B-π weak form factor with chiral current in the light-cone sum rules

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In this paper, we calculate the $B\to\pi$ transition form factor $f^+_{B\pi}(q^2)$ by including perturbative $O(\alpha_s)$ corrections to the twist-2 terms with chiral current in the light-cone QCD sum rule approach. The corrections to the product $f_B f^+_{B\pi}(q^2)$ in the leading twist approximation are found to be about 30%, while a similar magnitude corresponding to $O(\alpha_s)$ corrections for $f_B(q^2)$ in the two-point sum rule cancels them and results in small net corrections for $f^+_{B\pi}(q^2)$. Our results confirm the observations made in previous light-cone QCD sum rule studies.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is the appropriate theory for describing the strong interaction in a high energy region, however, the strong gauge coupling at low energy destroys the perturbative expansion method. The long distance properties of QCD, especially the hadronic matrix elements, can provide much important information for understanding and testing the standard model and beyond. The exclusive semileptonic decay $B\to\pi\bar{l}\nu_l$ can be used to determine the Cabibbo-Kobayashi-Maskawa (CKM) parameter $|V_{ub}|$ [1]. However, it requires a reliable calculation of the form factor $f^+_{B\pi}(q^2)$ defined by $\langle\pi(p)|\bar{b}\gamma_\mu u|B(p+q)\rangle = 2f^+_{B\pi}(q^2)p_\mu + [f^+_{B\pi}(q^2)+f^-_{B\pi}(q^2)]q_\mu$, with p and $p+q$ being the π- and B-meson four-momentum, respectively. $f^-_{B\pi}(q^2)$ plays a negligible role for semileptonic decays into the light leptons $l=e,\mu$.

In Ref. [2], the authors propose a formula called the QCD factorization approach for $B\to\pi\pi$, πK, and πD to deal with nonleptonic decays of the B meson. In this approach, the decay amplitudes are expressed in terms of the semileptonic form factors, hadronic light-cone distribution functions, and hard-scattering amplitudes. The semileptonic form factors, and the light-cone distribution functions are taken as input parameters and the hard-scattering amplitudes are calculated by perturbative QCD. Again, the precise knowledge of heavy-to-light form factors plays crucial roles. Among the existing approaches, such as QCD sum rules, chiral perturbation theory, heavy quark effective theory, and phenomenological quark models, the QCD light-cone sum rules (LCSR) approach is very prominent for calculating $f^+_{B\pi}(q^2)$ [3–5].

The light-cone QCD sum rule approach carries out operator product expansion near the light cone $x^2\approx 0$ instead of the short distance $x\approx 0$ while the nonperturbative matrices are parametrized by light-cone wave functions which classified according to their twist instead of the vacuum condensates. For a detailed discussion of this method one can see Ref. [6]. The LCSR for $f^+_{B\pi}(q^2)$ is valid at small and intermediate momentum transfer squared $q^2 \leqslant m_Q^2 - 2m_Q\chi$, where χ is a typical hadronic scale of roughly 500 MeV and independent of the heavy quark mass m_Q.

In this paper, we calculate the form factor $f^+_{B\pi}(q^2)$ (which is different from Refs. [7–10]) up to twist-4 light-cone functions by including perturbative α_s corrections for twist-2 terms using chiral current. Remarkably, the main uncertainties of the light-cone sum rules come from the light-cone wave functions. The chiral current approach has a striking advantage in that the twist-3 light-cone functions which are not known as well as the twist-2 light-cone functions eliminated are supposed to provide results with less uncertainties [11]. In fact, only the twist-2 wave function, which is dominant in contributions to the sum rules, has been investigated systematically. The update investigation of the twist-3 wave functions can be found in Ref. [12] and the calculations of the form factor $f^+_{B\pi}$ including the $\alpha(s)$ corrections to the twist-3 terms are performed in Ref. [13]. Although the QCD radiative corrections to the twist-2 term for $f^+_{B\pi}$ are proven small in Ref. [8], it is interesting to see whether or not this is the case for chiral currents.

The paper is organized as follows: correlator and sum rule are derived in Sec. II; the perturbative correlator is calculated to order α_s in Sec. III; light-cone amplitudes and numerical results are presented in Sec. IV; Sec. V is reserved for the conclusion.

II. CORRELATOR AND SUM RULE

Let us start with the following definition of $B\to\pi$ weak form factors $f_{B\pi}(q^2)$:

$$\langle\pi(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = 2f^+_{B\pi}(q^2)p_\mu + [f^+_{B\pi}(q^2)+f^-_{B\pi}(q^2)]q_\mu, \quad (1)$$

with q being the momentum transfer. Following Ref. [11], we choose a chiral current to calculate the correlator function,

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$$\Pi_\mu(p,q)=i\int d^4x e^{iqx}\langle\pi(p)|T\{\overline{u}(x)\times\gamma_\mu(1+\gamma_5)b(x),\overline{b}(0)i(1+\gamma_5)d(0)\}|0\rangle m_b$$
$$=\Pi[q^2,(p+q)^2]p_\mu+\tilde{\Pi}[q^2,(p+q)^2]q_\mu, \qquad (2)$$

which is different from that in Refs. [7–10]. Here we take the chiral limit $p^2=m_\pi^2=0$.

We can insert a complete series of intermediate states with the same quantum numbers as the current operator $\overline{b}i(1+\gamma_5)d$ in the correlator to obtain the hadronic representation. After isolating the pole term of the lowest pseudoscalar B meson, we get the result

$$\Pi_\mu^H(p,q)=\Pi^H[q^2,(p+q)^2]p_\mu+\tilde{\Pi}^H[q^2,(p+q)^2]q_\mu$$
$$=\frac{\langle\pi|\overline{u}\gamma_\mu b|B\rangle\langle B|\overline{b}\gamma_5 d|0\rangle m_b}{m_B^2-(p+q)^2}+\sum_H\frac{\langle\pi|\overline{u}\gamma_\mu(1+\gamma_5)|B_H\rangle\langle B_H|\overline{b}i(1+\gamma_5)d|0\rangle m_b}{m_{B_H}^2-(p+q)^2}. \qquad (3)$$

The intermediate states B_H contain not only pseudoscalar resonances of the masses greater than m_B, but also scalar resonances with $J^p=0^+$, corresponding to the operator $\overline{b}d$. Taking into account the definition $\langle B|\overline{b}i\gamma_5 d|0\rangle=m_B^2f_B/m_b$, we obtain

$$\Pi^H[q^2,(p+q)^2]=\frac{2f_{B\pi}^+(q^2)m_B^2f_B}{m_B^2-(p+q)^2}+\int_{s_0}^\infty\frac{\rho^H(s)}{s-(p+q)^2}ds,$$

$$\tilde{\Pi}^H[q^2,(p+q)^2]=\frac{(f_{B\pi}^+(q^2)+f_{B\pi}^-(q^2))m_B^2f_B}{m_B^2-(p+q)^2}+\int_{s_0}^\infty\frac{\tilde{\rho}^H(s)}{s-(p+q)^2}ds. \qquad (4)$$

Here the contributions of higher resonances and continuum states above the threshold s_0 are written in terms of dispersion integrations, and the spectral densities $\rho^H(s)$ and $\tilde{\rho}^H(s)$ can be approximated by the quark-hadron duality ansatz. We can avoid the pollution from scalar resonances with $J^p=0^+$ by choosing s_0 near the B meson threshold and our final results confirm this assumption.

In the following, we briefly outline the calculation of the correlator in QCD theory and work in the large space-like momentum regions $(p+q)^2-m_b^2\ll 0$ for the $b\overline{d}$ channel, and $q^2\ll m_b^2-O(1\ \text{GeV}^2)$ for the momentum transfer, which correspond to the small light-cone distance $x^2\approx 0$ and are required by the validity of the operator product expansion method. First, we write down the full b-quark propagator:

$$\langle 0|Tb(x)\overline{b}(0)|0\rangle=i\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\frac{\hat{k}+m}{k^2-m_b^2}-ig_s\times\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 dv\times\left[\frac{1}{2}\frac{\hat{k}+m}{(m_b^2-k^2)^2}G^{\mu\nu}(vx)\sigma_{\mu\nu}+\frac{1}{m_b^2-k^2}vx_\mu G^{\mu\nu}(vx)\gamma_\nu\right], \qquad (5)$$

here $G_{\mu\nu}$ is the gluonic field strength, g_s denotes the strong coupling constant. Substituting the above b-quark propagator and the corresponding π meson light-cone wave functions into Eq. (2) and completing the integrations over x and k, finally we obtain

$$\Pi[q^2,(p+q)^2]=2f_\pi m_b^2\int_0^1 du\left\{\frac{\varphi_\pi(u)}{m_b^2-(1-u)q^2-u(p+q)^2}+\frac{2ug_2(u)}{[m_b^2-(1-u)q^2-u(p+q)^2]^2}-\frac{8m_b^2[g_1(u)+G_2(u)]}{[m_b^2-(1-u)q^2-u(p+q)^2]^3}+\int D\alpha_i\times\frac{2\varphi_\perp(\alpha_i)+2\tilde{\varphi}_\perp(\alpha_i)-\varphi_\parallel(\alpha_i)-\tilde{\varphi}_\parallel(\alpha_i)}{m_b^2-(1-\alpha_1-u\alpha_3)q^2-(\alpha_1+u\alpha_3)(p+q)^2}\right\}, \qquad (6)$$

with $G_2(u)=-\int_0^u g_2(v)dv$ and $D\alpha_i=d\alpha_1 d\alpha_2 d\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$. Here φ_π is a π meson twist-2 light-cone wave function, and $g_1(u)$, $g_2(u)$, $\varphi_\perp(\alpha_i)$, $\tilde{\varphi}_\perp(\alpha_i)$, $\varphi_\parallel(\alpha_i)$, and $\tilde{\varphi}_\parallel(\alpha_i)$ are π meson twist-4 light-cone wave functions. Their detailed expressions are given in Sec. IV. Then we carry out the subtraction procedure of the continuum spectrum by the standard procedure and perform the Borel transformations with respect to $(p+q)^2$, and finally obtain the result

$$
\begin{aligned}
f_{B\pi}^{+}(q^2)=&\frac{m_b^2 f_\pi}{m_B^2 f_B}e^{m_B^2/M^2}\Bigg\{\int_{\triangle}^{1}du\, e^{-[m_b^2-q^2(1-u)/uM^2]}\\
&\times\left(\frac{\varphi_\pi(u)}{u}+\frac{2g_2(u)}{uM^2}-\frac{8m_b^2[g_1(u)+G_2(u)]}{u^3M^4}\right)\\
&+\int_0^1 dv\int D\alpha_i\frac{\theta(\alpha_1+v\alpha_3-\Delta)}{(\alpha_1+v\alpha_3)^2M^2}\\
&\times e^{-[m_b^2-(1-\alpha_1-v\alpha_3)q^2/M^2(\alpha_1+v\alpha_3)]}\\
&\times[2\varphi_\perp(\alpha_i)+2\tilde{\varphi}_{i\perp}(\alpha_i)-\varphi_\parallel(\alpha_i)-\tilde{\varphi}_\parallel(\alpha_i)]\Bigg\}.
\end{aligned}
\tag{7}
$$

Here $\triangle=(m_b^2-q^2)/(s_0-q^2)$ and s_0 denotes the subtraction of the continuum from the spectral integral. For technical details, one can see Refs. [6,11].

III. RADIATIVE CORRECTIONS IN ORDER α_s

In this section, we calculate the perturbative contribution up to α_s for twist-2 terms, while the corrections for twist-3 terms and beyond are neglected, as they are supposed to be small. Applying Borel transformation for the α_s correction terms is tedious, we can facilitate the calculation greatly by writing down the following dispersion integral relation:

$$
f_{B\pi}^{+}(q^2)=\frac{1}{2m_B^2 f_B}\int_{m_b^2}^{s_0}\rho^{\rm QCD}(q^2,s)e^{(m_B^2-s)/M^2}ds, \tag{8}
$$

where

$$
\rho^{\rm QCD}(q^2,s)=-\frac{f_\pi}{\pi}\int_0^1 du\,\varphi_\pi(u)\,{\rm Im}\,T(q^2,s,u). \tag{9}
$$

For example, with the zeroth order approximation, one can easily obtain

$$
{\rm Im}\,T_0(q^2,s,u)=-2\pi m_b^2\delta[m_b^2-(1-u)q^2-us]. \tag{10}
$$

To order α_s, the amplitude can be written as

$$
T(r_1,r_2,u)=T_0(r_1,r_2,u)+\frac{\alpha_s C_F}{4\pi}T_1(r_1,r_2,u). \tag{11}
$$

Here we introduce convenient dimensionless variables $r_1=q^2/m_b^2$ and $r_2=(p+q)^2/m_b^2$. There are six Feynman diagrams for determining the first order amplitude T_1 in perturbative expansion. For simplicity, we perform the calculation in Feynman gauge. In the calculation, both the ultraviolet and collinear divergences are regularized by dimensional regularization and renormalized in the modified minimal subtraction ($\overline{\rm MS}$) scheme with totally anticommuting γ_5. To be more precise, the collinear divergences in the hard amplitude are factored out and absorbed in the evolution of the light-cone wave function which is determined by the QCD evolution equation [14]. Finally we get the result

$$
\begin{aligned}
T_1(r_1,r_2,u)=&\frac{6(1+\rho)}{(1-\rho)^2}\left(1-\ln\frac{m_b^2}{\mu^2}\right)-\frac{4}{1-\rho}\{[G(\rho)-G(r_1)]\\
&+[G(\rho)-G(r_2)]\}+\frac{4}{(r_1-r_2)^2}\\
&\times\left(\frac{1-r_2}{u}[G(\rho)-G(r_1)]+\frac{1-r_1}{1-u}\right.\\
&\left.\times[G(\rho)-G(r_2)]\right)+2\frac{\rho+(1-\rho)\ln(1-\rho)}{\rho^2}\\
&-\frac{4}{1-\rho}\frac{(1-r_2)\ln(1-r_2)}{r_2}\\
&+2\frac{3-\rho}{(1-\rho)^2}-\frac{4}{(1-u)(r_1-r_2)}\\
&\times\left(\frac{(1-\rho)\ln(1-\rho)}{\rho}-\frac{(1-r_2)\ln(1-r_2)}{r_2}\right)
\end{aligned}
\tag{12}
$$

with

$$
\rho=r_1+u(r_2-r_1),\quad {\rm Li}_2(x)=-\int_0^x\frac{dt}{t}\ln(1-t),
$$

$$
G(\rho)={\rm Li}_2(\rho)+\ln^2(1-\rho)-\ln(1-\rho)\left(1-\ln\frac{m_b^2}{\mu^2}\right). \tag{13}
$$

As in the calculation of the nonleading order evolution kernel of the wave function $\varphi_\pi(u,\mu)$, we take the UV renormalization scale and the factorization scale of the collinear divergences to be equal [15–17]. Our results are of the same Dirac structure as that of Ref. [8] but with different weight.

The ($\overline{\rm MS}$) quark mass depends explicitly on the renormalization scale μ and implicitly on the renormalization scheme. A renormalization scheme independent definition of the quark mass within QCD perturbation theory is given by the pole mass which is denoted by m_b^*. As in Ref. [8], we replace $\hat{m}_b$ by m_b^* using the well-known one-loop relation:

$$
\hat{m}_b=m_b^*\left\{1+\frac{\alpha_S C_F}{4\pi}\left(-4+3\ln\frac{m_b^{*2}}{\mu^2}\right)\right\}. \tag{14}
$$

To $O(\alpha_s)$, this replacement adds a term,

$$
-\frac{4\rho}{(1-\rho)^2}\left(4-3\ln\frac{m_b^{*2}}{\mu^2}\right), \tag{15}
$$

to the renormalized amplitude T_1.

To proceed further according to Eq. (9) we calculate the imaginary part of the hard scattering amplitude for $r_2>1$ and $r_1<1$:

$$-\frac{1}{\pi}\mathrm{Im}\,T(r_1,r_2,u,\mu)=\frac{\alpha_s(\mu)C_F}{2\pi}\Bigg\{\delta(1-\rho)\Bigg[2\pi^2-6+3\ln\frac{m_b^{*2}}{\mu^2}-2\,\mathrm{Li}_2(r_1)+2\,\mathrm{Li}_2\left(\frac{1}{r_2}\right)+\ln^2 r_2+2\frac{(1-r_2)\ln(r_2-1)}{r_2}$$
$$-2\ln^2(1-r_1)+2\ln(1-r_1)-2\ln(1-r_1)\ln\frac{m_b^{*2}}{\mu^2}-2\ln^2(r_2-1)+2\ln(r_2-1)$$
$$-2\ln(r_2-1)\ln\frac{m_b^{*2}}{\mu^2}\Bigg]+\theta(\rho-1)\Bigg[8\frac{\ln(\rho-1)}{\rho-1}\bigg|_+ +2\left(\ln r_2+\frac{1}{r_2}-2-2\ln(r_2-1)+\ln\frac{m_b^{*2}}{\mu^2}\right)$$
$$\times\frac{1}{\rho-1}\bigg|_+ +2\frac{1}{r_2-\rho}\left(\frac{1}{\rho}-\frac{1}{r_2}\right)+\frac{1-\rho}{\rho^2}+2\frac{1-r_1}{(r_1-r_2)(r_2-\rho)}\left(\ln\frac{\rho}{r_2}-2\ln\frac{\rho-1}{r_2-1}\right)-4\frac{\ln\rho}{\rho-1}$$
$$-2\frac{r_2-1}{(r_1-r_2)(\rho-r_1)}\left(\ln\rho-2\ln(\rho-1)+1-\ln\frac{m_b^{*2}}{\mu^2}\right)\Bigg]+\theta(1-\rho)\Bigg[2\left(\ln r_2+\frac{1}{r_2}-2\ln(r_2-1)\right.$$
$$\left.-\ln\frac{m_b^{*2}}{\mu^2}\right)\frac{1}{\rho-1}\bigg|_+ -2\frac{1}{r_2-\rho}\,\frac{1-r_2}{r_2}-2\frac{1-r_1}{(r_1-r_2)(r_2-\rho)}\left(\ln r_2+1-2\ln(r_2-1)-\ln\frac{m_b^{*2}}{\mu^2}\right)\Bigg]\Bigg\}, \tag{16}$$

here, the operation $+$ is defined by

$$F(x)|_+=\lim_{\eta\to 0}\left(F(x)\,\theta(1-x-\eta)-\delta(1-x-\eta)\int_0^{1-\eta}F(y)dy\right), \tag{17}$$

and thus remove the spurious divergences. The above expressions have a little difference compared with the corresponding ones in Ref. [8] for coefficients of the $\delta(1-\rho)$ term.

Substituting Eq. (16) into Eqs. (8)–(9), we can obtain the desired sum rule in $O(\alpha_s)$ for the form factor $f^+_{B\pi}$ in the leading twist-2 approximation.

IV. LIGHT-CONE AMPLITUDES AND NUMERICAL RESULTS

Let us choose the input parameters entering the sum rule for $f^+_{B\pi}(q^2)$ first. To begin with, let us specify the pion wave functions. For the leading twist-2 wave function $\varphi_\pi(u,\mu)$, the asymptotic form is exactly given by perturbative QCD $\varphi_\pi(u,\mu\to\infty)=6u(1-u)$ [14,18], nonperturbative corrections can be included in a systematic way in terms of the approximate conformal invariance of QCD and expanded in terms of Gegenbauer polynomials $C_n^{3/2}(2u-1)$ with weight $u(1-u)$.

To leading order (LO),

$$\varphi_\pi(u,\mu)=6u(1-u)\sum_n a_n(\mu_0)\times\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_0^n/\beta_0}C_n^{3/2}(2u-1); \tag{18}$$

and to nonleading order (NLO) [19],

$$\varphi_\pi(u,\mu)=6u(1-u)\sum_n a_n(\mu_0)\exp\left(-\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)}d\alpha\,\frac{\gamma^n(\alpha)}{\beta(\alpha)}\right)\times\left(C_n^{3/2}(2u-1)+\frac{\alpha_s(\mu)}{4\pi}\sum_{k>n}d_n^k(\mu)C_k^{3/2}(2u-1)\right), \tag{19}$$

with $a_0=1$. Arguments based on conformal spin expansion allows one to neglect higher terms in this expansion and we take $n\leqslant 4$. The coefficients $a_2(\mu_0)=2/3$ and $a_4(\mu_0)=0.43$ at the scale $\mu_0=500$ MeV have been extracted from a two-point QCD sum rule for the moments of $\varphi_\pi(u)$ [4,18]. The coefficients $d_n^k(\mu)$ are due to mixing effects, induced by the fact that the polynomials $C_n^{3/2}(2u-1)$ weight by $u(1-u)$ are the eigenfunctions of the LO, but not of the (NLO) evolution kernel. The QCD beta-function β and the anomalous dimension γ^n of the n-th moment $a_n(\mu)$ of the wave function have to be taken in NLO [20]. We can substitute the corresponding values into the above equation and obtain

$$a_2(\mu_b)=0.35,\quad a_4(\mu_b)=0.18\ \ (\mathrm{LO});$$
$$a_2(\mu_b)=0.218,\quad a_4(\mu_b)=0.084\ \ (\mathrm{NLO}), \tag{20}$$

at the scale $\mu_b=\sqrt{m_B^2-m_b^2}\approx 2.4$ GeV, which characterizes the mean virtuality of the b quark. The new analysis of the experimental data on the $\gamma\gamma^*\pi$ and π electromagnetic form factor indicates that the twist-2 wave function is close to its asymptotic form [23]. In this paper, we use both nonasymp-

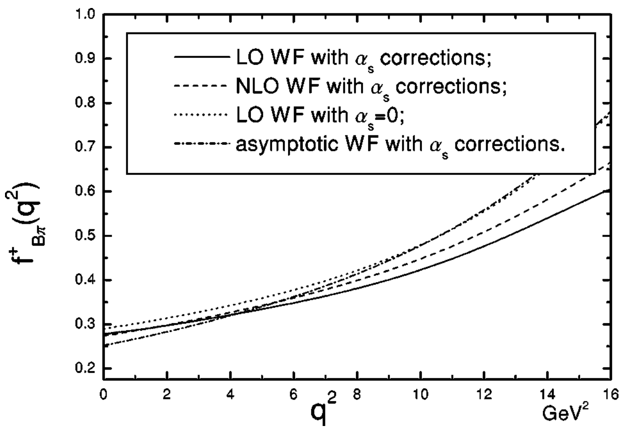

FIG. 1. $f^+_{B\pi}(q^2)$ with $s_0=33$ GeV^2,$m_b=4.7$ GeV.

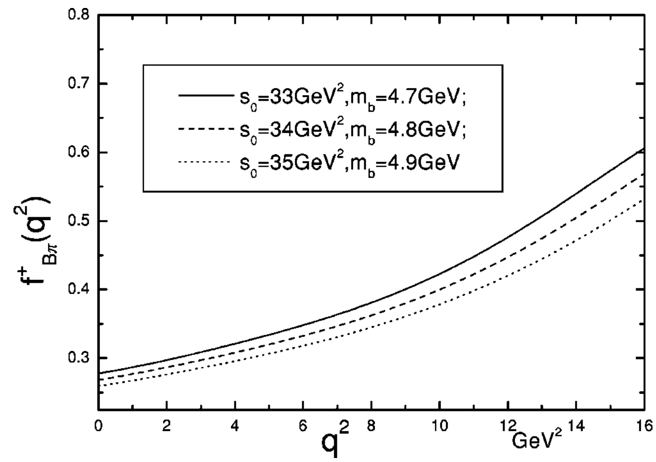

FIG. 2. $f^+_{B\pi}(q^2)$ in LO WF with α_s corrections.

totic and asymptotic form for the π twist-2 light-cone wave functions and compare the results.

The subleading twist-4 contributions are presently known only in zeroth order in α_s [21,22]. As the twist-3 contribution is eliminated, we need only the twist-4 wave functions:

$$\varphi_\perp(\alpha_i)=30\delta^2(\alpha_1-\alpha_2)\alpha_3^2[\tfrac{1}{3}+2\epsilon(1-2\alpha_3)],$$

$$\tilde{\varphi}_\perp(\alpha_i)=30\delta^2\alpha_3^2(1-\alpha_3)[\tfrac{1}{3}+2\epsilon(1-2\alpha_3)],$$

$$\varphi_\parallel(\alpha_i)=120\delta^2\epsilon(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3,$$

$$\tilde{\varphi}_\parallel(\alpha_i)=-120\delta^2\alpha_1\alpha_2\alpha_3[\tfrac{1}{3}+\epsilon(1-3\alpha_3)],$$

$$g_1(u)=\tfrac{5}{2}\varepsilon^2u^2\bar{u}^2+\tfrac{1}{2}\varepsilon\,\delta^2[u\bar{u}(2+13u\bar{u})+10u^3 \times\ln u(2-3u+\tfrac{6}{5}u^2)+10\bar{u}^3\ln\bar{u}(2-3\bar{u}+\tfrac{6}{5}\bar{u}^2)],$$

$$g_2(u)=\frac{10}{3}\delta^2u\bar{u}(u-\bar{u}), \tag{21}$$

with $\delta^2(\mu_b)=0.17$ GeV^2 and $\varepsilon(\mu_b)=0.36$. Unlike the case of the twist-2 wave functions, these twist-4 wave functions seem to be very difficult to test by experiment, for they usually are of negligible contributions in the sum rules.

Another important input is the decay constant of B meson f_B. To keep consistent, we have to calculate the two-point sum rule for f_B up to the corrections of order α_s. Here we use the two-loop expression for the running coupling constant with $N_f=4$ and $\overline{\Lambda}^{(4)}=234$ MeV corresponding to $\alpha_s(M_Z)=0.112$ [20] for comparing with the results in Ref. [8]. As the value of μ is concerned, we take the value 2.4 GeV which corresponds to the average virtuality of the correlation function which is given by the Borel mass parameter M^2. In the present case a chiral current correlator is adopted to delete the contributions from the twist-3 wave functions, we consider the following two-point correlator:

$$\Pi(q^2)=i\int d^4xe^{iqx}\langle 0|\bar{q}(x)(1+\gamma_5)b(x), \bar{b}(0)(1-\gamma_5)q(0)|0\rangle. \tag{22}$$

The standard manipulation yields three self-consistent sets of results:

$$f_B=218\ \text{MeV},\quad m_b=4.7\ \text{GeV},\quad s_0=33\ \text{GeV}^2;$$
$$f_B=212\ \text{MeV},\quad m_b=4.8\ \text{GeV},\quad s_0=34\ \text{GeV}^2;$$
$$f_B=206\ \text{MeV},\quad m_b=4.9\ \text{GeV},\quad s_0=35\ \text{GeV}^2. \tag{23}$$

The corresponding $\alpha_s=0$ results:

$$f_B=163\ \text{MeV},\quad m_b=4.7\ \text{GeV},\quad s_0=33\ \text{GeV}^2;$$
$$f_B=158\ \text{MeV},\quad m_b=4.8\ \text{GeV},\quad s_0=34\ \text{GeV}^2;$$
$$f_B=153\ \text{MeV},\quad m_b=4.9\ \text{GeV},\quad s_0=35\ \text{GeV}^2. \tag{24}$$

From the above results we can see that $f_B(\alpha_s=0)/f_B(\alpha_s\neq 0)\approx 76\%$, in other words, α_s corrections increase the value of f_B about 30%. They will be used as inputs in numerical analysis of the sum rule for $f^+_{B\pi}(q^2)$. As for the B meson mass m_B and the pion decay constant f_π, we take the present world average value $m_B=5.279$ GeV, and $f_\pi=0.132$ GeV. The continuum subtraction s_0 is about 33–35 GeV^2 and the pole mass for the b quark is taken as $m_b=4.7$–4.9 GeV. Here we make some comments about the continuum subtraction s_0. The special chiral current leads to cancellations between the condensates, the dominating contributions come from the perturbative parts and the nonperturbative parts only play tiny roles. The lowest pseudoscalar resonance appears at the energy threshold about $s=m_B^2\approx 28$ GeV^2. Though the B meson has a narrow decay width, the values taken in Ref. [11], $s_0=30$–33 GeV^2, are too low due to the large difference between the corresponding results for the values of f_B.

We exploit the sum rule numerically in the following:

$$f_Bf^+_{B\pi}(0)=60.5\ \text{MeV},\quad f^+_{B\pi}(0)=0.277,$$
$$m_b=4.7\ \text{GeV},\quad s_0=33\ \text{GeV}^2;$$
$$f_Bf^+_{B\pi}(0)=56.8\ \text{MeV},\quad f^+_{B\pi}(0)=0.268,$$

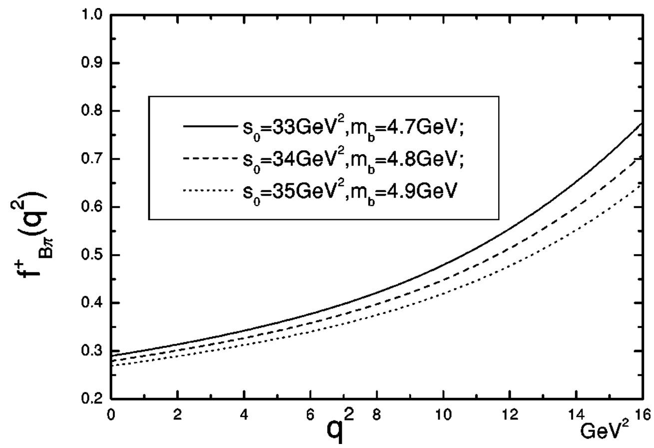

FIG. 3. $f^{+}_{B\pi}(q^2)$ in LO WF with $\alpha_s=0$.

$$m_b=4.8\ \text{GeV},\quad s_0=34\ \text{GeV}^2;$$

$$f_B f^{+}_{B\pi}(0)=53.4\ \text{MeV},\quad f^{+}_{B\pi}(0)=0.259,$$

$$m_b=4.9\ \text{GeV},\quad s_0=35\ \text{GeV}^2, \tag{25}$$

for $\alpha_s\neq 0$ in LO.

$$f_B f^{+}_{B\pi}(0)=59.6\ \text{MeV},\quad f^{+}_{B\pi}(0)=0.273,$$

$$m_b=4.7\ \text{GeV},\quad s_0=33\ \text{GeV}^2, \tag{26}$$

for $\alpha_s\neq 0$ in NLO.

$$f_B f^{+}_{B\pi}(0)=47.3\ \text{MeV},\quad f^{+}_{B\pi}(0)=0.290,$$

$$m_b=4.7\ \text{GeV},\quad s_0=33\ \text{GeV}^2;$$

$$f_B f^{+}_{B\pi}(0)=44.1\ \text{MeV},\quad f^{+}_{B\pi}(0)=0.279,$$

$$m_b=4.8\ \text{GeV},\quad s_0=34\ \text{GeV}^2;$$

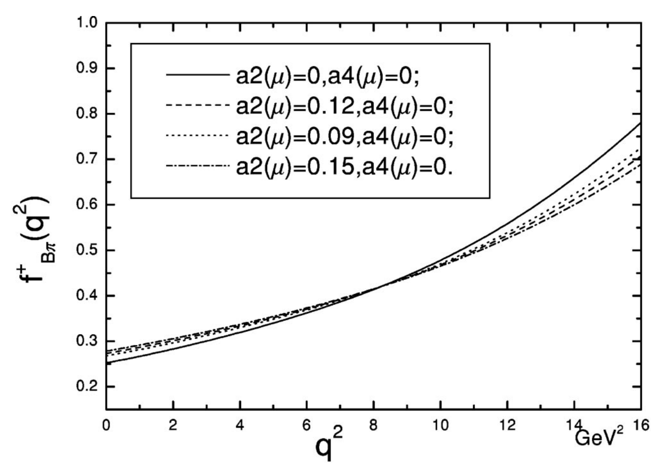

FIG. 4. $f^{+}_{B\pi}(q^2)$ with $s_0=33$ GeV^2,$m_b=4.7$ GeV; α_s corrections.

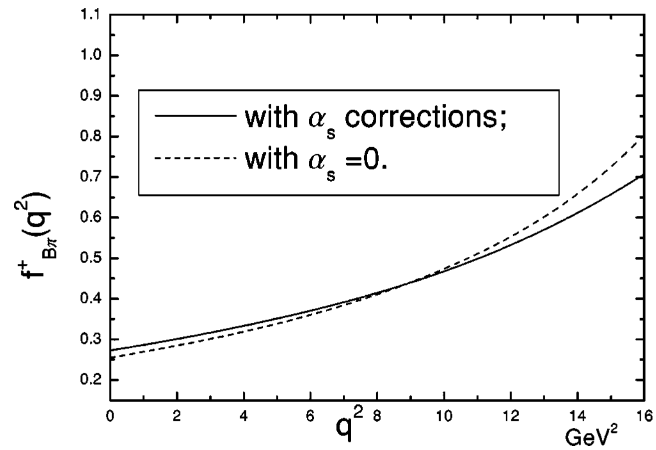

FIG. 5. $f^{+}_{B\pi}(q^2)$ with $a2(\mu)=0.12, a4(\mu)=0$.

$$f_B f^{+}_{B\pi}(0)=41.2\ \text{MeV},\quad f^{+}_{B\pi}(0)=0.269,$$

$$m_b=4.9\ \text{GeV},\quad s_0=35\ \text{GeV}^2, \tag{27}$$

for $\alpha_s=0$ in LO. From the above results we can see that $f_B f^{+}_{B\pi}(0)(\alpha_s\neq 0)/f_B f^{+}_{B\pi}(0)(\alpha_s=0)\approx 130\%$, in other words, α_s corrections increase the value of $f_B f^{+}_{B\pi}(0)$ about 30%. Due to the same corrections to the decay constant, the resulting net α_s corrections are very small, say, for $f^{+}_{B\pi}(0)$ less than 3%. The large correction for $f_B f^{+}_{B\pi}(0)$ is cancelled by the corresponding value for f_B. They are compatible with the values obtained in Ref. [8], for $\alpha_s=0, f^{+}_{B\pi}(0)=0.30$; for $\alpha_s\neq 0, f^{+}_{B\pi}(0)=0.27$. Our numerical results show that the vibrations for the form factor $f^{+}_{B\pi}(0)$ are about ± 0.01 around the center values, for $\alpha_s\neq 0, f^{+}_{B\pi}(0)=0.27$; for $\alpha_s=0, f^{+}_{B\pi}(0)=0.28$ with LO wave functions. It is shown in Fig. 1 that the form factor $f^{+}_{B\pi}(q^2)$ with α_s corrections lies below the uncorrected one for LO wave function (WF); the quantities of the α_s corrections increase with q^2, at $q^2=15$ GeV^2, numerically lesser than 20% for LO wave functions; the curve for NLO wave function lies a little above the corresponding one for LO wave function; the curve for asymptotic wave function with α_s corrections is almost the

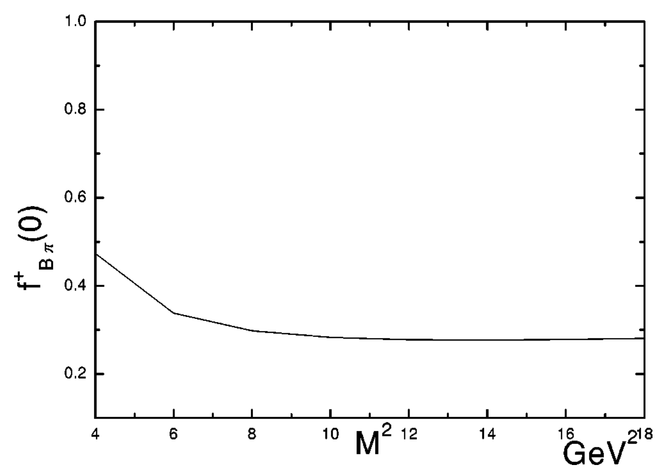

FIG. 6. $f^{+}_{B\pi}(0)$ as a function of Borel parameter M^2.

same as the uncorrected one for LO wave functions at $q^2 > 8$ GeV^2; the deviation of the curves for the α_s corrected LO wave function and asymptotic wave function from each other is notable. In Figs. 2 and 3, we plot the $f^+_{B\pi}(q^2)$ as a function of q^2 in a leading order π light-cone wave function with different boundary conditions. From the two figures, we can see that the vibrations for $f^+_{B\pi}(0)$ are small, numerically about ± 0.01 around the center values both for the α_s corrected and uncorrected form factor. In Fig. 4 we use the parameters obtained in Ref. [23] as input, from the figure we can see that the curve for $f^+_{B\pi}(q^2)$ with α_s corrections varies according to the π twist-2 light-cone wave functions, the largest deviation of the values from each other is less than 15%. In Fig. 5, we plot the $f^+_{B\pi}(q^2)$ with boundary condition $s_0 = 33$ GeV^2, $m_b = 4.7$ GeV both for α_s corrected and uncorrected form factor using the parameters obtained in Ref. [23]. Again, we can see that the net α_s correction is small. There is a platform for $f^+_{B\pi}(q^2)$ as a function of the Borel parameter M^2 for $M^2 = 8$–14 GeV^2 which verifies the value we took $M^2 = 12$ GeV^2 in calculation. For example, the product $f^+_{B\pi}(0)$ is plotted as a function of M^2 in Fig. 6. The uncertainties due to the Borel parameter M^2 can thus be diminished or eliminated.

V. CONCLUSION

To summarize, we have reexamined that the weak form factor $f^+_{B\pi}(q^2)$ up to $q^2 = 16$ GeV^2 for B decays into light pseudoscalar mesons by taking the contributions of α_s corrections to twist-2 terms in a light-cone QCD sum rule framework. Due to the special structure of the chiral current, the contributions of α_s corrections to twist-2 terms are of the same Dirac structure as that of Ref. [8] with different weight. As the contributions of twist-3 terms are eliminated, the uncertainties due to the twist-3 light-cone wave functions which are not understood as well as the twist-2 light-cone wave function are avoided. Furthermore, the possible pollution from wrong parity 0^+ mesons is deleted by suitable choice of continuum subtraction parameter s_0, the final results are supposed to be with less uncertainties. The results presented here will be beneficial to the precision extracting of the CKM matrix element $|V_{ub}|$ from the exclusive processes $B \to \pi \ell \tilde{\nu}_l (l = e, \mu)$, by confronting the theoretical predictions with the experimentally available data. Although the α_s corrections to $f_B f^+_{B\pi}$ are large, about 30%, the similar corrections to f_B canceled them, and the resulting net corrections to form factor $f^+_{B\pi}(q^2)$ are small. Our results are compatible with the observations made in Ref. [8]. Compared with the results obtained in Ref. [8], our results are with lesser uncertainties due to the elimination of the twist-3 light-cone wave functions.

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