Conventional forces can explain the anomalous acceleration of Pioneer 10

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Anderson *et al.* find the measured trajectories of Pioneer 10 and 11 spacecrafts deviate from the trajectories computed from known forces acting on them. This unmodeled acceleration (and the less well known, but similar, unmodeled torque) can be accounted for by non-isotropic radiation of spacecraft heat. Various forms of non-isotropic radiation were proposed by Katz, Murphy, and Scheffer, but Anderson *et al.* felt that none of these could explain the observed effect. This paper calculates the known effects in more detail and considers new sources of radiation, all based on spacecraft construction. These effects are then modeled over the duration of the experiment. The model reproduces the acceleration from its appearance at a heliocentric distance of 5 AU to the last measurement at 71 AU to within 10%. However, it predicts a larger decrease in acceleration between intervals I and III of the Pioneer 10 observations than is observed. This is a 2σ discrepancy from the average of the three analyses (SIGMA, CHASMP, and Markwardt). A more complex (but more speculative) model provides a somewhat better fit. Radiation forces can also plausibly explain the previously unmodeled torques, including the spindown of Pioneer 10 that is directly proportional to spacecraft bus heat, and the slow but constant spin-up of Pioneer 11. In any case, by accounting for the bulk of the acceleration, the proposed mechanism makes it much more likely that the entire effect can be explained without the need for new physics.

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I. INTRODUCTION

In [1], Anderson *et al.* compare the measured trajectory of spacecraft against the theoretical trajectory computed from known forces acting on the spacecraft. They find a small but significant discrepancy, referred to as the unmodeled or anomalous acceleration. It has an approximate magnitude of 8×10^{-8} cm s^{-2} directed approximately towards the Sun. Needless to say, *any* acceleration of *any* object that cannot be explained by conventional physics is of considerable interest. These spacecraft have been tracked very accurately over a period of many years, so the data are quite reliable, and the analysis, though complex, has been reproduced by Markwardt [2]. Explanations for the acceleration fall into two general categories—either new physics is needed or some conventional force has been overlooked.

One of the most likely candidates for the anomalous acceleration is non-isotropic radiation of spacecraft heat. This is an appealing explanation since the spacecraft dissipates about 2000 W total; if only 58 W of this total power was directed away from the Sun it could account for the acceleration. The bulk of the spacecraft heat is radiated from the two Radioisotope Thermoelectric Generators (RTGs), which convert the heat of decaying plutonium to electrical power to run the spacecraft. The remainder of the heat is radiated from various spacecraft components as a result of electrical power dissipation, and by a few small Radioisotope Heater Units (RHUs) which serve to keep crucial components warm. At least three mechanisms have been proposed that could convert heat radiation to net thrust–non-isotropic radiation from the RTGs themselves, heat from the RTGs reflected off the antenna, and non-isotropic radiation from the spacecraft bus. Anderson *et al.* reply with arguments against each of the proposed mechanisms.

Although less well known, Anderson *et al.* also report that both Pioneers experience anomalous angular accelerations. Pioneer 10 is spinning down at a rate corresponding to a torque of approximately 4.3×10^{-8} Newton-meters (N-m) (in 1986). This torque is slowly decreasing—for most of the data span, intervals I and III of Anderson, the torque is directly proportional to the power dissipated by the spacecraft bus (in interval II, it appears that gas leaks dominate the spin behavior). This proportionality, and the size of the effect, lead naturally to an explanation of non-symmetric radiation of bus heat, supplemented by somewhat larger gas leaks in interval II. Pioneer 11, when not maneuvering, was slowly and constantly spinning up. The authors speculate that the source could be gas leaks.

This paper argues once again that non-isotropic radiation is the most likely cause for both the unmodeled acceleration and at least some of the unmodeled torques. Each of the radiation asymmetries is reexamined, and a few previously unmodeled forces are included. Their sum is more than enough to account for the acceleration, and provides a plausible explanation for the unmodeled torques. Furthermore, we compare the acceleration induced by the proposed mechanisms with the measured data. We get reasonable, but not perfect, agreement over the whole data span. The main discrepancy is that the radiation thrust is predicted to decrease more quickly than the observed acceleration. The discrepancy is small (less than 1σ) from the analysis of Markwardt [2], but roughly a 2σ discrepancy from the average results of the three analyses.

Getting radiation forces right is notoriously difficult. Even for Cassini, whose construction is well known, the predicted and measured values differ by 50% [3]. However, the total force can be no larger than the sum of the possible components, though it can easily be less. Therefore the main job is

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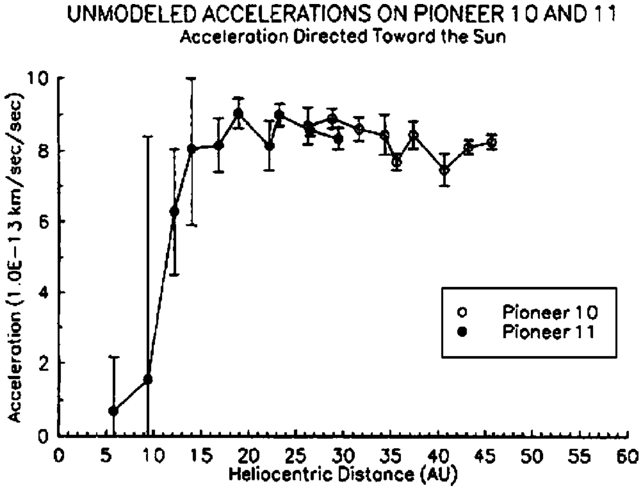

FIG. 1. Unmodeled acceleration as a function of distance from the Sun, by Anderson *et al.* [3].

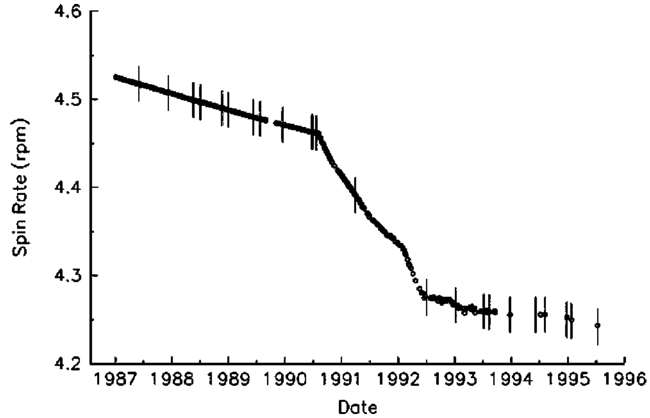

FIG. 3. Spin changes in Pioneer 10 [1]. The vertical lines indicate the times of maneuvers.

to show that enough force is available; any lesser result is easily explained.

II. THE ANOMALOUS ACCELERATION

As the Pioneer spacecraft recedes from the sun, solar forces decrease and only gravitational forces, and an occasional maneuver, should affect the trajectory of the spacecraft. Anderson, *et al.* noticed that a small additional acceleration needed to be added to make the measured data and computations match. This is the anomalous acceleration, which started to become noticeable about 5 astronomical units (AU) from the sun. It was roughly the same for Pioneer 10 and 11, as shown in Fig. 1.

Additional constraints come from the further study of Pioneer 10, since the data are of higher quality and the data span is long enough to provide significant constraints due to the radioactive decay of the heat sources. Figure 2, reproduced from [4], shows the measured acceleration from 1987 to 1998. (Although they have different horizontal axes, Fig. 2 largely follows Fig. 1 chronologically. Pioneer 10 was at 40 AU in 1987.) The authors divide the history into three intervals. Interval I is January 1987 to July of 1990, interval II from July of 1990 to July of 1992, and interval III is from July of 1992 to the June of 1998. The authors make this distinction by looking at the spin rate of the craft (see Fig. 3). In intervals I and III it was decreasing smoothly, but in interval II it decreased quickly and irregularly. They therefore consider the data from interval II to be less reliable than intervals I and III, since whatever affected the spin in interval II (probably gas leaks) may also have affected the acceleration.

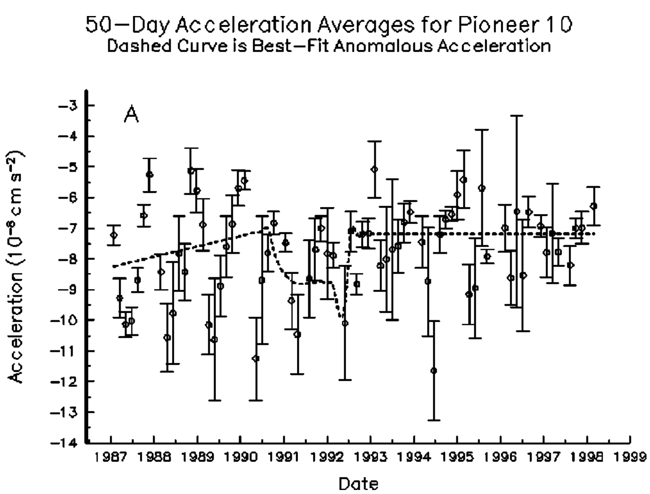

FIG. 2. Unmodeled acceleration and an empirical fit from Turyshev [4].

More recent analyses have refined these results somewhat, though the main conclusions remain unchanged. Three different analyses have been reported in the literature. SIGMA and CHASMP are two different trajectory modeling programs each with many possible analysis options. We use the best weighed least squares (WLS) results from each program, from [3]. Markwardt [2] wrote a new program with the explicit goal of an independent reanalysis.

Table I shows the most recent results from [3], which fits a constant, independent acceleration in each interval. Table II shows the results of Markwardt's reanalysis, which fits a constant plus a linear term to the data from 1987–1994. His best solution is

$$a(t) = -8.13\times10^{-8}\ \ \mathrm{cm/sec}^2 + 3.7\times10^{-17}t\ \ \mathrm{cm/sec}^3$$

where t is the time in seconds since the beginning of 1987. Accelerations are in units of 10^{-8} cm sec^{-2}. For convenience, we show the amount of directed power, in watts, that would be needed to account for each acceleration, assuming the 241 kg estimate of spacecraft mass from [3].

TABLE I. Summary of results from Anderson *et al.* [3].

Interval	SIGMA accel.	Equiv. watts	CHASMP accel.	Equiv. watts
Jan. 1987–July 1990	8.00 ± 0.01	57.8	8.25 ± 0.03	59.6
July 1992–July 1998	7.84 ± 0.01	56.7	7.91 ± 0.01	57.2

TABLE II. Summary of results from Markwardt [2].

Date	Accel.	Equiv. watts
Jan. 1987–Mar. 1994 (all data, constant acceleration)	7.70 ± 0.02	55.7
Jan. 1987–July 1990 (constant acceleration)	7.98 ± 0.02	57.7
Jan. 1987 (from linear fit)	8.13 ± 0.02	58.8
Jan. 1987–July 1990 (from linear fit)	7.93 ± 0.02	57.3
July 1992–July 1998 (extrapolated from linear fit)	7.14	51.6

Note that each program claims very small formal errors, but the programs differ from each other by far greater amounts. Therefore the errors are probably systematic, not random, and the differences between the programs are better estimates of the real uncertainties. If we create a meta-analysis by averaging over the 3 analyses (rather dubious, but it is the best we can do) we get an acceleration in interval I of 8.08 ± 0.12 (58.3 ± 0.87 watts). Agreement here is good, so this number can be regarded as fairly secure. The variation with time is less clear, with SIGMA and CHASMP showing a 2.00% and 4.12% decrease between interval I and interval III, and Markwardt finding a linear trend that predicts a 10.6% decrease in this interval. This gives a meta-prediction of $(5.55 \pm 3.63)\%$ decrease from intervals I to III.

Although much less publicized, there are other unmodeled forces acting on the craft as well. In the absence of external forces and/or spacecraft structure changes, the spin rate should not change. It does change, though, as shown in Figs. 3 and 4, from [3]. Note that Pioneer 10 is spinning down at a rate proportional to the bus power (in intervals I and III), and Pioneer 11 is spinning up, except at manuevers. From the viewpoint of fundamental physics, unexplained torques are as interesting as unexplained forces, and the two are of comparable size (13 W m and 57 W for Pioneer 10). Nonetheless, [3] assumes that the spin changes are caused by spacecraft systematics, but tries to show the acceleration changes are not. This distinction is driven partly by the data (the spin rates change at boundaries defined by spacecraft events, the acceleration does not) but also by a lack of any remotely plausible alternative. There are many (and interesting) theories that could cause acceleration (modified gravity, dark matter, and so on) but there are few proposed theories that could cause anomalous spins.

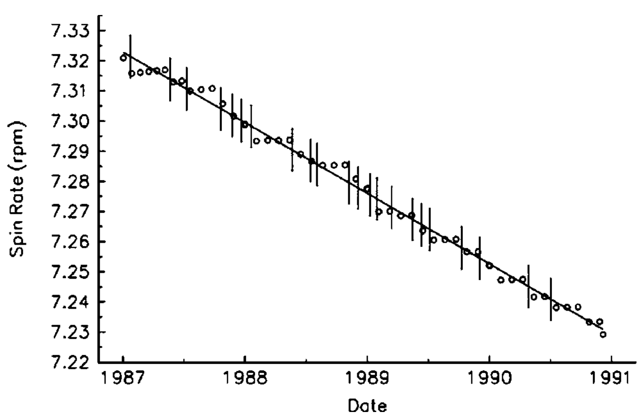

FIG. 4. Spin changes in Pioneer 11 [1]. The vertical lines indicate the times of maneuvers.

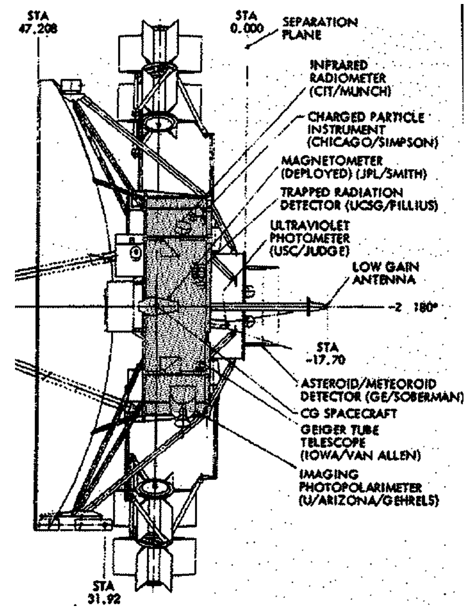

FIG. 5. Reproduction of Fig. 3.1-2 from [5]. A few lines were removed for clarity, and the main equipment compartment is shaded in.

III. PREVIOUS WORK

For the convenience of the reader, Sec. III A consists of direct quotes from [3], covering the relevant details of the Pioneer spacecraft, and Fig. 5, from [5]. Many other paper [5] and web [6,7] descriptions are available. In Sec. III C we summarize the existing literature on the hypothesis that nonisotropic radiation is responsible for the unmodeled acceleration.

A. General description of the Pioneer spacecraft, from [3]

The main equipment compartment is 36 cm deep. The hexagonal flat top and bottom have 71 cm long sides. Most of the scientific instruments' electronic units and internally mounted sensors are in an instrument bay ("squashed" hexagon) mounted on one side of the central hexagon.

At present only about 65 W of power is available to Pioneer 10 [8]. Therefore, all the instruments are no longer able to operate simultaneously. But the power subsystem continues to provide sufficient power to support the current spacecraft load: transmitter, receiver, command and data handling, and the Geiger Tube Telescope (GTT) science instrument. The sunward side of the spacecraft is the back, and the anti-sunward side, in the direction of motion, is the front [9].

B. Gas leaks

Gas leaks are always a prime suspect when unmodeled spacecraft accelerations are found. As the authors themselves say, ''Although this effect is largely unpredictable, many spacecraft have experienced gas leaks producing accelerations on the order of 10^{-7} cm/s^2'' [3]. Furthermore, the authors think that gas leaks are a significant part of the spin behavior. Why then do they think that gas leaks are not the source of the acceleration? They present four arguments:

The effect seems constant over long periods of time (many years).

The acceleration does not change as a result of thruster activity, as many gas leaks do (as valves seat and unseat).

The effect is roughly the same on two spacecrafts, Pioneer 10 and 11.

A force big enough to cause the acceleration would cause bigger spin changes than are observed unless it was directed along the spin axis.

In rebuttal, there are many possible sources of gas leaks, not all of which are variable or affected by thruster activity. (The same authors [3] speculate that a gas leak causes the spin-up of Pioneer 11, which is also constant and unaffected by maneuvers through 4 years.) Furthermore, the two spacecraft were intended to be identical, so an identical artifact such as a gas leak would not be surprising, and it might be aligned with the axis. In short, it would require an unusual gas leak, duplicated on each spacecraft, to cause the observed effect, but it is certainly allowed by physics.

In [3], the error budget for gas leaks is set as follows: First, take the biggest uncommanded spin-rate change, assume it was caused by gas leaks, assume the leak was at the spin thrusters, and then increase it a little. Thus they are setting the budget to the biggest known leak on this particular spacecraft. This is hardly a rigorous method for estimating the maximum possible size of an unknown leak, since there could be more than one leak, and locations other than the thrusters require a bigger leak for the same spin change. Furthermore, as the authors note, other spacecraft are known to have had larger leaks. Clearly at least some of the authors of [3] are not convinced by their own argument since they still suspect gas leaks as the cause of the unmodeled acceleration ([3], Sec. XII).

C. Non-isotropic radiation—previous work

Murphy suggests that the anomalous acceleration seen in the Pioneer 10 and 11 spacecrafts can be ''explained, at least in part, by non-isotropic radiative cooling of the spacecraft'' [10]. The main idea is that heat from the main and instrument compartments would radiate through the cooling louvers on the front of the craft. Anderson *et al.* argue in reply [11] that over the data span in question the louver doors were already closed (if the doors were open then the effect would surely be significant). They conclude ''the contribution of the thermal radiation to the Pioneer anomalous acceleration should be small.'' They also argue that the spacecraft power is decreasing, but the unmodeled acceleration is not. Scheffer [12] points out that the front of the spacecraft has a much higher emissivity than typical thermal blankets (even with the louvers closed), and therefore the majority of the heat will radiate from the front in any case. Anderson *et al.* [13] dispute this, based on the emissivity data in [5], which assigns a high emissivity to the thermal blanket. (These data are true but misleading—they specify the emissivity of the outer layer of the blanket. This is very different from the emissivity of the blanket as a whole, called the *effective* emissivity, which is quite low. The next section has a more in-depth discussion of this point.)

Katz [14] proposes that at least part of the acceleration is generated by radiation from the RTGs reflecting off the back of the antenna. Anderson *et al.* in [15] argue that this effect must be small since the antenna is end-on to the RTGs, and hence gets very little illumination.

Slusher (as credited by Anderson) proposed that the forward and backward surfaces of the RTGs may emit nonequally. Anderson *et al.* conclude there is no credible mechanism to explain the large difference in surfaces that would be required if this was to explain the whole effect.

IV. DISCUSSION

We consider asymmetrical radiation from five sources—the RTGs themselves, the two spacecraft compartments, RTG radiation reflected from the antenna, the radioisotope heater units (RHUs) on the spacecraft, and radiation from the feed that misses the antenna. We also consider one modeling error, a misestimation of the reflectivity of the antenna to solar radiation.

Consider thermal radiation from the spacecraft body with the louvers closed, as they have been since 9 AU. An extremely simple argument shows that the electrical power dissipated in the main spacecraft compartment must result in a significant amount of thrust. The Pioneer antenna points roughly at the Sun, and the instrument compartment is directly behind the antenna. Since the antenna blocks radiation in the sunward direction, the waste heat *must* be preferentially rejected anti-sunward. Referring to Fig. 5, a good scale model is a 60 W bulb about 4 cm behind a 25 cm diameter pie dish. The dish casts a huge shadow in the sunward direction, resulting in an average anti-sunward thrust.

However, the efficiency of conversion of heat to thrust is higher than this simple argument indicates. From [5], ''The Pioneer F/G thermal control concept consists of an insulated equipment compartment with passively controlled heat rejection via an aft [25] mounted louver system.'' Since even a closed louver is a much better radiator than thermal insulation, most of the radiation occurs from the front. It is as simple as that.

Instrument heat may also contribute to thrust, but possibly with less efficiency. This is because the instruments could possibly radiate at right angles to the spin axis through their observation ports, which are not covered with thermal blankets. Furthermore, the science compartment is much closer to the edge of the dish than the main compartment, so the dish will shadow much less of any thermal radiation generated by the science instruments.

We estimate the efficiency using the spacecraft construction. Assuming a uniform internal temperature, the power

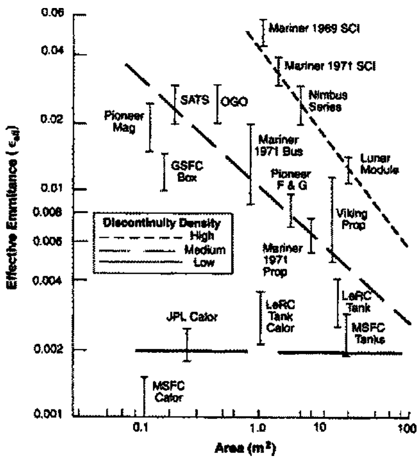

FIG. 6. Figure from [16]. Pioneer F became Pioneer 10.

emitted from each surface is proportional to the area times the effective emissivity of the surface. The front and back of the central equipment compartment have about 1.3 m^2 area, and the sides about 1.5 m^2 total. The sides and the rear of the compartment are covered with multilayer insulation (MLI) [5]. When calculating radiation from multi-layer insulation, the correct value to use is the ‘‘effective’’ emissivity, ϵ_{eff}, which accounts for the lower temperature of the outer layer [16]. (Anderson [13] points out that the outer layer of the MLI has an emissivity of 0.70 according to [5]. This is not a contradiction because the outer layer of the MLI is much colder than the interior—that is how MLI works.) From [16], the multilayer insulation used on Pioneer 10 has an effective emissivity of 0.007 to 0.01 (see Fig. 6). Assuming a value of 0.0085, and a 1998 internal temperature of 241 K [17], the main compartment will lose about 4 Watts total through the MLI on the sides and back. (Even this may be an overestimate. Two of the sides are facing 1 kW IR sources just 2 m away, and may even conduct heat *into* the compartment.) Allowing a few watts for conduction losses through wires and struts, perhaps 10% of the power (about 6 W) goes through the back, 10% through the sides, and the remaining 80% through the front. The back radiation will have a near zero efficiency (it squirts out from between the dish and the compartment at right angles to the flight path). Radiation from the side should be about 10% efficient, assuming Lambertian radiation and a 45° obstruction by the dish. Radiation from the front will be about 66% efficient, again assuming Lambertian emission. The overall efficiency of main bus radiation could therefore be as high as 54%.

Is it reasonable for the front of the main compartment to radiate the 47 W or so this requires? At an average temperature of 241 K, and assuming a flat surface, this would require an average emissivity of 0.19. From a picture of the Pioneer 10 replica in the National Air and Space Museum [18], the front of the spacecraft is rather complex, with considerable surface area (such as a rather large cylinder that connects to the booster) and a variety of surface finishes. There are also some fairly large instruments on the front of the spacecraft, such as the plasma analyzer [5]. Although the louver blades themselves have a low emissivity of 0.04 [5], a composite emissivity of 0.19 seems reasonable.

The main conclusion seems quite robust. Multi-layer insulation is specifically designed to reduce heat losses, whereas the louvers have at most one layer of obstruction even when closed, and by definition are riddled with discontinuities, which are a major source of heat leaks [16]. The lowest emissivity material on the front, 0.04, has at least 4 times the highest quoted emissivity of the sides and back. Surely, therefore, a majority of the heat will be radiated from the front of the spacecraft.

A. Feed pattern of the radio beam

An ideal radio feed antenna would illuminate its dish uniformly, with no wasted energy missing the dish. However, the feed is physically small and cannot create such a sharp-edged distribution, so some radiation always spills over the edge. Since the dish area is wasted if not fully illuminated, an optimum feed (for transmission) normally allows about 10% of the total power to miss the dish. This power is converted to sunward thrust with an efficiency of 0.7 since it is directed roughly at a 45° angle to the spin axis. The rest of the energy hits the antenna and is reflected sunward. If ϵ_{FEED} is the fraction of the energy that misses the antenna, and the transmitter is 8 W, then the net thrust towards the sun is

$$(8\ \ \mathrm{W})[\epsilon_{\mathrm{FEED}}\cdot 0.7-(1-\epsilon_{\mathrm{FEED}})].$$

This is negative as expected, since most of the radiation is sunward.

As a side note, the radio beam is circularly polarized and thus carries angular momentum away from the spacecraft. A circularly polarized beam of power P and wavelength λ will impart a torque T of

$$T=P\frac{\lambda}{hc}\hbar.$$

For the Pioneer spacecraft, $P=8$ W and $\lambda=13$ cm, so the torque is about 0.165 W m. This is about 1/100th of the total observed torque and can be neglected.

B. Radiation from the RHUs

From the diagrams in [5], 11 1-W (in 1972) radioisotope heater units are mounted to external components (thrusters and the sun sensor) to keep them sufficiently warm. The diagram is not very specific, but it appears that 10 of the units, those mounted to thrusters and the sun sensor, are behind the edge of the main dish. If we assume these radiate isotropically into the hemisphere behind the antenna, then they contribute the equivalent of 4 W of directed force in 1998. The remaining RHU is at the magnetometer and would not appear to contribute net thrust. RHU thermal radiation will decrease with a half-life of 88 years.

C. Asymmetrical radiation from the RTGs

The RTGs might contribute to the acceleration by radiating more to the front of the spacecraft than the rear. This might be caused by differing solar wind and/or dust environments, as proposed by Slusher. In [3] this is analyzed and estimated to be a small effect.

However, a more likely cause has recently been proposed by Herbert [19], who notes that solar UV radiation can bleach coatings such as that used on the RTGs, and hence make the sunward side a slightly worse radiator than the unexposed anti-sunward side. There is at least some experimental evidence to indicate this is plausible. Two commonly used coatings, Z93 and YB71, broadly similar to those on the Pioneer RTGs (metal oxide pigment and a silicate binder), were tested on the NASA Long Duration Exposure Facility (LDEF) which was left in orbit for several years and then retrieved by the space shuttle. The IR reflectance and absorbance characteristics of the coatings were measured after the exposure to orbital conditions. The Z93 coating was essentially unchanged as an IR emitter (see Fig. 49 of [20]), but the YB71 coating was a worse emitter, by up to few percent depending on the wavelength (Fig. 57 of [20]). Weighting this curve by the blackbody spectrum of a 440 K radiator gives about a 3.2% degradation in IR emission. This evidence is certainly not proof that this effect has occurred, since the exact coatings and environments differ (for example, the Earth's orbit encounters more charged particles, atomic oxygen, and contamination, in addition to the solar UV), but indicates that it is a plausible suspect.

In [3], RTG asymmetry is found to give about 6 W of thrust for a 1% asymmetry, which they take as the maximum plausible value. From the data above, this maximum is too low, since 3.2% has been observed on similar surfaces. In this paper we will assume that 1% bleaching occurred, resulting in 6 W of thrust, but up to 3% is certainly plausible in the absence of additional data.

Asymmetrical RTG radiation (to one side, not fore and aft) could also be the cause of the slow but constant spin-up observed on Pioneer 11. This would explain why the spin-up is almost constant in value and unaffected by manuevers.

D. Revisiting RTG reflection

Some of the waste heat from the RTGs will reflect from the back of the high gain antenna and be converted to thrust, as proposed by Katz [14]. Anderson *et al.* [15] argue that at most 30 W of radiation hit the antenna, and hence RTG reflection cannot account for the whole acceleration, which is true. Similarly, Slabinski [21], in an unpublished analysis, concluded that roughly 28 W of radiation hits the antenna, and hence the whole effect could not be explained. However, it is clear the effect is real, and can provide a significant fraction of the observed anomaly. Only the exact amount is in question.

The RTGs are not on-axis as viewed from the antenna. From Fig. 5 we see that the centerline of the RTGs is behind the center of the antenna. Measurements from this diagram indicate this distance is about 23.8 cm. (Slabinski [21] independently estimated 26 cm for this distance.) Another figure (not included here) from [5] shows the far end of the RTGs is 120.5 in. (or 3.06 m) from the center line. The near end of the RTGs will then be about 60 cm further in, or at about 2.46 m from the center. The antenna extends 1.37 m from the center, so the rim of the antenna is 69.8 cm off axis and 1.09 m away radially. Thus the edge of the antenna, where the illumination is by far the brightest, views the inner RTG at a 32.6° angle. This is far from on axis.

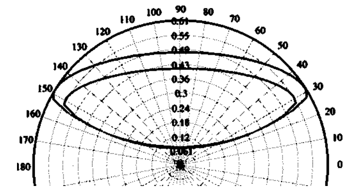

FIG. 7. Antenna size in spherical coordinates from RTGs. The radial axis is the angle from the center line in radians; the other axis is the angle around this line with the magnetometer defined as zero.

The fins of the RTGs radiate symmetrically, and all are visible from the antenna, so the center of this illumination will be 23.8 cm behind the antenna. The cylindrical center of the RTG is about 8.4 cm in radius [22] so this illumination will come from at about 15.4 cm behind the antenna. The fins have more area than the cylinder, so for this calculation we take a rough weighted average and assume a cylindrical Lambertian source 20 cm behind the antenna. We assume the inner RTG is centered 2.66 m from the center, and the outer RTG 2.91 m.

The area blocked by the antennas is shown in Fig. 7 in spherical coordinates. Numerical integration of the two areas shows about 12 W for the near RTG and 8 W for the far one if the total RTG power is 2000 W. This does not include radiation from the end caps or supporting rods.

Combining the analyses, we conclude that at least 20 W, but no more than 30 W, of radiation hits the antenna. In this paper we will use 25 W as the basis for further analysis. This energy is turned into thrust by two effects. First, the antenna shadows radiation which would otherwise go forward. An angle in the middle of the antenna is about 17° forward; this corresponds to an efficiency of 0.3 (the true efficiency is probably higher since the edge is both at a greater angle and more brightly illuminated). Next, the energy that hits the antenna must go somewhere. Some will be absorbed and re-radiated; some will bounce into space, and some will bounce and hit the instrument compartment, and be reflected or re-radiated from there. A detailed accounting seems difficult, but an overall efficiency of 0.6 to 0.9 seems reasonable (0.3 for shadowing and 0.3 to 0.6 for reflection and re-emission).

E. Total of all effects

Here we sum the maximum value of all the effects as of 1998. The total is more than enough to account for the acceleration, giving us the freedom to reduce some of the efficiencies if needed to fit the data. See Table III.

F. Antenna solar reflectivity

In this section we argue that a mismodeled solar reflection might account for the sudden onset of the anomalous force shown in Fig. 1. This argument is offered *only* as a possible explanation of the *onset* and initial decrease of the anoma-

TABLE III. Available thrust from different sources as of 1998.

Source of effect	Total power	Efficiency	Thrust	Decay
Radiation from RHUs	8	0.5	4	0.78%/year
Antenna shadow	25	0.3	7.5	0.68%/year
Antenna radiate	25	0.6	15	0.68%/year
RTG asymmetry	2000	0.009	18	0.68%/year
Feed pattern	0.8	0.7	0.6	0%
Radio beam	7.2	−1	−7.2	0%
Radiation, main bus	59	0.54	32	see text
Radiation, instruments	1	0.1	0.1	see text
Total			70.0	

lous acceleration; it is not relevant to the existence, magnitude, or source of the acceleration at later times since past about 30 AU the contribution from the solar radiation is negligible.

First, we show there is surely a possibility of error in these coefficients since the numbers for the two spacecraft disagree. We start with the data from Anderson, averaging the SIGMA and CHASMP values. We assume, following Anderson (Sec. VII B), that the trajectory was fit correctly but the mass used in the calculation was incorrect. In Table IV we correct the fitted values using the best available estimates for spacecraft mass, keeping the acceleration (and hence trajectory) the same. We would expect a nearly identical value of $\mathcal{K}$ for both spacecraft—they were the same size and painted with the same paint—but we observe $\mathcal{K}=1.66$ for one and $\mathcal{K}=1.77$ for the other, a 6.6% difference. One possible explanation is that the two spacecraft had different amounts of thermal radiation thrust, and the fitting procedure used an adjusted value of $\mathcal{K}$ to fit the observed trajectory.

Is it possible that the trajectory is right, but $\mathcal{K}$ is wrong? Anderson *et al.* [13] claim that any appreciable error in this value would have resulted in navigation errors, but the 6.6% difference between the Pioneers (surely not physically present) was easily absorbed into the fit (perhaps into the velocity increments at maneuvers, for example). This would certainly not be the first time that an excellent fit to the data was obtained with the wrong explanation.

The analysis in Anderson [3] reinforces this point. They tried to separate a constant anomalous force from the $1/r^2$ solar reflectivity, but found they were tightly correlated. On Ulysses, for example, the correlation between these two parameters was 0.888, so 90% of the change in one parameter could be explained by a spurious change in the other. If the hypothesis of this article is correct, the two parameters will be even harder to separate, since for Pioneer both the radiation contribution and the solar reflection are decreasing as the spacecraft recedes from the Sun (the total power is decreasing, and the efficiency will decrease as well as the louvers close).

TABLE IV. Solar reflectivity from Anderson *et al.* [3].

Spacecraft	Fitted $\mathcal{K}$	Mass used in fit	Actual mass	Resulting true $\mathcal{K}$
Pioneer 10	1.73	251.8	241	1.66
Pioneer 11	1.83	239.7	232	1.77

In the scenario of this paper, the acceleration has existed all along, and might even have been stronger closer to the Sun. When Pioneer was closer to the Sun, though, the fitting programs absorbed the extra acceleration by adjusting the value of $\mathcal{K}$ and perhaps other parameters such as the delta-v of maneuvers. As Pioneer receded from the Sun, and maneuvers became less frequent, adjustments to these parameters could no longer fit the trajectory properly. At this point the anomalous acceleration ''appears.'' This argument is not specific to radiation-induced acceleration—any small radial acceleration can be compensated for by adjusting the value of $\mathcal{K}$.

In this paper we model the effect of any error in $\mathcal{K}$ by introducing a fictitious force, whose value is simply the solar force on the spacecraft times the error in $\mathcal{K}$.

V. COMPARISON WITH EXPERIMENT

How well does this explanation account for the acceleration? The explanation has six adjustable parameters:

(i) ϵ_{RHU}, the fraction of RHU heat converted to thrust.

(ii) ϵ_{RTG}, the fraction of RTG heat converted to thrust. Includes both direct asymmetry and reflection from the antenna.

(iii) ϵ_{FEED}, the fraction of RF power that misses the antenna.

(iv) ϵ_{INST}, the fraction of instrument heat that is converted to thrust.

(v) ϵ_{BUS}, the fraction of main compartment heat that is converted to thrust.

(vi) K_{SOLAR}, the amount by which the solar reflection constant is underestimated. This cannot exceed about 0.2 since the true value can be no more than 2.0.

In theory all parameters are separable since they decay at different rates. In practice there are many similar solutions that cannot be distinguished by the existing data.

We compute the net thrust as follows: let d be the date in years. The total electrical power, in watts, is modeled as

$$E(d)=68+2.6\cdot(1998.5-d).$$

The RHU power, in watts, is

TABLE V. Instrument power 1987–2001. IPP=imaging photopolarimeter, TRD=trapped radiation detector, PA=plasma analyzer.

Dates	Watts	Notes
Jan. 1987–Oct. 1993	11.6	IPP off Oct. 1993
Oct. 1993–Nov. 1993	8.1	TRD off Nov. 1993
Nov. 1993–Sept. 1995	5.3	PA off Sept. 1995
Sept. 1995–present	0.8	Only Geiger active

$$RHU(d)=10.0\cdot 2^{-(d-1972)/88}.$$

The RTG heat dissipation, in watts, is

$$RTG(d)=2580\cdot 2^{-(d-1972)/88}-E(d).$$

We assume the distance from the Sun, measured in AU, increases linearly from 20 AU in 1980 to 78.5 AU in 2001:

$$r(d)=20+(d-1980)/21\cdot(78.5-20).$$

The power incident upon the antenna, in watts, is

$$SOLAR(d)=\pi(1.37\ \text{m})^2 f_{\odot}/r^2(d)$$

where $f_{\odot}=1367\ \text{W/m}^2(\text{AU})^2$ is the "solar radiation constant" at 1 AU. We use the expression from Sec. IV A for the radio thrust.

The power dissipated in the instrument compartment, $INST(d)$, is given in Table V from [23].

The other units that were turned off during this period (the Program Storage and Execution unit, and the Duration and Steering Logic) did not affect the instrument heat since they simply substituted one heat source in the main compartment for another.

The electrical power that does not go into the instruments or the radio beam goes into the main compartment:

$$BUS(d)=E(d)-INST(d)-8.0.$$

We sum the individual sources, then convert to acceleration by dividing by *c*, the speed of light, and *m*, the spacecraft mass (here 241 kg):

$$\begin{aligned} acc(d)=\frac{1}{c\cdot m}\{&\epsilon_{RHU}\cdot RHU(d)+\epsilon_{RTG}\cdot RTG(d)\\ &+(8\ \text{w})[\epsilon_{\text{FEED}}\cdot 0.7-(1-\epsilon_{\text{FEED}})]\\ &+\epsilon_{INST}\cdot INST(d)+\epsilon_{BUS}\cdot BUS(d)\\ &-K_{SOLAR}*SOLAR(d)\}.\end{aligned}$$

To examine the fit, we use the plots from [3,4], and try to fit them with our model. We make three fits. The first is a conservative fit, using only known and documented spacecraft characteristics. The second is the nominal fit, adding in effects such as RTG asymmetry that are plausible but not proven. The third is constructed to get the best possible fit to the data, but might be physically unrealistic.

The conservative fit uses only *known* and *documented* spacecraft characteristics. These are that the front of the spacecraft is a better radiator than the sides [5,16], and that the antenna will block and reflect some of the RTG radiation [3,5,14,21]. A good fit is obtained with the following:

$\epsilon_{RTG}=0.01$. 25 W hit the antenna, 30% blockage efficiency, and 50% reflection efficiency.

$\epsilon_{INST}=0.51$. Instruments same as main bus for simplicity.

$\epsilon_{BUS}=0.51$. About 80% the main bus heat goes out the front, with Lambertian efficiency.

This model correctly predicts 58.6 W in interval I, but predicts a decrease in interval III to 48.6 W. This is a 17% decrease as opposed to the 3% measured in Anderson and 10.6% of Markwardt. This model does not explain the onset at 5 AU, and overpredicts the rate of decrease, but it shows that at most 20% of the effect can be due to new physics. At the very least, 80% of the effect can be accounted for by entirely conventional physics, based on known, documented, and measured spacecraft construction.

The nominal fit adds radiation from the RHUs, asymmetrical radiation from the RTGs, feed spillover, and solar reflectance mis-modeling. These sources are all plausible but neither proven or disproven by any records or measurements found so far. The fit assigns the same efficiency to main compartment heat and instrument heat. This avoids much of the need to look at spacecraft construction details and instrument history, since the acceleration only depends on the total electrical power.

The additional sources allow a better fit since RHU and RTG heat decays more slowly than electrical heat, feed spillover does not decay at all, and we can now model the onset of the anomalous acceleration at 5 AU. Once again, many parameter choices give similar results. We get a reasonable fit over the entire data span with the following coefficients:

$\epsilon_{RHU}=0.5$, the RHUs radiate like point sources behind the antenna.

$\epsilon_{RTG}=0.016$. 0.3% RTG asymmetry, 30% blockage efficiency, and 50% reflection efficiency.

$\epsilon_{FEED}=0.1$. 10% of the feed power misses the antenna.

$\epsilon_{INST}=0.39$. Instrument heat radiates as main bus heat for simplicity.

$\epsilon_{BUS}=0.39$. About 60% of the main bus heat goes out the front, with Lambertian efficiency.

$K_{SOLAR}=0.2$. Antenna reflection estimates are too low by 0.2.

The fit to the data is shown in Figs. 8 and 9. The agreement seems reasonable in both regimes. In particular, the early anomalous acceleration between 15 and 40 AU is fit well by this model. In Fig. 8 two other models are shown, all assuming that a $1/r^2$ error of some sort (here solar constant mismodeling) is responsible for the onset. The middle trace assumes the acceleration is a pure exponential with an 88 year half-life. This is the form for a model that assumes RTG radiation (direct or reflected) is asymmetric but spacecraft electrical heat is radiated isotropically. Between 15 and 40 AU this model underpredicts the observed decrease, where the nominal model fits much better. This strongly favors a model where radiation from the spacecraft bus is a major

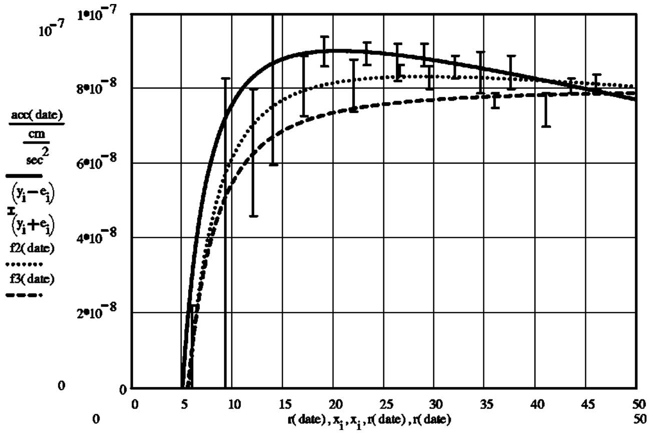

FIG. 8. Data from Fig. 1 (error bars), model prediction from this paper (solid line), pure 88 year half-life plus solar constant error (middle line), and constant acceleration plus solar constant error (lowest line).

contributor to the anomalous acceleration. The lower trace is a constant acceleration plus an error that scales as $1/r^2$. This shows that if the acceleration is indeed constant at large distances, a different explanation for the onset is required.

The fit from 1987 to 1998, shown in Fig. 9, also looks reasonable. We compare this model to the consensus of the most recent analyses [2,3]. Using the parameters above, the average sunward thrust is 58.0 W in interval I and 50.2 W in interval III. We can adjust the parameters to get the correct overall average, or the right acceleration in interval I, but in either case we would expect to see a 13.2% decrease from intervals I to III, where only a 5.6% decrease is observed. The 7.6% discrepancy is about 2 standard deviations out. Taken at face value, this makes it unlikely at about the 2% level that this hypothesis alone accounts for all the measured result. However, the reanalysis by Markwardt [2] has concluded that the data does not rule out a slowly decreasing force, at least if the decrease is an exponential with a half-life of more than 50 years. (This corresponds to a 9% decrease in the 6.75 year span between the midpoints of intervals I and III.) The decrease here is not strictly exponential, but is close in size and shape to that explicitly allowed by Markwardt.

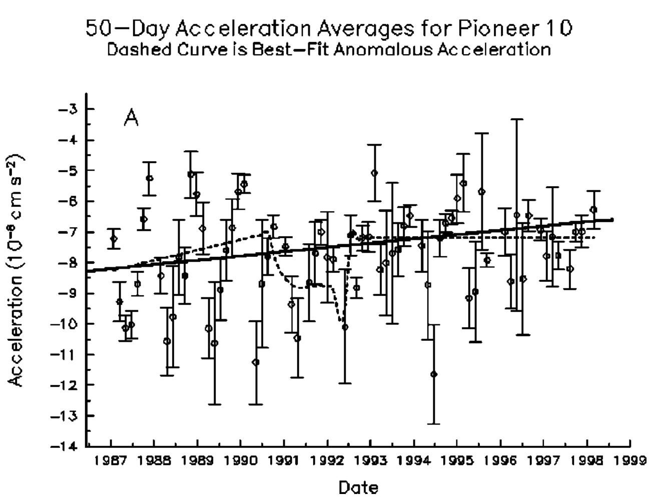

FIG. 9. Figure from [4], with fitted data added. The dotted line is Turyshev's empirical fit; the solid line is the model hypothesized in this paper.

More speculatively, an even better fit to the acceleration data can be obtained by assigning different efficiencies to instrument heat and main compartment heat. For example, we have the following:

$\epsilon_{RHU}=0.5$, the RHUs radiate like point sources behind the antenna.

$\epsilon_{RTG}=0.01425$. 0.3% RTG asymmetry, 30% blockage efficiency, and 60% reflection efficiency.

$\epsilon_{FEED}=0.1$. 10% of the feed power misses the antenna.

$\epsilon_{INST}=0.40$. About half the main bus heat radiated forwards with Lambertian efficiency.

$\epsilon_{BUS}=0.10$. Instruments radiate mostly to the side.

$K_{SOLAR}=0.2$. Antenna reflection estimates are too low by 0.2.

These efficiencies give a better fit, with only a 4.9% discrepancy (10.5% predicted versus 5.6% measured) on the I–III decline and a roughly equivalent fit at earlier times. This is only about 1.3 standard deviations from the consensus model, and an almost perfect fit for Markwardt. However, figuring the maximal reasonable difference between instrument efficiency and main compartment efficiency is difficult [13]. On the one hand, the two compartments are separate, the instrument bay is closer to the edge of the antenna, and it has side facing ports that extend though the thermal blankets. On the other hand, the two compartments are radiatively and conductively coupled. Without a much more detailed analysis it is very hard to determine the maximum plausible difference in efficiencies.

Finally, asymmetric radiation offers a parsimonious explanation for both the anomalous acceleration and the anomalous torque. Anderson *et al.* note that the Pioneer 10 spin-down torque is almost perfectly correlated with the main bus power. Radiation from the front of the craft, as proposed here, explains this. The needed emission geometry is numerically plausible—in 1986, there were 97 W available, and 13 W m of torque measured. Assuming the radiation is emitted 50 cm from the axis (the louver location), if the radiation was canted at an average angle of 15.5° from the normal to the surface, it could provide the observed torque. Such an angle would decrease the conversion of power into thrust by only 4%, leaving that argument intact. The louvers, covering the front surface and all canted to one side when closed, provide a natural explanation for the asymmetry required. One obvious objection to this explanation is that it predicts Pioneer 11 should be spinning down as well, instead of the spin-up that is actually observed. This is not a serious problem since the unknown spin-up mechanism, possibly gas leaks or RTG asymmetry, can easily overpower the small torque induced by main bus radiation.

In any case, the proposed explanation, by accounting for the bulk of the effect, makes it more likely that conventional physics can account for the entire unmodeled acceleration. Conventional explanations for the remaining discrepancy include other unmodeled effects such as gas leaks, inaccuracies in the simple thermal model, or the effects of a complex fitting procedure applied to noisy data.

VI. CONCLUSIONS AND FUTURE WORKS

No new physics is needed to explain the behavior of the Pioneer spacecraft. Either gas leaks or thermal radiation, or a combination of the two, could explain both the linear and angular accelerations that are measured.

A strong thermal effect is certainly present, based only on the construction of the Pioneers. Estimates show it can account for the magnitude of the unmodeled acceleration to within the errors, but it overpredicts the rate of change. The antenna shadowing of the main compartment radiation and the radiation from the RTGs falling on the antenna seem particularly robust sources of acceleration since they are only based on geometry. These effects alone account for more than half the acceleration. The other sources—RHU radiation, differential RTG radiation, and differential emissivity—depend more on construction details, but all seem plausible.

This explanation also explains some other puzzles: the values of acceleration of Pioneer 10 and 11 would be expected to be similar, but not identical, as observed. The acceleration and the observed torque (on Pioneer 10) share a common origin, and the torque is proportional to the main bus heat, as observed. Other spacecraft, built along the same general principles, would be expected to show a similar effect, but planets and other large bodies would not, as is observed.

The hypothesis here predicts an eventual, unambiguous decrease in the anomalous acceleration. If the acceleration remains constant, on the other hand, the hypothesis will be refuted. Extending the analysis of Markwardt to the whole Pioneer data span would be useful, since it currently stops at 1994 and it directly includes the possibility of a non-constant acceleration. Extending the analysis of Anderson by including post-1998 data would be helpful as well.

If Pioneer 10 remains operational, additional data may allow us to improve our understanding of the unmodeled acceleration. The difference between constant acceleration and the decrease predicted by the hypothesis of this paper grows quadratically with time. Since the beginning of data in 1987, by 2002 the two solutions differ by 4.4 cm/sec, or a Doppler shift of 0.58 Hz. Thus a single good 2002 measurement could tell the two hypotheses apart. Unfortunately the signal is now very weak, to the point where the standard JPL receivers have trouble locking onto the signal [7]. Careful recording of the return signal might probably work, though, with the frequency recovered through long averaging. Bigger telescopes such as Arecibo, the VLA, or Greenbank, might conceivably be pressed into service as well.

More detailed modeling, using the Pioneer materials, construction details, and history, could provide a much better estimate of the magnitude of this effect. A suitably detailed thermal model, measured in a cold vacuum chamber, would provide the strongest evidence for or against this hypothesis.

Longer term, other proposed experiments such as LISA [24] are designed specifically to reduce the systematics that bedevil retrospective analyses like Pioneer. (LISA is expected to be about 10^5 times better in this respect.) If the anomalous acceleration is not detected in these more precise experiments, then almost surely the unmodeled acceleration of Pioneer 10 is caused by overlooked prosaic sources such as those proposed here.

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[8] This is a ''theoretical value,'' which does not account for inverter losses, line losses, and such. It is interesting to note that at mission acceptance, the total ''theoretical'' power was 175 W.

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