Partial-wave analysis of two-hadron fragmentation functions

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We reconsider the option of extracting the transversity distribution by using interference fragmentation functions into two leading hadrons inside the same current jet. To this end, we perform a new study of two-hadron fragmentation functions. We derive new positivity bounds on them. We expand the hadron pair system in relative partial waves, so that we can naturally incorporate in a unified formalism specific cases already studied in the literature, such as the fragmentation functions arising from the interference between the *s*- and *p*-wave production of two mesons, as well as the production of a spin-one hadron. In particular, our analysis clearly distinguishes two different ways to access the transversity distribution in two-hadron semi-inclusive leptoproduction.

I. INTRODUCTION

Two-hadron fragmentation functions have been proposed for the first time in Refs. [1,2] and then systematically analyzed at leading twist in Ref. [3]. The interest in these functions is mainly justified by the search for a mechanism to single out the chiral-odd transversity distribution[1] in an alternative and technically simpler way than the Collins effect [5]. In fact, in semi-inclusive deep inelastic scattering (SIDIS) where two unpolarized hadrons are produced in the current fragmentation region, i.e., for the reaction $ep \rightarrow e'h_1h_2X$, it is indeed possible to build a leading-twist single-spin asymmetry (SSA) containing the factorized product of the transversity and a chiral-odd two-hadron fragmentation function [2,6]. In this process, the asymmetry occurs in the azimuthal angle between the two-hadron plane and the laboratory plane; the total momentum of the hadronic system does not need to have a transverse component, i.e., out of collinearity with respect to the virtual photon axis. Therefore, the intrinsic transverse momentum of the quark can be integrated away and no transverse momentum dependent functions are required, thus introducing simplifications both on the experimental and theoretical side [7] as compared to the Collins effect. Model calculations of such objects are feasible [8,6], but a more realistic approach is needed to estimate their size and measurability. Some of the two-hadron fragmentation functions are also naive time-reversal odd (T-odd) and originate from the interference between two production amplitudes with two different phases [5,3,9]. Therefore, in the literature these functions are usually referred to as interference fragmentation functions (IFF).

In an apparently independent context, semi-inclusive pro-duction of spin-1 hadrons (e.g., ρ, K^*, ϕ) has also been studied and proposed as a method to measure the transversity distribution [10–13]. To measure the polarization of the outgoing vector meson (e.g., ρ^0) it is necessary to measure the 4-momenta of the decay products (e.g., $\pi^+\pi^-$). Thus, the reaction $ep \rightarrow e'\rho^0X(\rho^0 \rightarrow \pi^+\pi^-)$ is just a part of the more general reaction $ep \rightarrow e'\pi^+\pi^-X$ (namely the part where the total invariant mass of the pion pair is equal to the ρ mass). However, up to now the relation between spin-1 fragmentation functions and two-hadron fragmentation functions has never been thoroughly examined, nor has it ever been specified clearly how to access the transversity distribution in the case of spin-1 fragmentation. The present work is motivated by the need to fill this gap.

Although in our work we focus mainly on SIDIS, two-hadron fragmentation functions can be measured also in e^+e^- annihilation, if hadron pairs belonging to the same jet are identified [14,15]. Some data are already available concerning two hadrons being produced via a spin-1 resonance [16–20].

The work is organized as follows. In Sec. II we will review the systematic analysis of semi-inclusive production of two unpolarized hadrons at leading twist. We will recover the results originally presented in Ref. [3]. We will devote particular attention to the connection with the helicity basis formalism (see, e.g., Refs. [21,12]) and for the first time we will deduce positivity bounds on IFF.

In Sec. III the whole problem is reconsidered by expanding in partial waves the two-hadron system in its center-of-mass frame. If we consider only low invariant masses, the expansion can be truncated to include the first two terms only, as hadron pairs are produced mainly in the *s*-wave channel or in the *p*-wave channel (via a spin-1 resonance). We can thus deduce a general unifying formalism that naturally incorporates the specific case of Ref. [2] in the subsector describing the interference between relative *s* and *p* waves, as well as the case of spin-1 hadron fragmentation [13] in the subsector of the relative *p* wave. In particular, we

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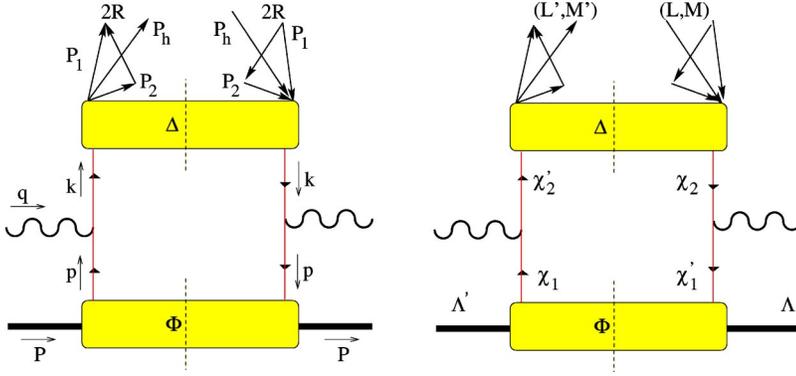[1]See Ref. [4] for a review on the topic.

FIG. 1. The usual quark handbag diagram contributing at leading twist to the semi-inclusive DIS into two leading hadrons: (a) hadron and parton momenta are shown, in particular the total momentum $P_h = P_1 + P_2$ and relative momentum $R = (P_1 - P_2)/2$ of the two-hadron system; (b) target helicity, parton chirality, and two-hadron partial wave indices are shown.

will identify a SSA where the transversity distribution appears in connection with a s-p IFF, and a SSA where the transversity is connected to a pure p-wave IFF. These two asymmetries are completely distinct; they could have different physical origins and different magnitudes.

In Sec. IV we complete our analysis by including the intrinsic partonic transverse momentum and $\vec{k}_T$-unintegrated fragmentation functions. Also in this case, in Sec. V we will present positivity bounds and will carry out the partial wave expansion. The results for the complete cross section for all combinations of beam and target polarizations are listed in Appendixes A and C. Finally, some conclusions are drawn in Sec. VI.

II. TWO-PARTICLE INCLUSIVE DEEP INELASTIC SCATTERING

In the following we will describe the kinematics and the details of the semi-inclusive production of two unpolarized hadrons in the context of the SIDIS process. However, we point out that the involved fragmentation functions can be used also in the case of reactions with a hadronic probe or in $e^+ e^-$ annihilation [14,15].

A. Kinematics and hadronic tensor

The process is schematically depicted in Fig. 1. An electron with momentum l scatters off a target nucleon with mass
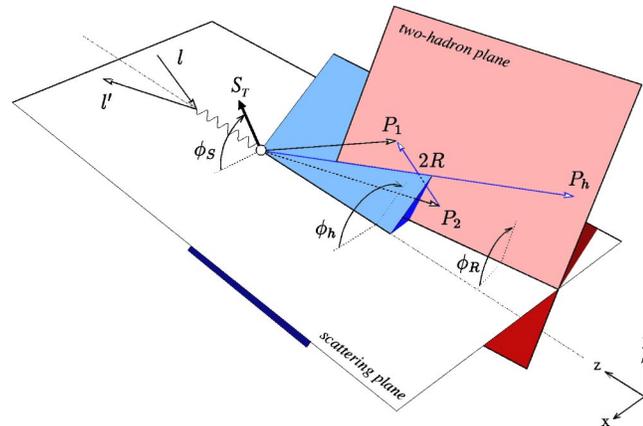

FIG. 2. Kinematics for the SIDIS of the lepton l on a transversely polarized target leading to two hadrons inside the same current jet.

M, polarization S, and momentum P via the exchange of a virtual hard photon with momentum $q = l - l'$ ($q^2 = -Q^2$). Inside the target, the photon hits a quark with momentum p, changing its momentum to $k = p + q$. The quark then fragments into a residual jet and two leading unpolarized hadrons with masses M_1, M_2, and momenta P_1 and P_2. We introduce the vectors $P_h = P_1 + P_2$ and $R = (P_1 - P_2)/2$. We describe a 4-vector a as $[a^-, a^+, \vec{a}_T]$, i.e., in terms of its light-cone components $a^\pm = (a^0 \pm a^3)/\sqrt{2}$ and the bidimensional vector $\vec{a}_T$. It is convenient to choose the $\hat{z}$ axis according to the condition $\vec{P}_T = \vec{P}_{hT} = 0$. In this case, the virtual photon has a nonvanishing transverse momentum $\vec{q}_T$. However, it is also customary to align the $\hat{z}$ axis opposite to the direction of the virtual photon, in which case the outgoing hadron has a nonvanishing transverse momentum $\vec{P}_{h\perp} = -z\vec{q}_T$. These two directions overlap up to corrections of order $1/Q$, which we will systematically neglect in the following. The y axis is chosen to point in the direction of the vector product $(-\vec{q} \times \vec{l}')$ [22] (see Fig. 2).

We define the variables $x = p^+/P^+$, which represents the light-cone fraction of target momentum carried by the initial quark, and $z = P_h^-/k^-$, the light-cone fraction of fragmenting quark momentum carried by the final hadron pair. Analogously, we define the light-cone fraction $\zeta = 2R^-/P_h^-$, which describes how the total momentum of the hadron pair is split into the two single hadrons.[2] The relevant momenta can be parametrized as

$$P^\mu = \left[\frac{M^2}{2P^+}, P^+, \vec{0} \right],$$

$$p^\mu = \left[\frac{p^2 + \vec{p}_T^2}{2xP^+}, xP^+, \vec{p}_T \right],$$

$$k^\mu = \left[\frac{P_h^-}{z}, \frac{z(k^2 + \vec{k}_T^2)}{2P_h^-}, \vec{k}_T \right], \quad (1)$$

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[2]Note that $-1 \leq \zeta \leq 1$, and $\zeta = 2\xi - 1$, with ξ defined in Ref. [3].

$$P_h^\mu = \left[P_h^-, \frac{M_h^2}{2P_h^-}, \vec{0} \right],$$

$$R^\mu = \left[\frac{\zeta}{2} P_h^-, \frac{(M_1^2 - M_2^2) - \frac{\zeta}{2} M_h^2}{2P_h^-}, \vec{R}_T \right].$$

Not all components of the 4-vectors are independent. In particular, here we observe that

$$R^2 = \frac{M_1^2 + M_2^2}{2} - \frac{M_h^2}{4},$$

$$R_T^2 = \frac{1}{2} \left[\frac{(1-\zeta)(1+\zeta)}{2} M_h^2 - (1-\zeta) M_1^2 - (1+\zeta) M_2^2 \right],$$

$$P_h \cdot R = \frac{M_1^2 - M_2^2}{2},$$ (2)

$$P_h \cdot k = \frac{M_h^2}{2z} + z \frac{k^2 + |\vec{k}_T|^2}{2},$$

$$R \cdot k = \frac{(M_1^2 - M_2^2) - \frac{\zeta}{2} M_h^2}{2z} + z\zeta \frac{k^2 + |\vec{k}_T|^2}{4} - \vec{k}_T \cdot \vec{R}_T.$$

The positivity requirement $R_T^2 \geq 0$ imposes the further constraint

$$M_h^2 \geq \frac{2}{1+\zeta} M_1^2 + \frac{2}{1-\zeta} M_2^2.$$ (3)

We shall first consider the case when the cross section is integrated over the transverse momentum of the virtual photon, $\vec{q}_T$, postponing the analysis of the complete case in Sec. IV. Until then, no transverse-momentum-dependent distribution and fragmentation functions will appear. The sevenfold differential cross section for two-particle-inclusive deep inelastic scattering (DIS) is

$$\frac{d^7\sigma}{d\zeta dM_h^2 d\phi_R dz dx dy d\phi_S} = \sum_a \frac{\alpha^2 y e_a^2}{32zQ^4} L_{\mu\nu} 2MW_a^{\mu\nu},$$ (4)

where $L_{\mu\nu}$ is the lepton tensor; $y = (E - E')/E$ is the fraction of beam energy transferred to the hadronic system and it is related to the lepton scattering angle in the center-of-mass (c.m.) frame; ϕ_R and ϕ_S are the azimuthal angles of $\vec{R}_T$ and $\vec{S}_T$ with respect to the lepton scattering plane (see Fig. 2), where $\vec{S}_T = (S_x, S_y)$ indicates the transverse components of the target spin S. At tree level, the hadronic tensor for a flavor a is given by

$$2MW_a^{\mu\nu} = 32z \ \text{Tr}[\Phi_a(x,S)\gamma^\mu \Delta_a(z,\zeta,M_h^2,\phi_R)\gamma^\nu]$$
$$+ \left(\begin{matrix} q \leftrightarrow -q \\ \mu \leftrightarrow \nu \end{matrix} \right),$$ (5)

where

$$\Phi_a(x,S) = \int d\vec{p}_T dp^- \Phi_a(p;P,S)|_{p^+ = xP^+},$$ (6)

$$\Delta_a(z,\zeta,M_h^2,\phi_R) = \frac{z}{32} \int d\vec{k}_T dk^+ \Delta_a(k;P_h,R)|_{k^- = P_h^-/z}.$$ (7)

The quark-quark correlator Φ describes the nonperturbative processes determining the distribution of parton a inside the spin-$\frac{1}{2}$ target (represented by the lower shaded area in Fig. 1) and, similarly, the correlator Δ symbolizes the fragmentation of quark a producing two tagged leading hadrons in a residual jet (upper shaded area in Fig. 1).

We are going to focus only on the leading twist contributions to the hadronic tensor of Eq. (5). A method to extract these contributions consists of projecting the so-called good light-cone components out of the quark field ψ. As is evident from the kinematics in the infinite momentum frame, the $+$ and the $-$ light-cone components are the dominant ones for the parton entering and exiting the hard vertex, respectively. They can be projected out by means of the operators $\mathcal{P}_\pm = \frac{1}{2}\gamma^\mp \gamma^\pm$. Any other component of ψ is automatically of higher twist. Therefore, the hadronic tensor (5) at leading twist looks like

$$2MW_a^{\mu\nu} = 32z\text{Tr}[\mathcal{P}_+ \Phi_a(x,S)\bar{\mathcal{P}}_+ \gamma^\mu \mathcal{P}_-$$
$$\times \Delta_a(z,\zeta,M_h^2,\phi_R)\bar{\mathcal{P}}_- \gamma^\nu]$$
$$= 32z[\mathcal{P}_+ \Phi_a(x,S)\gamma^+]_{ij}[\tfrac{1}{2}\gamma^- \gamma^\mu \mathcal{P}_-]_{jl}$$
$$\times [\tfrac{1}{2}\gamma^+ \gamma^\nu \mathcal{P}_+]_{mi}[\mathcal{P}_- \Delta_a(z,\zeta,M_h^2,\phi_R)\gamma^-]_{lm},$$ (8)

where $\bar{\mathcal{P}}_\pm \equiv \gamma^0 \mathcal{P}_\pm^\dagger \gamma^0$. In the last step the Dirac indices have been explicitly indicated. In the following, we will analyze each contribution to Eq. (8) separately.

B. The quark-quark correlator Φ

The leading-twist projection of the quark-quark correlator Φ can be parametrized in terms of the well known distribution functions [23,24][3]

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[3]Other common notations are $f_1^a(x) = a(x)$, $g_1^a(x) = \Delta a(x)$, $h_1^a(x) = \delta a(x), \Delta_T a(x)$ [4].

$$\mathcal{P}_+\Phi_a(x,S)\gamma^+=[f_1^a(x)+\lambda g_1^a(x)\gamma_5+2h_1^a(x)\gamma_5\slashed{S}_T]\mathcal{P}_+$$

$$=\begin{pmatrix} f_1^a+\lambda g_1^a & 0 & 0 & (S_x-iS_y)h_1^a \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (S_x+iS_y)h_1^a & 0 & 0 & f_1^a-\lambda g_1^a \end{pmatrix},$$

$$(9)$$

where $\lambda=MS^+/P^+$ is the light-cone helicity ($\mathcal{P}_+\Phi$ corresponds to the $\vec{p}_T$-integrated parametrization of Eq. (2) in Ref. [6]). It is possible to rewrite Eq. (9) in a more compact notation, namely in the chiral basis of the good quark fields $\psi_{\pm\,R/L}=\mathcal{P}_\pm\mathcal{P}_{R/L}\psi$, with $\mathcal{P}_{R/L}=(1\pm\gamma_5)/2$ [24],

$$[\mathcal{P}_+\Phi_a(x,S)\gamma^+]_{\chi_1'\chi_1}$$

$$=\begin{pmatrix} f_1^a(x)+\lambda g_1^a(x) & (S_x-iS_y)h_1^a(x) \\ (S_x+iS_y)h_1^a(x) & f_1^a(x)-\lambda g_1^a(x) \end{pmatrix}.$$

$$(10)$$

Finally, it is useful to project out also the target helicity density matrix $\rho_{\Lambda\Lambda'}$ by

$$[\mathcal{P}_+\Phi_a\gamma^+]_{\chi_1'\chi_1}=\rho_{\Lambda\Lambda'}[\mathcal{P}_+\Phi_a\gamma^+]_{\chi_1'\chi_1}^{\Lambda'\Lambda}, \qquad (11)$$

with

$$\rho_{\Lambda\Lambda'}=\frac{1}{2}\begin{pmatrix} 1+\lambda & S_x-iS_y \\ S_x+iS_y & 1-\lambda \end{pmatrix}, \qquad (12)$$

$$[\mathcal{P}_+\Phi_a\gamma^+]_{\chi_1'\chi_1}^{\Lambda'\Lambda}=\begin{pmatrix} f_1^a+g_1^a & 0 & 0 & 0 \\ 0 & f_1^a-g_1^a & 2h_1^a & 0 \\ 0 & 2h_1^a & f_1^a-g_1^a & 0 \\ 0 & 0 & 0 & f_1^a+g_1^a \end{pmatrix}.$$

$$(13)$$

In Eq. (13) the pair of indices (Λ,Λ') identifies each component of the 2×2 submatrices and indicates the spin state of the target; they are attached to each corresponding nucleon leg in the diagram of Fig. 1(b). The pair (χ_1,χ_1') identifies each submatrix and indicates the parton chirality; they are attached to the emerging quark legs in Fig. 1(b). Equation (13) satisfies general requirements, such as the angular momentum conservation $(\chi_1+\Lambda=\chi_1'+\Lambda')$, Hermiticity, and parity invariance. The chiral transposed matrix is also positive semidefinite, from which the well known Soffer bound [25], among others, is obtained:

$$f_1^a(x)\geq0, \quad f_1^a(x)\geq|g_1^a(x)|,$$

$$|h_1^a(x)|\leq\frac{1}{2}[f_1^a(x)+g_1^a(x)]. \qquad (14)$$

C. The quark-quark correlator Δ and positivity bounds

The most general parametrization of the quark-quark correlator $\Delta(k,P_h,R)$ entering Eq. (7), compatible with Hermiticity and parity invariance, is [3]

$$\Delta(k,P_h,R)=M_hC_1I+C_2\slashed{P}_h+C_3\slashed{R}+C_4\slashed{k}+\frac{C_5}{M_h}\sigma_{\mu\nu}P_h^\mu k^\nu$$

$$+\frac{C_6}{M_h}\sigma_{\mu\nu}R^\mu k^\nu+\frac{C_7}{M_h}\sigma_{\mu\nu}P_h^\mu R^\nu$$

$$+\frac{C_8}{M_h^2}\gamma_5\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_{h\,\nu}R_\rho k_\sigma, \qquad (15)$$

where the amplitudes $C_i(k^2,k\cdot P_h,k\cdot R,R^2)$ are dimensionless real scalar functions. By using Eqs. (15),(7) the leading-twist projection becomes

$$\mathcal{P}_-\Delta_a(z,\zeta,M_h^2,\phi_R)\gamma^-=\frac{1}{8\pi}\left(D_1^a(z,\zeta,M_h^2)+iH_1^{\sphericalangle a}(z,\zeta,M_h^2)\frac{\slashed{R}_T}{M_h}\right)\mathcal{P}_-=\frac{1}{8\pi}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & D_1^a & ie^{i\phi_R}\frac{|\vec{R}_T|}{M_h}H_1^{\sphericalangle a} & 0 \\ 0 & -ie^{-i\phi_R}\frac{|\vec{R}_T|}{M_h}H_1^{\sphericalangle a} & D_1^a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$(16)$$

where

$$D_1(z,\zeta,M_h^2) = \frac{z\pi}{4}\int d^2\vec{k}_T dk^2 d(2k\cdot P_h)$$

$$\times \delta\left(\vec{k}_T^2 + k^2 + \frac{M_h^2}{z^2} - \frac{2k\cdot P_h}{z}\right)$$

$$\times\left[C_2 + \frac{\zeta}{2}C_3 + \frac{1}{z}C_4\right], \qquad (17)$$

$$H_1^{\not\zeta}(z,\zeta,M_h^2) = \frac{z\pi}{4}\int d^2\vec{k}_T dk^2 d(2k\cdot P_h)$$

$$\times \delta\left(\vec{k}_T^2 + k^2 + \frac{M_h^2}{z^2} - \frac{2k\cdot P_h}{z}\right)$$

$$\times\left[\frac{1}{z}C_6 - C_7\right]. \qquad (18)$$

The prefactors have been chosen to have a better connection with the one-hadron results, i.e., after integrating over ζ,

M_h^2, and ϕ_R. In Eq. (16), $\mathcal{P}_-\Delta$ corresponds to the parametrization of Eq. (3) in Ref. [6].

The fragmentation function $H_1^{\not\zeta}$ is chiral odd and represents a possible partner to isolate the transversity distribution inside the cross section at leading twist [6]. Moreover, it is also odd with respect to naive time-reversal transformations (for brevity, T-odd) [3]. Noteworthy, it is the only example of leading-twist T-odd function surviving the integration upon the quark transverse momentum $\vec{k}_T$. It would be interesting to investigate it in order to understand what is the relevance of the transverse-momentum dependence in generating T-odd effects [26,27]. As a consequence, the $\vec{k}_T$-integrated $H_1^{\not\zeta}$ could have simpler evolution equations than the ones of the Collins function. Since $H_1^{\not\zeta}$ has the same operator structure as the transversity, it has been suggested that it could have the same evolution equations [28–30]. However, the situation is complicated by the presence of two hadrons [31],[4] except for the component of $H_1^{\not\zeta}$ describing the production of a spin-1 resonance (see Sec. III B).

Again, in the chiral basis for the good light-cone components Eq. (16) is simplified to

$$[\mathcal{P}_-\Delta_a(z,\zeta,M_h^2,\phi_R)\gamma^-]_{\chi_2'\chi_2} = \frac{1}{8\pi}\begin{pmatrix} D_1^a(z,\zeta,M_h^2) & ie^{i\phi_R}\frac{|\vec{R}_T|}{M_h}H_1^{\not\zeta a}(z,\zeta,M_h^2) \\ -ie^{-i\phi_R}\frac{|\vec{R}_T|}{M_h}H_1^{\not\zeta a}(z,\zeta,M_h^2) & D_1^a(z,\zeta,M_h^2) \end{pmatrix}, \qquad (19)$$

where (χ_2,χ_2') are the quark chiralities to be attached to the parton legs entering the Δ area in Fig. 1(b).

The matrix in Eq. (19) is positive semi-definite, from which the following bounds can be derived:

$$D_1^a(z,\zeta,M_h^2) \geq 0, \quad D_1^a(z,\zeta,M_h^2) \geq \frac{|\vec{R}_T|}{M_h}|H_1^{\not\zeta a}(z,\zeta,M_h^2)|. \qquad (20)$$

D. Cross section and transverse spin asymmetry

Using the previous results, we can now rewrite the leading-twist cross section for unpolarized two-hadron SIDIS in the helicity basis. In fact, after inserting Eqs. (11) and (19) inside Eq. (8), the cross section in Eq. (4) becomes

$$\frac{d^7\sigma}{d\zeta dM_h^2 d\phi_R dz dx dy d\phi_S}$$

$$= \sum_a \rho_{\Lambda\Lambda'}(S)[\mathcal{P}_+\Phi_a(x)\gamma^+]_{\chi_1'\chi_1}^{\Lambda'\Lambda}\left(\frac{d\sigma^{eq_a}}{dy}\right)^{\chi_1\chi_1';\chi_2\chi_2'}$$

$$\times[\mathcal{P}_-\Delta_a(z,\zeta,M_h^2,\phi_R)\gamma^-]_{\chi_2'\chi_2} \qquad (21)$$

where

$$\left(\frac{d\sigma^{eq_a}}{dy}\right)^{\chi_1\chi_1';\chi_2\chi_2'}$$

$$= \frac{e_a^2\alpha^2 y}{Q^4}L_{\mu\nu}\left(\frac{\gamma^-\gamma^\mu}{2}\mathcal{P}_-\right)^{\chi_1\chi_2}\left(\frac{\gamma^+\gamma^\nu}{2}\mathcal{P}_+\right)^{\chi_2'\chi_1'}$$

$$= \frac{2e_a^2\alpha^2}{Q^2 y}\begin{pmatrix} A(y)+\lambda_e\frac{C(y)}{2} & 0 & 0 & -B(y) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -B(y) & 0 & 0 & A(y)-\lambda_e\frac{C(y)}{2} \end{pmatrix} \qquad (22)$$

represents the elementary electron-quark scattering. Strictly speaking, this is not a scattering matrix, but a scattering am-

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[4]We thank D. Boer for pointing out this detail.

plitude times the conjugate of a different scattering amplitude [12]. However, for conciseness we follow the notation of Ref. [2]. The polarization of the incident beam is indicated with λ_e and

$$A(y)=1-y+\frac{y^2}{2}, \quad B(y)=1-y, \quad C(y)=y(2-y).$$
(23)

In Eq. (22), the indices (χ_1,χ_1') refer to the chiralities of the entering quarks and identify each submatrix, while (χ_2,χ_2') refer to the exiting quarks and point to the elements inside each submatrix. By expanding the sum over repeated indices in Eq. (21), we get the expression

$$\frac{d^7\sigma}{d\zeta dM_h^2 d\phi_R dz dx dy d\phi_S}$$

$$=\sum_a e_a^2 \frac{2\alpha^2}{4\pi Q^2 y}\Bigg\{A(y)f_1^a(x)D_1^a(z,\zeta,M_h^2)$$

$$+\lambda_e\lambda\frac{C(y)}{2}g_1^a(x)D_1^a(z,\zeta,M_h^2)$$

$$+B(y)\frac{|\vec{S}_T||\vec{R}_T|}{M_h}\sin(\phi_R+\phi_S)h_1^a(x)H_1^{\sphericalangle a}(z,\zeta,M_h^2)\Bigg\}.$$
(24)

For an unpolarized beam ($\lambda_e=0$, indicated with O) and a transversely polarized target ($\lambda=0$, indicated with T), Eq. (24) corresponds to Eq. (10) of Ref. [6] after integrating over all transverse momenta. The following SSA can be built:

$$A_{OT}^{\sin(\phi_R+\phi_S)}(y,x,z,M_h^2)$$

$$=\frac{\int d\phi_S d\phi_R d\zeta \sin(\phi_R+\phi_S)d^7\sigma_{OT}}{\int d\phi_S d\phi_R d\zeta d^7\sigma_{OO}}$$

$$=|\vec{S}_T|\frac{B(y)}{A(y)}\frac{\sum_a e_a^2 h_1^a(x)\int d\zeta\frac{|\vec{R}_T|}{2M_h}H_1^{\sphericalangle a}(z,\zeta,M_h^2)}{\sum_a e_a^2 f_1^a(x)\int d\zeta D_1^a(z,\zeta,M_h^2)},$$
(25)

which allows us to isolate the transversity h_1 at leading twist. Apart from the usual variables x, y, z, the only other variable to be measured is the angle $\phi_R+\phi_S$. Instead of using the scattering plane as a reference to measure azimuthal angles, it is sometimes convenient to use the directions of the beam and of the transverse component of the target spin. The new plane is rotated by the angle $\phi_S\equiv-\phi_l^S$ with respect to the scattering plane; therefore, we have $\phi_R\equiv\phi_R^S-\phi_l^S$ and $\phi_R+\phi_S\equiv\phi_R^S-2\phi_l^S$ [6].

The asymmetry described in Eq. (25) is the most general one at leading twist for the case of two-hadron production
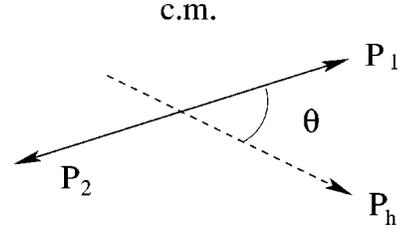

FIG. 3. The hadron pair in the c.m. frame; θ is the c.m. polar angle of the pair with respect to the direction of P_h in the target rest frame.

when an unpolarized lepton beam scatters off a transversely polarized target. No assumptions are made on the behavior of the fragmentation functions. However, as we shall see in the next section, it is useful and desirable to understand how different partial waves contribute to the above fragmentation functions.

III. PARTIAL-WAVE EXPANSION FOR THE TWO-HADRON SYSTEM

If the invariant mass M_h of the two hadrons is not very large, the pair can be assumed to be produced mainly in the relative s-wave channel, with a typical smooth distribution, or in the p-wave channel with a Breit-Wigner profile [32]. Therefore, it is useful to expand Eq. (16)—or equivalently Eq. (19)—in relative partial waves keeping only the first two harmonics. To this purpose, in the following we reformulate the kinematics in the c.m. frame of the two-hadron system. Then, the leading-twist projection for the quark-quark correlator Δ is conveniently expanded deducing a more detailed structure than Eq. (19). A set of new bounds is derived and the corresponding expression for the cross section is discussed.

In the c.m. frame the emission of the two hadrons occurs back to back. The direction identified by this emission forms an angle θ with the direction of P_h in the target rest frame (see Fig. 3). In this frame, the relevant variables become

$$P_h^\mu=\left[\frac{M_h}{\sqrt{2}}, \quad \frac{M_h}{\sqrt{2}}, \quad 0, \quad 0\right],$$

$$R^\mu=\left[\frac{\sqrt{M_1^2+|\vec{R}|^2}-\sqrt{M_2^2+|\vec{R}|^2}-2|\vec{R}|\cos\theta}{2\sqrt{2}},\right.$$

$$\frac{\sqrt{M_1^2+|\vec{R}|^2}-\sqrt{M_2^2+|\vec{R}|^2}+2|\vec{R}|\cos\theta}{2\sqrt{2}},$$

$$\left.|\vec{R}|\sin\theta\cos\phi_R,|\vec{R}|\sin\theta\sin\phi_R\right],$$

$$\zeta=\frac{2R^-}{P_h^-}=\frac{1}{M_h}(\sqrt{M_1^2+|\vec{R}|^2}-\sqrt{M_2^2+|\vec{R}|^2}-2|\vec{R}|\cos\theta),$$
(26)

where

$$|\vec{R}| = \frac{1}{2M_h}\sqrt{M_h^2 - 2(M_1^2 + M_2^2) + (M_1^2 - M_2^2)^2}. \quad (27)$$

The crucial remark is that in this frame ζ depends linearly on $\cos\theta$, i.e., $\zeta = a + b\cos\theta$, with a, b, functions only of the invariant mass. This suggests that any function of ζ can be conveniently expanded in the basis of Legendre polynomials in $\cos\theta$, as discussed in the following.

A. Partial-wave expansion of the quark-quark correlator Δ and positivity bounds

We first express the leading-twist quark-quark correlator (16) in terms of the c.m. variables. The connection between the two representations is defined as

$$\Delta(z,\cos\theta,M_h^2,\phi_R) \equiv \frac{2|\vec{R}|}{M_h}\Delta(z,\zeta,M_h^2,\phi_R), \quad (28)$$

to take into account the Jacobian of the transformation, $d\zeta = 2|\vec{R}|/M_h\, d\cos\theta$. Therefore

$$\mathcal{P}_-\Delta_a(z,\cos\theta,M_h^2,\phi_R)\gamma^-$$

$$= \frac{2|\vec{R}|}{8\pi M_h}\left(D_1^a(z,\zeta(\cos\theta),M_h^2) \right.$$

$$\left. + iH_1^{\ast a}(z,\zeta(\cos\theta),M_h^2)\frac{|\vec{R}|}{M_h}\sin\theta\, \hbar_{\phi_R} \right)\mathcal{P}_-, \quad (29)$$

where $\hbar_{\phi_R} = [0,0,\cos\phi_R,\sin\phi_R]$.

The fragmentation functions can be expanded in Legendre polynomials as

$$\frac{2|\vec{R}|}{M_h}D_1(z,\zeta(\cos\theta),M_h^2) = \sum_n D_{1,n}(z,M_h^2)P_n(\cos\theta),$$

$$\frac{2|\vec{R}|}{M_h}H_1^{\ast}(z,\zeta(\cos\theta),M_h^2) = \sum_n H_{1,n}^{\ast}(z,M_h^2)P_n(\cos\theta), \quad (30)$$

with

$$D_{1,n}(z,M_h^2) = \int_{-1}^1 d\cos\theta\, P_n(\cos\theta)\frac{2|\vec{R}|}{M_h}D_1(z,\zeta(\cos\theta),M_h^2),$$

$$H_{1,n}^{\ast}(z,M_h^2) = \int_{-1}^1 d\cos\theta\, P_n(\cos\theta)\frac{2|\vec{R}|}{M_h}$$

$$\times H_1^{\ast}(z,\zeta(\cos\theta),M_h^2). \quad (31)$$

We can truncate the expansion to the first three terms only ($n \leq 2$), which are the minimal set required to describe all the "polarization" states of the system in the c.m. frame for relative partial waves with $L = 0,1$. In fact, for $n = 0$ ($P_0 = 1$) the correponding term in the correlator does not depend on θ, it is "unpolarized." For $n = 1$, a term linear in $\cos\theta$ ($P_1 = \cos\theta$) describes the interference between an "unpolarized" hadron pair in s wave, for example on the left-hand side of Fig. 1(b), and a "longitudinally polarized" pair in p wave on the right-hand side. Whenever in the correlator we encounter a term linear in $\sin\theta$, we will interpret it as the interference between an "unpolarized" pair in s wave and a "transversely polarized" pair in p wave. Similarly, a term proportional to $\sin\theta\cos\theta$ indicates the interference between "longitudinally" and "transversely polarized" pairs always in a relative p wave. The last case corresponds to $n = 2$, which is interpreted as a "tensor polarization" still related to the intereference between pairs in a relative p wave. With notations that are consistent with previous arguments, the correlator (29) is expanded as

$$\mathcal{P}_-\Delta(z,\zeta(\cos\theta),M_h^2,\phi_R)\gamma^-$$

$$\sim \frac{1}{8\pi}\left[D_{1,0}(z,M_h^2) + D_{1,1}(z,M_h^2)\cos\theta \right.$$

$$+ D_{1,2}(z,M_h^2)\frac{1}{2}(3\cos^2\theta - 1) + i[H_{1,0}^{\ast}(z,M_h^2)$$

$$\left. + H_{1,1}^{\ast}(z,M_h^2)\cos\theta]\sin\theta\frac{|\vec{R}|}{M_h}\hbar_{\phi_R} \right]\mathcal{P}_-$$

$$\equiv \frac{1}{8\pi}\left[D_{1,OO}(z,M_h^2) + D_{1,OL}(z,M_h^2)\cos\theta \right.$$

$$+ D_{1,LL}(z,M_h^2)\frac{1}{4}(3\cos^2\theta - 1) + i[H_{1,OT}^{\ast}(z,M_h^2)$$

$$\left. + H_{1,LT}^{\ast}(z,M_h^2)\cos\theta]\sin\theta\frac{|\vec{R}|}{M_h}\hbar_{\phi_R} \right]\mathcal{P}_-. \quad (32)$$

Consequently, the same correlator in the chiral basis becomes

$$
[\mathcal{P}_-\Delta(z,\zeta,M_h^2,\phi_R)\gamma^-]_{\chi_2'\chi_2} \sim \frac{1}{8\pi}
\begin{pmatrix}
\begin{array}{l}
D_{1,OO}(z,M_h^2)+D_{1,OL}(z,M_h^2)\cos\theta \\[4pt]
\quad +D_{1,LL}(z,M_h^2)\frac{1}{4}(3\cos^2\theta-1)
\end{array}
&
\begin{array}{l}
ie^{i\phi_R}\dfrac{|\vec{R}|}{M_h}\sin\theta \\[4pt]
\quad\times[H_{1,OT}^{\not\star}(z,M_h^2)+H_{1,LT}^{\not\star}(z,M_h^2)\cos\theta]
\end{array}
\\[30pt]
\begin{array}{l}
-ie^{-i\phi_R}\dfrac{|\vec{R}|}{M_h}\sin\theta \\[4pt]
\quad\times[H_{1,OT}^{\not\star}(z,M_h^2)+H_{1,LT}^{\not\star}(z,M_h^2)\cos\theta]
\end{array}
&
\begin{array}{l}
D_{1,OO}(z,M_h^2)+D_{1,OL}(z,M_h^2)\cos\theta \\[4pt]
\quad +D_{1,LL}(z,M_h^2)\frac{1}{4}(3\cos^2\theta-1)
\end{array}
\end{pmatrix}.
\tag{33}
$$

It is useful to project out of Eq. (33) the information about the orbital angular momentum of the system, which is encoded in the angular distribution of the hadron pair. In fact, for $L\leqslant 1$ the decay matrix for the hadron pair is given by the following bilinear combination of spherical harmonics:

$$
\mathcal{D}_{MM'}^{LL'}(\theta,\phi_R)=Y_{LM}Y_{L'M'}^*
$$

$$
=\frac{1}{4\pi}
\left(
\begin{array}{c|ccc}
1 & -\sqrt{\frac{3}{2}}\sin\theta e^{i\phi_R} & \sqrt{3}\cos\theta & \sqrt{\frac{3}{2}}\sin\theta e^{-i\phi_R} \\[10pt]
\hline
-\sqrt{\frac{3}{2}}\sin\theta e^{-i\phi_R} & \frac{3}{2}\sin^2\theta & -\frac{3}{\sqrt{2}}\cos\theta\sin\theta e^{-i\phi_R} & -\frac{3}{2}\sin^2\theta e^{-2i\phi_R} \\[10pt]
\sqrt{3}\cos\theta & -\frac{3}{\sqrt{2}}\cos\theta\sin\theta e^{i\phi_R} & 3\cos^2\theta & \frac{3}{\sqrt{2}}\cos\theta\sin\theta e^{-i\phi_R} \\[10pt]
\sqrt{\frac{3}{2}}\sin\theta e^{i\phi_R} & -\frac{3}{2}\sin^2\theta e^{2i\phi_R} & \frac{3}{\sqrt{2}}\cos\theta\sin\theta e^{i\phi_R} & \frac{3}{2}\sin^2\theta
\end{array}
\right),
\tag{34}
$$

with $L,L'\leqslant 1$ and $|M^{(')}|\leqslant L^{(')}$. The upper left block corresponds to $L=L'=0$, i.e., to the system being in relative s wave. The lower right block instead corresponds to $L=L'=1$, i.e., to the system being in relative p wave, including all the contributions corresponding to different M,M' projections and their interferences. The off-diagonal blocks indicate, obviously, the interference between the s and p waves. Using the decay matrix, it is possible to represent the fragmentation in the basis of the quark chirality and of the pair orbital angular momentum. In fact, Eq. (33) can be written as

$$
[\mathcal{P}_-\Delta(z,\zeta,M_h^2,\phi_R)\gamma^-]_{\chi_2'\chi_2}=[\mathcal{P}_-\Delta(z,M_h^2)\gamma^-]_{M'M\chi_2'\chi_2}^{L'L}\mathcal{D}_{MM'}^{LL'}(\theta,\phi_R),
\tag{35}
$$

where

$$
[\mathcal{P}_-\Delta(z,M_h^2)\gamma^-]_{M'M\,\chi_2'\chi_2}^{L'L}=\frac{1}{8}
\begin{pmatrix}
A_{M'M}^{L'L} & B_{M'M}^{L'L} \\[6pt]
(B_{M'M}^{L'L})^\dagger & A_{M'M}^{L'L}
\end{pmatrix}
\tag{36}
$$

and

$$
A_{M'M}^{L'L}=
\left(
\begin{array}{c|ccc}
D_{1,OO}^s & 0 & \frac{2}{\sqrt{3}}D_{1,OL} & 0 \\[10pt]
\hline
0 & D_{1,OO}^p-\frac{1}{3}D_{1,LL} & 0 & 0 \\[10pt]
\frac{2}{\sqrt{3}}D_{1,OL} & 0 & D_{1,OO}^p+\frac{2}{3}D_{1,LL} & 0 \\[10pt]
0 & 0 & 0 & D_{1,OO}^p-\frac{1}{3}D_{1,LL}
\end{array}
\right),
\tag{37}
$$

$$
B_{M'M}^{L'L} = \begin{pmatrix} 0 & 0 & 0 & i\dfrac{2\sqrt{2}}{\sqrt{3}}\dfrac{|\vec{R}|}{M_h}H_{1,OT}^{\sphericalangle} \\ -i\dfrac{2\sqrt{2}}{\sqrt{3}}\dfrac{|\vec{R}|}{M_h}H_{1,OT}^{\sphericalangle} & 0 & -i\dfrac{2\sqrt{2}}{3}\dfrac{|\vec{R}|}{M_h}H_{1,LT}^{\sphericalangle} & 0 \\ 0 & 0 & 0 & i\dfrac{2\sqrt{2}}{3}\dfrac{|\vec{R}|}{M_h}H_{1,LT}^{\sphericalangle} \\ 0 & 0 & 0 & 0 \end{pmatrix}.
\tag{38}
$$

The fragmentation matrix $[\mathcal{P}_-\Delta(z,M_h^2)\gamma^-]_{M'M\chi_2'\chi_2}^{L'L}$ fulfills all the fundamental properties, namely Hermiticity, parity invariance [33], and angular momentum conservation ($\chi_2' + M = \chi_2 + M'$). The imaginary entries in its off-diagonal submatrix are T-odd fragmentation functions. It is worth noticing that with the projection (35) we gained a further information on the "unpolarized" state of the hadron pair. In fact, we see from the diagonal of Eq. (37) that the spherically symmetric state in the c.m. frame receives contributions from both the relative s and p waves, such that when performing the matrix multiplication of Eq. (35) we get

$$
D_{1,OO}(z,M_h^2) = \frac{1}{4}D_{1,OO}^s(z,M_h^2) + \frac{3}{4}D_{1,OO}^p(z,M_h^2).
\tag{39}
$$

However, in an actual cross section the two contributions are merged together and are kinematically indistinguishable, unless a specific hypothesis on the dependence upon the invariant mass M_h is assumed for the two different partial waves, e.g., a resonant contribution for the p wave and a continuum background for the s wave.

Finally, from the matrix (36) being positive semidefinite the following bounds are derived:[5]

$$
D_{1,OO}^s \geqslant 0, \quad D_{1,OO}^p \geqslant 0,
$$

$$
-\frac{3}{2}D_{1,OO}^p \leqslant D_{1,LL} \leqslant 3D_{1,OO}^p,
$$

$$
D_{1,OL} \leqslant \sqrt{\frac{3}{4}D_{1,OO}^s\left(D_{1,OO}^p + \frac{2}{3}D_{1,LL}\right)} \leqslant \frac{3}{2}D_{1,OO},
$$

$$
\frac{|\vec{R}|}{M_h}H_{1,OT}^{\sphericalangle} \leqslant \sqrt{\frac{3}{8}D_{1,OO}^s\left(D_{1,OO}^p - \frac{1}{3}D_{1,LL}\right)} \leqslant \frac{3}{2}D_{1,OO},
$$

---

[5]Note that the bounds involving the pure p-wave functions correspond to those obtained in Ref. [34].

$$
\frac{|\vec{R}|}{M_h}H_{1,LT}^{\sphericalangle} \leqslant \frac{3}{2\sqrt{2}}
$$

$$
\times \sqrt{\left(D_{1,OO}^p + \frac{2}{3}D_{1,LL}\right)\left(D_{1,OO}^p - \frac{1}{3}D_{1,LL}\right)}
$$

$$
\leqslant \frac{9}{8}D_{1,OO}.
\tag{40}
$$

B. Cross section and transverse spin asymmetries

Using Eq. (35) inside Eq. (21), we can take advantage of the full power of the analysis in the helicity formalism. In fact, the cross section can be expanded in the density matrices for the target helicity, for the chirality of the initial and fragmenting quark, and for the relative orbital angular momentum of the leading hadron pair [2]. Inserting the corresponding expressions (12),(13),(22),(36),(34), we get

$$
\frac{d^7\sigma}{d\zeta dM_h^2 d\phi_R dz dx dy d\phi_S}
$$

$$
= \sum_a \rho_{\Lambda\Lambda'}(S)[\mathcal{P}_+\Phi_a(x)\gamma^+]_{\chi_1'\chi_1}^{\Lambda'\Lambda}\left(\frac{d\sigma^{eq_a}}{dy}\right)^{\chi_1\chi_1':\chi_2\chi_2'}
$$

$$
\times [\mathcal{P}_-\Delta(z,M_h^2)\gamma^-]_{M'M\chi_2'\chi_2}^{L'L}\mathcal{D}_{MM'}^{LL'}(\theta,\phi_R)
$$

$$
= \sum_a e_a^2 \frac{\alpha^2}{2\pi Q^2 y}\left\{\left[A(y)f_1^a(x) + \lambda_e\lambda\frac{C(y)}{2}g_1^a(x)\right]\right.
$$

$$
\times\left[\frac{D_{1,OO}^s + 3D_{1,OO}^p}{4} + D_{1,OL}\cos\theta\right.
$$

$$
\left.+ D_{1,LL}\frac{1}{4}(3\cos^2\theta - 1)\right] + B(y)\frac{|\vec{S}_T||\vec{R}|}{M_h}
$$

$$
\times \sin(\phi_R + \phi_S)h_1^a(x)
$$

$$
\left.\times \sin\theta[H_{1,OT}^{\sphericalangle} + H_{1,LT}^{\sphericalangle}\cos\theta]\right\},
\tag{41}
$$

where all the fragmentation functions depend just on (z,M_h^2).

Replacing $\lambda = \lambda_e = |\vec{S}_T| = 0$ in the previous equation, we get the unpolarized cross section $d^7\sigma_{OO}$. However, it is particularly interesting to consider the case for an unpolarized beam and a transversely polarized target, i.e.,

$$\frac{d^7\sigma_{OT}}{d\zeta dM_h^2 d\phi_R dz dx dy d\phi_S}$$

$$= \sum_a e_a^2 \frac{\alpha^2}{2\pi Q^2 y} B(y) \frac{|\vec{S}_T||\vec{R}|}{M_h} \sin(\phi_R + \phi_S)$$

$$\times h_1^a(x) \sin\theta [H_{1,OT}^{\not\prec} + H_{1,LT}^{\not\prec} \cos\theta], \qquad (42)$$

because we can see that the transversity h_1 can be matched by two different chiral-odd, T-odd IFF: one ($H_{1,OT}^{\not\prec}$) pertaining to the interference between s- and p-wave states of the hadron pair, the other ($H_{1,LT}^{\not\prec}$) pertaining to the p wave only. The partial-wave analysis allows us for the first time to comprehend different theoretical analyses in a unifying framework. In fact, $H_{1,OT}^{\not\prec}$ corresponds to the hypothesis first formulated in Ref. [2], and later reconsidered in Ref. [6], where the necessary spin asymmetry is generated by the interference between two channels describing two leading pions in the relative s and p waves, respectively. As a simple cross-check, taking Eq. (42) and integrating the θ dependence away yields

$$\int_{-1}^{1} d\cos\theta \frac{d^7\sigma_{OT}}{d\cos\theta dM_h^2 d\phi_R d\phi_S dz dx dy}$$

$$= \int_{-1}^{1} d\cos\theta \frac{d^7\sigma_{OT}}{d\zeta dM_h^2 d\phi_R d\phi_S dz dx dy}$$

$$= \sum_a e_a^2 \frac{\alpha^2}{4Q^2 y} B(y) \frac{|\vec{S}_T||\vec{R}|}{M_h} \sin(\phi_R + \phi_S)$$

$$\times h_1^a(x) H_{1,OT}^{\not\prec}(z, M_h^2). \qquad (43)$$

This asymmetry corresponds to the one studied in Ref. [2], although in that paper several assumptions were made. Firstly, the IFF was factorized in a part dependent only on the variable z, designated as $\delta\hat{q}_I(z)$, and in a part containing the M_h-dependent π-π phase shifts, $\sqrt{6}\sin\delta_0\sin\delta_1\sin(\delta_0 - \delta_1)$. Secondly, the azimuthal angle of the target spin was taken $\phi_S = 0$, due to neglecting the scattering angle (see Fig. 2). The azimuthal angle of the hadron pair defined in Ref. [2] is $\phi = \pi/2 - \phi_R$. It is worth noting that the peculiar behavior in the invariant mass discussed in Ref. [2] relies on the assumption that only the π-π rescattering can generate the T-odd character of the IFF. It has been already shown, however, that a different model with more general assumptions leads to an unfactorized (z, M_h^2) dependence of the fragmentation function and to a completely different behavior of the SSA [6]. Therefore, it is of great interest to experimentally explore the production of two unpolarized hadrons, e.g., two pions, in the relevant kinematic range, namely, with an invariant mass around the ρ resonance.

As for the function $H_{1,LT}^{\not\prec}$, it naturally links with the analysis developed in the case of a spin-1 hadron fragmentation [13], because the two spinless hadrons, e.g., two pions, can be considered as the decay product of a spin-1 resonance, e.g., a ρ particle. The T-odd IFF arise from the interference between two different channels in the relative p wave. To the purpose of isolating an asymmetry containing the function $H_{1,LT}^{\not\prec}$, we show that integrating Eq. (42) upon θ in a different range, namely in the interval $[-\pi/2, \pi/2]$, yields

$$\int_{-\pi/2}^{\pi/2} d\theta \sin\theta \frac{d^7\sigma_{OT}}{d\cos\theta dM_h^2 d\phi_R dz dx dy d\phi_S}$$

$$= \int_{-\pi/2}^{\pi/2} d\theta \sin\theta \frac{d^7\sigma_{OT}}{d\zeta dM_h^2 d\phi_R dz dx dy d\phi_S}$$

$$= \sum_a e_a^2 \frac{\alpha^2}{4Q^2 y} B(y) \frac{|\vec{S}_T||\vec{R}|}{M_h} \sin(\phi_R + \phi_S) h_1^a(x)$$

$$\times \left[H_{1,OT}^{\not\prec}(z, M_h^2) + \frac{4}{3\pi} H_{1,LT}^{\not\prec}(z, M_h^2) \right], \qquad (44)$$

where both kinds of IFF appear at leading twist and can contribute to a SSA isolating the transversity h_1. Although spin-1 fragmentation functions have already been proposed in the past as possible chiral-odd partners for the transversity [10–13], to our knowledge this is the first time that the asymmetry where they occur is explicitly identified and a clear distinction from the s-p interference is made.

There are not yet quantitative model predictions for $H_{1,LT}^{\not\prec}$; on the other hand, since the p-wave production of two hadrons becomes significant only when it proceeds via a spin-1 resonance, we can expect that the shape of this function in the invariant mass corresponds to a Breit-Wigner curve peaked at the resonance mass. Moreover, it has the same features as a single-particle fragmentation function, unlike $H_{1,OT}^{\not\prec}$: its evolution equations can be expected to be analogous to that of the transversity [28–30]; it does not require a rescattering of the hadrons after they are produced and its physical origin could have something in common with the one of the Collins function. However, it should be noticed that in the case of the Collins function an essential role is played by the partonic transverse momentum, which in the case of $H_{1,LT}^{\not\prec}$ is replaced by the relative transverse momentum of the hadron pair. It would be interesting to elaborate on these topics since data for the electromagnetic ρ production and decay are already available in the diffractive regime [35–37], and they could be available in the DIS regime as well in the near future.

IV. EXPLICIT DEPENDENCE ON THE TRANSVERSE MOMENTA

For the sake of completeness, in this section we extend the previous results to the case where the transverse momenta are not integrated away. In this case, the cross section is ninefold and reads

$$\frac{d^9\sigma}{d\zeta dM_h^2 d\phi_R dz d\vec{P}_{h\perp} dx dy d\phi_S} = \sum_a \frac{\alpha^2 y e_a^2}{32 z Q^4} L_{\mu\nu} 2 M W_a^{\mu\nu}.$$

(45)

The hadronic tensor takes the form

$$2MW_a^{\mu\nu} = 32z\mathcal{I}[\mathrm{Tr}[\Phi_a(x,\vec{p}_T,S)$$

$$\times \gamma^\mu \Delta_a(z,\vec{k}_T,\zeta,M_h^2,\phi_R)\gamma^\nu]],$$

(46)

where we introduced the shorthand notation

$$\mathcal{I}[f] \equiv \int d\vec{p}_T d\vec{k}_T \delta(\vec{p}_T - \vec{P}_{h\perp}/z - \vec{k}_T)[f],$$

(47)

and where the transverse-momentum-dependent correlation functions are

$$\Phi_a(x,\vec{p}_T,S) = \int dp^- \Phi_a(p;P,S)|_{p^+=xP^+},$$

(48)

$$\Delta_a(z,\vec{k}_T,\zeta,M_h^2,\phi_R) = \frac{1}{32z}\int dk^+ \Delta_a(k;P_h,R)|_{k^-=P_h^-/z}.$$

(49)

The leading-twist projection of $W^{\mu\nu}$ proceeds in an analogous way to Eq. (8); we usually have [22]

$$\mathcal{P}_+\Phi_a(x,\vec{p}_T,S)\gamma^+ = \left\{ f_1^a(x,\vec{p}_T^2) + \frac{\epsilon_{T\rho\sigma}S_T^\rho p_T^\sigma}{M} f_{1T}^{\perp a}(x,\vec{p}_T^2) \right.$$

$$+ i h_1^{\perp a}(x,\vec{p}_T^2)\frac{\slashed{p}_T}{M}$$

$$+ \left[\lambda g_{1L}^a(x,\vec{p}_T^2) + \frac{\vec{p}_T\cdot\vec{S}_T}{M} g_{1T}^a(x,\vec{p}_T^2) \right]\gamma_5$$

$$+ \left[\lambda h_{1L}^{\perp a}(x,\vec{p}_T^2) + \frac{\vec{p}_T\cdot\vec{S}_T}{M} h_{1T}^{\perp a}(x,\vec{p}_T^2) \right]$$

$$\left. \times \gamma_5\frac{\slashed{p}_T}{M} + h_{1T}^a(x,\vec{p}_T^2)\gamma_5\slashed{S}_T \right\},$$

(50)

where $\epsilon_T^{\mu\nu} = \epsilon^{-+\mu\nu}$. Equation (50) corresponds to Eq. (2) of Ref. [6]. Again, similarly to Eq. (11) and the following equations, we project out the density matrix of the target helicity so that Eq. (50) in the basis of quark chirality and target helicity becomes

$$[\mathcal{P}_+\Phi_a\gamma^+]_{\chi_1'\chi_1}^{\Lambda'\Lambda}$$

$$= \begin{pmatrix} f_1^a + g_{1L}^a & \frac{|\vec{p}_T|}{M}e^{-i\phi_p}(g_{1T}^a + if_{1T}^{\perp a}) & \frac{|\vec{p}_T|}{M}e^{-i\phi_p}(h_{1L}^{\perp a} + ih_1^{\perp a}) & \frac{|\vec{p}_T|^2}{M^2}e^{-2i\phi_p}h_{1T}^{\perp a} \\ \frac{|\vec{p}_T|}{M}e^{i\phi_p}(g_{1T}^a - if_{1T}^{\perp a}) & f_1^a - g_{1L}^a & 2h_1^a & -\frac{|\vec{p}_T|}{M}e^{-i\phi_p}(h_{1L}^{\perp a} - ih_1^{\perp a}) \\ \frac{|\vec{p}_T|}{M}e^{i\phi_p}(h_{1L}^{\perp a} - ih_1^{\perp a}) & 2h_1^a & f_1^a - g_{1L}^a & -\frac{|\vec{p}_T|}{M}e^{-i\phi_p}(g_{1T}^a - if_{1T}^{\perp a}) \\ \frac{|\vec{p}_T|^2}{M^2}e^{2i\phi_p}h_{1T}^{\perp a} & -\frac{|\vec{p}_T|}{M}e^{i\phi_p}(h_{1L}^{\perp a} + ih_1^{\perp a}) & -\frac{|\vec{p}_T|}{M}e^{i\phi_p}(g_{1T}^a + if_{1T}^{\perp a}) & f_1^a + g_{1L}^a \end{pmatrix},$$

(51)

where ϕ_p is the azimuthal angle of $\vec{p}_T$. The matrix is Hermitian, respects parity invariance and conservation of total angular momentum. Introducing the dependence upon the quark transverse momentum $\vec{p}_T$ modifies the conditions for angular momentum and parity conservation, which now read, respectively,

$$\Lambda_1' + \chi_1' + l_{p_T} = \Lambda_1 + \chi_1,$$

$$[\mathcal{P}_+\Phi\gamma^+]_{\chi_1'\chi_1}^{\Lambda'\Lambda} = (-1)^{l_{p_T}}[\mathcal{P}_+\Phi\gamma^+]_{-\chi_1-\chi_1'}^{-\Lambda'-\Lambda}\Big|_{l_{p_T}\to -l_{p_T}},$$

(52)

where l_{p_T} denotes the units of angular momentum introduced by $\vec{p}_T$. The chiral transposed matrix is still positive definite, so that the bounds on the various distribution functions can be obtained [24].

The leading-twist projection of the fragmenting quark correlator is

$$\mathcal{P}_-\Delta_a(z,\vec{k}_T,\zeta,M_h^2,\phi_R)\gamma^-$$

$$= \frac{1}{8\pi}\left(D_1^a(z,\zeta,M_h^2,\vec{k}_T^2,\vec{k}_T\cdot\vec{R}_T) \right.$$

$$+ i\bar{H}_1^{\sphericalangle a}(z,\zeta,M_h^2,\vec{k}_T^2,\vec{k}_T\cdot\vec{R}_T)\frac{\slashed{R}_T}{M_h}$$

$$+ iH_1^{\perp a}(z,\zeta,M_h^2,\vec{k}_T^2,\vec{k}_T\cdot\vec{R}_T)\frac{\not{k}_T}{M_h}$$

$$+ G_1^{\perp a}(z,\zeta,M_h^2,\vec{k}_T^2,\vec{k}_T\cdot\vec{R}_T)\frac{\epsilon_T^{\mu\nu}R_{T\mu}k_{T\nu}}{M_h^2}\gamma_5 \Bigg)\mathcal{P}_-, \quad (53)$$

where the actual dependence of the fragmentation functions is the most general one possible [3]. In Eq. (53) $\mathcal{P}_-\Delta_a$ cor-

responds to Eq. (3) of Ref. [6]. New functions appear: $G_1^\perp$ is chiral even but T-odd, $H_1^\perp$ is chiral odd and T-odd and represents the analogue of the Collins effect for a two-hadron emission [3]. Upon integration over $d\vec{k}_T$, $G_1^\perp$ vanishes and the surviving parts of $\bar{H}_1^{\not\asymp}$ and $H_1^\perp$ merge into the function $H_1^{\not\asymp}$ of Eq. (16) keeping the same $\not{k}_T/M_h$ structure. In the chiral basis of the fragmenting quark, Eq. (53) becomes

$$[\mathcal{P}_-\Delta_a(z,\vec{k}_T,\zeta,M_h^2,\phi_R)\gamma^-]_{\chi_2'\chi_2} = \frac{1}{8\pi}\begin{pmatrix} D_1^a + \dfrac{\epsilon_T^{\mu\nu}R_{T\mu}k_{T\nu}}{M_h^2}G_1^{\perp a} & i\left(e^{i\phi_R}\dfrac{|\vec{R}_T|}{M_h}\bar{H}_1^{\not\asymp a}+e^{i\phi_k}\dfrac{|\vec{k}_T|}{M_h}H_1^{\perp a}\right) \\ -i\left(e^{-i\phi_R}\dfrac{|\vec{R}_T|}{M_h}\bar{H}_1^{\not\asymp a}+e^{-i\phi_k}\dfrac{|\vec{k}_T|}{M_h}H_1^{\perp a}\right) & D_1^a - \dfrac{\epsilon_T^{\mu\nu}R_{T\mu}k_{T\nu}}{M_h^2}G_1^{\perp a} \end{pmatrix}.$$

$$(54)$$

The following bounds are derived:

$$\frac{|\epsilon_T^{\mu\nu}R_{T\mu}k_{T\nu}|}{M_h^2}|G_1^{\perp a}| \leq D_1^a,$$

$$\frac{|\vec{R}_T|^2}{M_h^2}(\bar{H}_1^{\not\asymp a})^2 + \frac{|\vec{k}_T|^2}{M_h^2}(H_1^{\perp a})^2 + \frac{2\vec{k}_T\cdot\vec{R}_T}{M_h^2}\bar{H}_1^{\not\asymp a}H_1^{\perp a}$$

$$\leq (D_1^a)^2 - \frac{|\epsilon_T^{\mu\nu}R_{T\mu}k_{T\nu}|^2}{M_h^4}|G_1^{\perp a}|^2. \quad (55)$$

Expanding the cross section of Eq. (45) along the same lines leading to Eq. (21), we have

$$\frac{d^9\sigma}{d\zeta dM_h^2 d\phi_R d\vec{P}_{h\perp}dzdxdyd\phi_S}$$

$$= \sum_a \rho_{\Lambda\Lambda'}(S)\mathcal{I}\Bigg[[\mathcal{P}_+\Phi_a(x,\vec{p}_T)\gamma^+]_{\chi_1'\chi_1}^{\Lambda'\Lambda}$$

$$\times\left(\frac{d\sigma^{eq_a}}{dy}\right)^{\chi_1\chi_1';\chi_2\chi_2'}[\mathcal{P}_-\Delta_a(z,\vec{k}_T,\zeta,M_h^2,\phi_R)\gamma^-]_{\chi_2'\chi_2}\Bigg],$$

$$(56)$$

where $[\mathcal{P}_+\Phi_a\gamma^+]$ and $[\mathcal{P}_-\Delta_a\gamma^-]$ are given by Eqs. (51) and (54), respectively. The complete formula for the cross section is given in Appendix A.

V. PARTIAL-WAVE EXPANSION WITH TRANSVERSE MOMENTA

It is again useful to expand all the fragmentation functions of Eq. (53) in the relative partial waves of the hadron pair. The dependence on $\vec{k}_T\cdot\vec{R}_T$ makes the expansion more involved:

$$D_1 = D_{1,OO}+D_{1,OL}\cos\theta+D_{1,LL}\frac{1}{4}(3\cos^2\theta-1)$$

$$+\cos(\phi_k-\phi_R)\sin\theta(D_{1,OT}+D_{1,LT}\cos\theta)$$

$$+\cos(2\phi_k-2\phi_R)\sin^2\theta D_{1,TT},$$

$$G_1^\perp = G_{1,OT}^\perp+G_{1,LT}^\perp\cos\theta+\cos(\phi_k-\phi_R)\sin\theta G_{1,TT}^\perp,$$

$$\bar{H}_1^{\not\asymp} = \bar{H}_{1,OT}^{\not\asymp}+\bar{H}_{1,LT}^{\not\asymp}\cos\theta$$

$$+2\cos(2\phi_k-2\phi_R)\sin\theta\bar{H}_{1,TT}^{\not\asymp},$$

$$H_1^\perp = H_{1,OO}^\perp+H_{1,OL}^\perp\cos\theta+H_{1,LL}^\perp\frac{1}{4}(3\cos^2\theta-1)$$

$$+2\cos(\phi_k-\phi_R)\sin\theta(H_{1,OT}^\perp+H_{1,LT}^\perp\cos\theta)$$

$$+2\cos(2\phi_k-2\phi_R)\sin^2\theta H_{1,TT}^\perp$$

$$-\sin^2\theta\frac{|\vec{R}|}{|\vec{k}_T|}H_{1,TT}^{\not\asymp}, \quad (57)$$

where all the functions depend on $(z,\vec{k}_T^2,M_h^2)$. Then, similarly to Eq. (35), Eq. (54) can be further expanded in the basis of the pair orbital angular momentum as

$$[\mathcal{P}_-\Delta(z,\vec{k}_T,\zeta,M_h^2,\phi_R)\gamma^-]_{\chi_2'\chi_2}$$

$$=[\mathcal{P}_-\Delta(z,\vec{k}_T^2,M_h^2)\gamma^-]_{M'M\chi_2'\chi_2}^{L'L}\mathcal{D}_{MM'}^{LL'}(\theta,\phi_k,\phi_R).$$

$$(58)$$

The full expression of $[\mathcal{P}_-\Delta(z,\vec{k}_T^2,M_h^2)\gamma^-]$ is shown in Appendix B. The fully expanded differential cross section in the helicity basis of target, initial and final quark, as well as in the basis of orbital angular momentum of the hadron pair is then

$$\frac{d^9\sigma}{d\zeta dM_h^2 d\phi_R d\vec{P}_{h\perp}dzdxdyd\phi_S}$$

$$=\sum_a \rho_{\Lambda\Lambda'}(S)$$

$$\times\mathcal{I}\left[[\mathcal{P}_+\Phi_a(x,\vec{p}_T)\gamma^+]_{\chi_1'\chi_1}^{\Lambda'\Lambda}\left(\frac{d\sigma^{eq_a}}{dy}\right)^{\chi_1\chi_1';\chi_2\chi_2'}\right.$$

$$\left.\times[\mathcal{P}_-\Delta_a(z,\vec{k}_T,\zeta,M_h^2,\phi_R)\gamma^-]_{M'M\chi_2'\chi_2}^{L'L}\right]$$

$$\times\mathcal{D}_{MM'}^{LL'}(\theta,\phi_R). \qquad (59)$$

Its explicit expression is presented in Appendix C. The pure p-wave sector corresponds to the cross section for the production of a polarized spin-1 hadron and has already been fully studied in Refs. [13,38]. For the sake of completeness, we show it in Appendix C together with the formulas for the pure s and s-p interference sectors.

VI. CONCLUSIONS

In this paper we have reconsidered the option of extracting the transversity distribution h_1 at leading twist by using the analyzing power of the interference fragmentation functions (IFF) into two leading unpolarized hadrons inside the same current jet. As already shown in Ref. [6] for the process $ep^\uparrow\to e'h_1h_2X$ (and similarly, but not fully exploited, for the fragmentation into spin-1 hadrons [10–13]), the transversity distribution enters a single-spin asymmetry in the azimuthal angle ϕ_R of the hadron pair plane. The effect survives after the integration upon the transverse component of $P_h=P_1+P_2$; therefore, no transverse-momentum-dependent function is required. In principle, this is a clear advantage with respect to the Collins effect. However, more sophisticated calculations than the ones in Ref. [6] are needed to get a realistic estimate of the size of this asymmetry at the experimental scale. At present, it is not yet possible to judge which one between the IFF and the Collins function is better suited to extract h_1.

Here, we have reanalyzed the whole problem in the helic-

ity formalism by further expanding the IFF in the basis of the relative orbital angular momentum in the c.m. frame of the hadron pair. New positivity bounds have been derived. If the invariant mass of the pair is not large, the expansion can be limited to the first two modes, namely the relative s and p waves.

Off-diagonal elements in the chirality and in the orbital angular momentum L represent the IFF of Refs. [2] and [6], where the interference arises from the hadron pair being in a state with either s or p relative wave. Elements in the $L=L'=1$ sector correspond to the analysis of spin-1 hadron fragmentation [13]. Therefore, the present formalism represents a unifying framework for the problem of fragmentation into two unpolarized hadrons and can be used to correctly and exhaustively discuss the extraction of transversity from two-hadron leptoproduction.

In fact, after calculating the complete leading-twist cross section, we have identified a single spin asymmetry containing two distinct chiral-odd partners of the transversity. By integrating the asymmetry over different ranges of the c.m. polar angle of the hadron pair, the transversity h_1 can be extracted through the chiral-odd, T-odd fragmentation $H_{1,OT}^{\sphericalangle}$ (corresponding to the s-p interference of Ref. [2]) or through the chiral-odd, T-odd fragmentation $H_{1,LT}^{\sphericalangle}$ (corresponding to the p-p interference). This second option has been often neglected in the literature, despite the fact that the two functions have, in principle, a different dependence on the invariant mass and a different physical origin.

In conclusion, we believe that the fragmentation into two leading unpolarized hadrons can be a promising tool to measure the transversity distribution, as well as to achieve further comprehension of the hadronization mechanism.

APPENDIX A

In this appendix we write explicitly the cross section for two-hadron leptoproduction at leading order in $1/Q$ and with the inclusion of partonic transverse momenta. Moreover, we include also T-odd distribution functions, since recently there have been some indications that they are not forbidden by time invariance [26,39,27]. To simplify the notation, we introduce the projection $\vec{a}_T\wedge\vec{b}_T=a_i\epsilon_T^{ij}b_j$. Inserting in Eq. (45) the formulas for the target helicity density matrix, Eq. (12), for the distribution correlation matrix, Eq. (51), for the elementary scattering matrix, Eq. (22), and the two-hadron fragmentation matrix, Eq. (54), we obtain the following result:

$$d^9\sigma_{OO} = \sum_a \frac{\alpha^2 e_a^2}{2\pi Q^2 y}\left\{ A(y)\mathcal{I}[f_1 D_1] - B(y)\frac{|\vec{R}_T|}{M_h}\cos(\phi_h+\phi_R)\mathcal{I}\left[\frac{\vec{p}_T\cdot\hat{P}_{h\perp}}{M}h_1^\perp \bar{H}_1^{\sphericalangle}\right]\right.$$

$$+ B(y)\frac{|\vec{R}_T|}{M_h}\sin(\phi_h+\phi_R)\mathcal{I}\left[\frac{\hat{P}_{h\perp}\wedge\vec{p}_T}{M}h_1^\perp\bar{H}_1^{\sphericalangle}\right] - B(y)\cos(2\phi_h)\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{k}_T\cdot\hat{P}_{h\perp})-\vec{p}_T\cdot\vec{k}_T}{MM_h}h_1^\perp H_1^\perp\right]$$

$$\left. + B(y)\sin(2\phi_h)\mathcal{I}\left[\frac{(\vec{p}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{k}_T)+(\vec{k}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{p}_T)}{MM_h}h_1^\perp H_1^\perp\right]\right\}, \tag{A1}$$

$$d^9\sigma_{LO} = -\sum_a \frac{\alpha^2 e_a^2}{2\pi Q^2 y}|\lambda_e|C(y)\frac{|\vec{R}_T|}{M_h}\left\{\sin(\phi_h-\phi_R)\mathcal{I}\left[\frac{\vec{k}_T\cdot\hat{P}_{h\perp}}{M_h}f_1 G_1^\perp\right] + \cos(\phi_h-\phi_R)\mathcal{I}\left[\frac{\hat{P}_{h\perp}\wedge\vec{k}_T}{M_h}f_1 G_1^\perp\right]\right\}, \tag{A2}$$

$$d^9\sigma_{OL} = \sum_a \frac{\alpha^2 e_a^2}{2\pi Q^2 y}|S_L|\left\{-A(y)\frac{|\vec{R}_T|}{M_h}\sin(\phi_h-\phi_R)\mathcal{I}\left[\frac{\vec{k}_T\cdot\hat{P}_{h\perp}}{M_h}g_{1L}G_1^\perp\right] - A(y)\frac{|\vec{R}_T|}{M_h}\cos(\phi_h-\phi_R)\right.$$

$$\times\mathcal{I}\left[\frac{\hat{P}_{h\perp}\wedge\vec{k}_T}{M_h}g_{1L}G_1^\perp\right] + B(y)\frac{|\vec{R}_T|}{M_h}\sin(\phi_h+\phi_R)\mathcal{I}\left[\frac{\vec{p}_T\cdot\hat{P}_{h\perp}}{M}h_{1L}^\perp\bar{H}_1^{\sphericalangle}\right] + B(y)\frac{|\vec{R}_T|}{M_h}\cos(\phi_h+\phi_R)$$

$$\times\mathcal{I}\left[\frac{\hat{P}_{h\perp}\wedge\vec{p}_T}{M}h_{1L}^\perp\bar{H}_1^{\sphericalangle}\right] + B(y)\sin(2\phi_h)\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{k}_T\cdot\hat{P}_{h\perp})-\vec{p}_T\cdot\vec{k}_T}{MM_h}h_{1L}^\perp H_1^\perp\right]$$

$$\left. + B(y)\cos(2\phi_h)\mathcal{I}\left[\frac{(\vec{p}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{k}_T)+(\vec{k}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{p}_T)}{MM_h}h_{1L}^\perp H_1^\perp\right]\right\}, \tag{A3}$$

$$d^9\sigma_{LL} = \sum_a \frac{\alpha^2 e_a^2}{2\pi Q^2 y}|\lambda_e||S_L|C(y)\mathcal{I}[g_{1L}D_1], \tag{A4}$$

$$d^9\sigma_{OT} = \sum_a \frac{\alpha^2 e_a^2}{2\pi Q^2 y}|\vec{S}_T|A(y)\left\{\frac{|\vec{R}_T|}{M_h}\sin(\phi_R-\phi_S)\mathcal{I}\left[\frac{\vec{p}_T\cdot\vec{k}_T}{2MM_h}g_{1T}G_1^\perp\right]\right.$$

$$-\frac{|\vec{R}_T|}{M_h}\cos(\phi_R-\phi_S)\mathcal{I}\left[\frac{(\vec{p}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{k}_T)-(\vec{k}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{p}_T)}{2MM_h}g_{1T}G_1^\perp\right] - \frac{|\vec{R}_T|}{M_h}\sin(2\phi_h-\phi_R-\phi_S)$$

$$\times\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{k}_T\cdot\hat{P}_{h\perp})-\vec{p}_T\cdot\vec{k}_T}{2MM_h}g_{1T}G_1^\perp\right] - \frac{|\vec{R}_T|}{M_h}\cos(2\phi_h-\phi_R-\phi_S)$$

$$\times\mathcal{I}\left[\frac{(\vec{p}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{k}_T)+(\vec{k}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{p}_T)}{2MM_h}g_{1T}G_1^\perp\right] + \sin(\phi_h-\phi_S)\mathcal{I}\left[\frac{\vec{p}_T\cdot\hat{P}_{h\perp}}{M}f_{1T}^\perp D_1\right]$$

$$\left. + \cos(\phi_h-\phi_S)\mathcal{I}\left[\frac{\hat{P}_{h\perp}\wedge\vec{p}_T}{M}f_{1T}^\perp D_1\right]\right\} + \sum_a \frac{\alpha^2 e_a^2}{2\pi Q^2 y}|\vec{S}_T|B(y)\left\{\sin(\phi_h+\phi_S)\mathcal{I}\left[\frac{\vec{k}_T\cdot\hat{P}_{h\perp}}{M_h}h_1 H_1^\perp\right]\right.$$

$$+ \cos(\phi_h+\phi_S)\mathcal{I}\left[\frac{\hat{P}_{h\perp}\wedge\vec{k}_T}{M_h}h_1 H_1^\perp\right] + \frac{|\vec{R}_T|}{M_h}\sin(\phi_R+\phi_S)\mathcal{I}[h_1\bar{H}_1^{\sphericalangle}]$$

$$+ \sin(3\phi_h-\phi_S)\mathcal{I}\left[\frac{4(\vec{p}_T\cdot\hat{P}_{h\perp})^2(\vec{k}_T\cdot\hat{P}_{h\perp})-2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)-\vec{p}_T^2(\vec{k}_T\cdot\hat{P}_{h\perp})}{2M^2 M_h}h_{1T}^\perp H_1^\perp\right]$$

$$+ \cos(3\phi_h-\phi_S)\mathcal{I}\left[\left(\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})^2(\hat{P}_{h\perp}\wedge\vec{k}_T)+2(\vec{k}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{p}_T)}{2M^2 M_h}-\frac{\vec{p}_T^2(\hat{P}_{h\perp}\wedge\vec{k}_T)}{2M^2 M_h}\right)h_{1T}^\perp H_1^\perp\right]$$

$$+ \frac{|\vec{R}_T|}{M_h}\sin(2\phi_h+\phi_R-\phi_S)\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})^2-\vec{p}_T^2}{2M^2}h_{1T}^\perp\bar{H}_1^{\perp}\right]+\frac{|\vec{R}_T|}{M_h}\cos(2\phi_h+\phi_R-\phi_S)\mathcal{I}\left[\frac{(\vec{p}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{p}_T)}{2M^2}h_{1T}^\perp\bar{H}_1^{\perp}\right]\Bigg\},$$

$$\text{(A5)}$$

$$d^9\sigma_{LT}=\sum_a\frac{\alpha^2 e_a^2}{2\pi Q^2 y}|\vec{S}_T|C(y)\Bigg\{\cos(\phi_h-\phi_S)\mathcal{I}\left[\frac{\vec{p}_T\cdot\hat{P}_{h\perp}}{M}g_{1T}D_1\right]-\sin(\phi_h-\phi_S)\mathcal{I}\left[\frac{\hat{P}_{h\perp}\wedge\vec{p}_T}{M}g_{1T}D_1\right]$$

$$-\frac{|\vec{R}_T|}{M_h}\cos(\phi_R-\phi_S)\mathcal{I}\left[\frac{\vec{p}_T\cdot\vec{k}_T}{2MM_h}f_{1T}^\perp G_1^\perp\right]+\frac{|\vec{R}_T|}{M_h}\cos(2\phi_h-\phi_R-\phi_S)\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{k}_T\cdot\hat{P}_{h\perp})-\vec{p}_T\cdot\vec{k}_T}{2MM_h}f_{1T}^\perp G_1^\perp\right]$$

$$-\frac{|\vec{R}_T|}{M_h}\sin(\phi_R-\phi_S)\mathcal{I}\left[\frac{(\vec{p}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{k}_T)-(\vec{k}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{p}_T)}{2MM_h}f_{1T}^\perp G_1^\perp\right]$$

$$+\frac{|\vec{R}_T|}{M_h}\sin(2\phi_h-\phi_R-\phi_S)\mathcal{I}\left[\frac{(\vec{p}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{k}_T)+(\vec{k}_T\cdot\hat{P}_{h\perp})(\hat{P}_{h\perp}\wedge\vec{p}_T)}{2MM_h}f_{1T}^\perp G_1^\perp\right]\Bigg\}. \quad\text{(A6)}$$

In the case of $d^9\sigma_{OT}$, i.e., for an unpolarized beam and a transversely polarized target, the full expression of the cross section corresponds to the one in Eq. (10) of Ref. [6], apart for a different overall factor, due to slightly different definitions of the hadron tensor and of the fragmentation functions, and the use of M_h instead of M_1 (M_2) in the denominators, due to a different definition of the expansion (15).

APPENDIX B

The full expression of $[\mathcal{P}_-\Delta(z,\vec{k}_T^2,M_h^2)\gamma^-]_{M'M\chi_2'\chi_2}^{L'L}$ in Eq. (58) is

$$[\mathcal{P}_-\Delta(z,\vec{k}_T^2,M_h^2)\gamma^-]_{M'M\chi_2'\chi_2}^{L'L}=\frac{1}{8}\begin{pmatrix} A_{M'M}^{L'L} & B_{M'M}^{L'L} \\ (B_{M'M}^{L'L})^\dagger & C_{M'M}^{L'L} \end{pmatrix}, \quad\text{(B1)}$$

where

$$A_{M'M}^{L'L}=\begin{pmatrix} D_{1,OO}^s & -\sqrt{\frac{2}{3}}e^{i\phi}\left(D_{1,OT}+i\frac{|\vec{k}_T||\vec{R}|}{M_h^2}G_{1,OT}^\perp\right) & \frac{2}{\sqrt{3}}D_{1,OL} & \sqrt{\frac{2}{3}}e^{-i\phi}\left(D_{1,OT}-i\frac{|\vec{k}_T||\vec{R}|}{M_h^2}G_{1,OT}^\perp\right) \\ -\sqrt{\frac{2}{3}}e^{-i\phi}\left(D_{1,OT}-i\frac{|\vec{k}_T||\vec{R}|}{M_h^2}G_{1,OT}^\perp\right) & D_{1,OO}^p-\frac{1}{3}D_{1,LL} & -\frac{\sqrt{2}}{3}e^{-i\phi}\left(D_{1,LT}-i\frac{|\vec{k}_T||\vec{R}|}{M_h^2}G_{1,LT}^\perp\right) & -\frac{2}{3}e^{-2i\phi}\left(2D_{1,TT}-i\frac{|\vec{k}_T||\vec{R}|}{M_h^2}G_{1,TT}^\perp\right) \\ \frac{2}{\sqrt{3}}D_{1,OL} & -\frac{\sqrt{2}}{3}e^{i\phi}\left(D_{1,LT}+i\frac{|\vec{k}_T||\vec{R}|}{M_h^2}G_{1,LT}^\perp\right) & D_{1,OO}^p+\frac{2}{3}D_{1,LL} & \frac{\sqrt{2}}{3}e^{-i\phi}\left(D_{1,LT}-i\frac{|\vec{k}_T||\vec{R}|}{M_h^2}G_{1,LT}^\perp\right) \\ \sqrt{\frac{2}{3}}e^{i\phi}\left(D_{1,OT}+i\frac{|\vec{k}_T||\vec{R}|}{M_h^2}G_{1,OT}^\perp\right) & -\frac{2}{3}e^{2i\phi}\left(2D_{1,TT}+i\frac{|\vec{k}_T||\vec{R}|}{M_h^2}G_{1,TT}^\perp\right) & \frac{\sqrt{2}}{3}e^{i\phi}\left(D_{1,LT}+i\frac{|\vec{k}_T||\vec{R}|}{M_h^2}G_{1,LT}^\perp\right) & D_{1,OO}^p-\frac{1}{3}D_{1,LL} \end{pmatrix}$$

$$\text{(B2)}$$

$$B_{M'M}^{L'L}=i\frac{|\vec{k}_T|}{M_h}\begin{pmatrix} e^{i\phi}H_{1,OO}^{\perp s} & -\frac{2\sqrt{2}}{\sqrt{3}}e^{2i\phi}H_{1,OT}^\perp & \frac{2}{\sqrt{3}}e^{i\phi}H_{1,OL}^\perp & \frac{2\sqrt{2}}{\sqrt{3}}\left(\frac{|\vec{R}|}{|\vec{k}_T|}\bar{H}_{1,OT}^{\perp}+H_{1,OT}^\perp\right) \\ -\frac{2\sqrt{2}}{\sqrt{3}}\left(\frac{|\vec{R}|}{|\vec{k}_T|}\bar{H}_{1,OT}^{\perp}+H_{1,OT}^\perp\right) & e^{i\phi}\left(H_{1,OO}^{\perp p}-\frac{1}{3}H_{1,LL}^\perp\right) & -\frac{2\sqrt{2}}{3}\left(\frac{|\vec{R}|}{|\vec{k}_T|}\bar{H}_{1,LT}^{\perp}+H_{1,LT}^\perp\right) & -\frac{8}{3}e^{i\phi}\left(\frac{|\vec{R}|}{|\vec{k}_T|}H_{1,TT}^{\perp}+H_{1,TT}^\perp\right) \\ \frac{2}{\sqrt{3}}e^{i\phi}H_{1,OL}^\perp & -\frac{2\sqrt{2}}{\sqrt{3}}e^{2i\phi}H_{1,TT}^\perp & e^{i\phi}\left(H_{1,OO}^{\perp p}+\frac{2}{3}H_{1,LL}^\perp\right) & \frac{2\sqrt{2}}{3}\left(\frac{|\vec{R}|}{|\vec{k}_T|}\bar{H}_{1,LT}^{\perp}+H_{1,LT}^\perp\right) \\ \frac{2\sqrt{2}}{\sqrt{3}}e^{2i\phi}H_{1,OT}^\perp & -\frac{8}{3}e^{3i\phi}H_{1,TT}^\perp & \frac{2\sqrt{2}}{\sqrt{3}}e^{2i\phi}H_{1,TT}^\perp & e^{i\phi}\left(H_{1,OO}^{\perp p}-\frac{1}{3}H_{1,LL}^\perp\right) \end{pmatrix}, \quad\text{(B3)}$$

and $\phi\equiv\phi_k-\phi_R$. The matrix (B1) respects Hermiticity, angular momentum conservation, and parity invariance. Due to the explicit dependence upon the transverse momentum $\vec{k}_T$, the conditions for angular momentum and parity conservation read

$$M+\chi_2'=M'+\chi_2+l_{k_T},$$

$$[\mathcal{P}_-\Delta\gamma^-]_{M'M\chi_2'\chi_2}^{L'L}=(-1)^{l_{k_T}}[\mathcal{P}_-\Delta\gamma^-]_{-M'-M-\chi_2'-\chi_2}^{L'L}, \quad\text{(B4)}$$

where l_{k_T} denotes the units of angular momentum introduced by $\vec{k}_T$. From the last constraint it is possible to derive the lower right block, i.e., $C_{M'M}^{L'L} = (-1)^{l_{k_T}} A_{-M'-M}^{L'L}$.

Again, as in the case of Eq. (39), we have

$$H_{1,OO}(z,\vec{k}_T^2,M_h^2) = \frac{1}{4} H_{1,OO}^{\perp s}(z,\vec{k}_T^2,M_h^2) + \frac{3}{4} H_{1,OO}^{\perp p}(z,\vec{k}_T^2,M_h^2)$$
(B5)

and the functions $H_{1,OO}^{\perp s}, H_{1,OO}^{\perp p}$ are kinematically indistinguishable unless some hypothesis is made on their M_h^2 dependence. The $L = L' = 1$ sector of Eqs. (B2),(B3) has been studied in the case of spin-1 fragmentation [13]. The interference $(L=0, L'=1)$ sector has never been analyzed in this form, namely, including the explicit dependence on $\vec{k}_T$. Finally, from $[\mathcal{P}_-\Delta\gamma^-]_{M'M \chi_2'\chi_2}^{L'L}$ being positive semidefinite, it is possible to derive bounds on each of the displayed fragmentation functions.

APPENDIX C:

In this appendix we explicitly present the complete cross section for the production of two unpolarized hadrons in relative s and p waves, at leading order in $1/Q$, including transverse momenta and T-odd distribution and fragmentation functions.

The cross section is obtained by replacing Eqs. (12),(51),(22),(B1),(34) in Eq. (59). It is convenient to introduce the following combination of fragmentation functions:

$$H_{1,OT}^{\not\asymp} = \bar{H}_{1,OT}^{\not\asymp} + \frac{|\vec{k}_T|}{|\vec{R}|} H_{1,OT}^{\perp},$$
(C1)

$$H_{1,LT}^{\not\asymp} = \bar{H}_{1,LT}^{\not\asymp} + \frac{|\vec{k}_T|}{|\vec{R}|} H_{1,LT}^{\perp},$$
(C2)

$$H_{1,TT}^{\not\asymp} = \bar{H}_{1,TT}^{\not\asymp} + \frac{|\vec{k}_T|}{|\vec{R}|} H_{1,TT}^{\perp}.$$
(C3)

1. Unpolarized lepton beam and unpolarized target

We have

$$d^9\sigma_{OO} = \sum_a \frac{\alpha^2 e_a^2}{2\pi sxy^2} A(y) \left\{ \mathcal{I}\left[f_1 \left(\frac{1}{4} D_{1,OO}^s + \frac{3}{4} D_{1,OO}^p \right) \right] + \cos\theta \mathcal{I}[f_1 D_{1,OL}] + \frac{1}{3}(3\cos^2\theta - 1) \mathcal{I}\left[f_1 \left(\frac{3}{4} D_{1,LL} \right) \right] \right.$$

$$+ \sin\theta\cos(\phi_h - \phi_R) \mathcal{I}\left[\frac{\vec{k}_T \cdot \hat{P}_{h\perp}}{M_h} f_1 \left(-\frac{M_h}{|\vec{k}_T|} D_{1,OT} \right) \right] - \sin 2\theta\cos(\phi_h - \phi_R)$$

$$\times \mathcal{I}\left[\frac{\vec{k}_T \cdot \hat{P}_{h\perp}}{M_h} f_1 \left(-\frac{M_h}{2|\vec{k}_T|} D_{1,LT} \right) \right] - \sin^2\theta\cos(2\phi_h - 2\phi_R) \mathcal{I}\left[\frac{2(\vec{k}_T \cdot \hat{P}_{h\perp})^2 - \vec{k}_T^2}{M_h^2} f_1 \left(-\frac{M_h^2}{|\vec{k}_T|^2} D_{1,TT} \right) \right] \right\}$$

$$+ \sum_a \frac{\alpha^2 e_a^2}{2\pi sxy^2} B(y) \left\{ -\cos 2\phi_h \mathcal{I}\left[\frac{2(\vec{p}_T \cdot \hat{P}_{h\perp})(\vec{k}_T \cdot \hat{P}_{h\perp}) - \vec{p}_T \cdot \vec{k}_T}{MM_h} h_1^\perp \left(\frac{1}{4} H_{1,OO}^{\perp s} + \frac{3}{4} H_{1,OO}^{\perp p} \right) \right] - \frac{1}{3}(3\cos^2\theta - 1) \right.$$

$$\times\cos 2\phi_h \mathcal{I}\left[\frac{2(\vec{p}_T \cdot \hat{P}_{h\perp})(\vec{k}_T \cdot \hat{P}_{h\perp}) - \vec{p}_T \cdot \vec{k}_T}{MM_h} h_1^\perp \left(\frac{3}{4} H_{1,LL}^\perp \right) \right] + \sin\theta\cos(\phi_h + \phi_R) \mathcal{I}\left[\frac{\vec{p}_T \cdot \hat{P}_{h\perp}}{M} h_1^\perp \left(-\frac{|\vec{R}|}{M_h} H_{1,OT}^{\not\asymp} \right) \right]$$

$$+ \sin 2\theta\cos(\phi_h + \phi_R) \mathcal{I}\left[\frac{\vec{p}_T \cdot \hat{P}_{h\perp}}{M} h_1^\perp \left(-\frac{|\vec{R}|}{2M_h} H_{1,LT}^{\not\asymp} \right) \right] + \sin^2\theta\cos 2\phi_R \mathcal{I}\left[\frac{\vec{p}_T \cdot \vec{k}_T}{MM_h} h_1^\perp \left(-\frac{|\vec{R}|}{|\vec{k}_T|} H_{1,TT}^{\not\asymp} \right) \right]$$

$$+ \sin\theta\cos(3\phi_h - \phi_R) \mathcal{I}\left[\frac{4(\vec{k}_T \cdot \hat{P}_{h\perp})^2(\vec{p}_T \cdot \hat{P}_{h\perp}) - 2(\vec{k}_T \cdot \hat{P}_{h\perp})(\vec{p}_T \cdot \vec{k}_T) - \vec{k}_T^2(\vec{p}_T \cdot \hat{P}_{h\perp})}{2MM_h^2} h_1^\perp \left(-\frac{2M_h}{|\vec{k}_T|} H_{1,OT}^\perp \right) \right]$$

$$+ \sin 2\theta\cos(3\phi_h - \phi_R) \mathcal{I}\left[\frac{4(\vec{k}_T \cdot \hat{P}_{h\perp})^2(\vec{p}_T \cdot \hat{P}_{h\perp}) - 2(\vec{k}_T \cdot \hat{P}_{h\perp})(\vec{p}_T \cdot \vec{k}_T) - \vec{k}_T^2(\vec{p}_T \cdot \hat{P}_{h\perp})}{2MM_h^2} h_1^\perp \left(-\frac{M_h}{|\vec{k}_T|} H_{1,LT}^\perp \right) \right]$$

$$+ \sin^2\theta\cos(4\phi_h - 2\phi_R) \mathcal{I}\left[\left(\frac{[\vec{k}_T^2 - 4(\vec{k}_T \cdot \hat{P}_{h\perp})^2][\vec{p}_T \cdot \vec{k}_T - 4(\vec{k}_T \cdot \hat{P}_{h\perp})(\vec{p}_T \cdot \hat{P}_{h\perp})]}{2MM_h^3} - \frac{8(\vec{k}_T \cdot \hat{P}_{h\perp})^3(\vec{p}_T \cdot \hat{P}_{h\perp})}{2MM_h^3} \right) \right.$$

$$\left. \times h_1^\perp \left(-\frac{2M_h^2}{|\vec{k}_T|^2} H_{1,TT}^\perp \right) \right] \right\}.$$
(C4)

2. Polarized lepton beam and unpolarized target

We have

$$d^9\sigma_{LO} = -\sum_a \frac{\alpha^2 e_a^2}{2\pi sxy^2}\lambda_e C(y)\left\{\sin\theta\sin(\phi_h-\phi_R)\mathcal{I}\left[\frac{\vec{k}_T\cdot\hat{P}_{h\perp}}{M_h}f_1\left(\frac{|\vec{R}|}{M_h}G_{1,OT}^\perp\right)\right] + \sin 2\theta\sin(\phi_h-\phi_R)\right.$$

$$\times\mathcal{I}\left[\frac{\vec{k}_T\cdot\hat{P}_{h\perp}}{M_h}f_1\left(\frac{|\vec{R}|}{2M_h}G_{1,LT}^\perp\right)\right] + \sin^2\theta\sin(2\phi_h-2\phi_R)\mathcal{I}\left[\frac{2(\vec{k}_T\cdot\hat{P}_{h\perp})^2-\vec{k}_T^2}{M_h^2}f_1\left(\frac{|\vec{R}|}{2|\vec{k}_T|}G_{1,TT}^\perp\right)\right]\right\}. \tag{C5}$$

3. Unpolarized lepton beam and longitudinally polarized target

We have

$$d^9\sigma_{OL} = -\sum_a \frac{\alpha^2 e_a^2}{2\pi sxy^2}|S_L|A(y)\left\{\sin\theta\sin(\phi_h-\phi_R)\mathcal{I}\left[\frac{\vec{k}_T\cdot\hat{P}_{h\perp}}{M_h}g_{1L}\left(\frac{|\vec{R}|}{M_h}G_{1,OT}^\perp\right)\right] + \sin 2\theta\sin(\phi_h-\phi_R)\right.$$

$$\times\mathcal{I}\left[\frac{\vec{k}_T\cdot\hat{P}_{h\perp}}{M_h}g_{1L}\left(\frac{|\vec{R}|}{2M_h}G_{1,LT}^\perp\right)\right] + \sin^2\theta\sin(2\phi_h-2\phi_R)\mathcal{I}\left[\frac{2(\vec{k}_T\cdot\hat{P}_{h\perp})^2-\vec{k}_T^2}{M_h^2}g_{1L}\left(\frac{|\vec{R}|}{2|\vec{k}_T|}G_{1,TT}^\perp\right)\right]\right\}$$

$$-\sum_a \frac{\alpha^2 e_a^2}{2\pi sxy^2}|S_L|B(y)\left\{\sin 2\phi_h\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{k}_T\cdot\hat{P}_{h\perp})-\vec{p}_T\cdot\vec{k}_T}{MM_h}h_{1L}^\perp\left(\frac{1}{4}H_{1,OO}^{\perp s}+\frac{3}{4}H_{1,OO}^{\perp p}\right)\right] - \frac{1}{3}(3\cos^2\theta-1)\right.$$

$$\times\sin 2\phi_h\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{k}_T\cdot\hat{P}_{h\perp})-\vec{p}_T\cdot\vec{k}_T}{MM_h}h_{1L}^\perp\left(\frac{3}{4}H_{1,LL}^\perp\right)\right] + \sin\theta\sin(\phi_h+\phi_R)\mathcal{I}\left[\frac{\vec{p}_T\cdot\hat{P}_{h\perp}}{M}h_{1L}^\perp\left(-\frac{|\vec{R}|}{M_h}H_{1,OT}^{\not\sim}\right)\right]$$

$$+\sin 2\theta\sin(\phi_h+\phi_R)\mathcal{I}\left[\frac{\vec{p}_T\cdot\hat{P}_{h\perp}}{M}h_{1L}^\perp\left(-\frac{|\vec{R}|}{2M_h}H_{1,LT}^{\not\sim}\right)\right] + \sin^2\theta\sin 2\phi_R\mathcal{I}\left[\frac{\vec{p}_T\cdot\vec{k}_T}{MM_h}h_{1L}^\perp\left(-\frac{|\vec{R}|}{|\vec{k}_T|}H_{1,TT}^{\not\sim}\right)\right]$$

$$+\sin\theta\sin(3\phi_h-\phi_R)\mathcal{I}\left[\frac{4(\vec{k}_T\cdot\hat{P}_{h\perp})^2(\vec{p}_T\cdot\hat{P}_{h\perp})-2(\vec{k}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)-\vec{k}_T^2(\vec{p}_T\cdot\hat{P}_{h\perp})}{2MM_h^2}h_{1L}^\perp\left(-\frac{2M_h}{|\vec{k}_T|}H_{1,OT}^\perp\right)\right]$$

$$+\sin 2\theta\sin(3\phi_h-\phi_R)\mathcal{I}\left[\frac{4(\vec{k}_T\cdot\hat{P}_{h\perp})^2(\vec{p}_T\cdot\hat{P}_{h\perp})-2(\vec{k}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)-\vec{k}_T^2(\vec{p}_T\cdot\hat{P}_{h\perp})}{2MM_h^2}h_{1L}^\perp\left(-\frac{M_h}{|\vec{k}_T|}H_{1,LT}^\perp\right)\right]$$

$$+\sin^2\theta\sin(4\phi_h-2\phi_R)\mathcal{I}\left[\left(\frac{[\vec{k}_T^2-4(\vec{k}_T\cdot\hat{P}_{h\perp})^2][\vec{p}_T\cdot\vec{k}_T-4(\vec{k}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\hat{P}_{h\perp})]}{2MM_h^3}-\frac{8(\vec{k}_T\cdot\hat{P}_{h\perp})^3(\vec{p}_T\cdot\hat{P}_{h\perp})}{2MM_h^3}\right)\right.$$

$$\left.\left.\times h_{1L}^\perp\left(-\frac{2M_h^2}{|\vec{k}_T|^2}H_{1,TT}^\perp\right)\right]\right\}. \tag{C6}$$

4. Polarized lepton beam and longitudinally polarized target

We have

$$d^9\sigma_{LL} = \sum_a \frac{\alpha^2 e_a^2}{2\pi sxy^2}\lambda_e|S_L|C(y)\left\{\mathcal{I}\left[g_{1L}\left(\frac{1}{4}D_{1,OO}^s+\frac{3}{4}D_{1,OO}^p\right)\right] + \cos\theta\mathcal{I}[g_{1L}D_{1,OL}] + \frac{1}{3}(3\cos^2\theta-1)\mathcal{I}\left[g_{1L}\left(\frac{3}{4}D_{1,LL}\right)\right]\right.$$

$$-\sin\theta\cos(\phi_h-\phi_R)\mathcal{I}\left[\frac{\vec{k}_T\cdot\hat{P}_{h\perp}}{M_h}g_{1L}\left(-\frac{M_h}{|\vec{k}_T|}D_{1,LT}\right)\right] - \sin 2\theta\cos(\phi_h-\phi_R)\mathcal{I}\left[\frac{\vec{k}_T\cdot\hat{P}_{h\perp}}{M_h}g_{1L}\left(-\frac{M_h}{2|\vec{k}_T|}D_{1,LT}\right)\right]$$

$$-\sin^2\theta\cos(2\phi_h-2\phi_R)\mathcal{I}\left[\frac{2(\vec{k}_T\cdot\hat{P}_{h\perp})^2-\vec{k}_T^2}{M_h^2}g_{1L}\left(-\frac{M_h^2}{|\vec{k}_T|^2}D_{1,TT}\right)\right]\right\}. \tag{C7}$$

5. Unpolarized lepton beam and transversely polarized target

We have

$$
d^9\sigma_{OT} = \sum_a \frac{\alpha^2 e_a^2}{2\pi s x y^2} |\vec{S}_T| A(y) \left\{ \sin\theta \sin(\phi_R - \phi_S) \mathcal{I}\left[\frac{(\vec{p}_T \cdot \vec{k}_T)}{2MM_h} g_{1T}\left(\frac{|\vec{R}|}{M_h} G_{1,OT}^{\perp} \right) \right] - \sin\theta \sin(2\phi_h - \phi_R - \phi_S) \right.
$$

$$
\times \mathcal{I}\left[\frac{2(\vec{p}_T \cdot \hat{P}_{h\perp})(\vec{k}_T \cdot \hat{P}_{h\perp}) - \vec{p}_T \cdot \vec{k}_T}{2MM_h} g_{1T}\left(\frac{|\vec{R}|}{M_h} G_{1,OT}^{\perp} \right) \right] + \sin 2\theta \sin(\phi_R - \phi_S) \mathcal{I}\left[\frac{(\vec{p}_T \cdot \vec{k}_T)}{2MM_h} g_{1T}\left(\frac{|\vec{R}|}{2M_h} G_{1,LT}^{\perp} \right) \right]
$$

$$
- \sin 2\theta \sin(2\phi_h - \phi_R - \phi_S) \mathcal{I}\left[\frac{2(\vec{p}_T \cdot \hat{P}_{h\perp})(\vec{k}_T \cdot \hat{P}_{h\perp}) - \vec{p}_T \cdot \vec{k}_T}{2MM_h} g_{1T}\left(\frac{|\vec{R}|}{2M_h} G_{1,LT}^{\perp} \right) \right] - \sin^2\theta \sin(\phi_h - 2\phi_R + \phi_S)
$$

$$
\times \mathcal{I}\left[\frac{2(\vec{k}_T \cdot \hat{P}_{h\perp})(\vec{p}_T \cdot \vec{k}_T) - \vec{k}_T^2(\vec{p}_T \cdot \hat{P}_{h\perp})}{2MM_h^2} g_{1T}\left(\frac{|\vec{R}|}{2|\vec{k}_T|} G_{1,TT}^{\perp} \right) \right] - \sin^2\theta \sin(3\phi_h - 2\phi_R - \phi_S)
$$

$$
\times \mathcal{I}\left[\left(\frac{4(\vec{k}_T \cdot \hat{P}_{h\perp})^2(\vec{p}_T \cdot \hat{P}_{h\perp}) - 2(\vec{k}_T \cdot \hat{P}_{h\perp})(\vec{p}_T \cdot \vec{k}_T)}{2MM_h^2} - \frac{\vec{k}_T^2(\vec{p}_T \cdot \hat{P}_{h\perp})}{2MM_h^2} \right) g_{1T}\left(\frac{|\vec{R}|}{2|\vec{k}_T|} G_{1,TT}^{\perp} \right) \right] + \sin(\phi_h - \phi_S)
$$

$$
\times \mathcal{I}\left[\frac{\vec{p}_T \cdot \hat{P}_{h\perp}}{M} f_{1T}^{\perp}\left(\frac{1}{4} D_{1,OO}^s + \frac{3}{4} D_{1,OO}^p \right) \right] + \cos\theta \mathcal{I}[f_{1T}^{\perp} D_{1,OL}] + \frac{1}{3}(3\cos^2\theta - 1)\sin(\phi_h - \phi_S)
$$

$$
\times \mathcal{I}\left[\frac{\vec{p}_T \cdot \hat{P}_{h\perp}}{M} f_{1T}^{\perp}\left(\frac{3}{4} D_{1,LL} \right) \right] - \sin\theta \sin(\phi_R - \phi_S) \mathcal{I}\left[\frac{(\vec{p}_T \cdot \vec{k}_T)}{2MM_h} f_{1T}^{\perp}\left(-\frac{M_h}{|\vec{k}_T|} D_{1,OT} \right) \right] - \sin\theta \sin(2\phi_h - \phi_R - \phi_S)
$$

$$
\times \mathcal{I}\left[\frac{2(\vec{p}_T \cdot \hat{P}_{h\perp})(\vec{k}_T \cdot \hat{P}_{h\perp}) - \vec{p}_T \cdot \vec{k}_T}{2MM_h} f_{1T}^{\perp}\left(-\frac{M_h}{|\vec{k}_T|} D_{1,OT} \right) \right] - \sin 2\theta \sin(\phi_R - \phi_S) \mathcal{I}\left[\frac{(\vec{p}_T \cdot \vec{k}_T)}{2MM_h} f_{1T}^{\perp}\left(-\frac{M_h}{2|\vec{k}_T|} D_{1,LT} \right) \right]
$$

$$
- \sin 2\theta \sin(2\phi_h - \phi_R - \phi_S) \mathcal{I}\left[\frac{2(\vec{p}_T \cdot \hat{P}_{h\perp})(\vec{k}_T \cdot \hat{P}_{h\perp}) - \vec{p}_T \cdot \vec{k}_T}{2MM_h} f_{1T}^{\perp}\left(-\frac{M_h}{2|\vec{k}_T|} D_{1,LT} \right) \right] + \sin^2\theta \sin(\phi_h - 2\phi_R + \phi_S)
$$

$$
\times \mathcal{I}\left[\frac{2(\vec{k}_T \cdot \hat{P}_{h\perp})(\vec{p}_T \cdot \vec{k}_T) - \vec{k}_T^2(\vec{p}_T \cdot \hat{P}_{h\perp})}{2MM_h^2} f_{1T}^{\perp}\left(-\frac{M_h^2}{|\vec{k}_T|^2} D_{1,TT} \right) \right] - \sin^2\theta \sin(3\phi_h - 2\phi_R - \phi_S)
$$

$$
\times \mathcal{I}\left[\left(\frac{4(\vec{k}_T \cdot \hat{P}_{h\perp})^2(\vec{p}_T \cdot \hat{P}_{h\perp}) - 2(\vec{k}_T \cdot \hat{P}_{h\perp})(\vec{p}_T \cdot \vec{k}_T)}{2MM_h^2} - \frac{\vec{k}_T^2(\vec{p}_T \cdot \hat{P}_{h\perp})}{2MM_h^2} \right) f_{1T}^{\perp}\left(-\frac{M_h^2}{|\vec{k}_T|^2} D_{1,TT} \right) \right] \right\} + \sum_a \frac{\alpha^2 e_a^2}{2\pi s x y^2} B(y)
$$

$$
\times \left\{ \cos 2\phi_h \mathcal{I}\left[\frac{2(\vec{p}_T \cdot \hat{P}_{h\perp})(\vec{k}_T \cdot \hat{P}_{h\perp}) - \vec{p}_T \cdot \vec{k}_T}{MM_h} h_1\left(\frac{1}{4} H_{1,OO}^{\perp s} + \frac{3}{4} H_{1,OO}^{\perp p} \right) \right] + \frac{1}{3}(3\cos^2\theta - 1)\sin(\phi_h + \phi_S) \right.
$$

$$
\times \mathcal{I}\left[\frac{\vec{k}_T \cdot \hat{P}_{h\perp}}{M_h} h_1\left(\frac{3}{4} H_{1,LL}^{\perp} \right) \right] - \sin\theta \sin(\phi_R + \phi_S) \mathcal{I}\left[h_1\left(-\frac{|\vec{R}|}{M_h} H_{1,OT}^{\sphericalangle} \right) \right] - \sin 2\theta \sin(\phi_R + \phi_S) \mathcal{I}\left[h_1\left(-\frac{|\vec{R}|}{2M_h} H_{1,LT}^{\sphericalangle} \right) \right]
$$

$$
+ \sin^2\theta \sin(\phi_h - 2\phi_R - \phi_S) \mathcal{I}\left[\frac{\vec{k}_T \cdot \hat{P}_{h\perp}}{M_h} h_1\left(-\frac{|\vec{R}|}{|\vec{k}_T|} H_{1,TT}^{\sphericalangle} \right) \right] - \sin\theta \sin(2\phi_h - \phi_R + \phi_S) \mathcal{I}\left[\frac{2(\vec{k}_T \cdot \hat{P}_{h\perp})^2 - \vec{k}_T^2}{2M_h^2} \right.
$$

$$
\times h_1\left(-\frac{2M_h}{|\vec{k}_T|} H_{1,OT}^{\perp} \right) \right] - \sin 2\theta \sin(2\phi_h - \phi_R + \phi_S) \mathcal{I}\left[\frac{2(\vec{k}_T \cdot \hat{P}_{h\perp})^2 - \vec{k}_T^2}{2M_h^2} h_1\left(-\frac{M_h}{|\vec{k}_T|} H_{1,LT}^{\perp} \right) \right]
$$

$$
- \sin^2\theta \sin(3\phi_h - 2\phi_R - \phi_S) \mathcal{I}\left[\frac{4(\vec{k}_T \cdot \hat{P}_{h\perp})^3 - 3\vec{k}_T^2(\vec{k}_T \cdot \hat{P}_{h\perp})}{2M_h^3} h_1\left(-\frac{2M_h^2}{|\vec{k}_T|^2} H_{1,TT}^{\perp} \right) \right]
$$

$$
+ \cos 2\phi_h \mathcal{I}\left[\frac{2(\vec{p}_T \cdot \hat{P}_{h\perp})(\vec{k}_T \cdot \hat{P}_{h\perp}) - \vec{p}_T \cdot \vec{k}_T}{MM_h} h_{1T}^{\perp}\left(\frac{1}{4} H_{1,OO}^{\perp s} + \frac{3}{4} H_{1,OO}^{\perp p} \right) \right] + \frac{1}{3}(3\cos^2\theta - 1)\sin(3\phi_h - \phi_S)
$$

$$\times \mathcal{I}\left[\frac{4(\vec{p}_T\cdot\hat{P}_{h\perp})^2(\vec{k}_T\cdot\hat{P}_{h\perp})-2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)-\vec{p}_T^2(\vec{k}_T\cdot\hat{P}_{h\perp})}{2M^2M_h}h_{1T}^\perp\left(\frac{3}{4}H_{1,LL}^\perp\right)\right]-\sin\theta\sin(2\phi_h+\phi_R-\phi_S)$$

$$\times\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})^2-\vec{p}_T^2}{2M^2}h_{1T}^\perp\left(-\frac{|\vec{R}|}{M_h}H_{1,OT}^{\not\prec}\right)\right]-\sin2\theta\sin(2\phi_h+\phi_R-\phi_S)\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})^2-\vec{p}_T^2}{2M^2}h_{1T}^\perp\left(-\frac{|\vec{R}|}{2M_h}H_{1,LT}^{\not\prec}\right)\right]$$

$$-\sin^2\theta\sin(\phi_h+2\phi_R-\phi_S)\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\vec{k}_T)(\vec{p}_T\cdot\hat{P}_{h\perp})-(\vec{k}_T\cdot\hat{P}_{h\perp})\vec{p}_T^2}{2M^2M_h}h_{1T}^\perp\left(-\frac{|\vec{R}|}{|\vec{k}_T|}H_{1,TT}^{\not\prec}\right)\right]+\sin\theta\sin(4\phi_h-\phi_R-\phi_S)$$

$$\times\mathcal{I}\left[\left(\frac{\vec{k}_T^2[2(\vec{p}_T\cdot\hat{P}_{h\perp})^2-\vec{p}_T^2]}{4M^2M_h^2}-2(\vec{k}_T\cdot\hat{P}_{h\perp})\frac{4(\vec{p}_T\cdot\hat{P}_{h\perp})^2(\vec{k}_T\cdot\hat{P}_{h\perp})-2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)-\vec{p}_T^2(\vec{k}_T\cdot\hat{P}_{h\perp})}{4M^2M_h^2}\right)\right.$$

$$\left.\times h_{1T}^\perp\left(-\frac{2M_h}{|\vec{k}_T|}H_{1,OT}^\perp\right)\right]+\sin2\theta\sin(4\phi_h-\phi_R-\phi_S)\mathcal{I}\left[\left(\frac{\vec{k}_T^2[2(\vec{p}_T\cdot\hat{P}_{h\perp})^2-\vec{p}_T^2]}{4M^2M_h^2}-2(\vec{k}_T\cdot\hat{P}_{h\perp})\right.\right.$$

$$\left.\times\frac{4(\vec{p}_T\cdot\hat{P}_{h\perp})^2(\vec{k}_T\cdot\hat{P}_{h\perp})-2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)-\vec{p}_T^2(\vec{k}_T\cdot\hat{P}_{h\perp})}{4M^2M_h^2}\right)h_{1T}^\perp\left(-\frac{M_h}{|\vec{k}_T|}H_{1,LT}^\perp\right)\right]+\sin^2\theta\sin(5\phi_h-2\phi_R-\phi_S)$$

$$\times\mathcal{I}\left[\left(\frac{2\vec{k}_T^2(\vec{k}_T\cdot\hat{P}_{h\perp})[2(\vec{p}_T\cdot\hat{P}_{h\perp})^2-\vec{p}_T^2]}{4M^2M_h^3}+[\vec{k}_T^2-4(\vec{k}_T\cdot\hat{P}_{h\perp})^2]\right.\right.$$

$$\left.\left.\times\frac{4(\vec{p}_T\cdot\hat{P}_{h\perp})^2(\vec{k}_T\cdot\hat{P}_{h\perp})-2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)-\vec{p}_T^2(\vec{k}_T\cdot\hat{P}_{h\perp})}{4M^2M_h^3}\right)h_{1T}^\perp\left(-\frac{2M_h^2}{|\vec{k}_T|^2}H_{1,TT}^\perp\right)\right]\right\}. \tag{C8}$$

6. Polarized lepton beam and transversely polarized target

We have

$$d^9\sigma_{LT}=\sum_a\frac{\alpha^2e_a^2}{2\pi sxy^2}\lambda_e|\vec{S}_T|C(y)\left\{\cos(\phi_h-\phi_S)\mathcal{I}\left[\frac{\vec{p}_T\cdot\hat{P}_{h\perp}}{M}g_{1T}\left(\frac{1}{4}D_{1,OO}^s+\frac{3}{4}D_{1,OO}^p\right)\right]+\frac{1}{3}(3\cos^2\theta-1)\cos(\phi_h-\phi_S)\right.$$

$$\times\mathcal{I}\left[\frac{\vec{p}_T\cdot\hat{P}_{h\perp}}{M}g_{1T}\left(\frac{3}{4}D_{1,LL}\right)\right]-\sin2\theta\cos(\phi_R-\phi_S)\mathcal{I}\left[\frac{(\vec{p}_T\cdot\vec{k}_T)}{2MM_h}g_{1T}\left(-\frac{M_h}{2|\vec{k}_T|}D_{1,LT}\right)\right]-\sin2\theta\cos(2\phi_h-\phi_R-\phi_S)$$

$$\times\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{k}_T\cdot\hat{P}_{h\perp})-\vec{p}_T\cdot\vec{k}_T}{2MM_h}g_{1T}\left(-\frac{M_h}{2|\vec{k}_T|}D_{1,LT}\right)\right]-\sin^2\theta\cos(\phi_h-2\phi_R+\phi_S)$$

$$\times\mathcal{I}\left[\frac{2(\vec{k}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)-\vec{k}_T^2(\vec{p}_T\cdot\hat{P}_{h\perp})}{2MM_h^2}g_{1T}\left(-\frac{M_h^2}{|\vec{k}_T|^2}D_{1,TT}\right)\right]-\sin^2\theta\cos(3\phi_h-2\phi_R-\phi_S)$$

$$\times\mathcal{I}\left[\left(\frac{4(\vec{k}_T\cdot\hat{P}_{h\perp})^2(\vec{p}_T\cdot\hat{P}_{h\perp})-2(\vec{k}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)}{2MM_h^2}-\frac{\vec{k}_T^2(\vec{p}_T\cdot\hat{P}_{h\perp})}{2MM_h^2}\right)g_{1T}\left(-\frac{M_h^2}{|\vec{k}_T|^2}D_{1,TT}\right)\right]-\sin2\theta\cos(\phi_R-\phi_S)$$

$$\times\mathcal{I}\left[\frac{(\vec{p}_T\cdot\vec{k}_T)}{2MM_h}f_{1T}^\perp\left(\frac{|\vec{R}|}{2M_h}G_{1,LT}^\perp\right)\right]+\sin2\theta\cos(2\phi_h-\phi_R-\phi_S)\mathcal{I}\left[\frac{2(\vec{p}_T\cdot\hat{P}_{h\perp})(\vec{k}_T\cdot\hat{P}_{h\perp})-\vec{p}_T\cdot\vec{k}_T}{2MM_h}f_{1T}^\perp\left(\frac{|\vec{R}|}{2M_h}G_{1,LT}^\perp\right)\right]$$

$$-\sin^2\theta\cos(\phi_h-2\phi_R+\phi_S)\mathcal{I}\left[\frac{2(\vec{k}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)-\vec{k}_T^2(\vec{p}_T\cdot\hat{P}_{h\perp})}{2MM_h^2}f_{1T}^\perp\left(\frac{|\vec{R}|}{2|\vec{k}_T|}G_{1,TT}^\perp\right)\right]+\sin^2\theta\cos(3\phi_h-2\phi_R-\phi_S)$$

$$\times\mathcal{I}\left[\left(\frac{4(\vec{k}_T\cdot\hat{P}_{h\perp})^2(\vec{p}_T\cdot\hat{P}_{h\perp})-2(\vec{k}_T\cdot\hat{P}_{h\perp})(\vec{p}_T\cdot\vec{k}_T)}{2MM_h^2}-\frac{\vec{k}_T^2(\vec{p}_T\cdot\hat{P}_{h\perp})}{2MM_h^2}\right)f_{1T}^\perp\left(\frac{|\vec{R}|}{2|\vec{k}_T|}G_{1,TT}^\perp\right)\right]\right\}. \tag{C9}$$

The pure p-wave sector of the previous cross sections corresponds to the results of spin-1 production presented in Refs. [13,38], once we apply the following identifications:

$$\frac{3}{4}D_{1,OO}^p = D_1, \quad \frac{3}{4}D_{1,LL} = D_{1LL}, \quad -\frac{M_h}{2|\vec{k}_T|}D_{1,LT} = D_{1LT}, \quad -\frac{M_h^2}{|\vec{k}_T|^2}D_{1,TT} = D_{1TT},$$

$$\frac{|\vec{R}|}{2M_h}G_{1,LT}^\perp = G_{1LT}, \quad \frac{|\vec{R}|}{2|\vec{k}_T|}G_{1,TT}^\perp = G_{1TT}, \quad \frac{3}{4}H_{1,OO}^{\perp\,p} = H_1^\perp, \quad \frac{3}{4}H_{1,LL}^\perp = H_{1LL}^\perp, \tag{C10}$$

$$-\frac{|\vec{R}|}{2M_h}H_{1,LT}^{\vDash} = H_{1LT}, \quad -\frac{M_h}{|\vec{k}_T|}H_{1,LT}^\perp = H_{1LT}^\perp, \quad -\frac{|\vec{R}|}{|\vec{k}_T|}H_{1,TT}^{\vDash} = H_{1TT}, \quad -\frac{2M_h^2}{|\vec{k}_T|^2}H_{1,TT}^\perp = H_{1TT}^\perp.$$

Note, however, that while the functions on the left-hand side contain a dependence on z as well as on the invariant mass M_h^2, the functions on the right-hand side depend only on z: it is required to assume that the spin-1 functions behave as resonances (Breit-Wigner shapes) in the invariant mass.

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