PHYSICAL REVIEW D **67**, 091503(R) (2003)

Pinning down the kaon form factors in $K^+ \to \mu^+ \nu_\mu \gamma$ decay

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(Received 17 February 2003; published 22 May 2003)

We find that the normal muon polarization in the decay $K \to \mu \nu_\mu \gamma$ is very sensitive to the values of the kaon vector F_V and axial-vector F_A form factors. It is shown that the ongoing KEK-E246 experiment can definitely determine the signs of the sum of the form factors if their difference is fixed from other considerations. This method can also verify the form factor values and signs obtained from the $K^+ \to l^+ \nu_l e^+ e^-$ decays. A new experiment with sensitivity to the normal and transverse muon polarizations of about 10^{-4} will provide a unique possibility to determine the F_V and F_A values with a few percent accuracy.

DOI: 10.1103/PhysRevD.67.091503 PACS number(s): 13.20.Eb, 12.39.Fe, 13.20.−v

Quantum chromodynamics (QCD) describes strong processes at high energies with remarkable precision. Processes at low energies are usually described by the effective Lagrangian of the light mesons. The matching of the corresponding coupling constants is very involved and is often replaced simply by the requirement of reasonable fitting of low-energy experimental data.

In this paper, we study the normal muon polarization in the decay $K^+ \to \mu^+ \nu_\mu \gamma$ ($K_{\mu 2\gamma}$) in order to investigate the possibility of extracting the values of the vector and axial-vector kaon form factors F_V and F_A. Because of the lack of understanding of the QCD low-energy structure, there is no definite prediction for the values of the F_V and F_A form factors: the calculation of them is a model-dependent procedure. So, the measurement of these form factors would provide a possibility to select among various candidates for the correct description of the QCD low-energy limit.

Introducing three unit vectors

$$\vec{e}_L = \frac{\vec{p}_\mu}{|\vec{p}_\mu|}, \quad \vec{e}_N = \frac{\vec{p}_\mu \times (\vec{q} \times \vec{p}_\mu)}{|\vec{p}_\mu \times (\vec{q} \times \vec{p}_\mu)|}, \quad \vec{e}_T = \frac{\vec{q} \times \vec{p}_\mu}{|\vec{q} \times \vec{p}_\mu|},$$

with p_μ and q being the four-momenta of μ^+ and γ, respectively, one can define the longitudinal (P_L), normal (P_N) and transverse (P_T) components of the muon polarization as the corresponding contributions to the squared matrix element of the $K_{\mu 2\gamma}$ decay,

$$|M|^2 = \rho_0 [1 + (P_L \vec{e}_L + P_N \vec{e}_N + P_T \vec{e}_T) \cdot \vec{\xi}],$$

with $\vec{\xi}$ being a unit vector along the muon spin and ρ_0 is

$$\rho_0(x,y) = \tfrac{1}{2} e^2 G_F^2 |V_{us}|^2 (1-\lambda) \{ f_{\rm IB}(x,y) + f_{\rm SD}(x,y) + f_{\rm IBSD}(x,y) \},$$

where the internal bremsstrahlung (IB), structure dependent (SD) and interference contributions (IBSD) are given as follows [1–3]:

$$f_{\rm IB} = \frac{4 m_\mu^2 |f_K|^2}{\lambda x^2} \left[x^2 + 2(1-r_\mu) \left(1 - x - \frac{r_\mu}{\lambda} \right) \right], \tag{1}$$

$$f_{\rm SD} = m_K^4 x^2 \left[|F_V + F_A|^2 \frac{\lambda^2}{1-\lambda} \left(1 - x - \frac{r_\mu}{\lambda} \right) + |F_V - F_A|^2 (y - \lambda) \right], \tag{2}$$

$$f_{\rm IBSD} = -4 m_K m_\mu^2 \left[{\rm Re}[f_K (F_V + F_A)^*] \left(1 - x - \frac{r_\mu}{\lambda} \right) - {\rm Re}[f_K (F_V - F_A)^*] \frac{1 - y + \lambda}{\lambda} \right]. \tag{3}$$

Here we used the standard notations $\lambda = (x + y - 1 - r_\mu)/x$, $r_\mu = m_\mu^2/m_K^2$, and $x = 2E_\gamma/m_K$, $y = 2E_\mu/m_K$ with E_γ, E_μ being the photon and muon energies in the kaon rest frame, respectively; G_F is the Fermi constant, V_{us} is the corresponding element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, f_K is the kaon decay constant. In terms of these variables the differential decay width reads

$$d\Gamma(\vec{\xi}) = \frac{m_K}{32(2\pi)^3} |M(x,y,\vec{\xi})|^2 \, dx \, dy.$$

The normal muon polarization P_N is equal to the following asymmetry in the partial decay width:

$$P_N = \frac{d\Gamma(\vec{e}_N) - d\Gamma(-\vec{e}_N)}{d\Gamma(\vec{e}_N) + d\Gamma(-\vec{e}_N)} \equiv \frac{\rho_N}{\rho_0}, \tag{4}$$

and at the tree level one has [1–3]

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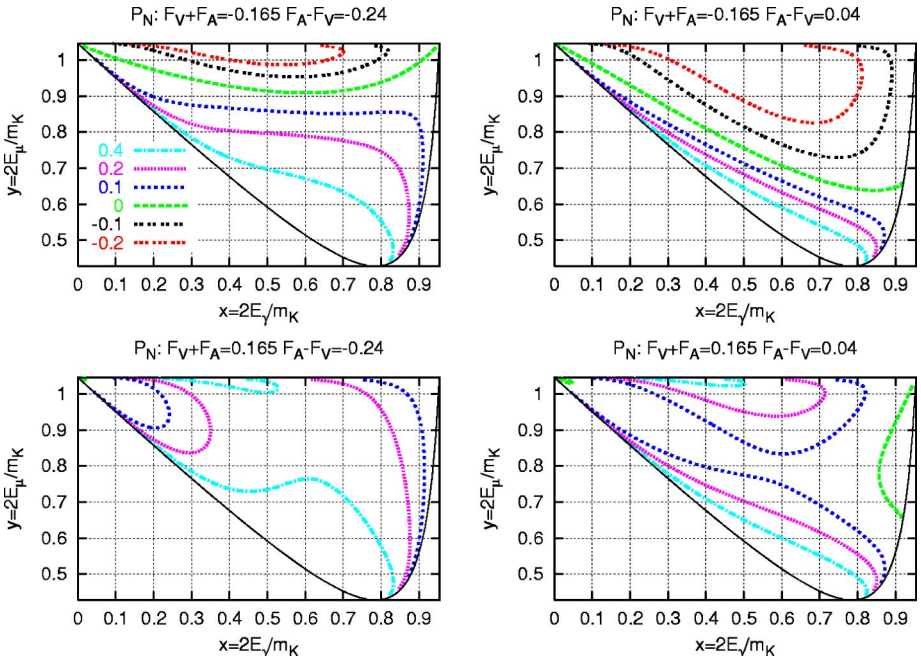

FIG. 1. P_N distribution for different F_V+F_A and F_A-F_V. In the cases of opposite sign of the sum of the form factors, P_N also has opposite sign at large y value.

$$\rho_N(x,y)=e^2G_F^2|V_{us}|^2\frac{(1-\lambda)\sqrt{\lambda y-\lambda^2-r_\mu}}{m_K\lambda\sqrt{y^2-4r_\mu}}$$
$$\times\left\{\frac{4m_\mu^3|f_K|^2}{\lambda x}(x+y-2\lambda)-m_K^4m_\mu\lambda x^2\right.$$
$$\times\left[|F_V+F_A|^2\frac{\lambda}{1-\lambda}\left(1-x-\frac{r_\mu}{\lambda}\right)+|F_V-F_A|^2\right]$$
$$-2m_K^3m_\mu\left[\mathrm{Re}[f_K(F_V+F_A)^*]\right.$$
$$\times\left(\frac{(r_\mu-\lambda)(1-x-r_\mu)}{1-\lambda}+\lambda x(1-x)\right)$$
$$\left.\left.-\mathrm{Re}[f_K(F_V-F_A)^*](y-2r_\mu)\right]\right\}.$$

Since in the Standard Model both ρ_0 and ρ_N are of the same order, P_N is of order one.

Only the absolute values of the sum and difference of the kaon form factors can be determined from the Dalitz plot distribution of the $K_{e2\gamma}$ decay width, since the term f_{IBSD} [see Eq. (3)] is small. In $K_{\mu2\gamma}$ both the linear and quadratic terms in F_A and F_V contribute at comparable levels, making it possible to measure the signs as well as the magnitudes of the form factors. Unfortunately, in the region where the linear terms grow, the dominant contribution to $K_{\mu2\gamma}$ (the IB term which depends only on f_K) also increases, significantly reducing the sensitivity of $K_{\mu2\gamma}$ experiments to these form factors. In practice, the situation is even worse, since in these experiments only the absolute value of the sum of the kaon form factors has been measured with good accuracy, while their difference still has only lower and upper bounds [4,5]:

$$|F_V+F_A|=0.165\pm0.013, \tag{5}$$

$$-0.24<F_A-F_V<0.04. \tag{6}$$

The Dalitz plot distributions of P_N for several values of the form factors satisfying Eqs. (5) and (6) are presented in Fig. 1. One can see that the normal polarization is very sensitive to the signs of F_V and F_A, especially at large y. There in the case of opposite signs of the sum of the form factors the P_N values also have opposite signs.

Recently, both vector and axial-vector form factors have been measured in $K^+\to\mu^+\nu_\mu e^+e^-$ and $K^+\to e^+\nu_e e^+e^-$ decays [6]. These decays are generalizations of $K_{l2\gamma}$ for the case of a virtual photon in the final state, so F_V and F_A are believed to be the same in all these processes. The combined fit for both four-body decay experiments results in

$$F_V=-0.112\pm0.018, \quad F_A=-0.035\pm0.020. \tag{7}$$

These values are in a good agreement with $\mathcal{O}(p^4)$ predictions [5,7] of the chiral perturbation theory (ChPT)

$$F_V=-0.096, \quad F_A=-0.041. \tag{8}$$

The distributions of P_N over the Dalitz plot evaluated for the measured (7) and predicted (8) values of the form factors exhibit behavior similar to the upper right plot in Fig. 1.

Actually, form factors F_V and F_A are not constants, but functions of the momentum of the lepton pair, $Q^2\equiv(p_K-q)^2=m_K^2(1-x)$. In ChPT Q^2-dependence emerges due to higher order corrections, which have not been calculated yet. These corrections generally decrease the chances to determine F_V and F_A in $K_{l2\gamma}$ experiments with reasonable precision. The Q^2 dependence can be estimated [7] supposing that the dominant contribution comes from the exchange of the first strange hadronic vector and axial-vector resonances with masses m_V (the K^* mass) and m_A (the K_1 mass)

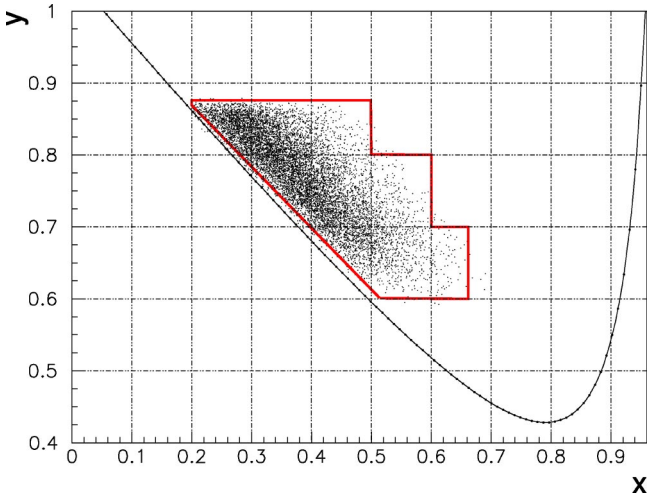

FIG. 2. The Dalitz plot with $K_{\mu 2\gamma}$ events accumulated in the E246 experiment [9]. The region of the Dalitz plot confined by the solid lines was used for the calculation of the P_N values.

$$F_{V,A}(x)=\frac{F_{V,A}}{1-\frac{m_K^2}{m_{V,A}^2}(1-x)}. \tag{9}$$

The results from the recent $K\to l\nu e^+e^-$ experiment [6] favor this x dependence over constant form factors. Unfortunately, the statistics accumulated in this experiment did not allow definite confirmation or rejection of x dependence. Until now Q^2 dependence remains unknown theoretically and new experiments with higher statistics are needed to fix it experimentally.

For the following analysis we need to estimate the level of statistical precision in the measurement of P_N which can be achieved in the currently running and forthcoming experiments. Generally, with the analyzing power of the detector α and the kinematical attenuation factor f, the expected sensitivity to P_N in some region $\mathcal{R}$ of the Dalitz plot can be estimated as

$$\delta P_N(\mathcal{R})\simeq\frac{1}{\alpha f\sqrt{N_{K_{\mu 2\gamma}}(\mathcal{R})}}, \tag{10}$$

where $N_{K_{\mu 2\gamma}}(\mathcal{R})$ is the number of $K_{\mu 2\gamma}$ events in the region $\mathcal{R}$. This should be compared to the expected value of the normal muon polarization

$$P_N(\mathcal{R})=\frac{\int_{\mathcal{R}}\rho_N(x,y)dx\,dy}{\int_{\mathcal{R}}\rho_0(x,y)dx\,dy}. \tag{11}$$

The ongoing E246 experiment at KEK [8] dedicated to a measurement of P_T in the decay $K^+\to\pi^0\mu^+\nu$ has only a limited sensitivity of about 10^{-2} to both P_T and P_N in $K_{\mu 2\gamma}$ [9]. About 2×10^5 $K_{\mu 2\gamma}$ events (see Fig. 2) are expected to be accumulated in the region of the Dalitz plot where the IB term dominates. Integrating over the region confined by thick solid line in Fig. 2, we obtain

$$P_N(F_V+F_A=0.165,F_A-F_V=-0.24)=0.360,$$

$$P_N(F_V+F_A=0.165,F_A-F_V=0.04)=0.210,$$

$$P_N(F_V+F_A=-0.165,F_A-F_V=-0.24)=0.318,$$

$$P_N(F_V+F_A=-0.165,F_A-F_V=0.04)=0.166.$$

With values $\alpha\simeq0.3$ and $f\simeq0.65$ adopted in the experiment, the statistical sensitivity obtained using Eq. (10) is estimated to be $\sim1.2\times10^{-2}$. Hence, the analysis of the E246 data will determine the signs of the kaon form factors for sure if the difference between the F_V and F_A values is fixed from other considerations.

The sensitivity of the E246 experiment to the form factor values can be estimated from the difference between P_N values obtained for experimental form factors (7) and for the ChPT predictions (8):

$$P_N(F_V=-0.112,F_A=-0.035)=0.145,$$

$$P_N(F_V=-0.096,F_A=-0.041)=0.161.$$

This yields $\Delta P_N=1.6\times10^{-2}$ which is comparable to the expected statistical error of E246. One can conclude that these values can be distinguished at the level of about 1σ, but only if the systematic error will be as small as 1%.

We note in passing that the ambiguity associated with unknown Q^2 dependence reduces the sensitivity of the E246 experiment to the form factors, if Q^2 corrections are not fixed. To deal with this one can, for example, fit the experimental data with constant $F_{V,A}$ form factors and with form factors given by Eq. (9). If the latter turns out to be more favorable (as in the case of the $K\to l\nu e^+e^-$ experiment [6]), then P_N should be calculated assuming the same dependence. In this case

$$P_N(F_V=-0.112,F_A=-0.035)=0.128,$$

$$P_N(F_V=-0.096,F_A=-0.041)=0.147.$$

A new experiment at the Japanese Hadron Facility (JHF) [11] in which a statistical sensitivity of $\lesssim10^{-4}$ for both P_N and P_T in $K_{\mu 2\gamma}$ was proposed in [10]. The main features of this experiment include a high resolution measurement of neutral particles from $K_{\mu 3}$ and $K_{\mu 2\gamma}$ decays, an active muon polarimeter which provides information about stopped

TABLE I. The P_N values in several regions of Dalitz plot calculated according to Eq. (11) and their sensitivity to small changes of the form factor values from the ChPT predictions $F_V=-0.096$, $F_A=-0.041$: $\Delta P_N^{+a}=P_N[(1+a\times10^{-2})(F_V+F_A),F_A-F_V]-P_N[F_V+F_A,F_A-F_V]$; $\Delta P_N^{-a}=P_N[F_V+F_A,(1+a\times10^{-2})(F_A-F_V)]-P_N[F_V+F_A,F_A-F_V]$, where a takes values 1.0 and 3.0. The photon energy cut $E_\gamma>20$ MeV and muon energy cut $E_\mu>200$ MeV are adopted; the values of δP_N are the statistical errors estimated according to Eq. (10) with $\alpha\simeq0.3$ and $f\simeq0.8$ in corresponding regions of the Dalitz plot.

Region	P_N	$\delta P_N\times10^5$	$\Delta P_N^{+1.0}\times10^5$	$\Delta P_N^{-1.0}\times10^5$	$\Delta P_N^{+3.0}\times10^5$	$\Delta P_N^{-3.0}\times10^5$
$y>0.9,\ 0°<\theta<60°$	0.090	4.2	−0.88	−3.1	−2.6	−9.2
$y>0.9,\ 60°<\theta<110°$	0.002	4.4	−8.8	−5.8	−26.5	−17.5
$y<0.9,\ 0°<\theta<60°$	0.198	6.0	−2.3	−10.6	−6.9	−31.9
$y<0.9,\ 60°<\theta<110°$	0.019	7.4	−22.2	−23.4	−66.7	−70.2
$y>0.9,\ 110°<\theta<160°$	−0.099	8.2	−77.9	−15.7	−233.9	−47.2
$y<0.9,\ 110°<\theta<160°$	−0.166	16.6	−121.2	−68.3	−364.0	−205.2

muons (stopping point, momentum), positron direction, and also detects photons, and a highly efficient photon veto system covering nearly 4π solid angle. This approach allows $K_{\mu2\gamma}$ events for all θ angles between photon and muon momenta to be accumulated due to efficient photon veto detection. This system eliminates $K_{\pi2}$ decays which are estimated to be the main background source at large θ. Thus, the region of large x and y becomes available for studying P_N. In this experiment, the number of $K_{\mu2\gamma}$ events in the signal region with $E_\gamma>20$ MeV and $E_\mu>200$ MeV is estimated to be about 3×10^{10} for a one year running period and beam intensity of about 10^7K^+/sec. With $\alpha\simeq0.3$ and $f\simeq0.8$ this would provide a very low statistical error $\delta P_T\sim2.5\times10^{-5}$ over the entire Dalitz plot. This experiment is expected to control the systematic error at a level comparable to the statistical uncertainty. However, in the measurement of P_N, where expected non-zero values of normal asymmetry could be a few tens percent, the accuracy will be diminished due to the error associated with the polarimeter analyzing power and uncertainty in the level of background present in $K_{\mu2\gamma}$ events from other kaon decays and accidentals. It is rather difficult to measure P_N with an error less than a few percent, and therefore this restricts the accuracy in extraction of the form factor values. In order to significantly improve the accuracy for extraction of F_V and F_A values we looked at the relative changes of P_N values in several Dalitz plot regions.

Let us evaluate the accuracy which can be obtained for F_V and F_A in this experiment. Changing F_V+F_A and F_A-F_V by 1% and 3% around the ChPT predictions we examine the corresponding effect on the normal polarization for different regions of the Dalitz plot. The results are presented in Table I. As seen from this table, the behavior of P_N differs dramatically when either the sum or difference of the form factors is changed by 1% from its ChPT value. While P_N is almost unchanged for Dalitz region $y>0.9,\ 0°<\theta<60°$, its values are very sensitive to small changes in form factors in the regions with large θ (lines 5 and 6 in Table I). Statistical errors of P_N are small enough to clearly distinguish this effect. It looks possible that even 1%-deviations of the form factors values from the ChPT predictions (8) can be measured in the experiment [10].

It is worth noting that the P_N value depends on the ratios $F_{V,A}/f_K$ rather than on the form factors themselves. The current uncertainty in the f_K value is also about 1% [5], thus the method presented above indeed permits to achieve the same precision in the measurement of F_V and F_A.

Finally, if the Q^2 dependence of the form factors is not fixed theoretically, it reduces the accuracy of the determination of the form factors. Similarly to the experiment [6] one can fit the data by constant form factors, by form factors depending on x as in Eq. (9), or by form factors with polynomial dependence on x with unknown coefficients to be determined from the best fit. Alternatively, Q^2 dependence may be taken into account by extracting F_V and F_A for a set of the P_N values obtained by integrating over several narrow bins in x. In any case Q^2 dependence will be fixed to some extent by one of these procedures. This certainly decreases the sensitivity of the measurements of the P_N to the kaon form factors. Nevertheless, taking into account the large statistics expected in the proposed experiment one may hope to achieve the accuracy of a few percent in determination of F_V and F_A.

In the analysis presented above, we have demonstrated that the normal muon polarization P_N in $K_{\mu2\gamma}$ decay is a very effective observable to pin down the kaon form factors F_V and F_A. It was found that the distribution of the P_N over the Dalitz plot is very sensitive to the values of the kaon form factors. The best sensitivity is exhibited in the region of large angles θ between the outgoing muon and photon. The measurement of P_N with the accuracy of about 10^{-2} pins down the signs of the form factors while 10^{-4} is required in order to pin down their values also.

The longitudinal P_L and transverse P_T muon polarizations are also very sensitive to the values of the kaon form factors. P_L arises at the tree level and, though its measurement with high precision in the experiments discussed above is very difficult, could be used as a cross check observable. P_T arises only at the one-loop level (so-called FSI contribution), thus only the signs of the form factors could be determined for sure. In a forthcoming paper we will show that in the proposed experiment at JHF, reasonable accuracy could be achieved with statistics expected to be reached of $\gtrsim10^9$ $K_{\mu2\gamma}$ events at large θ.

It should be noted that the signs of the pion form factors F_V^π and F_A^π also have not been measured yet [5]. The experi-

mental situation there is similar to kaons, though F_V^π has a definite CVC prediction and the pion form factors are expected to be almost constant over the whole Dalitz plot and P_N value ranges from 0.1 to 1. From the analysis given above, we can suggest that the measurement of the lepton polarization in $\pi_{l2\gamma}$ might allow the pion form factors to be pinned down. This issue will be considered elsewhere.

We would like to thank J. Imazato and J. Macdonald for valuable comments. The work is supported in part by RFBR grant 02-02-17398 and by the program SCOPES of the Swiss National Science Foundation, Project No. 7SUPJ062239. The work of F.B. is supported in part by CRDF grant RP1-2364-MO-02. The work of D.G. is supported in part by the INTAS YSF 2001/2-142.

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