Only Cabibbo-favored decays are shown.

Decay	Amplitude	Decay	Amplitude
$\overline{B}^0\to\Sigma_c^0\overline{n}$	$-(b_2+b_E)$[a]	$B^-\to\Sigma_c^0\overline{p}$	$-(b_1+b_2)$[b]
$\overline{B}^0\to\Sigma_c^+\overline{p}$	$(b_E-b_1)/\sqrt{2}$	$B^-\to\Xi_c'^0\overline{\Sigma}^-$	$(b_1+b_2)/\sqrt{2}$
$\overline{B}^0\to\Xi_c'^0\overline{\Lambda}$	$(b_2-b_1+3b_E)/(2\sqrt{3})$	$\overline{B}_s^0\to\Xi_c'^0\overline{\Xi}^0$	$b_2/\sqrt{2}$
$\overline{B}^0\to\Xi_c'^0\overline{\Sigma}^0$	$(b_1+b_2+b_E)/2$	$\overline{B}_s^0\to\Sigma_c^+\overline{\Sigma}^-$	$b_1/\sqrt{2}$
$\overline{B}^0\to\Xi_c'^+\overline{\Sigma}^-$	$-b_E/\sqrt{2}$	$\overline{B}_s^0\to\Sigma_c^0\overline{\Sigma}^0$	$-b_1/\sqrt{2}$
$\overline{B}^0\to\Omega_c^0\overline{\Xi}^0$	b_E	$\overline{B}_s^0\to\Sigma_c^0\overline{\Lambda}$	$(b_1+2b_2)/\sqrt{6}$

[a]The branching ratio for this mode is predicted to be $\mathcal{O}(10^{-7}\sim10^{-6})$ in a pole model [15].

[b]A branching ratio of $(0.45^{+0.26}_{-0.19}\pm0.07\pm0.12)\times10^{-4}$ is measured for $B^-\to\Sigma_c^0\overline{p}$ by the Belle Collaboration [19]. This sets a 90% C.L. upper limit 0.93×10^{-4}, to be compared with the 90% C.L. upper limit 0.8 $\times10^{-4}$ set by the CLEO Collaboration [20]. A prediction based on the pole model of Ref. [15] agrees with these limits.

TABLE VI. $SU(3)_f$ predictions of the amplitudes for $\overline{B}\to(3^*$ charmed baryon + antidecuplet antibaryon) (Cabibbo-favored decays). Note that $\mathcal{A}(\overline{B}^0\to\Xi_c^+\overline{\Sigma}^{*-})=0$.

Decay	Amplitude	Decay	Amplitude
$\overline{B}^0\to\Lambda_c^+\overline{\Delta}^-$	$-c/\sqrt{3}$	$\overline{B}^0\to\Xi_c^0\overline{\Sigma}^{*0}$	$c/\sqrt{6}$
$B^-\to\Lambda_c^+\overline{\Delta}^{--}$	$-c$[a]	$B^-\to\Xi_c^0\overline{\Sigma}^{*-}$	$c/\sqrt{3}$
$\overline{B}_s^0\to\Lambda_c^+\overline{\Sigma}^{*-}$	$-c/\sqrt{3}$	$\overline{B}_s^0\to\Xi_c^0\overline{\Xi}^{*0}$	$c/\sqrt{3}$

[a]Branching ratios of $(1.87^{+0.43}_{-0.40}\pm0.28\pm0.49)\times10^{-4}$ and $(2.4\pm0.6^{+0.19}_{-0.17}\pm0.6)\times10^{-4}$ are observed for $B^-\to\Lambda_c^+\overline{p}\pi^-$ by the Belle [19] and CLEO [20] Collaborations, respectively. Since $\overline{\Delta}^{--}$ decays almost exclusively to $\overline{p}\pi^-$, the branching ratio for $B^-\to\Lambda_c^+\overline{\Delta}^{--}$ should be less than $\mathcal{B}(B^-\to\Lambda_c^+\overline{p}\pi^-)$. The pole model of Ref. [15] predicts $\mathcal{B}(B^-\to\Lambda_c^+\overline{\Delta}^{--})=1.9\times10^{-5}$.

cays to $6+8$, 3^*+10^*, and $6+10^*$, respectively. These are equivalent to the decompositions presented in Ref. [10], but we find the present notation convenient for seeing what happens when we assume that the exchange amplitudes are small. We do not show the amplitudes for Cabibbo-suppressed decays (which can be looked up in [10]), since these decays generally involve Ξ_c', Ω_c or $\overline{B}_s^0$, none of which is easy to observe or produce in experiments. Furthermore, the branching ratios for these decays are expected to be only a few percent of those for the Cabibbo-favored ones.

III. TRIANGLE RELATIONS

In all the processes we consider, the charmed baryon has spin 1/2. Since the decaying particle has spin 0, and parity is not conserved in the decay, there are two independent amplitudes, labeled by the helicity of the charmed baryon. The following triangle relations are valid for each. The parity-conserving (PC) and parity-violating (PV) amplitudes are linear combinations of the two helicity amplitudes. In some models (see, e.g., [13]), one of the amplitudes (e.g., PV) may be absent or suppressed with respect to the other. In the absence of final-state phases, one can show that the triangle formed by the square roots of three decay rates has zero area if and only if the PC and PV amplitudes for the three decay processes form similar triangles. Indeed, zero final-state phases and similar PC and PV triangles are two necessary and sufficient conditions for the triangle formed by the square roots of three decay rates to have zero area. The proof is given below.

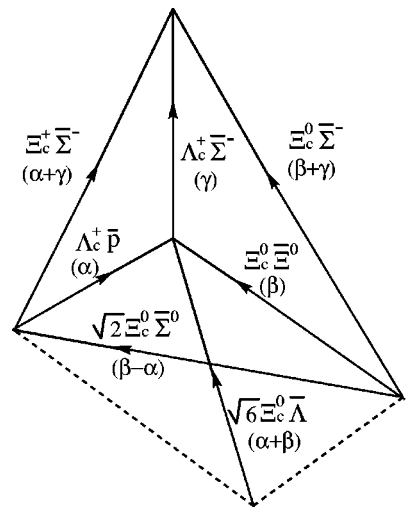

FIG. 2. Triangles for $\mathcal{A}(\overline{B}^0\to\Lambda_c^+\overline{p})=\alpha$ and related amplitudes described in Table III. Note that $\alpha=a_1+a_E$, $\beta=-a_2$ and $\gamma=-a_1$.

Suppose that $s_i^2=|c_i|^2+|v_i|^2$ $(i=1,2,3)$ are the decay rates for three processes, with c_i and v_i being the PC and PV amplitudes, respectively. Assuming that these amplitudes satisfy triangle relations $c_1+c_2=c_3$ and $v_1+v_2=v_3$, we have

$$
\begin{aligned}
s_3^2 &= s_1^2+s_2^2+2\mathrm{Re}(c_1c_2^*+v_1v_2^*)\\
&\leqslant s_1^2+s_2^2+2(|c_1||c_2|+|v_1||v_2|)\\
&\leqslant s_1^2+s_2^2+2\sqrt{|c_1|^2+|v_1|^2}\sqrt{|c_2|^2+|v_2|^2}\\
&=(s_1+s_2)^2,
\end{aligned}
$$

where the second inequality is due to the Cauchy-Schwarz inequality. Obviously, the equality $s_3=s_1+s_2$ holds if and only if there are no relative phases both between c_1 and c_2

TABLE VII. $SU(3)_f$ predictions of the amplitudes for $\overline{B}\to(6$ charmed baryon + antidecuplet antibaryon) (Cabibbo-favored decays).

Decay	Amplitude	Decay	Amplitude
$\overline{B}^0\to\Sigma_c^{++}\overline{\Delta}^{--}$	$-d_E$[a]	$B^-\to\Sigma_c^+\overline{\Delta}^{--}$	$d/\sqrt{2}$
$\overline{B}^0\to\Sigma_c^+\overline{\Delta}^-$	$(2d_E+d)/\sqrt{6}$	$B^-\to\Sigma_c^0\overline{\Delta}^-$	$-d/\sqrt{3}$
$\overline{B}^0\to\Sigma_c^0\overline{\Delta}^0$	$-(d_E+d)/\sqrt{3}$	$B^-\to\Xi_c'^0\overline{\Sigma}^{*-}$	$-d/\sqrt{6}$
$\overline{B}^0\to\Xi_c'^+\overline{\Sigma}^{*-}$	$(2/3)^{1/2}d_E$	$\overline{B}_s^0\to\Sigma_c^+\overline{\Sigma}^{*-}$	$d/\sqrt{6}$
$\overline{B}^0\to\Xi_c'^0\overline{\Sigma}^{*0}$	$-(2d_E+d)/(2\sqrt{3})$	$\overline{B}_s^0\to\Sigma_c^0\overline{\Sigma}^{*0}$	$-d/\sqrt{6}$
$\overline{B}^0\to\Omega_c^0\overline{\Xi}^{*0}$	$-d_E/\sqrt{3}$	$\overline{B}_s^0\to\Xi_c'^0\overline{\Xi}^{*0}$	$-d/\sqrt{6}$

[a]The branching ratio for $\overline{B}^0\to\Sigma_c^{++}\overline{\Delta}^{--}$ should be less than that for $\overline{B}^0\to\Sigma_c^{++}\overline{p}\pi^-$. The latter is measured to be $(2.38^{+0.63}_{-0.55}\pm0.41\pm0.62)\times10^{-4}$ and $(3.7\pm0.8\pm0.7\pm1.0)\times10^{-4}$ by the Belle [19] and CLEO [20] Collaborations, respectively.

and between v_1 and v_2, and the relation $|c_1|/|c_2| = |v_1|/|v_2|$ is satisfied. One then has $|c_1|/|v_1| = |c_2|/|v_2| = |c_3|/|v_3|$.

In what follows we shall assume that, by studying decay distributions, one has been able to separate out the individual rates for parity-conserving and parity-violating transitions, or the individual rates for charmed baryon helicities $\pm 1/2$. In the case of amplitude equalities (rather than triangle relations), total rates as well as individual ones will of course be equal.

A. 3*+8 final states

The Cabibbo-favored amplitudes of Table III are denoted by arrows in Fig. 2. For each helicity or partial wave, three independent complex amplitudes will be specified completely, up to an irrelevant overall phase, by five lengths of these vectors, leaving two predictions for rates. There will be a discrete ambiguity corresponding to the folding of two adjacent triangles about their common side. (We do not show the corresponding figure for Cabibbo-suppressed decays.) We now discuss some individual triangle relations associated with this construction. These triangles, if shown to have non-zero area, will indicate non-zero relative final-state phases between their contributing amplitudes.

As a consequence of the isospin of the weak Hamiltonian for $b\to c\overline{u}d$, two invariant isospin amplitudes, with $I=1/2$ and $I=3/2$, govern $\overline{B}\to\Xi_c\overline{\Sigma}$. The three decay processes then obey a triangle relation:

$$\mathcal{A}(B^-\to\Xi_c^0\overline{\Sigma}^-) = \sqrt{2}\mathcal{A}(\overline{B}^0\to\Xi_c^0\overline{\Sigma}^0) + \mathcal{A}(\overline{B}^0\to\Xi_c^+\overline{\Sigma}^-). \tag{17}$$

This relation is somewhat challenging in view of the need to reconstruct the $\overline{\Sigma}^0$ through its $\overline{\Lambda}\gamma$ decay. However, it involves only non-strange B mesons, which are the focus of current studies at e^+e^- colliders.

Three triangle relations involve the observed $\overline{B}^0\to\Lambda_c^+\overline{p}$ decay:

$$\sqrt{2}\mathcal{A}(\overline{B}^0\to\Lambda_c^+\overline{p}) + \mathcal{A}(\overline{B}^0\to\Xi_c^0\overline{\Sigma}^0) = \sqrt{3}\mathcal{A}(\overline{B}^0\to\Xi_c^0\overline{\Lambda}), \tag{18}$$

$$\mathcal{A}(\overline{B}^0\to\Lambda_c^+\overline{p}) + \mathcal{A}(\overline{B}_s^0\to\Xi_c^0\overline{\Xi}^0) = \sqrt{6}\mathcal{A}(\overline{B}^0\to\Xi_c^0\overline{\Lambda}), \tag{19}$$

$$\mathcal{A}(\overline{B}^0\to\Lambda_c^+\overline{p}) + \mathcal{A}(\overline{B}_s^0\to\Lambda_c^+\overline{\Sigma}^-) = \mathcal{A}(\overline{B}^0\to\Xi_c^+\overline{\Sigma}^-). \tag{20}$$

The first one is particularly useful since it involves only $\overline{B}^0$ decays. The last two relations involve the detection of a $\overline{B}_s^0$ decay, requiring either a dedicated run at KEKB or PEP-II (currently running below the $B_s^0\overline{B}_s^0$ threshold) or an experiment at a hadron collider.

In the Cabibbo-suppressed sector two isospin relations stem from the $I=1/2$, $I_3=-1/2$ nature of the weak Hamiltonian:

$$\mathcal{A}(B^-\to\Xi_c^0\overline{p}) = \mathcal{A}(\overline{B}^0\to\Xi_c^0\overline{n}) + \mathcal{A}(\overline{B}^0\to\Xi_c^+\overline{p}), \tag{21}$$

$$\mathcal{A}(\overline{B}_s^0\to\Xi_c^+\overline{\Sigma}^-) = -\sqrt{2}\mathcal{A}(\overline{B}_s^0\to\Xi_c^0\overline{\Sigma}^0). \tag{22}$$

The first of these involves only non-strange B's and no $\overline{\Sigma}^0$'s. Two additional triangle relations may be written, both involving $\overline{B}_s^0$ decays. Since these involve Cabibbo-suppressed decays of the less easily produced $\overline{B}_s^0$, the corresponding triangles may not be so easy to construct.

B. 6+8 final states

The isospin triangles in these processes are

$$\mathcal{A}(B^-\to\Sigma_c^0\overline{p}) = \sqrt{2}\mathcal{A}(\overline{B}^0\to\Sigma_c^+\overline{p}) + \mathcal{A}(\overline{B}^0\to\Sigma_c^0\overline{n}), \tag{23}$$

which involves an antineutron, and

$$\mathcal{A}(B^-\to\Xi_c'^0\overline{\Sigma}^-) = \sqrt{2}\mathcal{A}(\overline{B}^0\to\Xi_c'^0\overline{\Sigma}^0) + \mathcal{A}(\overline{B}^0\to\Xi_c'^+\overline{\Sigma}^-), \tag{24}$$

which involves the Ξ_c' states. These were not observed until quite recently [21] since they decay to $\Xi_c\gamma$. A simple isospin relation

$$\mathcal{A}(\overline{B}_s^0\to\Sigma_c^+\overline{\Sigma}^-) = -\mathcal{A}(\overline{B}_s^0\to\Sigma_c^0\overline{\Sigma}^0) \tag{25}$$

involves $\overline{B}_s^0$ decays. Several amplitude triangles not involving isospin can be formed from the relations for $6+8$ decays, but they involve particles which are not especially easy to produce ($\overline{B}_s^0$) or detect (Ξ_c').

There are several ways to check whether the exchange amplitude b_E is much smaller than b_1 or b_2. For example, the decay $\overline{B}^0\to\Xi_c'^+\overline{\Sigma}^-$ occurs only via the exchange amplitude, so it would be suppressed in comparison with the other decays to $\Xi_c'\overline{\Sigma}$. Similarly, the decay $\overline{B}^0\to\Omega_c^0\overline{\Xi}^0$ would be suppressed. If, indeed, b_E is found to be suppressed, a useful amplitude triangle based on the two independent amplitudes b_1 and b_2 could be formed:

$$2\sqrt{3}\mathcal{A}(\overline{B}^0\to\Xi_c'^0\overline{\Lambda}) + \mathcal{A}(B^-\to\Sigma_c^0\overline{p}) = 2\sqrt{2}\mathcal{A}(\overline{B}^0\to\Sigma_c^+\overline{p}). \tag{26}$$

Other such triangles can also be formed, but they generally involve $\overline{B}_s^0$ decays.

C. 3*+10* final states

Here a single amplitude describes all decays. The relation

$$\mathcal{A}(B^-\to\Lambda_c^+\overline{\Delta}^{--}) = \sqrt{3}\mathcal{A}(B^0\to\Lambda_c^+\overline{\Delta}^-) \tag{27}$$

is a consequence of the pure isospin ($I=3/2$) of the final state. The decays $\overline{B}\to\Xi_c\overline{\Sigma}^*$ involve both $I=1/2$ and $I=3/2$, but these amplitudes are related to one another since $\overline{B}^0\to\Xi_c^+\overline{\Sigma}^{*-}$ is forbidden. This process could have proceeded only via an exchange amplitude, but the final charmed baryon is antisymmetric in its light quarks, which cannot couple to the symmetrized quarks in the final antidecuplet antibaryon. Thus the isospin relation

$$\mathcal{A}(B^-\to\Xi_c^0\overline{\Sigma}^{*-})=\sqrt{2}\mathcal{A}(\overline{B}^0\to\Xi_c^0\overline{\Sigma}^{*0})+\mathcal{A}(\overline{B}^0\to\Xi_c^+\overline{\Sigma}^{*-}) \quad (28)$$

is implemented as

$$\mathcal{A}(B^-\to\Xi_c^0\overline{\Sigma}^{*-})=\sqrt{2}\mathcal{A}(\overline{B}^0\to\Xi_c^0\overline{\Sigma}^{*0}). \quad (29)$$

There are no triangle relations, and no tests for a vanishing exchange amplitude since it never contributes in the first place.

D. 6+10* final states

There are a number of isospin triangles involving the charge states of $\overline{B}\to\Sigma_c\overline{\Delta}$. One example for which detection of final states may be particularly favorable is

$$\mathcal{A}(\overline{B}^0\to\Sigma_c^{++}\overline{\Delta}^{--})+\sqrt{3}\mathcal{A}(B^-\to\Sigma_c^0\overline{\Delta}^-)$$
$$=\sqrt{3}\mathcal{A}(\overline{B}^0\to\Sigma_c^0\overline{\Delta}^0). \quad (30)$$

Another useful relation involves the two charge states of $B^-\to\Sigma_c\overline{\Delta}$:

$$\sqrt{2}\mathcal{A}(\overline{B}^0\to\Sigma_c^+\overline{\Delta}^{--})+\sqrt{3}\mathcal{A}(B^-\to\Sigma_c^0\overline{\Delta}^-)=0. \quad (31)$$

In order that the isospin triangles have non-zero area, both d and d_E must be nonvanishing and have a nontrivial relative phase. A good test for $d_E=0$ is to check whether the decay $\overline{B}^0\to\Sigma_c^{++}\overline{\Delta}^{--}$ is suppressed in comparison with other $\overline{B}\to\Sigma_c\overline{\Delta}$ decays. When $d_E=0$, all the rates for processes in Table VII are either zero or related to one another by simple factors.

Another isospin triangle involving the charge states of $\overline{B}^0\to\Xi_c\overline{\Sigma}^*$ is

$$\mathcal{A}(B^-\to\Xi'^0_c\overline{\Sigma}^{*-})=\sqrt{2}\mathcal{A}(\overline{B}^0\to\Xi'^0_c\overline{\Sigma}^{*0})$$
$$+\mathcal{A}(\overline{B}^0\to\Xi'^+_c\overline{\Sigma}^{*-}). \quad (32)$$

However, experimentally it is not easy to construct.

IV. DISCUSSION AND SUMMARY

The recent observation of a two-body baryon-antibaryon B decay [16] is likely to be the first in a series of such decays. We have shown that these processes are capable of providing information on two main questions which have been of interest in B meson decays for some years: (1) Are there significant final-state interaction phases between different decay amplitudes characterized by the same weak phases? (2) Are processes involving the spectator quark (such as the exchange amplitudes described here by the suffix E) suppressed in comparison with other amplitudes in which the spectator does not enter into the weak Hamiltonian? We have described a number of tests of both these questions which may be feasible in the near future. In particular, if amplitude triangles formed of total rates for three processes appear to have zero area, we have shown that relative final-state phases must vanish *and* that partiy-conserving and parity-violating transition amplitudes must be in the same proportion in all three processes. Tests for the smallness of exchange amplitudes can be performed by several comparisons of rates in Tables III, V, and VII. Other tests may require separation of helicity amplitudes before being fully implemented.

Given the value of the observed branching ratio for $\overline{B}^0\to\Lambda_c^+\overline{p}$ [16], which was based on an integrated luminosity of 78.2 fb^{-1}, several times the present data sample may be needed to see some of the related decay modes, but the triangle construction in Fig. 2 suggests that at least some other decay modes to a charmed baryon and an octet antibaryon may be observable with comparable branching ratios. Combined with the predictions of Ref. [15] and the assumption of suppression of the exchange amplitudes, Table V indicates that a few other processes (such as $\overline{B}^0\to\Sigma_c^+\overline{p}$, $B^-\to\Xi'^0_c\overline{\Sigma}^-$, $\overline{B}_s^0\to\Sigma_c^+\overline{\Sigma}^-$ and $\overline{B}_s^0\to\Sigma_c^0\overline{\Sigma}^0$) may have branching ratios of about the same order as the already observed decay $B^-\to\Sigma_c^0\overline{p}$. Another decay in Table VI, $B^-\to\Lambda_c^+\overline{\Delta}^{--}$, should also be observable if its branching ratio is of order 10^{-5} as predicted by the pole model of Ref. [15].

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APPENDIX: QUARK COMPOSITION OF BARYONS

In our convention (Ξ_c^0,Ξ_c^+), $(\overline{p},\overline{n})$, $(\overline{\Sigma}^-,\overline{\Sigma}^0,\overline{\Sigma}^+)$, $(\overline{\Xi}^0,\overline{\Xi}^+)$, $(\overline{\Delta}^{--},\overline{\Delta}^-,\overline{\Delta}^0,\overline{\Delta}^+)$, $(\overline{\Sigma}^{*-},\overline{\Sigma}^{*0},\overline{\Sigma}^{*+})$ and $(\overline{\Xi}^{*0},\overline{\Xi}^{*+})$ are in iso-multiplets. We recall that $I_-u=d$, $I_-\overline{d}=-\overline{u}$. Our convention for the $\overline{B}$ mesons is: $B^-=-b\overline{u}$, $\overline{B}^0=b\overline{d}$, $\overline{B}_s^0=b\overline{s}$.

(1) Antitriplet charmed baryons:

$$\Lambda_c^+=(cud-cdu)/\sqrt{2},$$
$$\Xi_c^+=(csu-cus)/\sqrt{2},$$
$$\Xi_c^0=(csd-cds)/\sqrt{2}.$$

(2) Sextet charmed baryons:

$$\Sigma_c^{++}=cuu,$$
$$\Sigma_c^+=(cud+cdu)/\sqrt{2},$$
$$\Sigma_c^0=cdd,$$
$$\Xi'^+_c=(cus+csu)/\sqrt{2},$$
$$\Xi'^0_c=(cds+csd)/\sqrt{2},$$
$$\Omega_c^0=css.$$

(3) Octet antibaryons:

$$\overline{p}=(\overline{udu}-\overline{duu})/\sqrt{2},$$

$$\overline{n}=(\overline{dud}-\overline{udd})/\sqrt{2},$$

$$\overline{\Sigma}^{-}=(\overline{suu}-\overline{usu})/\sqrt{2},$$

$$\overline{\Sigma}^{0}=(\overline{usd}-\overline{sud}+\overline{dsu}-\overline{sdu})/2,$$

$$\overline{\Sigma}^{+}=(\overline{sdd}-\overline{dsd})/\sqrt{2},$$

$$\overline{\Xi}^{0}=(\overline{uss}-\overline{sus})/\sqrt{2},$$

$$\overline{\Xi}^{+}=(\overline{sds}-\overline{dss})/\sqrt{2},$$

$$\overline{\Lambda}=(2\overline{uds}-2\overline{dus}-\overline{dsu}+\overline{sdu}-\overline{sud}+\overline{usd})/\sqrt{12}.$$

(4) Antidecuplet antibaryons:

$$\overline{\Delta}^{--}=-\overline{uuu},$$

$$\overline{\Delta}^{-}=(\overline{uud}+\overline{udu}+\overline{duu})/\sqrt{3},$$

$$\overline{\Delta}^{0}=-(\overline{udd}+\overline{dud}+\overline{ddu})/\sqrt{3},$$

$$\overline{\Delta}^{+}=\overline{ddd},$$

$$\overline{\Sigma}^{*-}=(\overline{uus}+\overline{usu}+\overline{suu})/\sqrt{3},$$

$$\overline{\Sigma}^{*0}=-(\overline{uds}+\overline{usd}+\overline{dus}+\overline{dsu}+\overline{sud}+\overline{sdu})/\sqrt{6},$$

$$\overline{\Sigma}^{*+}=(\overline{dds}+\overline{dsd}+\overline{sdd})/\sqrt{3},$$

$$\overline{\Xi}^{*0}=-(\overline{uss}+\overline{sus}+\overline{ssu})/\sqrt{3},$$

$$\overline{\Xi}^{*+}=(\overline{dss}+\overline{sds}+\overline{ssd})/\sqrt{3},$$

$$\overline{\Omega}^{+}=\overline{sss}.$$

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