

Electroweak and finite width corrections to top quark decays into transverse and longitudinal W bosons

H. S. Do, S. Groote, J. G. Körner, and M. C. Mauser

Institut für Physik der Johannes-Gutenberg-Universität, Staudinger Weg 7, 55099 Mainz, Germany

(Received 17 September 2002; published 14 May 2003)

We calculate the electroweak and finite width corrections to the decay of an unpolarized top quark into a bottom quark and a W -gauge boson where the helicities of the W are specified as longitudinal, transverse plus and transverse minus. Together with the $O(\alpha_s)$ corrections these corrections may become relevant for the determination of the mass of the top quark through angular decay measurements.

DOI: 10.1103/PhysRevD.67.091501

PACS number(s): 14.65.Ha, 12.15.Lk

I. INTRODUCTION

Because the Kobayashi-Maskawa (KM) matrix element V_{tb} is very close to 1 the dominating decay mode of the top quark is into the channel $t \rightarrow X_b + W^+$. The helicity content of the W boson (transverse plus, transverse minus and longitudinal) in this decay can be probed through a measurement of the shape of the lepton spectrum in the decay of the W boson as was recently done by the Collider Detector at Fermilab (CDF) Collaboration [1]. Of particular interest is the size of the longitudinal contribution which encodes the physics of the spontaneous symmetry breaking of electroweak symmetry. The transverse-plus contribution vanishes at the Born term level. Any significant deviation from this value would point to sizable radiative corrections or a non-SM ($V+A$) admixture in the weak $t \rightarrow b$ current transition.

A first measurement of the helicity content of the W boson was recently carried through by the CDF Collaboration [1]. Their result is

$$\Gamma_L/\Gamma = 0.91 \pm 0.37(\text{stat}) \pm 0.13(\text{syst}), \quad (1)$$

where Γ_L denotes the rates into the longitudinal polarization state of the W boson and Γ is the total rate. The CDF Collaboration has also quoted a value for the transverse-plus contribution Γ_+/Γ which was obtained by fixing the longitudinal contribution to its standard model (SM) value $\Gamma_L/\Gamma = 0.7$. They obtained

$$\Gamma_+/\Gamma = 0.11 \pm 0.15. \quad (2)$$

The measured values of the two normalized partial helicity rates are well within SM expectations which, at the Born term level, are $\Gamma_L/\Gamma \approx 0.70$ and $\Gamma_+/\Gamma = 0$. However, the errors on this measurement are too large to allow for a significant test of the SM predictions. The errors will be much reduced when larger data samples become available in the future from Fermilab Tevatron run II, and, at a later stage, from the CERN Large Hadron Collider (LHC). The measurement errors can eventually be reduced to the (1–2)% level [2]. If such a level of accuracy can in fact be reached it is important to take into account the radiative and finite width corrections to the different helicity rates.

The radiative corrections to the top width are rather large. Relative to the $m_b = 0$ Born term rate they amount to

–8.54% (QCD one loop) [3–8], +1.54% (electroweak one loop) [3,9,10] and –2.05% (–2.16%) (QCD two loop; approximate) [11] ([12]). The $m_b \neq 0$ and finite width corrections reduce the Born width by 0.27% [13–15] (for $m_b = 4.8$ GeV [16]) and 1.55% [17], respectively. Given the fact that the radiative and finite width corrections to the total rate are sizable it is important to know also the respective corrections to the partial longitudinal and transverse rates. The QCD one-loop corrections to the partial helicity rates were given in [13–15]. In this paper we provide the missing one-loop electroweak (EW) corrections and the finite width (FW) corrections to the partial helicity rates. Note that to leading order the finite width correction is also a one-loop effect.

The angular decay distribution for the decay process $t \rightarrow X_b + W^+$ followed by $W^+ \rightarrow l^+ + \nu_l$ (or by $W^+ \rightarrow \bar{q} + q$) is given by (see, e.g. [14])

$$\frac{d\Gamma}{d\cos\theta} = \frac{3}{8}(1 + \cos\theta)^2\Gamma_+ + \frac{3}{8}(1 - \cos\theta)^2\Gamma_- + \frac{3}{4}\sin^2\theta\Gamma_L, \quad (3)$$

where Γ_+ , Γ_- and Γ_L denote the partial rates into transverse-plus, transverse-minus and longitudinal W bosons. Integrating over $\cos\theta$ one recovers the total rate

$$\Gamma = \Gamma_+ + \Gamma_- + \Gamma_L. \quad (4)$$

One can also define a forward-backward asymmetry by considering the rate in the forward hemisphere Γ_F and in the backward hemisphere Γ_B . The forward-backward asymmetry A_{FB} is then given by

$$A_{FB} = \frac{\Gamma_F - \Gamma_B}{\Gamma_F + \Gamma_B} = \frac{3}{4} \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_- + \Gamma_L}. \quad (5)$$

The angular decay distribution is described in cascade fashion, i.e. the polar angle θ is measured in the W rest frame where the lepton pair or the quark pair emerges back to back. The angle θ denotes the polar angle between the original W^+ momentum direction and the antilepton l^+ (or the antiquark \bar{q}) in the W rest frame. The various contributions in Eq. (3) are reflected in the shape of the lepton energy spectrum in the rest frame of the top quark. From the angular factors in

Eq. (3) it is clear that the contribution of Γ_+ makes the lepton spectrum harder while Γ_- softens the spectrum where the hardness or softness is gauged relative to the longitudinal contribution. The only surviving contribution in the forward direction $\theta=0$ comes from Γ_+ . The fact that Γ_+ is predicted to be quite small implies that the lepton spectrum will be soft. The CDF measurement of the helicity content of the W^+ in top decays was in fact done by fitting the values of the helicity rates to the shape of the lepton's energy spectrum.

The Born term contributions to the normalized helicity rates are given by $\Gamma_+/\Gamma_0=0$, $\Gamma_-/\Gamma_0=(2x^2)/(1+2x^2)$ ($=0.297$) and $\Gamma_L/\Gamma_0=1/(1+2x^2)$ ($=0.703$), where Γ_0 is the total Born term rate and $x=m_W/m_t$ (see, e.g. [14]). Numerical values for the normalized helicity rates have been added in parentheses using $m_t=175$ GeV and $m_W=80.419$ GeV.

We begin with the electroweak one-loop corrections. They consist of the four tree diagram contributions shown in Fig. 1 and the one-loop contributions which are too numerous to be depicted in this paper. In the Feynman-'t Hooft gauge one has to calculate 18 different massive one-loop three-point functions as well as the many massive one-loop two-point functions needed in the renormalization program. We have recalculated all one-loop contributions analytically and have checked them numerically with the help of a XLOOPS/GINAC package that automatically calculates one-loop three-point

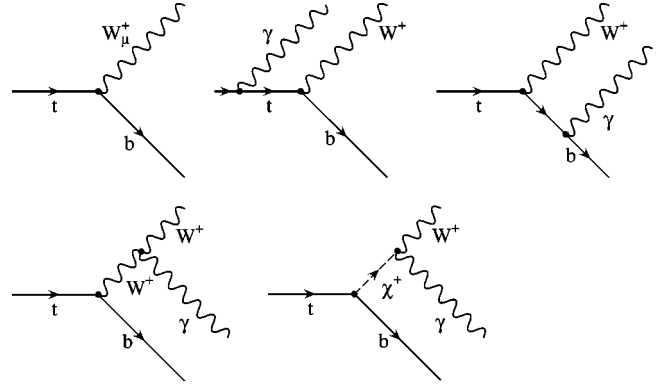


FIG. 1. Born and electroweak tree-graph contributions to $t \rightarrow b + W^+ (\gamma)$. χ^+ denotes the charged Goldstone boson.

functions [18]. Our one-loop results agree with the results of [3]. The results are too lengthy to be reproduced here in analytical form. Analytical results will be given in a forthcoming publication [19].

The tree-graph contributions $t \rightarrow b + W^+ + \gamma$ are determined in terms of the current transition matrix element $M^\mu = \langle b, \gamma | J^\mu | t \rangle$. Upon squaring the current transition matrix element one obtains the hadron tensor $H^{\mu\nu} = M^\mu M^{\nu\dagger}$. We have calculated the hadron tensor in the Feynman-'t Hooft gauge and obtain

$$\begin{aligned}
 H^{\mu\nu} = & e^2 \frac{(p_t \cdot k)(p_b \cdot k)}{(q \cdot k)^2} \left(\frac{Q_t}{p_t \cdot k} - \frac{Q_b}{p_b \cdot k} \right)^2 \left\{ -\frac{p_t \cdot k}{p_b \cdot k} [m_b^2(k^\mu p_t^\nu + k^\nu p_t^\mu - k \cdot p_t g^{\mu\nu}) + i(\epsilon^{\alpha\beta\mu\nu} p_b \cdot p_t - \epsilon^{\alpha\beta\gamma\nu} p_b^\mu p_{t,\gamma} \right. \\
 & + \epsilon^{\alpha\beta\gamma\mu} p_b^\nu p_{t,\gamma}) k_{\alpha} p_{b,\beta} - i(\epsilon^{\alpha\beta\mu\nu} k \cdot p_t - \epsilon^{\alpha\beta\gamma\nu} k^\mu p_{t,\gamma} + \epsilon^{\alpha\beta\gamma\mu} k^\nu p_{t,\gamma}) k_{\alpha} p_{b,\beta}] + \frac{p_b \cdot k}{p_t \cdot k} [m_t^2(k^\mu p_b^\nu + k^\nu p_b^\mu - k \cdot p_b g^{\mu\nu} \\
 & - i\epsilon^{\alpha\beta\mu\nu} k_{\alpha} p_{b,\beta}) - (p_t \cdot k)(p_t^\mu p_b^\nu + p_t^\nu p_b^\mu - p_t \cdot p_b g^{\mu\nu} - i\epsilon^{\alpha\beta\mu\nu} p_{t,\alpha} p_{b,\beta}) + (p_t \cdot k)(k^\mu p_b^\nu + k^\nu p_b^\mu - k \cdot p_b g^{\mu\nu} \\
 & - i\epsilon^{\alpha\beta\mu\nu} k_{\alpha} p_{b,\beta})] - (p_t \cdot p_b)(k^\mu p_t^\nu + k^\nu p_t^\mu - k \cdot p_b g^{\mu\nu} - i\epsilon^{\alpha\beta\mu\nu} k_{\alpha} p_{b,\beta}) + (p_t \cdot p_b)(k^\mu p_t^\nu + k^\nu p_t^\mu - k \cdot p_t g^{\mu\nu}) + (k \cdot p_t) \\
 & \times ((p_b + k)^\mu p_t^\nu + (p_b + k)^\nu p_t^\mu + (p_b + k) \cdot p_t g^{\mu\nu}) - (k \cdot p_b)(2p_t^\mu p_t^\nu - m_t^2 g^{\mu\nu}) + (k \cdot p_t)(2p_b^\mu p_b^\nu - m_b^2 g^{\mu\nu}) - i(\epsilon^{\alpha\beta\mu\nu} k \cdot p_t \\
 & \left. + \epsilon^{\alpha\beta\gamma\mu} k^\nu p_{t,\gamma} - \epsilon^{\alpha\beta\gamma\nu} k^\mu p_{t,\gamma}) p_{b,\alpha} p_{t,\beta} + i(\epsilon^{\alpha\beta\mu\nu} m_t^2 + \epsilon^{\alpha\beta\gamma\mu} p_t^\nu p_{t,\gamma} - \epsilon^{\alpha\beta\gamma\nu} p_t^\mu p_{t,\gamma}) k_{\alpha} p_{b,\beta} \right\} + \frac{1}{2} B^{\mu\nu} \Delta_{\text{SPF}}, \quad (6)
 \end{aligned}$$

where

$$B^{\mu\nu} = 2(p_t^\mu p_b^\nu + p_t^\nu p_b^\mu - p_b \cdot p_t g^{\mu\nu} + i\epsilon^{\mu\nu\alpha\beta} p_{b,\alpha} p_{t,\beta}) \quad (7)$$

is the Born term contribution and the soft photon factor Δ_{SPF} is given by

$$\begin{aligned}
 \Delta_{\text{SPF}} = & -e^2 \left(\frac{Q_t^2 m_t^2}{(p_t \cdot k)^2} + \frac{Q_b^2 m_b^2}{(p_b \cdot k)^2} + \frac{Q_W^2 m_W^2}{(q \cdot k)^2} - \frac{2Q_t Q_b p_t \cdot p_b}{(p_t \cdot k)(p_b \cdot k)} \right. \\
 & \left. - \frac{2Q_t Q_W p_t \cdot q}{(p_t \cdot k)(q \cdot k)} + \frac{2Q_b Q_W p_b \cdot q}{(p_b \cdot k)(q \cdot k)} \right). \quad (8)
 \end{aligned}$$

$Q_t=2/3$, $Q_b=-1/3$ and $Q_W=1$ are the electric charges of the top quark, the bottom quark and the W boson, respectively, in units of the elementary charge e . The momenta of the top quark, the bottom quark, the gauge boson and the photon are denoted by p_t , p_b , q and k , respectively. From momentum conservation one has $p_t = p_b + q + k$. For convenience and generality we have kept $m_b \neq 0$ in Eq. (6). It is noteworthy that by setting $Q_t=Q_b=1$, $Q_W=0$, $e^2=g_s^2$, and by multiplying by the color factor $N_C C_F=4$ one recovers the QCD tree graph contribution treated, e.g. in [15].

The transverse-plus, transverse-minus and longitudinal components of the hadron tensor can be obtained with the help of the projectors $(\mathbb{P}_{U+L}^{\mu\nu} - \mathbb{P}_L^{\mu\nu} + \mathbb{P}_F^{\mu\nu})/2$, $(\mathbb{P}_{U+L}^{\mu\nu} - \mathbb{P}_L^{\mu\nu}$

$-P_F^{\mu\nu})/2$ and $P_L^{\mu\nu}$ [14,15], respectively, where

$$P_{U+L}^{\mu\nu} = -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_W^2}, \quad (9)$$

$$P_L^{\mu\nu} = \frac{m_W^2}{m_t^2} \frac{1}{|\vec{q}|^2} \left(p_t^\mu - \frac{p_t \cdot q}{m_W^2} q^\mu \right) \left(p_t^\nu - \frac{p_t \cdot q}{m_W^2} q^\nu \right) \quad (10)$$

and

$$P_F^{\mu\nu} = -\frac{1}{m_t} \frac{1}{|\vec{q}|} i \epsilon^{\mu\nu\alpha\beta} p_{t,\alpha} q_\beta. \quad (11)$$

The three components of the hadron tensor are then integrated over two-dimensional phase space. The infrared and collinear singularities are regularized with the help of a (small) photon and bottom quark mass, respectively. The logarithms of the auxiliary photon and bottom quark masses can be seen to cancel against the corresponding logarithms from the one-loop contributions in all three helicity components. Details of the calculation will be published elsewhere [19].

We use the so-called G_F -renormalization scheme for the electroweak corrections where G_F , M_W and M_Z are used as input parameters [3,9]. The G_F scheme is the appropriate renormalization scheme for processes with mass scales which are much larger than M_W as in the present case. The radiative corrections are substantially larger in the so-called α scheme where α , G_F and M_Z are used as input parameters [3,9]. In our numerical results we shall also present α -scheme results along with the numerical G_F -scheme results.

Before we present our numerical results on the electroweak corrections we briefly discuss the finite width corrections to the Born term rates. The finite width corrections are obtained by replacing the q^2 integration over the δ function $\delta(q^2 - m_W^2)$ by an integration over the Breit-Wigner resonance curve where the integration is done within the phase space limits $0 < q^2 < m_t^2$. The necessary replacement is given by

$$\int_0^{m_t^2} dq^2 \delta(q^2 - m_W^2) \rightarrow \int_0^{m_t^2} dq^2 \frac{m_W \Gamma_W}{\pi} \frac{1}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \quad (12)$$

where Γ_W is the width of the W boson ($\Gamma_W = 2.12$ GeV).

We are now in the position to present our numerical results. Including the QCD one-loop corrections $\Delta\Gamma_i(\text{QCD})$ taken from [14], the electroweak one-loop corrections $\Delta\Gamma_i(\text{EW})^1$ and the finite width corrections $\Delta\Gamma_i(\text{FW})$

¹Since our numerical results are normalized to the Born term rate the (small) renormalization of the KM matrix element V_{ib} [10] does not affect our numerical results. We set $m_H = 115$ GeV. Our results are only very weakly dependent on the Higgs boson mass.

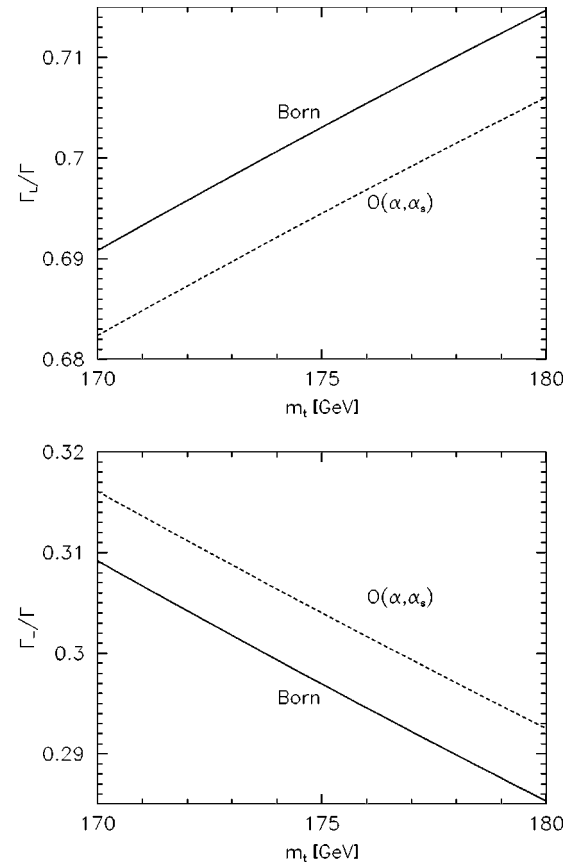


FIG. 2. Top mass dependences of the ratios Γ_L/Γ and Γ_-/Γ . Full line: Born term. Dashed line: Corrections including (QCD, electroweak (EW; G_F scheme), finite-width (FW) and $(m_b \neq 0)$ Born term corrections.

$= \Gamma_i(\text{FW}) - \Gamma_i(\text{zero width})$ calculated in this paper, and the $m_b \neq 0$ corrections to the partial Born term rates $\Delta\Gamma_i(m_b \neq 0)$ [14] we write

$$\Gamma_i = \Gamma_i(\text{Born}) + \Delta\Gamma_i(\text{QCD}) + \Delta\Gamma_i(\text{EW}) + \Delta\Gamma_i(\text{FW}) + \Delta\Gamma_i(m_b \neq 0). \quad (13)$$

It is convenient to normalize the partial rates to the total Born term rate Γ_0 . The normalized partial rates will be denoted by a hat. Thus we write $\hat{\Gamma}_i = \Gamma_i/\Gamma_0$ ($i = +, -, L$). For the transverse-minus and longitudinal rates we factor out the normalized partial Born rates $\hat{\Gamma}_i$ and write ($i = -, L$)

$$\hat{\Gamma}_i = \hat{\Gamma}_i(\text{Born}) [1 + \delta_i(\text{QCD}) + \delta_i(\text{EW}) + \delta_i(\text{FW}) + \delta\Gamma_i(m_b \neq 0)] \quad (14)$$

where $\delta_i = \Gamma_0 \Delta\Gamma_i/\Gamma_i(\text{Born})$. Writing the result in this way helps to quickly assess the percentage changes brought about by the various corrections.

Numerically one has

$$\hat{\Gamma}_- = 0.297 [1 - 0.0656(\text{QCD}) + 0.0206(\text{EW}) - 0.0197(\text{FW}) - 0.00172(m_b \neq 0)] \quad (15)$$

$$=0.297(1-0.0664) \quad (16)$$

and

$$\hat{\Gamma}_L = 0.703[1 - 0.0951(\text{QCD}) + 0.0132(\text{EW}) - 0.0138(\text{FW}) - 0.00357(m_b \neq 0)] \quad (17)$$

$$= 0.703(1 - 0.0993). \quad (18)$$

It is quite remarkable that the electroweak corrections tend to cancel the finite width corrections in both cases.

In the case of the transverse-plus rate the partial Born term rate cannot be factored out because of the fact that $\Gamma_+(\text{Born})$ is zero. In this case we present our numerical result in the form

$$\hat{\Gamma}_+ = \Delta\hat{\Gamma}_+(\text{QCD}) + \Delta\hat{\Gamma}_+(\text{EW}) + \Delta\hat{\Gamma}_+(m_b \neq 0), \quad (19)$$

$$\begin{aligned} \hat{\Gamma}_+ &= 0.000927(\text{QCD}) + 0.0000745(\text{EW}) \\ &\quad + 0.000358(m_b \neq 0) \\ &= 0.00136. \end{aligned} \quad (20)$$

Note that the finite width correction is zero in this case. Numerically the correction to $\hat{\Gamma}_+$ is only $O(0.1\%)$. If top quark decays reveal a violation of the SM ($V-A$) current structure that exceeds the 1% level, the violations must have a non-SM origin. Due to the fact that $\hat{\Gamma}_+$ is so small the forward-backward asymmetry A_{FB} is dominantly determined by $\hat{\Gamma}_-$ and $\hat{\Gamma}_L$. We find $A_{FB} = -0.2270$.

To conclude our numerical discussion we also list our numerical results for the electroweak corrections in the α scheme. In the notation of Eqs. (15), (17), (20) we obtain electroweak corrections of 0.0545(EW), 0.0474(EW) and 6.8810^{-5} (EW) which are $\approx 62\%$ larger than the corresponding corrections in the G_F scheme.

In Fig. 2 we show the top mass dependence of the ratio Γ_L/Γ and Γ_-/Γ . The Born term and the corrected curves are practically straight line curves. Since the electroweak and the finite width effects practically cancel the curves are very close to the QCD corrected curves presented in [14,15]. The horizontal displacement of the respective curves is ≈ 3.5 GeV and ≈ 3.0 GeV for Γ_L/Γ and Γ_-/Γ . One would thus make the corresponding mistakes in the top mass determination from a measurement of the angular structure functions if the Born term curves were used instead of the corrected curves. If we take $m_t = 175$ GeV as a central value a 1% relative error on the structure function measurement would allow us to determine the top quark mass with ≈ 3 GeV and ≈ 1.2 GeV accuracy, depending on whether the angular measurement was done on the longitudinal (L) or transverse-minus ($-$) mode. The latter result also holds true for the forward-backward asymmetry A_{FB} since the transverse-plus rate is negligible.

In summary, we have calculated the electroweak radiative and finite width corrections to the three diagonal structure functions that occur in the polar angle distribution of the decay $t \rightarrow b + W^+ (\rightarrow l^+ + \nu_l)$. These will be relevant for a determination of the top quark mass. We have not taken into account possible effects coming from the finite width of the top quark. These should be smaller than those calculated here for the W width because $\Gamma_t < \Gamma_W$. Also there can be QED and QCD cross-talk between the production and decay processes that could spoil the factorization-based angular decay pattern discussed in this paper. Both of these effects deserve further studies.

ACKNOWLEDGMENTS

S. Groote and M.C. Mauser were supported by the DFG (Germany) through the Graduiertenkolleg ‘‘Eichtheorien’’ at the University of Mainz.

-
- [1] CDF Collaboration, T. Affolder *et al.* Phys. Rev. Lett. **84**, 216 (2000).
 - [2] Future Electroweak Physics at the Fermilab Tevatron: Report of the TeV2000 Study Group, edited by D. Amidei and R. Brock, Fermilab-Pub-96/046, <http://www-theory.fnal.gov/TeV2000.html>
 - [3] A. Denner and T. Sack, Nucl. Phys. **B358**, 46 (1991).
 - [4] J. Liu and Y.-P. Yao, Int. J. Mod. Phys. A **6**, 4925 (1991).
 - [5] A. Czarnecki, Phys. Lett. B **252**, 467 (1990).
 - [6] C.S. Li, R.J. Oakes, and T.C. Yuan, Phys. Rev. D **43**, 3759 (1991).
 - [7] M. Jezabek and J.H. Kühn, Nucl. Phys. **B314**, 1 (1989).
 - [8] A. Ghinculov and Y.P. Yao, Mod. Phys. Lett. A **15**, 925 (2000).
 - [9] G. Eilam, R.R. Mendel, R. Migneron, and A. Soni, Phys. Rev. Lett. **66**, 3105 (1991).
 - [10] A. Barroso, L. Brücher, and R. Santos, Phys. Rev. D **62**, 096003 (2000).
 - [11] A. Czarnecki and K. Melnikov, Nucl. Phys. **B544**, 520 (1999).
 - [12] K.G. Chetyrkin, R. Harlander, T. Seidensticker, and M. Steinhauser, Phys. Rev. D **60**, 114015 (1999).
 - [13] M. Fischer, S. Groote, J.G. Körner, B. Lampe, and M.C. Mauser, Phys. Lett. B **451**, 406 (1999).
 - [14] M. Fischer, S. Groote, J.G. Körner, and M.C. Mauser, Phys. Rev. D **63**, 031501 (2001).
 - [15] M. Fischer, S. Groote, J.G. Körner, and M.C. Mauser, Phys. Rev. D **65**, 054036 (2002).
 - [16] A.A. Penin and A.A. Pivovarov, Nucl. Phys. **B549**, 217 (1999); Phys. Lett. B **443**, 264 (1998).
 - [17] M. Jezabek and J.H. Kühn, Phys. Rev. D **48**, 1910 (1993); **49**, 4970(E) (1994).
 - [18] C. Bauer and H.S. Do, Comput. Phys. Commun. **144**, 154 (2002).
 - [19] H.S. Do, S. Groote, J.G. Körner, and M.C. Mauser (unpublished).