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Aspects of causality and unitarity and comments on vortexlike configurations in an Abelian model with a Lorentz-breaking term

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The gauge-invariant Chern-Simons-type Lorentz- and CPT-breaking term is here reassessed and a spin-projector method is adopted to account for the breaking (vector) parameter. Issues such as causality, unitarity, spontaneous gauge-symmetry breaking, and vortex formation are investigated, and consistency conditions on the external vector are identified.

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I. INTRODUCTION

Symmetries are fundamental guides when one intends to systematize the study of any theory. In this sense, Lorentz and CPT invariances acquire supreme importance in modern quantum field theory, both symmetries being respected by the standard model for particle physics. A standard model description, where possible violations of such invariances are considered, was developed by Colladay and Kostelecky [1,2] and by Coleman and Glashow [3,4]. The main term that incorporates these features involves the gauge field and has the Chern-Simons form

$$\Sigma_{CS}=-\frac{1}{4}\int dx^4 \epsilon^{\mu\nu\alpha\beta}c_\mu A_\nu F_{\alpha\beta}, \tag{1}$$

where c_μ is a constant four-vector that selects a space-time direction [5–8]. One can easily show that such a term originates in a vacuum optical activity. Astrophysical results [9,10], nevertheless, contradict this possibility, putting very restrict limits on the magnitude of the c_μ four-vector.

An interesting discussion originated from the investigation of the possibility that this Chern-Simons part be radiatively generated from the fermionic sector of ordinary QED whenever an axial term, $b_\mu\bar{\Psi}\gamma^\mu\gamma^5\Psi$, that violates Lorentz and CPT symmetries, is included [11–22]. The discussion took place around some questions: Does this generated term depend on the regularization scheme? May the vanishing of this term be a result of gauge invariance and unitarity requirements? Do the astrophysical observations impose limits on the radiative correction generated by the axial term in the fermionic sector?

As shown in Ref. [18], and argued in Ref. [20], the finite radiative correction Δc_μ comes from cancellation of divergences and therefore is regularization dependent. The condition for gauge invariance can be stated in a weak way, since b_μ is a constant four-vector: it is the action that must be invariant under this transformation and not necessarily the Lagrangian density. It also means that it is not necessary to be considered a source for the violating term. In Ref. [20], it was shown that an indetermination in the radiative correction Δc_μ is not relevant for the physical content of the theory, since considering an effective constant

$$c_\mu^{eff}=(c+\Delta c+\delta c)_\mu, \tag{2}$$

where δc_μ is a finite counterterm (given some normalization condition), one can always adjust the counterterm in order to obtain the experimentally observed result.

We are then left with a careful analysis of limit situations, to which the four-vector c_μ could be submitted, in order to verify if there is physical consistency in some of these cases. In Ref. [8], the quantization consistency of an Abelian theory with the inclusion of Σ_{CS} is thoroughly analyzed. The authors study the implications on the unitarity and causality of the theory in cases where, for small magnitudes, c_μ is timelike and spacelike. The analysis shows that the behavior of these gauge field theories depends drastically on the space-time properties of c_μ. According to Ref. [8], for a purely spacelike c_μ, one finds a well-behaved Feynman propagator for the gauge field, and unitarity and microcausality are maintained. On the other hand, a timelike c_μ spoils unitarity and causality.

In this work, we analyze the possibility of having consistency of the quantization of an Abelian theory which incorporates the Lorentz- and CPT-violating term of Eq. (1), whenever gauge spontaneous symmetry breaking (SSB) takes place. The analysis is carried out by pursuing the investigation of unitarity and causality as read off from the gauge-field propagators. We therefore propose a discussion at tree approximation, without going through the canonical quantization procedure for field operators. In this investigation, we concentrate on the analysis of the residue matrices at each pole of the propagators. Basically, we check the positivity of the eigenvalues of the residue matrix associated to a given simple pole in order that unitarity may be undertaken. Higher-order poles unavoidably plague the theory with

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ghosts; this is why our analysis of the residues is restricted to the case of the simple poles. We shall find that only for c_μ spacelike both causality and unitarity can be ascertained. On the other hand, considering that SSB is interesting in such a situation (since the mass generation mechanism induced by the Higgs scalar presupposes that the theory is Lorentz invariant), we obtain that, once Lorentz symmetry is violated, there is the possibility of evading this mechanism, such that a gauge boson mass is not generated even if SSB of the local U(1) symmetry takes place.

In order to improve our comprehension of the physics presented by this theory, we also study vortexlike configurations, analyzing the influence of the direction selected by c_μ in space-time. The presence of the Chern-Simons term produces interesting modifications on the equations of motion that yield a vortex formation.

This work is outlined as follows: in Sec. II, we study the SSB and present our method to derive the gauge-field propagators. In Sec. III, we set our discussion on the poles and residues of the propagators. We study the formation of vortices in Sec. IV, and, finally, in Sec. V, we present our concluding comments.

II. GAUGE-HIGGS MODEL

We propose to carry out our analysis by starting off from the action

$$\Sigma=\int d^4x\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+(D_\mu\varphi)^*D^\mu\varphi-V(\varphi)\right\}+\Sigma_\psi+\Sigma_{cs}, \tag{3}$$

where Σ_ψ is some Fermionic action (we do not introduce fermions in our considerations here),

$$\Sigma_{cs}=-\frac{\mu}{4}\int d^4x\varepsilon^{\mu\nu\kappa\lambda}v_\mu A_\nu F_{\kappa\lambda} \tag{4}$$

is the Chern-Simons-like term, μ is a mass parameter, and v_μ is an arbitrary four vector of unit length which selects a preferred direction in the space-time ($c_\mu=\mu v_\mu$). The potential, V, given by

$$V(\varphi)=m^2|\varphi|^2+\lambda|\varphi|^4, \tag{5}$$

is the most general Higgs-like potential in four dimensions. Setting suitably the parameters such that the φ field acquires a nonvanishing vacuum expectation value (VEV), the mass spectrum of the photon would get shift upon the spontaneous breaking of local gauge symmetry by means of such VEV. The Higgs field is minimally coupled to the electromagnetic by means of its covariant derivative under U(1)-local gauge symmetry: namely,

$$D_\mu\varphi=\partial_\mu\varphi+ieQA_\mu\varphi. \tag{6}$$

This symmetry is spontaneously broken, and the new vacuum is given by

$$\langle 0|\varphi|0\rangle=a, \tag{7}$$

where

$$a=\left(-\frac{m^2}{2\lambda}\right)^{1/2};\quad m^2<0. \tag{8}$$

As usual, we adopt the polar parametrization

$$\varphi=\left(a+\frac{\sigma}{\sqrt{2}}\right)e^{i\rho/\sqrt{2}a}, \tag{9}$$

where σ e ρ are the scalar quantum fluctuations. Since we are actually interested in the analysis of the excitation spectrum, we choose to work in the unitary gauge, which is realized by setting $\rho=0$. Then, the bilinear gauge action is given as below:

$$\Sigma_g=\int d^4x\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{\mu}{4}v_\mu A_\nu F_{\kappa\lambda}\varepsilon^{\mu\nu\kappa\lambda}+\frac{M^2}{2}A_\mu A^\mu\right\}, \tag{10}$$

where $M^2=2e^2Q^2a^2$.

Its is noteworthy to stress that the SSB introduces the mass term M^2 in addition to the topological Lorentz-breaking term. μ. As we shall see throughout this section, this term will simply shift the pole induced by v^μ. If no SSB takes place, then $a=0$ and we reproduce the particle spectrum given in Ref. [8]. Another relevant issue to be tackled along this section regards the residues of the propagators at the poles, which inform about the eventual existence of negative-norm one-particle states. Later on, suitable conditions on the parameters of the model will be adopted in order that tachyon and ghost modes be suppressed from the spectrum.

We now rewrite the linearized action (10) in a more convenient form: namely,

$$\Sigma_g=\frac{1}{2}\int d^4xA^\mu\mathcal{O}_{\mu\nu}A^\nu, \tag{11}$$

where $\mathcal{O}_{\mu\nu}$ is the wave operator. The propagator is given by

$$\langle 0|T[A_\mu(x)A_\nu(y)]|0\rangle=i(\mathcal{O}^{-1})_{\mu\nu}\delta^4(x-y). \tag{12}$$

The wave operator can be written in terms of spin-projection operators as follows:

$$\mathcal{O}_{\mu\nu}=(\Box+M^2)\theta_{\mu\nu}+M^2\omega_{\mu\nu}+\mu S_{\mu\nu}, \tag{13}$$

where $\theta_{\mu\nu}$ and $\omega_{\mu\nu}$ are respectively the transverse and longitudinal projector operators

$$\theta_{\mu\nu}=g_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box},\quad \omega_{\mu\nu}=\frac{\partial_\mu\partial_\nu}{\Box}, \tag{14}$$

and

$$S^{\mu\nu}=\varepsilon^{\mu\nu\kappa\lambda}v_\kappa\partial_\lambda.$$

In order to invert the wave operator, one needs to add up other two new operators, since the above ones do not form a closed algebra, as the expression below indicates:

TABLE I. Multiplicative table fulfilled by θ, ω, S, Λ, and Σ. The products are supposed to obey the order "row times column."

	$\theta^{\alpha}{}_{\nu}$	$\omega^{\alpha}{}_{\nu}$	$S^{\alpha}{}_{\nu}$	$\Lambda^{\alpha}{}_{\nu}$	$\Sigma^{\alpha}{}_{\nu}$	$\Sigma_{\nu}{}^{\alpha}$
$\theta_{\mu\alpha}$	$\theta_{\mu\nu}$	0	$S_{\mu\nu}$	$\Lambda_{\mu\nu}-\frac{\lambda}{\Box}\Sigma_{\nu\mu}$	$\Sigma_{\mu\nu}-\lambda\omega_{\mu\nu}$	0
$\omega_{\mu\alpha}$	0	$\omega_{\mu\nu}$	0	$\frac{\lambda}{\Box}\Sigma_{\nu\mu}$	$\lambda\omega_{\mu\nu}$	$\Sigma_{\nu\mu}$
$S_{\mu\alpha}$	$S_{\mu\nu}$	0	$f_{\mu\nu}$	0	0	0
$\Lambda_{\mu\alpha}$	$\Lambda_{\mu\nu}-\frac{\lambda}{\Box}\Sigma_{\mu\nu}$	$\frac{\lambda}{\Box}\Sigma_{\mu\nu}$	0	$v^2\Lambda_{\mu\nu}$	$v^2\Sigma_{\mu\nu}$	$\lambda\Lambda_{\mu\nu}$
$\Sigma_{\mu\alpha}$	0	$\Sigma_{\mu\nu}$	0	$\lambda\Lambda_{\mu\nu}$	$\lambda\Sigma_{\mu\nu}$	$\Lambda_{\mu\nu}\Box$
$\Sigma_{\alpha\mu}$	$\Sigma_{\nu\mu}-\lambda\omega_{\mu\nu}$	$\lambda\omega_{\mu\nu}$	0	$v^2\Sigma_{\nu\mu}$	$v^2\Box\omega_{\mu\nu}$	$\lambda\Sigma_{\nu\mu}$

$$S_{\mu\alpha}S^{\alpha}{}_{\nu}=[v^2\Box-\lambda^2]\theta_{\mu\nu}-\lambda^2\omega_{\mu\nu}-\Box\Lambda_{\mu\nu}+\lambda(\Sigma_{\mu\nu}+\Sigma_{\nu\mu}) \equiv f_{\mu\nu}, \tag{15}$$

with

$$\Sigma_{\mu\nu}=v_{\mu}\partial_{\nu}, \quad \lambda\equiv\Sigma^{\mu}_{\mu}=v_{\mu}\partial^{\mu}, \quad \Lambda_{\mu\nu}=v_{\mu}v_{\nu}. \tag{16}$$

These results indicate that two new operators, namely, Σ and Λ, must be included in order to have an operator algebra with closed multiplicative rule. The operator algebra is displayed in Table I.

Using the spin-projector algebra displayed in Table I, the propagator may be obtained after a lengthy algebraic manipulation. Its explicit form in momentum space is

$$\langle A_{\mu}A_{\nu}\rangle=\frac{i}{D}\Bigg\{-(k^2-M^2)\theta_{\mu\nu}+\left(\frac{D}{M^2}-\frac{\mu^2(v\cdot k)^2}{(k^2-M^2)}\right)\omega_{\mu\nu} -i\mu S_{\mu\nu}-\frac{\mu^2k^2}{(k^2-M^2)}\Lambda_{\mu\nu} +\frac{\mu^2(v\cdot k)}{(k^2-M^2)}(\Sigma_{\mu\nu}+\Sigma_{\nu\mu})\Bigg\}, \tag{17}$$

where $D(k)=(k^2-M^2)^2+\mu^2v^2k^2-\mu^2(v\cdot k)^2$.

Now, that the gauge-field propagator is known, we are ready to discuss the particle content of the theory. We take the viewpoint that the elementary stable particles displayed in the spectrum of a Lagrangian model appear as the poles of the field propagators. However, there are issues like causality and unitarity that have to be analyzed once the poles have been identified. This matter shall be next discussed.

The result above enables us to set our discussion on the nature of the excitations present in the spectrum. At first sight, the denominator (k^2-M^2) appearing in connection with the operators ω, Λ, Σ, once multiplying the overall denominator D, could be the origin for dangerous multiple poles that plague the quantum spectrum with ghosts. For this reason, a careful study of this question is worthwhile. With this purpose, it is advisable to split our discussion into three cases: timelike, null (lightlike), and spacelike v_{μ}.

In the case v_{μ} is timelike, one can readily check that there will always be possible to find momenta k_{μ} such that $k^2=M^2$ appears as a double pole in the transverse sector (θ and S) and a triple pole in the ω, Λ, and Σ sectors. This shows that in these situations nonphysical states are present that correspond to negative norm particle states. There is no need therefore to discuss the residue matrix at these poles.

In the case v_{μ} is lightlike, it can be seen that tachyonic poles (that are simple poles) always appear; this also invalidates the model in this quantum version, for supraluminal excitations are always present in the spectrum.

However, if v_{μ} is a spacelike vector, no higher-order pole comes out; it can be shown that $k^2=M^2$ is not a zero of $D(k)$. It only appears as a simple pole for the ω, Λ, and Σ sectors. So, the model exhibits nontachyonic massive excitations associated to three simple poles: two of them coming from $D(k)$, the other being $k^2=M^2$. The fact that only the spacelike case is physically acceptable confirms the detailed study carried out by Adam and Klinkhammer in the work of Ref. [8]. Nevertheless, we should still investigate the residue at these poles so as to be sure that no ghosts are present. This shall carefully be done in the next section.

III. UNITARITY ANALYSIS IN THE SPACELIKE CASE

Our present task consists in the check of the character of the poles present for v_{μ} spacelike. Knowing that three different poles show up, we have to go through the study of the residue matrix of the vector propagator at each of its (timelike) poles $k^2=M^2$, $k^2=\tilde{m}_1^2$, and $k^2=\tilde{m}_2^2$, where $\tilde{m}_1$ and $\tilde{m}_2$ correspond to the zeroes of $D(k)$, that is, M^2, $\tilde{m}_1^2$, and $\tilde{m}_2^2$ are the physical masses at the tree approximation.

To infer about the physical nature of the simple poles, we have to calculate the eigenvalues of the residue matrix for each of these poles. This is done in the sequel. Before quoting our results, we should say that, without loss of generality, we fix our external spacelike vector as given by $v^{\mu}=(0;0,0,1)$. The momentum propagator, k^{μ}, is actually a Fourier-integration variable, so we are allowed to pick a representative momentum whenever $k^2>0$. We pursue our analysis of the residues by taking $k^{\mu}=(k^0;0,0,k^3)$.

With $k_0^2=m_1^2$, we have that

$$m_1^2=\frac{2(M^2+k_3^2)+\mu^2+\mu\sqrt{\mu^2+4(M^2+k_3^2)}}{2}; \tag{18}$$

the residue matrix reads as below:

$$R_1=\frac{1}{\sqrt{\mu^2+4(M^2+k_3^2)}}\times\begin{pmatrix}0&0&0&0\\0&m_1^2-(M^2+k_3^2)&i\mu m_1&0\\0&-i\mu m_1&m_1^2-(M^2+k_3^2)&0\\0&0&0&0\end{pmatrix}. \tag{19}$$

We calculate its eigenvalues and find only a single nonvanish eigenvalue:

$$\lambda=\frac{2|m_1|}{\sqrt{\mu^2+4(M^2+k_3^2)}}>0. \tag{20}$$

The same procedure and the same conclusions hold through for the second zero of $D(k)$ ($k^2=\tilde{m}_2^2$ with $k_0^2=m_2^2$):

$$m_2^2=\frac{2(M^2+k_3^2)+\mu^2-\mu\sqrt{\mu^2+4M^2}}{2}; \tag{21}$$

there comes out a unique nonvanishing eigenvalue ($\lambda=2|m_2|\sqrt{\mu^2+4(M^2+k_3^2)}>0$) as above.

The calculations above confirm the results found by the autors of Ref. [8]: for a spacelike v^μ, the poles of $D(k)$ respect causality (they are not tachyonic) and correspond to physically acceptable one-particle states with one degree of freedom, since the residue matrix exhibits a single positive eigenvalue

Finally, we are left with the consideration of the pole $k_0^2=(M^2+k_3^2)$. The residue matrix reads as follows:

$$R_M=\begin{pmatrix}\frac{k_3^2}{M^2}&0&0&\frac{|k_3|(M^2+k_3^2)^{1/2}}{M^2}\\0&0&0&0\\0&0&0&0\\\frac{|k_3|(M^2+k_3^2)^{1/2}}{M^2}&0&0&\frac{(M^2+k_3^2)}{M^2}\end{pmatrix}, \tag{22}$$

and again we have obtained only a nonvanish eigenvalue: $\lambda=(1/M^2)(M^2+2k_3^2)>0$. This opens up a very interesting conclusion: the M^2 pole, appearing in the longitudinal sector ($w_{\mu\nu}$), describes a physically realizable scalar mode. We are before a very peculiar result: The vector potential accommodates three physical excitations (with masses m_1^2, m_2^2, and M^2), each of them carrying a single degree of freedom; so, the external background influences the gauge field by drastically changing its physical content: instead of describing a three-degree of freedom massive excitation, it rather describes three different massive excitations, each carrying one physical degree of freedom.

We would like to report on one more possibility. As we know, the Higgs mechanism for mass generation for gauge bosons presupposes Lorentz invariance of the theory. This is no longer our case. So, we want to exhibit that, for a fixed background spacelike vector, v^μ, there may appear massless modes depending on the direction of the wave propagation. Indeed, the condition for a massless pole, $D(k)=0$ with $k^2=0$, can be written as

$$c\cdot k=\pm M^2. \tag{23}$$

Taking a spacelike c_μ of the form $c_\mu=(0;\vec{c})$, the condition above reads

$$\vec{c}\cdot\vec{k}=\mp M^2.$$

With $K^2=0$, $|\vec{k}|=k^0$, whenever $k^0\rangle 0$; then, we see that

$$\vec{c}\cdot\hat{k}=-\frac{M^2}{k^0}.$$

So, given $\vec{c}$, we can always find a k^μ such that $k^2=0$ is compatible with the condition above; for this to take place, the propagation must be along a direction with an angle bigger than 90°. The conclusion is that, according to the direction of the wave propagation, a massless pole shall always show up. This confirms the breaking of isotropy and illustrates that, despite spontaneous breaking of a local symmetry, massless excitations may be present in the spectrum.

After the technical details exposed previously, we should clarify better our analysis of the unitarity. In the paper of Ref. [8], the authors raise the question of the unitarity and they conclude that, exclusively for a spacelike v^μ, the Hamiltonian admits a semipositive self-adjoint extension, giving therefore rise to a unitary time evolution operator.

Here, the unitarity alluded to is not in the sense of a self-adjoint extension, but rather in the framework of the Hilbert space of particle states. Our analysis reveals the existence of one-particle states with negative norm square, i.e., one-particle ghost states, whenever v^μ is time or lightlike. On the other hand, when v^μ is spacelike, the poles of the vector propagator are physically acceptable and the model may be adopted as a consistent theory.

IV. A DISCUSSION ON VORTEXLIKE CONFIGURATIONS

Once our discussion on the consistency of the quantum-mechanical properties of the model has been settled down, we would like to address to an issue of a classical orientation, namely, the reassessment of vortexlike configurations in the presence of Lorentz-breaking term as the one we tackle here.

In our case, with the Chern-Simons-like term included, we get, from the action (3), the equations of motion

$$D^{\mu}D_{\mu}\varphi=-m^2\varphi-2\lambda\varphi|\varphi|^2 \tag{24}$$

and

$$\partial_{\nu}F^{\mu\nu}=ie(\varphi\partial^{\mu}\varphi^*-\varphi^*\partial^{\mu}\varphi)+2e^2A^{\mu}|\varphi|^2 + \mu\varepsilon^{\mu\nu\kappa\lambda}v_{\nu}\partial_{\kappa}A_{\lambda}, \tag{25}$$

so that we can explicitly derive the modified Maxwell equations

$$\nabla\cdot\mathbf{E}=-ie(\varphi\dot{\varphi}^*-\varphi^*\dot{\varphi})+2e^2|\varphi|^2\Phi-\mu\mathbf{v}\cdot\mathbf{B}, \tag{26}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}, \tag{27}$$

and

$$\nabla\cdot\mathbf{B}=0 \tag{28}$$

$$-\frac{\partial\mathbf{E}}{\partial t}+\nabla\times\mathbf{B}=ie(\varphi\nabla\varphi^*-\varphi^*\nabla\varphi)-2e^2|\varphi|^2\mathbf{A}-\mu v_0\mathbf{B} + \mu\mathbf{v}\times\mathbf{E}. \tag{29}$$

Before going on to analyses vortex configurations, we would like to handle the modified Maxwell equations above [Eqs. (26)–(29)] to understand that there is no room for a magnetic monopole once the Lorentz-breaking Chern-Simons term is switched on. For this purpose, we remove the charged scalar field and see that the presence of a static monopole immediately leads to

$$v_0B=v\times E. \tag{30}$$

Now, by applying the operator $\nabla\cdot$ to this equation, we come to a direct contradiction with Eq. (28). So, the modified Maxwell equations (26)–(29) do not support the presence of a Dirac-like magnetic monopole.

To analyze the vortex-type solutions, we consider a scalar field in two-dimensional space. The asymptotic solution that is proposed to be a circle (S^1),

$$\varphi=ae^{in\theta};\quad (r\to\infty), \tag{31}$$

where r and θ are polar coordinates in the plane, a is a constant and n is an integer. The gauge field assumes the form

$$\mathbf{A}=\frac{1}{e}\nabla(n\theta);\quad (r\to\infty), \tag{32}$$

or, in term of its components,

$$A_r\to 0,\quad A_{\theta}\to-\frac{n}{er};\quad (r\to\infty), \tag{33}$$

will be analyzed with our solution of the field Φ.

The breaking of Lorentz covariance prevents us from setting A_{μ} as a pure gauge at infinity, as usually done for the Nilsen-Olesen vortex. This means that $A^0=\Phi(r)$, as $r\to\infty$. The asymptotic behavior of Φ shall be fixed by the field equations, as shown in the sequel. Returning to our problem, in this situation, the magnetic field presents a cylindrical symmetry and

$$\varphi=\chi(r)e^{in\theta}. \tag{34}$$

To avoid singularity for $r\to 0$ and to keep an asymptotic solution, we make

$$\lim_{r\to 0}\chi(r)=0 \tag{35}$$

and

$$\lim_{r\to\infty}\chi(r)=a. \tag{36}$$

In the static case, Eq. (24), after summing over the components, becomes

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\chi}{dr}\right)-\left[\left(\frac{n}{r}+eA\right)^2+m^2+2\lambda\chi^2-e^2\Phi^2\right]\chi=0, \tag{37}$$

while the modified Maxwell equations take the form

$$\nabla^2\Phi+2e^2\chi^2\Phi-\mu\mathbf{v}\cdot\mathbf{B}=0 \tag{38}$$

and

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rA)\right)+2e\chi^2\left(\frac{n}{r}-eA\right)-\mu v_3\frac{d\Phi}{dr}=0. \tag{39}$$

In the asymptotic region, Eqs. (38) and (39) become

$$\nabla^2\Phi-2a^2e^2\Phi=0 \tag{40}$$

and

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rA)\right)-2e^2a^2A-\mu v_3\frac{d\Phi}{dr}=0, \tag{41}$$

where $\mathbf{B}$ has been set to zero, for $\mathbf{A}$ is a gradient at infinity. We then find

$$\Phi=Ce^{-\sqrt{2a^2e^2}r} \tag{42}$$

and

$$A(r)=CK_1(\sqrt{2}a|e|r)-i\sqrt{2}\mu v_3aeK_1(\sqrt{2}a|e|r) \times\int rdrI_1(\sqrt{2}a|e|r)e^{-\sqrt{2}a|e|r}. \tag{43}$$

So, both Φ and A falls down to zero exponentially in the asymptotic region. Note that asymptotically the complex scalar field $\varphi=\chi(r)e^{in\theta}$ goes to a nontrivial vacuum and becomes $\varphi=ae^{in\theta}$. Then the topology of the vacuum manifold is S^1. A relevant discussion at this point is the issue of the stability of the vortex configuration we have identified. This question has to be answered if we have some elements about the energy of the system. Following the results of the work of Ref. [9], we understand that, once v^{μ} is chosen to be spacelike (and, according to the results of our discussion in Sec. III, this is the unique sensible situation), the energy is limited from below, which assigns to our vortex the status of stable configuration.

More generally than in the case of Nielsen-Olesen vortices [23], Eq. (38) plays an important role as long as the electric field is concerned. If the magnetic field vortex (supposed such that $\mathbf{B}=B\hat{\mathbf{z}}$) is orthogonal to the external vector $\mathbf{v}$, then $\Phi=0$ is always a trivial solution that is compatible with the whole set of field equations.

However, whenever $\mathbf{v}\cdot\mathbf{B}\neq 0$, Φ must necessarily be nontrivial, and an electric field appears along with the magnetic flux. If this is the situation, in the asymptotic region Φ falls off exponentially, as exhibited in Eq. (42).

The appearance of an electrostatic field attached to the magnetic vortex, whenever $\mathbf{v}\cdot\mathbf{B}\neq 0$, is not surprising. Its origin may be traced back to the Lorentz-breaking term: indeed, being a Chern-Simons-like term, the electrostatic problem induces a magnetic field and the magnetostatic regime demands an electric field too. So, a nonvanishing Φ, therefore a nontrivial $\mathbf{E}$ response to the Chern-Simons Lorentz-breaking term.

V. CONCLUDING COMMENTS

The main purpose of our work is the investigation of two aspects: the first one is the quantization consistency of an Abelian model with violation of Lorentz and CPT symmetries, contemporarily with the spontaneous breaking of gauge symmetry. The other one concerns the study of classical vortexlike configurations eventually present in such a model.

The analysis carried out with the help of the propagators, derived thanks to an algebra of extended spin operators, reveals that unitarity is always violated for v^{μ} timelike and null. Whenever the external vector is spacelike, physically consistent excitations can be found that present a single degree of freedom each.

The analysis of the classical vortexlike configurations shows some interesting aspects. First, if the magnetic field vortex is orthogonal to the plane which contains the constant vector v^{μ}, then a trivial solution for the scalar potential, $\Phi=0$, is allowed. In this case, the vortex configuration will be similar to the one of the usual Abelian model. However, if $\mathbf{v}\cdot\mathbf{B}\neq 0$, we have a nontrivial solution for Φ and an electric field appears in connection with the magnetic flux. As we have already pointed out, the appearance of an electric field attached to the magnetic vortex is not surprising. It is the counterpart of what happens in a Chern-Simons theory in three dimensions, where the electrostatic problem induces a magnetic field and the magnetostatic regime demands an electric field too.

In connection with this phenomenon, the analysis of the dynamics of electrically charged particles, magnetic monopoles, and neutrinos in the region outside the vortex core becomes a well-motivated idea, for the presence of the electric field interferes now (at least for charged particles and monopoles) and alter our knowledge about the concentration of the particles in the region dominated by the vortex.

Finally, in view of the interesting results presented by Berger and Kosteleký in the paper of Ref. [24], it would be a relevant task to incorporate the (gauge-invariant) Lorentz-breaking term in the action (1), in a supersymmetric framework and therefore to study the gaugino counterpart of the action term given by Eq. (4). Results in this direction shall soon be presented elsewhere [25,26].

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