Dark energy, dissipation, and the coincidence problem

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In a recent paper we showed that a quintessence scalar field plus a dissipative matter fluid can drive late cosmic accelerated expansion and simultaneously solve the coincidence problem. In this Brief Report we extend this result to the cases when the scalar field is replaced either by a Chaplygin gas or a tachyonic fluid.

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The low luminosity of supernovae type Ia at high redshifts strongly suggests that our present Universe experiences a period of accelerated expansion (see, e.g., Refs. [1,2] and references therein), something at variance with the long-lived Einstein–de Sitter cosmological model [3]. This combined with that the position of the first acoustic peak of the cosmic microwave background (CMB) is compatible with a critical density universe (i.e., $\Omega=1$) and that estimations of the mass density ($\Omega_m \sim 0.3$) indicate that in addition to luminous and dark matter some other component (usually referred to as "dark energy") must contribute to the critical density value. Moreover, the latter component must, on the one hand, entail a negative pressure to drive the accelerated expansion and, on the other hand, cluster only weakly so that the structure formation scenario does not get spoiled [4].

In principle, the obvious dark energy candidate should be a small cosmological constant. However, on the one hand, there are serious theoretical problems regarding its small value (many orders of magnitude below the one predicted by any straightforward quantum field theory) and, on the other hand, it is unable to give a satisfactory answer to the embarrassing question: "Why are the vacuum and matter energy densities of precisely the same order today?" (One should bear in mind that the former remains constant with expansion while the latter redshifts approximately as a^{-3}.) This is the coincidence problem.

To overcome this problem recourse was repeatedly made to a self-interacting scalar field ϕ with an equation of state $p_\phi=(\gamma_\phi-1)\rho_\phi$, where γ_ϕ is a time-varying quantity restricted to the range [0,1] so that (i) p_ϕ is always negative, and (ii) its energy density is much lower than that of matter (and radiation) at early times but comparable to the latter at recent times [5]. Thus, the usual strategy was to assume some potential $V(\phi)$ leading to the desired behavior. As shown by Padmanabhan, it is a straightforward matter to design a suitable potential [6].

In a recent paper we demonstrated that a mixture of a perfect matter fluid and quintessence field, interacting with each other just gravitationally, cannot drive acceleration and simultaneously solve the coincidence problem. However when the matter fluid is dissipative enough (i.e., it possesses a sufficiently large bulk viscous pressure π), the coincidence problem can be solved (i.e., ρ_m/ρ_ϕ tends to some constant of order unity) and the Universe has a late accelerated expansion irrespective of the assumed potential $V(\phi)$ (cf. Ref. [1]). The proof can be sketched as follows.

The Friedmann equation plus the conservation equations for matter and quintessence in a Friedmann-Robertson-Walker universe dominated by these two components (noninteracting with one another), in terms of the density parameters, are

$$1=\Omega_m+\Omega_\phi+\Omega_k, \tag{1}$$

$$\dot{\Omega}=(3\gamma-2)H(\Omega-1)\Omega, \tag{2}$$

$$\dot{\Omega}_\phi=[2+(3\gamma-2)\Omega-3\gamma_\phi]\Omega_\phi H, \tag{3}$$

where $\Omega\equiv\Omega_m+\Omega_\phi$, and γ stands for the overall baryotropic index $\gamma=(\gamma_m\Omega_m+\gamma_\phi\Omega_\phi)/\Omega$, with $\gamma_{m,\phi}\equiv 1+(p_{m,\phi}/\rho_{m,\phi})$, and such that $1\leqslant\gamma_m\leqslant 2$ and $0\leqslant\gamma_\phi<1$ (it should be noted that in general γ_m and γ_ϕ may vary with time).

From the above equations it is immediately seen that for $\Omega=1$ Eq. (3) implies that $\dot{\Omega}_\phi>0$. Consequently, at large times $\Omega_\phi\to 1$ and $\Omega_m\to 0$, i.e., the accelerated expansion $[q\equiv-\ddot{a}/(aH^2)<0\Rightarrow\gamma_\phi<2/3]$ and the coincidence problem cannot be solved simultaneously within this approach. Moreover, for the solution $\Omega=1$ to be stable the overall baryotropic index must comply with the upper bound $\gamma<2/3$ which is uncomfortably low.

However, things fare differently when one assumes the matter fluid dissipative. Indeed, Eqs. (2) and (3) generalize to

$$\dot{\Omega}=\left[3\left(\gamma+\frac{\pi}{\rho}\right)-2\right]H(\Omega-1)\Omega, \tag{4}$$

$$\dot{\Omega}_\phi=\left\{2+\left[3\left(\gamma+\frac{\pi}{\rho}\right)-2\right]\Omega-3\gamma_\phi\right\}H\Omega_\phi. \tag{5}$$

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We now may have $\dot{\Omega}_\phi < 0$ as well as $\Omega_m \to \Omega_{m0} \neq 0$ and $\Omega_\phi \to \Omega_{\phi 0} \neq 0$ for late time so long as the stationary condition

$$\gamma_m + \frac{\pi}{\rho_m} = \gamma_\phi = -\frac{2\dot{H}}{3H^2} \tag{6}$$

is satisfied. Besides, the constraint $\gamma < 2/3$ is replaced by $\gamma + (\pi/\rho) < 2/3$, which is somewhat easier to satisfy since the second law of thermodynamics implies that π must be negative for expanding fluids (see, e.g., Ref. [7]).

For spatially flat FRW universes, the asymptotic stability of the stationary solution Ω_{m0} and $\Omega_{\phi 0}$ can be studied from Eq. (5). By slightly perturbing Ω_ϕ it follows that the solution is stable (and therefore an attractor) provided the quantity $\gamma_m + \pi/\rho_m - \gamma_\phi < 0$ and tends to zero as $t \to \infty$. This coincides with the stationary condition (6).

For $\Omega \neq 1$ (i.e., when $k \neq 0$), it is expedient to introduce the ansatz $\epsilon = \epsilon_0 + \delta$ in Eqs. (4) and (5), where $\epsilon_0 \equiv (\Omega_m / \Omega_\phi)_0 \sim \mathcal{O}(1)$ and $|\delta| \ll \epsilon_0$. One finds that

$$\dot{\delta} = -\frac{3}{\Omega_\phi}\left(\frac{2}{3} - \gamma_\phi\right)\Omega_k H(\epsilon_0 + \delta). \tag{7}$$

As a consequence, the stationary solution will be stable for open FRW universes ($\Omega_k > 0$). For closed FRW universes one has to go beyond the linear analysis.

A realization of these ideas is offered in Ref. [1]. There it is seen that the space parameter is ample enough that no fine-tuning is required to have late acceleration together with the fact that both density parameters tend to constant values compatible with observation. Moreover, it is well known that for a wide class of dissipative dark energy models the attractor solutions are themselves attracted towards a common asymptotic behavior. This "superattractor" regime provides a model of the recent universe that also exhibits an excellent fit to the high redshift supernovae data luminosity and no age conflict [8].

Soon after our proof was published some other mechanisms to provide late cosmic acceleration were proposed. Here we mention two: (i) the Chaplygin gas [9] and (ii) tachyonic matter [10].

(i) The Chaplygin gas corresponds to a fluid with equation of state given by

$$p_{ch} = -\frac{A}{\rho_{ch}}, \tag{8}$$

where A is a positive-definite constant. This equation has the attractive features of providing a negative pressure and a speed of sound always real and positive—something not shared by quintessence fields. Support for this exotic component comes from higher dimensional theories [11]. Likewise, Bento *et al.* demonstrated that Eq. (8) can be derived from a Lagrangian density of Born-Infeld type [12].

Assuming that this fluid does not interact with any other component it follows that its energy density evolves as

$$\rho_{ch} = \sqrt{A + \frac{B}{a^6}}, \tag{9}$$

with B a constant. This dependence has the appealing feature of leading to $\rho_{ch} \propto a^{-3}$ at early times (dust-type behavior), and $\rho_{ch} = -p_{ch} = \sqrt{A}$ at late times (cosmological constantlike behavior). It is obvious, however, that a universe filled with just this gas, or combined with a perfect fluid dark matter component, should rely upon fine-tuning to solve the coincidence problem and start accelerating at low redshift.

With the help of Eq. (9) it is immediately seen that the equation of state (8) can be written as $p_{ch} = (\gamma_{ch} - 1)\rho_{ch}$ where

$$\gamma_{ch} = \frac{B}{B + Aa^6} \tag{10}$$

lies in the range [0,1]. As a consequence, the same argument used in Ref. [1] regarding the coincidence problem when the dark component was a quintessential scalar field also applies when the latter is replaced by the Chaplygin gas.

At late time, the dynamics is governed by attractor condition (6). So, from Eqs. (6) and (10) it follows that

$$\frac{B}{B + Aa^6} = -\frac{2\dot{H}}{3H^2} \tag{11}$$

which for $1 \ll Aa^6/B$ yields the expansion rate

$$H^2 \simeq H_0^2 e^{B/2Aa^6}, \tag{12}$$

where H_0 is an integration constant. In this regime, for $B > 0$, the Chaplygin gas leads to a de Sitter phase and the energy density of the gas behaves as $\rho \approx \sqrt{A}$. Then from the Friedmann equation one has $3H_0^2 \approx \rho_m + \sqrt{A}$, while the attractor condition (6) gives the viscous pressure $\pi \approx -\gamma_m \rho_m = -\gamma_m(3H_0^2 - \sqrt{A})$. For $B < 0$ Eq. (6) implies that $\dot{H} > 0$, so the Chaplygin gas gives rise to a superaccelerated expansion.

It has been argued that this exotic fluid not only plays the role of dark energy but also makes nonbaryonic dark matter redundant in the sense that dark matter and dark energy would just be different manifestations of the Chaplygin gas [13]. Thus, one may think that under such circumstances this scenario evades the coincidence problem. However, this is not the case. In this unified scenario the coincidence problem is only slightly alleviated. Indeed assuming a spatially flat universe, the nonbaryonic dark component would account for about ninety-six percent of the critical density and the baryonic matter (luminous and nonluminous) would account for about four percent. While these figures are not of exactly the same order, they are not so different either. They may be seen as nearly coincident, if one bears in mind that at the present time one expects—in view of Eq. (9)—the dark energy component to be nearly constant while the baryonic matter redshifts as a^{-3}.

(ii) The tachyonic matter was introduced by Sen [10] and soon after its cosmological consequences were explored—

see e.g., Ref. [14]. In particular Feinstein showed that a never-ending power law inflation may be achieved provided the tachyonic potential were given by an inverse square law, $V(\varphi)\propto 1/\varphi^2$ [15]. Obviously this toy model may also serve for the purpose of late acceleration.

The effective tachyonic fluid is described by the Lagrangian $\mathcal{L}=-V(\varphi)\sqrt{1-\partial_a\varphi\partial^a\varphi}$ where the equation of motion of the field φ in a FRW background takes the form

$$\ddot{\varphi}+3H\dot{\varphi}(1-\dot{\varphi}^2)+\frac{dV(\varphi)}{d\varphi}\frac{1-\dot{\varphi}^2}{V(\varphi)}=0.$$

The corresponding energy density and pressure are given by

$$\rho_\varphi=\frac{V(\varphi)}{\sqrt{1-\dot{\varphi}^2}} \quad \text{and} \quad p_\varphi=-V(\varphi)\sqrt{1-\dot{\varphi}^2}, \tag{13}$$

respectively. They are linked by $p_\varphi=(\gamma_\varphi-1)\rho_\varphi$ where $\gamma_\varphi=\dot{\varphi}^2$ and is limited to the interval [0,1]. Again, as in the Chaplygin gas case, the proof sketched above regarding the quintessence field also applies when the dark component is tachyonic matter, irrespective of its potential.

If at late time the scale factor obeys a power-law evolution $a(t)\propto t^\alpha$, then the attractor condition (6)—with the subscript ϕ replaced by φ—implies that $\alpha=2/3\gamma_\varphi$. Since for tachyonic matter the adiabatic index is just $\dot{\varphi}^2$, we get [6]

$$\varphi(t)=\left(\frac{2}{3\alpha}\right)^{1/2}t+\varphi_0, \tag{14}$$

as well as

$$V(\varphi)=2\alpha\left(1-\frac{2}{3\alpha}\right)^{1/2}(\varphi-\varphi_0)^{-2}, \tag{15}$$

where $\alpha>2/3$. Also, from Eq. (6), one can obtain the viscous pressure in the late regime

$$\pi=\left(\frac{2}{3\alpha}-\gamma_m\right)\rho_m<0. \tag{16}$$

In summary, the proof offered in Ref. [1] naturally extends itself to two other dark energy candidates, namely the Chaplygin gas and the tachyonic effective fluid. Again, in both cases the dissipative pressure follows by invoking the attractor condition. If future observations come to show any of them (quintessence, Chaplygin gas or tachyonic matter) as the right answer to the accelerated expansion, it could be viewed as a strong indirect support for the existence of a large dissipative pressure at cosmic scales.

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