Helicity-dependent twist-two and twist-three generalized parton distributions in light-front QCD

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We investigate the helicity-dependent twist-two and twist-three generalized parton distributions in light-front Hamiltonian QCD for a massive dressed quark target. Working in the kinematical region $\xi < x < 1$, we obtain the splitting functions for the evolutions of twist-two quark and gluon distributions in a straightforward way. For the twist-three distribution, we find that all contributions are proportional to the quark mass and thus the twist-three distribution is directly related to the chiral symmetry breaking dynamics in light-front QCD. We also show that the off-forward Wandzura-Wilczek (WW) relation is violated in perturbative QCD for a massive dressed quark. We calculate the quark mass correction to the WW relation in the off-forward case and show that it is related to $h_1(x)$ in the forward limit. We extract the ''genuine twist-three'' part of the matrix element in the forward case and verify the Burkhardt-Cottingham and Efremov-Leader-Teryaev sum rules.

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I. INTRODUCTION

The generalized parton distributions (GPDs) [1] have been studied intensively in recent years. GPDs are hybrids of the usual parton distributions measured in inclusive processes such as deep inelastic scattering (DIS) and form factors measured in elastic exclusive processes. In general, they can be expressed as *off-forward* matrix elements of light-cone *bilocal* operators. In the forward limit, GPDs reduce to normal parton distributions, which can then be expressed as *forward* matrix elements of light-cone bilocal operators. The moments of GPDs over the parton momentum fraction *x* give form factors, which are given in terms of off-forward matrix elements of *local* operators. Thus, GPDs have a much richer structure and they connect various processes, both inclusive and exclusive. They provide new and important information about the structure of the hadron. They can be probed in deeply virtual Compton scattering (DVCS) and hard exclusive production of vector mesons (for recent reviews on GPDs and hard exclusive reactions; see [2–4] and references therein).

The GPDs have been investigated recently in the light-cone formalism by several authors, and an overlap representation of the plus component in terms of light-cone wave functions has been given [5,6]. GPDs have also been constructed using light-cone model wave functions [7]. The transverse and the minus components are somewhat complicated since they involve the constraint field ψ_-. They are usually called ''bad'' components, since the operators in these cases involve interaction terms and are higher twist objects. In other words, they involve direct quark-gluon dynamics. To interpret the experimental results for DVCS on a proton target in the presently accessible Q^2 range [8–11], it is of primordial importance to understand the effect of higher twist components. The perpendicular or twist-three component of the operator has been investigated using the Wandzura-Wilczek approximation [12], where the explicit interaction-dependent parts of the operator as well as the quark mass terms were neglected. In this case, the twist-three matrix element can be expressed solely in terms of twist-two GPDs. In the forward limit, these relations reduce to the Wandzura-Wilczek relation for the transversely polarized structure function g_T [13]. There is no theoretical justification for this approximation. In the forward case, recent experimental results for the transverse polarized structure function indicate that the deviation from the Wandzura-Wilczek approximated form is substantial in some kinematic range for a nucleon target [14]. Therefore, it is interesting to make a full calculation of the off-forward twist-three matrix element, within the context of perturbative QCD, taking into account the explicit interaction dependence of the operator, the mass as well as the intrinsic transverse momentum of the partons. A convenient tool is based on the light-front Hamiltonian description of composite systems utilizing many-body wave functions. Instead of a hadron target, here we consider a simpler target like a quark dressed with a gluon and calculate the off-forward matrix elements within the context of perturbative QCD. The two-particle wave function is given in terms of the light-front QCD Hamiltonian [15]. This approach has been used extensively in the recent years to calculate polarized and unpolarized distribution functions in DIS, twist two [16,17], twist three [18,19], and twist four [20], as well as the transversity distribution [21], and the transverse-momentum-dependent distributions [22]. Recently we have also extended it to calculate the off-forward matrix elements of light-front bilocal vector operators [23]. We verified the helicity sum rule in perturbation theory and showed the effect of quark mass in the twist-three matrix element, which is absent in the forward limit.

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In this work, we investigate the helicity-dependent generalized distributions, which are the off-forward matrix elements of light-front axial vector operators, in light-front Hamiltonian perturbation theory for a dressed quark target. We restrict ourselves to the region $\xi<\overline{x}<1$ of the generalized distributions, where ξ is the skewedness. In this region, the contributions come from the overlaps of two-body wave functions upto $O(\alpha_s)$. We obtain the splitting functions corresponding to the evolution of the twist-two helicity-dependent distributions in a straightforward way. In the twist-three distribution, we show the contributions from the quark-gluon interaction-dependent part, the quark mass, and the intrinsic transverse-momentum-dependent parts of the operator explicitly. We find that all three contributions are proportional to quark mass. We find that the Wandzura-Wilczek (WW) relation is not satisfied in perturbative QCD in the off-forward case for a massive quark, analogously to its forward counterpart. Our results also show that the twist-three distribution is directly related to the dynamical effect of chiral symmetry breaking in light-front QCD. We calculate the mass correction to the WW relation in the off-forward case. This contribution is related to $h_1(x)$ in the forward limit. Using the quark mass correction to the WW relation, we also obtain the "genuine twist-three" part of the matrix element. We show that the first and second moments of this are zero, which give the Burkhardt-Cottingham (BC) [24] and Efremov-Leader-Teryaev (ELT) [25] sum rules, respectively.

The plan of the paper is as follows. In Sec. II, we investigate the twist-two helicity-dependent distributions, involving both the quark and the gluon operators, for a dressed quark state. In Sec. III, we calculate the off-forward matrix element of the transverse component of the axial vector current. We investigate the WW relation in the off-forward case and the quark mass correction to it in Sec. IV. A summary and discussion are given in Sec. V. The light-front spinors for longitudinally and transversely polarized quarks are given in Appendix A. An outline of the derivation of the quark mass term in the WW approximation is given in Appendix B.

II. HELICITY-DEPENDENT TWIST-TWO DISTRIBUTIONS

A. Quark distribution

The twist-two helicity-dependent generalized distribution is given by

$$\tilde{F}^{+}_{\lambda\lambda'}=\int\frac{dz^-}{8\pi}e^{i\overline{xP}^+z^-/2}\langle P'\lambda'|\overline{\psi}\left(-\frac{z^-}{2}\right)\gamma^+\gamma^5\psi\left(\frac{z^-}{2}\right)|P\lambda\rangle. \tag{2.1}$$

Here, P,P' are the four-momenta and λ,λ' are the helicities of the initial and final states, respectively.

We work in the so called symmetric frame [5,6]. The momentum of the initial state is P^μ and that of the final state is P'^μ. The average momentum between the initial and final state is then $\overline{P}^\mu=(P^\mu+P'^\mu)/2$. The momentum transfer is given by $\Delta^\mu=P'^\mu-P^\mu$, $P'_\perp=-P_\perp=\Delta_\perp/2$, and skewedness $\xi=-\Delta^+/2\overline{P}^+$. Without any loss of generality, we take $\xi>0$. We also get $\Delta^-=\xi\overline{P}^2/\overline{P}^+$.

The above matrix element is conventionally parametrized in terms of the helicity-dependent distributions $\tilde{H}(x,\xi,t)$ and $\tilde{E}(x,\xi,t)$, where t is the invariant momentum transfer. The matrix element can also be expressed in terms of overlaps of light-front wave functions. The operator is given by

$$\int\frac{dz^-}{8\pi}e^{i\overline{xP}^+z^-/2}\overline{\psi}\left(-\frac{z^-}{2}\right)\gamma^+\gamma^5\psi\left(\frac{z^-}{2}\right)$$
$$=\frac{1}{4\pi}\int dz^-e^{i\overline{xP}^+z^-/2}\psi^{+\dagger}\left(-\frac{z^-}{2}\right)\gamma^5\psi^+\left(\frac{z^-}{2}\right). \tag{2.2}$$

Here $\psi^+=\Lambda^+\psi$ and $\Lambda^\pm=1/2\gamma^0\gamma^\pm$. The above expression is given in the light-front gauge $A^+=0$, where the path-ordered exponential between the fermion fields in the bilocal operator is unity. For simplicity we suppress the flavor indices. In the two-component representation, we have the dynamical fermion field

$$\psi^+(z)=\sum_\lambda\chi_\lambda\int\frac{dp^+d^2p^\perp}{2(2\pi)^3\sqrt{p^+}}[b(p,\lambda)e^{-iqz}$$
$$+d^\dagger(p,\lambda)e^{ipz}] \tag{2.3}$$

and the dynamical gauge field

$$A^i(z)=\sum_\lambda\int\frac{dq^+d^2q^\perp}{2(2\pi)^3q^+}[\epsilon^i_\lambda a(q,\lambda)e^{-iqz}+\text{H.c.}]. \tag{2.4}$$

Here, χ_λ is the eigenstate of σ_z in the two-component spinor of ψ^+. We have used the light-front γ matrix representation:

$$\gamma^0=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\quad \gamma^3=\begin{pmatrix}0 & i\\ i & 0\end{pmatrix},\quad \gamma^i=\begin{pmatrix}-i\tilde{\sigma}^i & 0\\ 0 & i\tilde{\sigma}^i\end{pmatrix}, \tag{2.5}$$

with $\tilde{\sigma}^1=\sigma^2$ and $\tilde{\sigma}^2=-\sigma^1$. $\epsilon^i(\lambda)$ is the polarization vector of the transverse gauge field.

The Fock space expansion of the operator is given by

$$O^{+5}=2\sum_{s,s'}\int\frac{dk^+d^2k^\perp}{2(2\pi)^3\sqrt{k^+}}\int\frac{dk'^+d^2k'^\perp}{2(2\pi)^3\sqrt{k'^+}}$$
$$\times[\delta(2\overline{xP}^+-k'^+-k^+)b^\dagger(k,s)b(k',s')$$
$$+\delta(2\overline{xP}^++k'^++k^+)d(k,-s)d^\dagger(k',-s')$$
$$+\delta(2\overline{xP}^++k^+-k'^+)d(k,-s)b(k',s')$$
$$+\delta(2\overline{xP}^++k'^+-k^+)b^\dagger(k,s)d^\dagger(k',-s')]\chi^\dagger_s\sigma_3\chi_{s'}. \tag{2.6}$$

We have $k^+>0$, $k'^+>0$, $k^+-k'^+=p^+-p'^+=2\xi\overline{p}^+$. In the kinematical region $\xi<\overline{x}<1$, only the first term in Eq. (2.6) contributes [6]. We restrict ourselves to this kinematical region.

We take the state $|P,\sigma\rangle$ of momentum P and helicity σ to be a dressed quark consisting of bare states of a quark and a quark plus a gluon:

$$|P,\sigma\rangle=\phi_1 b^\dagger(P,\sigma)|0\rangle + \sum_{\sigma_1,\lambda_2}\int\frac{dk_1^+ d^2k_1^\perp}{\sqrt{2(2\pi)^3 k_1^+}}\int\frac{dk_2^+ d^2k_2^\perp}{\sqrt{2(2\pi)^3 k_2^+}} \times\sqrt{2(2\pi)^3P^+}\,\delta^3(P-k_1-k_2) \times\phi_2(P,\sigma|k_1,\sigma_1;k_2,\lambda_2)b^\dagger(k_1,\sigma_1)a^\dagger(k_2,\lambda_2)|0\rangle. \tag{2.7}$$

Here $a^\dagger$ and $b^\dagger$ are bare gluon and quark creation operators, respectively, and ϕ_1 and ϕ_2 are the multiparton wave functions. They are the probability amplitudes to find, respectively, one bare quark and one quark plus gluon inside the dressed quark state. Up to one loop, if one considers all kinematical regions, there will be nonvanishing contributions from the overlap of three-particle and one-particle sectors of the state; this situation is similar to QED [5]. In the kinematical region we are considering, this kind of overlap is absent and it is sufficient to consider dressing only by a single gluon. The state is normalized as

$$\langle P',\lambda'|P,\lambda\rangle=2(2\pi)^3P^+\delta_{\lambda,\lambda'}\delta(P^+-P'^+)\delta^2(P^\perp-P'^\perp). \tag{2.8}$$

ϕ_1 actually gives the normalization constant of the state [16]:

$$|\phi_1|^2=1-\frac{\alpha_s}{2\pi}C_f\int_\epsilon^{1-\epsilon}dx\frac{1+x^2}{1-x}\log\frac{Q^2}{\mu^2}, \tag{2.9}$$

within order α_s. Here ϵ is a small cutoff on x.

The matrix element becomes

$$\tilde{F}^+=\sqrt{1-\xi^2}\Bigg[\psi_1^*\psi_1\delta(1-\overline{x}) + \sum_{s_1,s_2,\lambda}\int d^2q_\perp\psi^{*\uparrow}_{2s_1,\lambda}\left(\frac{\overline{x}-\xi}{1-\xi},q^\perp+\frac{1-\overline{x}}{1-\xi^2}\Delta^\perp\right) \times\chi^\dagger_{s_1}\sigma^3\chi_{s_2}\psi^\uparrow_{2s_2\lambda}\left(\frac{\overline{x}+\xi}{1+\xi},q^\perp\right)\Bigg]. \tag{2.10}$$

We have introduced Jacobi momenta $x_i,q_i^\perp$ such that $\Sigma_i x_i=1$ and $\Sigma_i q_i^\perp=0$, and the boost invariant wave functions

$$\psi_1=\phi_1,\quad \psi_2(x_i,q_i^\perp)=\sqrt{P^+}\phi(k_i^+,k_i^\perp). \tag{2.11}$$

The first term in Eq. (2.10) is the contribution from the single particle sector and the second term is the contribution of the two-particle sector of the state. Using the light-front QCD Hamiltonian, the two-particle wave function is given in terms of ψ_1 as

$$\psi^{\sigma,a}_{2\sigma_1,\lambda}(x,q^\perp)=-\frac{x(1-x)}{(q^\perp)^2}T^a\frac{1}{\sqrt{(1-x)}}\frac{g}{\sqrt{2(2\pi)^3}}\chi^\dagger_{\sigma_1} \times\left[2\frac{q^\perp}{1-x}+\frac{\tilde{\sigma}^\perp\cdot q^\perp}{x}\tilde{\sigma}^\perp-im\tilde{\sigma}^\perp\frac{(1-x)}{x}\right] \times\chi_\sigma\epsilon_\lambda^{\perp *}\psi_1, \tag{2.12}$$

where g is the coupling constant, T^a is the usual ($\frac{1}{2}$ of Gell-Mann) color matrix and m is the bare mass of the quark. In the denominator of the above expression, we have neglected terms of order m^2 compared to $(q^\perp)^2$. Using Eq. (2.1) and Eq. (2.12), we see that the linear mass terms which cause helicity flip are suppressed in the matrix element. The terms quadratic in mass do not flip helicity, but they are suppressed also. We calculate the helicity nonflip part.

Using Eq. (2.12) we get

$$\tilde{F}^+=\sqrt{1-\xi^2}\left[\delta(1-\overline{x})+\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\frac{(1+\overline{x}^2-2\xi^2)}{(1-\overline{x})(1-\xi^2)}\right] \times\psi_1^*\psi_1, \tag{2.13}$$

where $C_f=(N^2-1)/2N$ for $SU(N)$. We have cut off the transverse momentum integral at some scale Q, and μ is the factorization scale separating hard and soft dynamics [18]. Also, we have taken $|\Delta^\perp|$ to be small. For convenience, we take $\Delta^2=0$. It is important to note that the entire α_s dependency in Eq. (2.13) comes from the state and the operator is independent of the interaction. The single particle matrix element receives a contribution up to $O(\alpha_s)$ from the normalization of the state. Taking into account the normalization contribution, we get[1]

$$\tilde{F}^+=\sqrt{1-\xi^2}\left[\delta(1-\overline{x})+\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\left(\frac{3}{2}\delta(1-\overline{x}) + \frac{(1+\overline{x}^2-2\xi^2)}{(1-\overline{x})_+(1-\xi^2)}\right)\right]. \tag{2.14}$$

The end point singularity at $\overline{x}=1$ is canceled by the contribution from the normalization of the state to the single particle matrix element, as in the helicity-independent case [23]. The splitting function can easily be extracted from the above expression:

$$\tilde{P}_{qq}=C_f\frac{1+\overline{x}^2-2\xi^2}{(1-\overline{x})_+(1-\xi^2)}. \tag{2.15}$$

This agrees with the known result of [26] (when ξ in Ref. [26] is replaced by 2ξ).

[1]Here $1/(1-x)_+$ is the usual (principal value) plus prescription.

Turning next to the helicity flip part of the matrix element, we find that it arises solely from the mass term in the expression Eq. (2.12) for the two-particle wave function. The form of the wave function shows that this contribution is suppressed.

The helicity-dependent off-forward matrix element is conventionally parametrized in terms of the generalized quark distributions,

$$\tilde{F}^{+}_{\lambda\lambda'} = \frac{1}{\bar{P}^{+}} \bar{U}_{\lambda'}(P') \left[\tilde{H}_q(\bar{x},\xi,t)\gamma^{+}\gamma^{5} + \tilde{E}_q(\bar{x},\xi,t)\frac{\gamma^5 \Delta^{+}}{2M}\right] U_{\lambda}(P), \qquad (2.16)$$

where $U_\lambda(P)$ is the quark spinor in our case. The light-front spinors for longitudinally polarized quarks are given in Appendix A. Using Eq. (A3), and also the fact that the linear mass-dependent helicity flip terms give a suppressed contribution to the matrix element, we obtain that $\tilde{E}$ is suppressed (it has no logarithmic divergent part), provided $\Delta^{\perp}$ is small. We therefore get

$$\tilde{H}(\bar{x},\xi,t) = \frac{1}{2}\left[\delta(1-\bar{x}) + \frac{\alpha_s}{2\pi} C_f \log\frac{Q^2}{\mu^2}\left(\frac{3}{2}\delta(1-\bar{x}) + \frac{(1+\bar{x}^2-2\xi^2)}{(1-\bar{x})_{+}(1-\xi^2)}\right)\right]. \qquad (2.17)$$

The forward limit is easily obtained by putting $\xi=0$:

$$\tilde{H}(\bar{x},0,0) = \frac{1}{2}\left[\delta(1-\bar{x}) + \frac{\alpha_s}{2\pi} C_f \log\frac{Q^2}{\mu^2}\left(\frac{3}{2}\delta(1-\bar{x}) + \frac{(1+\bar{x}^2)}{(1-\bar{x})_{+}}\right)\right]. \qquad (2.18)$$

The above expression can be identified with $g_1(x)$ for a dressed quark target, as calculated in [17]. This gives the intrinsic helicity distribution for a quark dressed with a gluon in perturbation theory.

B. Gluon distribution

In this section, we calculate the gluon distribution

$$\tilde{F}^{+}_{g\lambda'\lambda} = -\frac{i}{4\pi\bar{x}\bar{P}^{+}}\int dz^{-} e^{i\bar{P}^{+}z^{-}\bar{x}/2} \times \langle P'\lambda'| F^{+\alpha}\left(-\frac{z^-}{2}\right)\tilde{F}^{+}_{\alpha}\left(\frac{z^-}{2}\right)|P\lambda\rangle, \qquad (2.19)$$

where

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}, \qquad \epsilon^{+1-2}=2. \qquad (2.20)$$

We use the light-front gauge $A^{+}=0$.

The Fock space expansion of the relevant part of the operator is given by

$$O_g = \frac{4i}{[2(2\pi)^3]^2}\sum_{\lambda}\lambda\int dk_1^{+}d^2k_1^{\perp}\int dk_2^{+}d^2k_2^{\perp} \times a^{\dagger}(k_1,\lambda)a(k_2,\lambda)\delta(2\bar{x}\bar{P}^{+}-k_1^{+}-k_2^{+}). \qquad (2.21)$$

Here, λ is the gluon helicity. We calculate the matrix element for a quark state dressed with a gluon. The Fock space expansion of the state is given by Eq. (2.7). The single particle sector does not contribute to the matrix element and the only contribution comes from the two-particle sector.

The matrix element is given by

$$\tilde{F}^{+}_{g} = \frac{1}{\bar{x}}\sum_{\lambda}\lambda\int d^2q^{\perp}\psi_2^{*}\left(\frac{1-\bar{x}}{1-\xi},q^{\perp}\right) \times \psi_2\left(\frac{1-\bar{x}}{1+\xi},q^{\perp}+\frac{1-\bar{x}}{(1-\xi^2)}\Delta^{\perp}\right)\sqrt{\bar{x}^2-\xi^2}. \qquad (2.22)$$

We have suppressed the quark helicity dependence of the wave functions and the sum over them. Using the full form of the two-particle wave function, we find that the helicity flip terms proportional to the quark mass give a suppressed contribution and the helicity nonflip part is given by

$$\tilde{F}^{+}_{g} = \frac{\sqrt{1-\xi^2}}{\bar{x}}\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\left[1-\frac{(1-\bar{x})^2}{(1-\xi^2)}\right], \qquad (2.23)$$

where the first (second) term in the square brackets comes from the state with gluon helicity $+1$ (-1). So we have

$$\tilde{F}^{+}_{g} = \frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\frac{[1-(1-\bar{x})^2-\xi^2]}{x\sqrt{1-\xi^2}}. \qquad (2.24)$$

Using the parametrization of $\tilde{F}^{+}_{g}$ in terms of $\tilde{H}_g$ and $\tilde{E}_g$, one can write

$$\tilde{F}^{+}_{g\lambda'\lambda} = \frac{1}{\bar{P}^{+}}\bar{U}_{\lambda'}(P')\left[\tilde{H}_g(\bar{x},\xi,t)\gamma^{+}\gamma^{5} + \tilde{E}_g(\bar{x},\xi,t)\frac{\gamma^5\Delta^{+}}{2M}\right]U_{\lambda}(P). \qquad (2.25)$$

The fact that the helicity flip part of the matrix element is suppressed means that $\tilde{E}_g$ is also suppressed. So we get

$$\tilde{H}_g(\bar{x},\xi,t) = \frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\frac{[1-(1-\bar{x})^2-\xi^2]}{x(1-\xi^2)}. \qquad (2.26)$$

The splitting function can easily be extracted and is given by

$$\tilde{P}_{qg} = C_f\frac{[1-(1-\bar{x})^2-\xi^2]}{\bar{x}(1-\xi^2)}, \qquad (2.27)$$

which again agrees with [26] when making the replacement in ξ of [26] by 2ξ. Also, in the forward limit, Eq. (2.26) gives

$$\tilde{H}_g(\bar{x},0,0)=\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\frac{[1-(1-\bar{x})^2]}{\bar{x}}. \quad (2.28)$$

or

$$\tilde{H}_g(1-\bar{x},0,0)=\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}(1+\bar{x}). \quad (2.29)$$

This gives the gluon intrinsic helicity distribution for a dressed quark target. In Eq. (2.29), we have taken $1-\bar{x}$ as the momentum fraction of the gluon, in order to compare with [17].

III. TWIST-THREE DISTRIBUTION

We now calculate the twist-three (transverse) component of the helicity-dependent off-forward distribution in perturbation theory. The matrix element of the transverse component is given by

$$\tilde{F}^{\perp}_{\lambda'\lambda}=\int\frac{dz^-}{8\pi}e^{i\bar{P}^+z^-\bar{x}/2}\langle P'\lambda'|\bar{\psi}\left(-\frac{z^-}{2}\right)\gamma^{\perp}\gamma^5\psi\left(\frac{z^-}{2}\right)|P\lambda\rangle. \quad (3.1)$$

We calculate the above matrix element for a transversely polarized dressed quark state. As before, we work in the light-front gauge $A^+=0$. The bilocal operator in this case can be written as

$$O^{\perp 5}=\bar{\psi}\left(-\frac{z^-}{2}\right)\gamma^{\perp}\gamma^5\psi\left(\frac{z^-}{2}\right)=\psi^{+\dagger}\left(-\frac{z^-}{2}\right)\alpha^{\perp}\gamma^5\psi^-\left(\frac{z^-}{2}\right)+\psi^{-\dagger}\left(-\frac{z^-}{2}\right)\alpha^{\perp}\gamma^5\psi^+\left(\frac{z^-}{2}\right). \quad (3.2)$$

The operator involves the constrained field $\psi^-(z^-/2)$ and therefore it is called higher twist. In the light-front gauge, ψ^- can be eliminated using the constraint equation

$$\psi^-=\frac{1}{i\partial^+}[\alpha^{\perp}\cdot(i\partial^{\perp}+gA^{\perp})+\gamma^0 m]\psi^+, \quad (3.3)$$

where the operator $1/\partial^+$ is defined, using the antisymmetric boundary condition, as

$$\frac{1}{\partial^+}f(x^-)=\frac{1}{4}\int_{-\infty}^{\infty}dy^-\epsilon(x^--y^-)f(y^-). \quad (3.4)$$

The operator, in terms of the dynamical fields, can be written as

$$O^{\perp 5}=O^{\perp}_m+O^{\perp}_{k^{\perp}}+O^{\perp}_g, \quad (3.5)$$

where

$$O^{\perp}_m=m\Phi^{\dagger}\frac{\sigma^1}{i\partial^+}\Phi+m\left(\frac{-\sigma^1}{i\partial^+}\Phi^{\dagger}\right)\Phi, \quad (3.6)$$

$$O^{\perp}_{k^{\perp}}=\Phi^{\dagger}\left(-\frac{z^-}{2}\right)(-\partial^2+i\sigma_3\partial^1)\frac{1}{i\partial^+}\Phi\left(\frac{z^-}{2}\right)+\left[(\partial^2+i\sigma_3\partial^1)\frac{1}{i\partial^+}\Phi^{\dagger}\left(-\frac{z^-}{2}\right)\right]\Phi\left(\frac{z^-}{2}\right). \quad (3.7)$$

$$O^{\perp}_g=g\Phi^{\dagger}\left(-\frac{z^-}{2}\right)\frac{1}{i\partial^+}(iA^2+\sigma_3A^1)\Phi\left(\frac{z^-}{2}\right)+g\left[\frac{1}{-i\partial^+}\Phi^{\dagger}\left(-\frac{z^-}{2}\right)(-iA^2+\sigma_3A^1)\right]\Phi\left(\frac{z^-}{2}\right). \quad (3.8)$$

Here, Φ is the two-component fermion field

$$\psi^+=\begin{bmatrix}\Phi\\0\end{bmatrix}. \quad (3.9)$$

The Fock space expansion of Φ is given by Eq. (2.3), with χ_λ being the two-component spinor. The operator has three parts: $O^{\perp}_m$ is the quark mass contribution, $O^{\perp}_{k^{\perp}}$ is the quark transverse momentum contribution, and $O^{\perp}_g$ is the quark-gluon interaction effect. The light-front expression clearly shows each contribution separately in the light-front gauge.

The longitudinally polarized dressed quark state is given in Eq. (2.7). The transversely polarized state is expressed in terms of the helicity states as

$$|k^+,k^{\perp},s^1\rangle=\frac{1}{\sqrt{2}}(|k^+,k^{\perp},\uparrow\rangle\pm|k^+,k^{\perp},\downarrow\rangle), \quad (3.10)$$

with $s^1=\pm m_R$, where m_R is the renormalized mass of the quark. Without any loss of generality, we take the state to be polarized along the x direction.

The contributions to the matrix element coming from the three parts of the operator are given by

$$\tilde{F}^1_m=\frac{m}{\bar{P}^+}\left[\delta(1-\bar{x})\psi_1^*\psi_1+\sum_{\sigma,\sigma'}\int d^2q^{\perp}\frac{\bar{x}}{\bar{x}^2-\xi^2}\times\psi_2^*\left(\frac{\bar{x}-\xi}{1-\xi},q^{\perp}+\frac{1-\bar{x}}{1-\xi^2}\Delta^{\perp}\right)\times\chi^{\dagger}_{\sigma}\sigma^1\chi_{\sigma'}\psi_2\left(\frac{\bar{x}+\xi}{1+\xi},q^{\perp}\right)\right]. \quad (3.11)$$

We have suppressed the quark helicity dependence of the wave function. Using the explicit form of the two-particle wave function,

$$\tilde{F}^1_m=\frac{m}{\overline{P}^+}\frac{1}{\sqrt{1-\xi^2}}\psi_1^*\psi_1 \times\left[\delta(1-\overline{x})+\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\left(\frac{2\overline{x}(\overline{x}-2\xi^2)}{(1-\overline{x})(\overline{x}^2-\xi^2)}\right)\right], \tag{3.12}$$

$$\tilde{F}^1_{k^\perp}=-i\sum_{\sigma,\sigma'}\int d^2q^\perp\psi_2^*\left(\frac{\overline{x}-\xi}{1-\xi},q^\perp+\frac{1-\overline{x}}{1-\xi^2}\Delta^\perp\right)\times\psi_2\left(\frac{\overline{x}+\xi}{1+\xi},q^\perp\right)\frac{q^2}{\overline{P}^+}\frac{\xi}{\overline{x}^2-\xi^2} +\sum_{\sigma,\sigma'}\int d^2q^\perp\psi_2^*\left(\frac{\overline{x}-\xi}{1-\xi},q^\perp+\frac{1-\overline{x}}{1-\xi^2}\Delta^\perp\right)\times\chi_\sigma^\dagger\frac{(\sigma^3q^1)}{\overline{P}^+}\chi_{\sigma'}\psi_2\left(\frac{\overline{x}+\xi}{1+\xi},q^\perp\right)\frac{\overline{x}}{\overline{x}^2-\xi^2}. \tag{3.13}$$

This gives

$$\tilde{F}^1_{k^\perp}=-\frac{m}{\overline{P}^+}\frac{1}{\sqrt{1-\xi^2}}C_f\log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}\frac{(1-\overline{x})(\overline{x}^2+\xi^2+2\overline{x}\xi^2)}{(\overline{x}^2-\xi^2)(1-\xi^2)}. \tag{3.14}$$

The interaction part gives the overlap contribution in terms of two- and one-particle wave functions and is given by

$$\tilde{F}^1_g=C_f\log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}\frac{m}{2\overline{P}^+}\frac{1}{\sqrt{1-\xi^2}}\delta(1-\overline{x}). \tag{3.15}$$

As before, we have taken $\Delta^\perp$ to be small. The interaction gives a contribution only at the end point $\overline{x}=1$. Considering the normalization contribution to the single-particle matrix element, we get the total contribution

$$\tilde{F}^1=\frac{m}{\overline{P}^+}\frac{1}{\sqrt{1-\xi^2}}\left[\delta(1-\overline{x})+C_f\log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}\times\left(2\delta(1-\overline{x})+\frac{1+2\overline{x}(1-\xi^2)-\overline{x}^2}{(1-\overline{x})_+(1-\xi^2)}\right)\right]. \tag{3.16}$$

Here, we have also considered the contribution of the normalization condition to the single-particle matrix element, which cancels the end point singularity, as in the twist-two case. Here, m is the bare quark mass. The above expression has no singularity at $\overline{x}=\xi$. In light-front theory, the linear mass term appearing in the light-front QCD Hamiltonian is renormalized as [27]

$$m_R=m\left(1+\frac{3}{4\pi}\alpha_sC_f\log\frac{Q^2}{\mu^2}\right). \tag{3.17}$$

Here m_R is the renormalized mass of the quark. The linear mass terms in the light-front QCD Hamiltonian are associated with explicit chiral symmetry breaking [28]. Also, from the above results, we find that the three contributions, including the quark transverse momentum effect and the quark gluon interaction effect, are proportional to the quark mass, which shows that the twist-three distribution is directly related to the dynamical effect of chiral symmetry breaking. In terms of the renormalized mass, we get

$$\tilde{F}^1=\frac{m_R}{\overline{P}^+}\frac{1}{\sqrt{1-\xi^2}}\left[\delta(1-\overline{x})+C_f\log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}\times\left(\frac{1}{2}\delta(1-\overline{x})+\frac{1+2\overline{x}(1-\xi^2)-\overline{x}^2}{(1-\overline{x})_+(1-\xi^2)}\right)\right]. \tag{3.18}$$

In the forward limit, this gives

$$\tilde{F}^1=\frac{m_R}{\overline{P}^+}\left[\delta(1-\overline{x})+C_f\log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}\times\left(\frac{1}{2}\delta(1-\overline{x})+\frac{(1+2\overline{x}-\overline{x}^2)}{(1-\overline{x})_+}\right)\right]. \tag{3.19}$$

By comparing the right-hand side (RHS) of the above equation with the transversely polarized structure function g_T for a dressed quark target [18], one obtains that

$$\tilde{F}^1=\frac{2m_R}{\overline{P}^+}g_T=\frac{2S_T}{\overline{P}^+}g_T, \tag{3.20}$$

since for a transversely polarized dressed quark $m_R=S_T$ (see Appendix A).

IV. EXAMINATION OF THE WANDZURA-WILCZEK RELATION IN PERTURBATION THEORY

The twist-three matrix element is parametrized as [29]

$$\tilde{F}^\perp=\frac{1}{\overline{P}^+}\overline{U}(P')\left(\gamma^\perp\gamma^5\tilde{H}+\frac{\Delta^\perp}{2M}\gamma^5\tilde{E}+\frac{\Delta^\perp\gamma^5}{2M}\tilde{G}_1+\gamma^\perp\gamma^5\tilde{G}_2 +\Delta^\perp\frac{\gamma^+}{\overline{P}^+}\gamma^5\tilde{G}_3+i\epsilon^{\perp\nu}\Delta_\nu\frac{\gamma^+}{\overline{P}^+}\tilde{G}_4\right)U(P). \tag{4.1}$$

The light-front spinors for a transversely polarized quark are given by Eq. (A4) in Appendix A. Using Eqs. (4.1),(A5),(A6) we get

$$\tilde{F}^1=\frac{2M}{\sqrt{1-\xi^2}\overline{P}^+}(\tilde{H}+\tilde{G}_2), \tag{4.2}$$

which in the forward limit gives $(2S_T/\overline{P}^+)g_T$, since $\tilde{H}(x,0,0)=g_1(x)$ and $\tilde{G}_2(x,0,0)=g_2(x)$ in the forward limit. Comparing with the result in the previous section, we see that Eq. (4.2) is in agreement with our result for a dressed quark.

Using Eqs. (2.17) and (3.18) we get

$$\tilde{G}_2(\bar{x},\xi,t)=\frac{1}{2}C_f\log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}\left[-\delta(1-\bar{x})+\frac{2(\bar{x}+\xi^2)}{(1-\xi^2)}\right], \tag{4.3}$$

which in the forward limit gives

$$\tilde{G}_2(\bar{x},0,0)=\frac{1}{2}C_f\log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}[-\delta(1-\bar{x})+2\bar{x}]. \tag{4.4}$$

The above expression agrees with g_2 for a transversely polarized dressed quark target [18].

In the Wandzura-Wilczek approximation, where the quark mass as well as the quark-gluon interaction terms are neglected, the twist-three matrix element is given in terms of twist-two matrix elements as [12]

$$\tilde{F}^{WW}_\mu(x,\xi)=\bar{U}(P')\left[\frac{\Delta^\mu\gamma^5}{2M}\tilde{E}(x,\xi)-\frac{\Delta^\mu}{2\xi}\gamma^+\gamma^5\tilde{H}(x,\xi)\right]U(P)$$
$$+\int_{-1}^{1}du\tilde{G}_\mu(u,\xi)W_+(x,u,\xi)$$
$$+i\epsilon_{\perp\mu k}\int_{-1}^{1}duG^k(u,\xi)W_-(x,u,\xi), \tag{4.5}$$

where

$$G^\mu(u,\xi)=\bar{U}(P')\left[\gamma^\mu_\perp(H+E)(u,\xi)+\frac{\Delta^\mu}{2\xi}\frac{1}{M}\right.$$
$$\times\left(u\frac{\partial}{\partial u}+\xi\frac{\partial}{\partial\xi}\right)E(u,\xi)-\frac{\Delta^\mu}{2\xi}\gamma^+$$
$$\left.\times\left(u\frac{\partial}{\partial u}+\xi\frac{\partial}{\partial\xi}\right)(H+E)(u,\xi)\right]U(P), \tag{4.6}$$

$$\tilde{G}^\mu(u,\xi)=\bar{U}(P')\left[\gamma^\mu_\perp\gamma^5\tilde{H}(u,\xi)+\frac{\Delta^\mu}{2}\frac{\gamma_5}{M}\right.$$
$$\times\left(1+u\frac{\partial}{\partial u}+\xi\frac{\partial}{\partial\xi}\right)\tilde{E}(u,\xi)$$
$$\left.-\frac{\Delta^\mu}{2\xi}\gamma^+\gamma_5\left(u\frac{\partial}{\partial u}+\xi\frac{\partial}{\partial\xi}\right)\tilde{H}(u,\xi)\right]U(P). \tag{4.7}$$

$W_\pm(x,u,\xi)$ are the Wandzura-Wilczek kernels given by

$$W_\pm(x,u,\xi)=\left[\theta(x>\xi)\frac{\theta(u>x)}{(u-\xi)}-\theta(x<\xi)\frac{\theta(u<x)}{(u-\xi)}\right]$$
$$\pm\left[\theta(x>-\xi)\frac{\theta(u>x)}{(u+\xi)}\right.$$
$$\left.-\theta(x<-\xi)\frac{\theta(u<x)}{(u+\xi)}\right]. \tag{4.8}$$

Using the light-front spinors given in Appendix A, we find, for $x>\xi$ and $\Delta^\perp=\Delta^1$, $\Delta^2=0$, that the WW relation for $\tilde{F}^\perp_{WW}$ reduces to

$$\tilde{F}^1_{WW}=\frac{2m_R}{\bar{P}^+\sqrt{1-\xi^2}}\int du\tilde{H}(u,\xi)\frac{\theta(x-\xi)\theta(u-x)}{u-\xi}. \tag{4.9}$$

In the forward limit, the RHS becomes $(2m_R/P^+)\int_x^1 dy g_1(y)/y$, which gives the well known Wandzura-Wilczek relation for the transversely polarized DIS structure function g_T. The twist-three vector distribution $F^\perp$ is similarly expressed in terms of a Wandzura-Wilczek relation; however, it vanishes in the forward limit. Using the expression for $\tilde{H}$ for a massive dressed quark in perturbation theory,

$$\tilde{F}^1_{WW}=\frac{2m_R}{\bar{P}^+\sqrt{1-\xi^2}}\frac{1}{2}\theta(\bar{x}-\xi)\left\{\frac{\theta(1-\xi)}{(1-\xi)}\right.$$
$$+\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\left[\frac{3}{2}\frac{\theta(1-\xi)}{1-\xi}+\frac{1}{(1-\xi^2)}\right.$$
$$\left.\left.\times\left[(1+\xi)\log\left(\frac{1-\xi}{\bar{x}-\xi}\right)-1+\bar{x}\right]\right]\right\}. \tag{4.10}$$

Comparing the above expression with Eq. (3.18), we see that the WW relation is not satisfied for a dressed quark state in perturbation theory, as in the forward case. This is not surprising because in the WW approximation the mass of the quark as well as the explicit interaction dependence of the operator are neglected, whereas we have obtained the full result in perturbative QCD for a massive quark.The effect of quark transverse momentum in g_T was investigated in a covariant parton model approach in [30]. Also, it is known that in the forward limit the WW relation is violated in perturbation theory [18]. However, the BC sum rule is satisfied [31,18,32].

The quark mass effect can be incorporated in the derivation of the off-forward WW relation [29]. This gives an additional contribution to $\tilde{F}^\perp$ which is of the form (see Appendix B)

$$\tilde{F}^\perp_{mass}=\frac{2m}{\bar{P}^+}\left[-\frac{\bar{x}}{\bar{x}^2-\xi^2}f^\perp(\bar{x},\xi,\Delta)\right.$$
$$\left.+\int_{\bar{x}}^1 dy\frac{y^2+\xi^2}{(y^2-\xi^2)^2}f^\perp(y,\xi,\Delta)\right] \tag{4.11}$$

for $\xi<\bar{x}<1$, where

$$f^\perp(\bar{x},\xi,\Delta)=\frac{1}{2}\int\frac{dz^-}{2\pi}e^{-i\bar{P}^+z^-\bar{x}/2}$$
$$\times\langle P'\lambda'|\bar{\psi}\left(-\frac{z^-}{2}\right)i\sigma^{+\perp}\gamma^5\psi\left(\frac{z^-}{2}\right)|P\lambda\rangle. \tag{4.12}$$

We use the parametrization [34]

$$\frac{1}{2}\int \frac{dz^-}{4\pi} e^{-i\bar{P}^+ z^- \bar{x}/2}\langle P'\lambda'|\bar{\psi}\left(-\frac{z^-}{2}\right) i\sigma^{+j}\gamma^5\psi\left(\frac{z^-}{2}\right)|P\lambda\rangle$$
$$=\frac{1}{\bar{P}^+}\bar{U}(P',\lambda')\left[H_T^q i\sigma^{+j}\gamma^5+\tilde{H}_T^q\frac{i\epsilon^{+j\alpha\beta}\Delta_\alpha P_\beta}{M^2}+E_T^q i\frac{\epsilon^{+j\alpha\beta}\Delta_\alpha\gamma_\beta}{2M}+\tilde{E}_T^q\frac{i\epsilon^{+j\alpha\beta}P_\alpha\gamma_\beta}{M}\right]U(P,\lambda). \tag{4.13}$$

Here M is the mass of the state. We calculate the above matrix element for a transversely polarized dressed quark state in perturbation theory. Using the relations of light-cone spinors and also using the normalization of the transversely polarized state, we obtain

$$H_T^q=\frac{1}{2}\left[\delta(1-\bar{x})+C_f\log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}\times\left(\frac{3}{2}\delta(1-\bar{x})+\frac{2(\bar{x}-\xi^2)}{(1-\bar{x})_+(1-\xi^2)}\right)\right], \tag{4.14}$$

which in the forward limit gives $h_1(x)$ for a dressed quark:

$$h_1(\bar{x})=\frac{1}{2}\left[\delta(1-\bar{x})+C_f\log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}\times\left(\frac{3}{2}\delta(1-\bar{x})+\frac{2\bar{x}}{(1-\bar{x})_+}\right)\right]. \tag{4.15}$$

Next, we investigate the mass corrections to the WW relation and the "genuine twist-three contribution" to the matrix element in somewhat more detail. In the forward limit, $\tilde{F}^\perp$ corresponds to g_T. We can write, in the forward limit,

$$g_T(x)=\int_x^1 dy\frac{g_1(y)}{y}+\frac{m}{M}\left(\frac{h_1(x)}{x}-\int_x^1 dy\frac{h_1(y)}{y^2}\right)+g_T^g(x), \tag{4.16}$$

where $g_T^g(x)$ is the so-called genuine twist-three contribution to g_T. If we neglect this, we get the WW relation with the quark mass correction,

$$g_T(x)=\int_x^1 dy\frac{g_1(y)}{y}+\frac{m}{M}\left(\frac{h_1(x)}{x}-\int_x^1 dy\frac{h_1(y)}{y^2}\right). \tag{4.17}$$

Here m is the quark mass and M is the mass of the target. It is very important to note that in a perturbative calculation m has to be renormalized. Taking the nth moment of both sides of Eq. (4.16) we get

$$g_2^n=-\frac{n}{n+1}g_1^n+\frac{m}{M}\frac{n}{n+1}h_1^{n-1}+g_{Tg}^n, \tag{4.18}$$

where $a^n=\int_0^1 dx\, x^n a(x)$. Using the expressions for $g_1(x)$, $g_2(x)$, and $h_1(x)$, the moments can be directly calculated:

$$g_2^n=\frac{1}{2}\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\left(-\frac{n}{n+2}\right), \tag{4.19}$$

$$g_1^n=\frac{1}{2}\left[1+\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\times\left(-\frac{1}{2}+\frac{1}{(n+1)(n+2)}-2\sum_{j=2}^{n+1}\frac{1}{j}\right)\right], \tag{4.20}$$

$$h_1^n=\frac{1}{2}\left[1+\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\left(\frac{3}{2}-2\sum_{j=1}^{n+1}\frac{1}{j}\right)\right]. \tag{4.21}$$

Using these and also the renormalization of the quark mass given by Eq. (3.17), we obtain

$$g_{Tg}^n=\frac{1}{2}\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\left[-\frac{n}{n+2}+\frac{n}{n+1}\left(\frac{3}{2}-\frac{2n+3}{(n+1)(n+2)}\right)\right]. \tag{4.22}$$

For $n=0$, the RHS of the above equation gives zero, which proves the BC sum rule. For $n=1$, the RHS of Eq. (4.22) also yields zero, which gives the Efremov-Leader-Teryaev sum rule with the correction due to the quark mass.

Next, we use Eq. (4.16) to extract the "genuine twist-three" part of g_T.

The $O(\alpha_s)$ part of g_T can be separated into two parts,

$$g_T^{(1)}=g_{TA}^{(1)}+g_{TB}^{(1)}. \tag{4.23}$$

Here $g_{TA}^{(1)}$ is the WW part with the mass corrections and $g_{TB}^{(1)}$ is the "genuine twist three part." Using Eq. (4.16) and also the expressions for $g_1(x),h_1(x)$, we get

$$g_{TB}^{(1)}=\frac{1}{2}\frac{\alpha_s}{2\pi}C_f\log\frac{Q^2}{\mu^2}\left[\frac{1}{2}\delta(1-x)-\frac{3}{2}-\log x\right]. \tag{4.24}$$

It is interesting to compare the RHS of the above equation with the forward limit of Eq. (3.15). This shows that Eq. (3.15) does not give the full "genuine" twist-three contribution but only a part of it. Also from the above expression it is easy to check that the first and second moments of $g_{TB}^{(1)}$ are zero.

We stress that the quark mass plays a very important role in the twist-three matrix element, and also, in our case, it is essential to obtain a transversely polarized state, since $S_T=m_R$, the renormalized mass of the quark. Our result shows that the twist-three generalized distribution is directly related to the chiral symmetry breaking dynamics in light-front QCD.

V. SUMMARY AND DISCUSSION

To summarize, in this work we have investigated the off-forward matrix elements of the light-cone axial vector operator. We have calculated the matrix elements of the plus and transverse components of the operator for a dressed quark in light-front Hamiltonian perturbation theory. This approach allows us to express the distributions in terms of light-front wave functions. We have restricted ourselves to the kinematical region $\xi<\bar{x}<1$. In this case, the overlaps of three-particle and one-particle wave functions are absent. We obtained the splitting functions for the evolution of the helicity-dependent twist-two quark and gluon distributions in a straightforward way. We showed that the singularity at $\bar{x}=1$ is canceled by the contribution from the normalization of the state, as in the helicity-independent case calculated earlier. The twist-two distributions reduce to the quark and gluon intrinsic helicity distributions for a dressed quark target in the forward limit. The twist-three distribution is expressed entirely in terms of the dynamical fields in the light-front gauge. This calculation shows that for the twist-two distributions the entire interaction dependence comes from the state, whereas the operator has free field structure, but in the case of twist three both the operator and the state introduce interaction dependence. The operator has three parts, an explicit mass-dependent term, a quark-gluon interaction term, and a term containing the quark transverse momentum effect. The calculation of this matrix element for a transversely polarized dressed quark shows that all the three contributions are proportional to the quark mass. Using the renormalized quark mass m_R in light-front Hamiltonian perturbation theory, we found that in the forward limit $\tilde{F}^{\perp}$ is proportional to $S_T g_T$, where S_T is the transverse polarization of the state, $S_T=m_R$ in our case. It is known that in light-front Hamiltonian QCD chirality is the same as helicity, and the terms that cause helicity flip in the light-front QCD Hamiltonian are explicit chiral symmetry breaking terms. These terms are linear in the quark mass. It is interesting to note that the quadratic mass terms do not flip helicity; however, they are suppressed here. Therefore, we concluded that $\tilde{F}^{\perp}$ is directly related to the chiral symmetry breaking dynamics in light-front QCD and the quark mass plays an important role. In particular, a finite mass is necessary to have a transversely polarized quark state. We have calculated the same off-forward matrix element in the Wandzura-Wilczek approximation and found that the actual result for a massive dressed quark deviates from the WW approximated form. The violation of the WW relation for g_T for a massive quark is known in perturbation theory and our result reduces to g_T for a massive dressed quark in the forward limit. It is to be noted that in the case of nucleons, the quark intrinsic transverse momentum effects and the quark-gluon coupling dynamics play a more complicated role and the pure quark mass effects in perturbative QCD may be suppressed by m/M where M is the hadron mass. We have also calculated the quark mass correction to the off-forward WW relation which in the forward limit reduces to a term proportional to $h_1(x)$. We extracted the "genuine twist-three part" of g_T and showed that both the BC and ELT sum rules are satisfied.

It is known that in the kinematical region $0<\bar{x}<\xi$, a contribution comes from the overlap of the three and one-particle wave functions. The GPDs in this region have a different type of evolution (Brodsky-Lepage). It will be interesting to investigate the GPDs for a dressed quark in this kinematical region using this approach and to check the various moment relations in the whole range of $\bar{x}$, $0<\bar{x}<1$. Another interesting topic for future work is to investigate the $\Delta^{\perp}$ dependence of the GPD's in the frame $\xi=0$.

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APPENDIX A: LIGHT-FRONT SPINORS

The light-front spinors for a longitudinally polarized quark of mass M and momentum P and helicity up and down, respectively, are given by [35]

$$U_{\uparrow}(P)=\frac{1}{\sqrt{2P^+}}\begin{pmatrix} P^+ + M \\ P^1+iP^2 \\ P^+ - M \\ P^1+iP^2 \end{pmatrix},$$

$$U_{\downarrow}(P)=\frac{1}{\sqrt{2P^+}}\begin{pmatrix} -P^1+iP^2 \\ P^+ + M \\ P^1-iP^2 \\ -P^+ + M \end{pmatrix}. \tag{A1}$$

Using these, we get

$$\bar{U}_{\uparrow}(P')\gamma^+\gamma^5 U_{\uparrow}(P)=2\sqrt{1-\xi^2}\bar{P}^+,$$

$$\bar{U}_{\uparrow}(P')\gamma^5 U_{\uparrow}(P)=\frac{2\xi M}{\sqrt{1-\xi^2}}. \tag{A2}$$

Also,

$$\bar{U}_{\uparrow}(P')\gamma^+\gamma^5 U_{\downarrow}(P)=0,$$

$$\bar{U}_{\uparrow}(P')\gamma^5 U_{\downarrow}(P)=\frac{-\Delta^1+i\Delta^2}{\sqrt{1-\xi^2}}. \tag{A3}$$

Light-front spinors for a transversely polarized quark are given by [36]

$$U_{\uparrow}(P)=\frac{1}{\sqrt{2P^+}}\begin{pmatrix} M+P^+-iP^2\\ -P^1\\ P^1\\ -M+P^++iP^2\end{pmatrix},$$

$$U_{\downarrow}(P)=\frac{1}{\sqrt{2P^+}}\begin{pmatrix} -M+P^+-iP^2\\ -P^1\\ P^1\\ M+P^++iP^2\end{pmatrix}. \tag{A4}$$

Using these, we get the components of the polarization vector $S^\mu=\frac{1}{2}\overline{U}(P)\gamma^\mu\gamma^5U(p)$: $S^+=0$, $S^2=0$, $S^1=M$, $S^-=2(P^1/P^+)M$.

Also,

$$\overline{U}_{\uparrow}(P')\gamma^\perp\gamma^5U_{\uparrow}(P)=\frac{2M}{\sqrt{1-\xi^2}},$$

$$\overline{U}_{\uparrow}(P')\gamma^5U_{\uparrow}(P)=0, \tag{A5}$$

and

$$\overline{U}_{\uparrow}(P')\gamma^+\gamma^5U_{\uparrow}(P)=0,$$

$$\overline{U}_{\uparrow}(P')\gamma^1U_{\uparrow}(P)=\frac{\xi\Delta^1}{\sqrt{1-\xi^2}}$$

$$\overline{U}_{\uparrow}(P')U_{\uparrow}(P)=\frac{2M}{\sqrt{1-\xi^2}}$$

$$\overline{U}_{\uparrow}(P')\gamma^+U_{\uparrow}(P)=2\overline{P}^+\sqrt{1-\xi^2}. \tag{A6}$$

APPENDIX B: MASS TERM IN WW RELATION

In this appendix, we give an outline of the derivation of Eq. (4.11). Using the approach described in [29], and taking into account the quark mass m, one gets

$$\tilde{F}^\alpha(x,\xi,t)=\int\frac{d\lambda}{2\pi}e^{-i\lambda x}\langle P'S'|\overline{\psi}\left(-\frac{z^-}{2}\right)\gamma^\alpha\gamma^5\psi\left(\frac{z^-}{2}\right)|PS\rangle$$
$$=M^\alpha(x,\xi,t)+X^\alpha(x,\xi,t), \tag{B1}$$

where $\lambda=\frac{1}{2}\overline{P}^+z^-$, $M^\alpha(x,\xi,t)$ is the mass term, and $X^\alpha(x,\xi,t)$ are all the other terms considered in the WW approximation. Here we concentrate on the mass term given by [33]

$$M^\alpha(x,\xi,t)=-im\int\frac{d\lambda}{2\pi}e^{-i\lambda x}\lambda\int_0^1du\,u[e^{i(1-u)\xi\lambda}$$
$$+e^{-i(1-u)\xi\lambda}]\frac{1}{2}\langle P'S'|\overline{\psi}\left(-\frac{uz^-}{2}\right)$$
$$\times i\sigma^{+\alpha}\gamma^5\psi\left(\frac{uz^-}{2}\right)|PS\rangle. \tag{B2}$$

If we define

$$f^\alpha(x,\xi,\Delta)=\frac{1}{2}\int\frac{dz^-}{2\pi}e^{-i\overline{P}^+z^-x/2}\langle P'S'|\overline{\psi}\left(-\frac{z^-}{2}\right)$$
$$\times i\sigma^{+\alpha}\gamma^5\psi\left(\frac{z^-}{2}\right)|PS\rangle, \tag{B3}$$

we get, from Eq. (B2),

$$M^\alpha(x,\xi,t)=\frac{m}{\overline{P}^+}\frac{\partial}{\partial x}\int_0^1du\left[f^\alpha\left(\frac{x+(1-u)\xi}{u},\xi,\Delta\right)\right.$$
$$\left.+f^\alpha\left(\frac{x-(1-u)\xi}{u},\xi,\Delta\right)\right]. \tag{B4}$$

Changing the variable, $y=[x\pm(1-u)\xi]/u$ we obtain, for $x>\xi$,

$$M^\alpha(x,\xi,t)=\frac{2m}{\overline{P}^+}\left[-\frac{x}{x^2-\xi^2}f^\alpha(x,\xi,\Delta)\right.$$
$$\left.+\int_x^1dy\frac{y^2+\xi^2}{(y^2-\xi^2)^2}f^\alpha(y,\xi,\Delta)\right]. \tag{B5}$$

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